"LBONet: Supervised Spectral Descriptors for Shape Analysis": Supplementary Material

Oguzhan Yigit, Richard C. Wilson, *Senior Member, IEEE*

***Abstract*—The supplementary materials include additional gradient formulations that were implemented but omitted from the main paper to avoid redundancy. Since all gradients follow straightforward calculus principles, we have omitted the detailed derivation steps for brevity.**

I. Gradients

A. RiemannNet

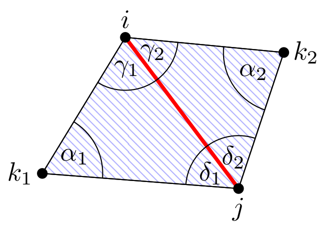

Fig. 1. The edge flap surrounding edge ij, highlighted in red, illustrates how adjusting the Riemannian metric affects all adjacent angles and distances. Consequently, these changes are incorporated into the gradients we introduce.

$$\frac{\partial A_{ii}}{\partial d_{ij}} = \frac{1}{8}\Bigg(\Bigg(-\frac{d_{ij}^3}{\sin^2(\alpha_1)2T_1} + \cot(\alpha_1)2d_{ij}\Bigg) + \Bigg(\frac{d_{ik}^3\cos(\delta_1)}{\sin^2(\gamma_1)2T_1}\Bigg) - \Bigg(\frac{d_{ij}^3}{\sin^2(\alpha_2)2T_2} + \cot(\alpha_2)2d_{ij}\Bigg) + \Bigg(\frac{d_{ik}^3\cos(\delta_2)}{\sin^2(\gamma_2)2T_2}\Bigg)\Bigg), \tag{1}$$

$$\frac{\partial A_{jj}}{\partial d_{ij}} = \frac{1}{8}\Bigg(\Bigg(-\frac{d_{ij}^3}{\sin^2(\alpha_1)2T_1} + \cot(\alpha_1)2d_{ij}\Bigg) + \Bigg(\frac{d_{jk}^3\cos(\gamma_1)}{\sin^2(\delta_1)2T_1}\Bigg) - \Bigg(\frac{d_{ij}^3}{\sin^2(\alpha_2)2T_2} + \cot(\alpha_2)2d_{ij}\Bigg) + \Bigg(\frac{d_{jk}^3\cos(\gamma_2)}{\sin^2(\delta_2)2T_2}\Bigg)\Bigg), \tag{2}$$

$$\frac{\partial A_{k_1k_1}}{\partial d_{ij}} = \frac{1}{8}\Bigg(\Bigg(-\frac{d_{ij}^3\cos(\gamma_1)}{\sin^2(\delta_1)2T_1}\Bigg) + \Bigg(\frac{d_{ij}^3\cos(\delta_1)}{\sin^2(\gamma_1)2T_1}\Bigg)\Bigg) \tag{3}$$

$$\frac{\partial A_{k_2k_2}}{\partial d_{ij}} = \frac{1}{8}\Bigg(\Bigg(-\frac{d_{ij}^3\cos(\gamma_2)}{\sin^2(\delta_2)2T_2}\Bigg) + \Bigg(\frac{d_{ij}^3\cos(\delta_2)}{\sin^2(\gamma_2)2T_2}\Bigg)\Bigg) \tag{4}$$

B. ALBONet

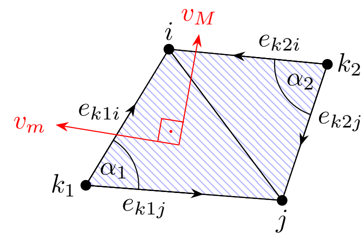

Fig. 2. An orthonormal reference frame on a face, defined by the principal curvature directions v_m and v_M (highlighted in red), is shown. Scaling these directions by a factor a a modifies the anisotropy of the face, which in turn scales the dot product between the corresponding edge and the direction.

$$\frac{\partial W_{ii}}{\partial a_{1f}} = \frac{1}{2}\langle e_{ki}v_M\rangle^2(cot(\alpha_1) + cot(\alpha_2)) \tag{5}$$

$$\frac{\partial W_{ij}}{\partial a_{1f}} = \frac{1}{2}\frac{\langle e_{ki}v_M\rangle\langle e_{kj}v_M\rangle}{sin(\alpha_1)} \tag{6}$$

$$\frac{\partial W_{ii}}{\partial a_{2f}} = \frac{1}{2}\langle e_{kj}v_m\rangle^2(cot(\alpha_1) + cot(\alpha_2)) \tag{7}$$

$$\frac{\partial W_{ij}}{\partial a_{2f}} = \frac{1}{2}\frac{\langle e_{ki}v_m\rangle\langle e_{kj}v_m\rangle}{sin(\alpha_1)} \tag{8}$$

C. Anisotropic RiemannNet

$$\begin{aligned}\frac{\partial W_{ii}}{\partial d_{ij}} &= -\frac{1}{2}(a_1\langle e_{ki}v_M\rangle^2 + a_2\langle e_{ki}v_m\rangle^2)\frac{d_{ij}}{2T}\frac{1}{sin(\delta)^2} \\ &\quad - \frac{(d_{ik}cos(\gamma))}{2T}\frac{1}{sin(\alpha)^2} \\ &\quad + (a_1 2sin(acos(\langle e_{ki}v_M\rangle))\langle e_{ki}v_M\rangle\frac{d_{ik}cos(\gamma)}{2T} \\ &\quad + a_2 2sin(acos(\langle e_{ki}v_m\rangle))\langle e_{ki}v_m\rangle\frac{d_{ik}cos(\gamma)}{2T})(cot(\alpha) \\ &\quad + cot(\gamma))\end{aligned} \tag{9}$$

$$\frac{\partial W_{jj}}{\partial d_{ij}} = -\frac{1}{2}(a_1\langle e_{kj}v_M\rangle^2 + a_2\langle e_{kj}v_m\rangle^2)\frac{d_{ij}}{2T}\frac{1}{sin(\delta)^2} - \frac{(d_{jk}cos(\alpha))}{2T}\frac{1}{sin(\gamma)^2} + (a_1 2sin(acos(\langle e_{kj}v_M\rangle))\langle e_{kj}v_M\rangle\frac{d_{ij}}{2T} + a_2 2sin(acos(\langle e_{kj}v_m\rangle))\langle e_{kj}v_m\rangle\frac{d_{jk}}{2T})(cot(\alpha) + cot(\delta)) \tag{10}$$

$$\frac{\partial W_{ij}}{\partial d_{ij}} = -\frac{1}{2}sin(\gamma_1)((a_1\langle e_{kj}v_M\rangle - sin(acos(\langle e_{kj}v_M\rangle))\frac{d_{ik}cos(\delta_1)}{2T} + -sin(acos(\langle e_{ki}v_M\rangle))\frac{-d_{jk}cos(\alpha)}{2T}\langle e_{ki}v_M\rangle) + (a_2\langle e_{kj}v_m\rangle - sin(acos(\langle e_{kj}v_m\rangle))\frac{d_{ik}cos(\delta_1)}{2T} + -sin(acos(\langle e_{ki}v_m\rangle))\frac{-d_{jk}cos(\alpha)}{2T}\langle e_{ki}v_m\rangle)) - \frac{1}{2}((a_1(\langle e_{ki}v_M\rangle)\langle e_{kj}v_M\rangle) + a_2(\langle e_{ki}v_m\rangle\langle e_{kj}v_m\rangle))\frac{-d_{ij}}{2T}\frac{cos(\delta)}{sin(\delta)^2} \tag{11}$$

$$\frac{\partial W_{kk}}{\partial d_{ij}} = -\frac{1}{2}((a_1\langle e_{kj}v_M\rangle^2 + a_2\langle e_{kj}v_m\rangle^2)\frac{d_{jk}cos(\alpha}{2T}\frac{1}{sin(\gamma)^2} + (\frac{d_{ik}cos(\delta)}{2T}\frac{1}{\alpha^2}) + (a_1 2sin(acos(\langle e_{kj}v_M\rangle))\langle e_{kj}v_M\rangle^2\frac{d_{jk}cos(\alpha)}{2T} + a_2 2sin(acos(\langle e_{kj}v_m\rangle))\langle e_{kj}v_m\rangle^2\frac{d_{jk}cos(\alpha)}{2T})(\alpha+\gamma) \tag{12}$$