Weekly Train Timetabling integrating Stop Planning for High-speed Rail Lines

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ABSTRACT

In high-speed rail (HSR) train planning and scheduling, traditional approaches often focus on passenger demand over short periods, such as one or two hours or a single day, while overlooking demand fluctuations over an entire week. This study proposes an integrated model for weekly train timetabling and stop planning, aiming to optimize both train stops and schedules across different times of day and days of the week. To improve computational efficiency for large-scale, real-world applications, a Lagrangian relaxation algorithm is developed. Case studies based on Chinese HSR lines demonstrate that the proposed model and algorithm outperform both the commercial solver CPLEX and the conventional sequential approach of line planning followed by timetabling. The weekly timetable generated by the proposed algorithm significantly reduces train and passenger traveling costs by improving traveling speeds and the proportion of passengers traveling within their preferred periods compared to current practical timetables, making it widely applicable to a wide range of HSR lines.

Keywords: Train timetabling, weekly timetable, stop planning, passenger demand, Lagrangian relaxation, time-space network

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1 INTRODUCTION

High-speed rail (HSR) offers convenient and efficient transportation for passengers. However, many HSR lines are increasingly facing resource limitations due to rising demand driven by urban population growth and economic expansion. As a result, improving resource utilization and aligning train services more effectively with passenger demand have long been central goals in optimizing HSR train operations.

HSR train operations are typically optimized through several stages. During the line planning stage, decisions are made regarding origin and terminal stations, stopping patterns, and frequencies of the trains. In the train timetabling stage, departure and arrival times at each station are determined. These stages are followed by vehicle and crew scheduling to ensure seamless service execution. Among these, line planning and timetabling are critical, as they directly influence how effectively services can respond to fluctuating passenger demand.

With continuous economic growth in many regions, the variation between passenger demand patterns on different days have become increasingly evident on numerous HSR lines. Among these, variations between weekdays and weekends are the most prominent and have become a major focus for many railway companies. For instance, weekday travel is typically dominated by commuter traffic between urban centers and surrounding areas, while weekends often see spikes in tourism-related demand. Along busy HSR lines, business travelers frequently account for high weekday volumes. These variations lead to distinct demand patterns between weekdays and weekends, and even among individual days such as Friday, Saturday, and Sunday. Moreover, different origin-destination (OD) pairs exhibit distinct daily and weekly demand trends.

Such weekly demand fluctuations pose significant challenges to classical optimization models for train planning and scheduling, which have traditionally focused on short periods, such as one or two hours, or have only addressed fluctuations within a single day. To manage these complex weekly patterns, several alternative strategies have been explored. A common practice among railway companies is to manually adjust train schedules, adding or removing trains to create separate timetables for weekdays and weekends. However, when OD pairs have varying demand peaks, simply increasing or reducing train numbers can lower service quality for some OD pairs. Moreover, these adjustments are typically based on experience and lack scientific optimization, a limitation highlighted in recent studies.

Another alternative is to develop daily-varying timetables, which more accurately reflect demand changes across the week. While this can improve demand matching, it results in highly irregular schedules that may confuse passengers, making it harder for them to remember departure times, plan transfers, or reschedule trips. The challenges posed by complex weekly demand patterns to classical models, along with the limitations of current weekly scheduling practices, form the core motivation for this study.

Most train timetabling models either repeat trains within each period, resulting in periodic trains and timetables (Kroon et al., 2014; Zhang & Nie, 2016), or run trains only during certain periods of the day, resulting in aperiodic trains and timetables (Cacchiani et al., 2010; Cacchiani et al., 2015).

With predetermined line plans as input, integer variables have been formulated in some studies to determine specific arrival and departure times at stations (Barrena et al., 2014; Gong et al., 2021). Other studies employ binary variables in time-space networks to determine train routes (Tian & Niu, 2020; Yao et al., 2023). Given the strong interdependence between line planning and timetabling, feedback strategies have been developed to integrate the two stages. For instance, Burggraeve et al. (2017) proposed an algorithm where the output timetable evaluates the quality of the line plan. Similarly, in Yan & Goverde (2019), train frequency constraints are adjusted during line planning to generate corresponding timetables, with the optimal timetable selected as the output. Integrated optimization approaches for line planning and timetabling have also been explored, often solved through decomposition algorithms such as Lagrangian relaxation (Yue et al., 2015) or ADMM (Alternating Direction Method of Multipliers; Zhang et al., 2021). These methods have also been extended to other stages, such as platform assignment (Xu et al., 2021) and vehicle scheduling (Liao et al., 2021).

As passenger demand becomes more complex, improving the alignment of train services with demand variation has become critical. Schmidt & Schöbel (2010) proposed methods to integrate passenger routing into train scheduling models, and in their subsequent work, two algorithms were introduced to co-optimize timetables and passenger routes (Schmidt & Schöbel, 2014). Addressing passenger routing, Borndörfer et al. (2017) presented four assumptions: passengers may search for multiple routes within a travel time limit, choose only the shortest route, select the shortest route with capacity constraints, or follow a unique route for each OD pair, factoring in capacity limits. In more recent studies, complex passenger demand scenarios have necessitated specialized train operation strategies, such as skip-stop and short-turning strategies (Zhao et al., 2020; Yuan et al., 2021), which require consideration of passenger routing during train planning to enhance service quality. Other approaches integrate passenger routing into objective functions, optimizing metrics like minimizing transfer station congestion (Yin et al., 2021), passenger waiting times (Dong et al., 2020; Bucak & Demirel, 2022), and transfer times (Xu et al., 2021).

Despite these advancements, several gaps remain in addressing the complex weekly demand variations in HSR scenarios. Traditional models typically optimize line plans and timetables for short periods or single days. Traditional models typically focus on short time horizons or single-day planning, which is insufficient when demand patterns vary significantly across the week. Furthermore, maintaining consistent stopping patterns and schedules across days, which is important for passenger convenience, adds another layer of complexity. This makes it difficult to implement fully daily-varying schedules or to apply a peak-day schedule with simple reductions for other days.

To bridge these gaps and better align train operations with weekly fluctuations in passenger demand, this study proposes the Weekly Train Timetabling Integrating Stop Planning (WTTSP) model, along with a customized Lagrangian relaxation algorithm. The key contributions of this study are as follows:

(1) . The model determines train arrival and departure times across a seven-day period. While this extended planning horizon increases decision variables and constraints, it enables better alignment with complex, real-world demand variations across different days and time periods.

(2). The model integrates stop planning and passenger routing into the weekly timetable optimization. By incorporating seating capacity constraints and OD-specific travel needs, the model ensures consistency between supply and demand.

(3). A weekly train operation network is constructed, converting the integrated optimization problem into a joint routing problem for both trains and passengers. This transformation, which includes headway and capacity constraints, enables a more tractable and scalable formulation.

(4). A customized Lagrangian relaxation algorithm is developed to solve the WTTSP model in practical HSR scenarios. The algorithm relaxes key constraints (e.g., safety headway, seating capacity limits) into the objective function and decomposes the problem into independent routing subproblems for trains and passenger groups. A specialized solution sequence and path-search strategy based on cross-day and cross-period scheduling constraints further enhance efficiency.

The structure of this paper is as follows: Section 2 defines the WTTSP problem and presents key concepts, including the weekly train operation network. Section 3 presents the mathematical formulation of the WTTSP model. Section 4 details the customized Lagrangian relaxation algorithm. Section 5 describes computational experiments, including small-scale scenarios to test the algorithm and large-scale scenarios based on Chinese HSR cases. Finally, Section 6 concludes the paper with a discussion of findings and future research directions.

2 PROBLEM DESCRIPTION

2.1 Weekly Train Timetabling Integrating Stop Planning Problem

Weekly train operations and passenger travel times are structured into two dimensions: **days** and **periods**. A full week consists of seven days, where some days may share identical timetables due to similar passenger demand patterns (e.g., Tuesday through Thursday), while others may differ (e.g., Friday and Saturday). Each day is further divided into consecutive periods, excluding maintenance hours during which trains do not operate (typically from midnight to 6:00 AM in China). For instance, a day might be divided into nine 2-hour periods, such as 6:00-8:00, 8:00-10:00, and so on. Based on a train's schedule and stopping patterns across different days and periods, it can be classified into one of three **weekly modes**:

Periodic Trains: These trains run at the same time across all days and periods with fixed schedules and stops, ensuring high regularity in the timetable and facilitating passenger transfers. A **periodic train group** consists of multiple trains that operate on various days and periods, sharing identical stops and departure times within their respective periods (e.g., departing at 6:15 in the 6:00–8:00 period, 8:15 in the 8:00–10:00 period, etc.). Once the stop and schedule for any train within a periodic group is determined, the stops and schedules of all other trains in the group are fixed. Figure 1 (1) and (2) show examples of periodic trains and their groups in a weekly timetable.

Daily Trains: These trains operate with the same schedule and stops every day, helping passengers memorize timetables and simplifying crew and vehicle scheduling. Periodic trains are a specific type of daily trains that run in every period. A **daily train group** includes trains with identical stops and daily schedules. Once the stop and schedule of any train within this group is determined, the stops and schedules of all other trains in the group are also fixed. Figure 1 (3) illustrates examples of daily trains and their groups in a weekly timetable.

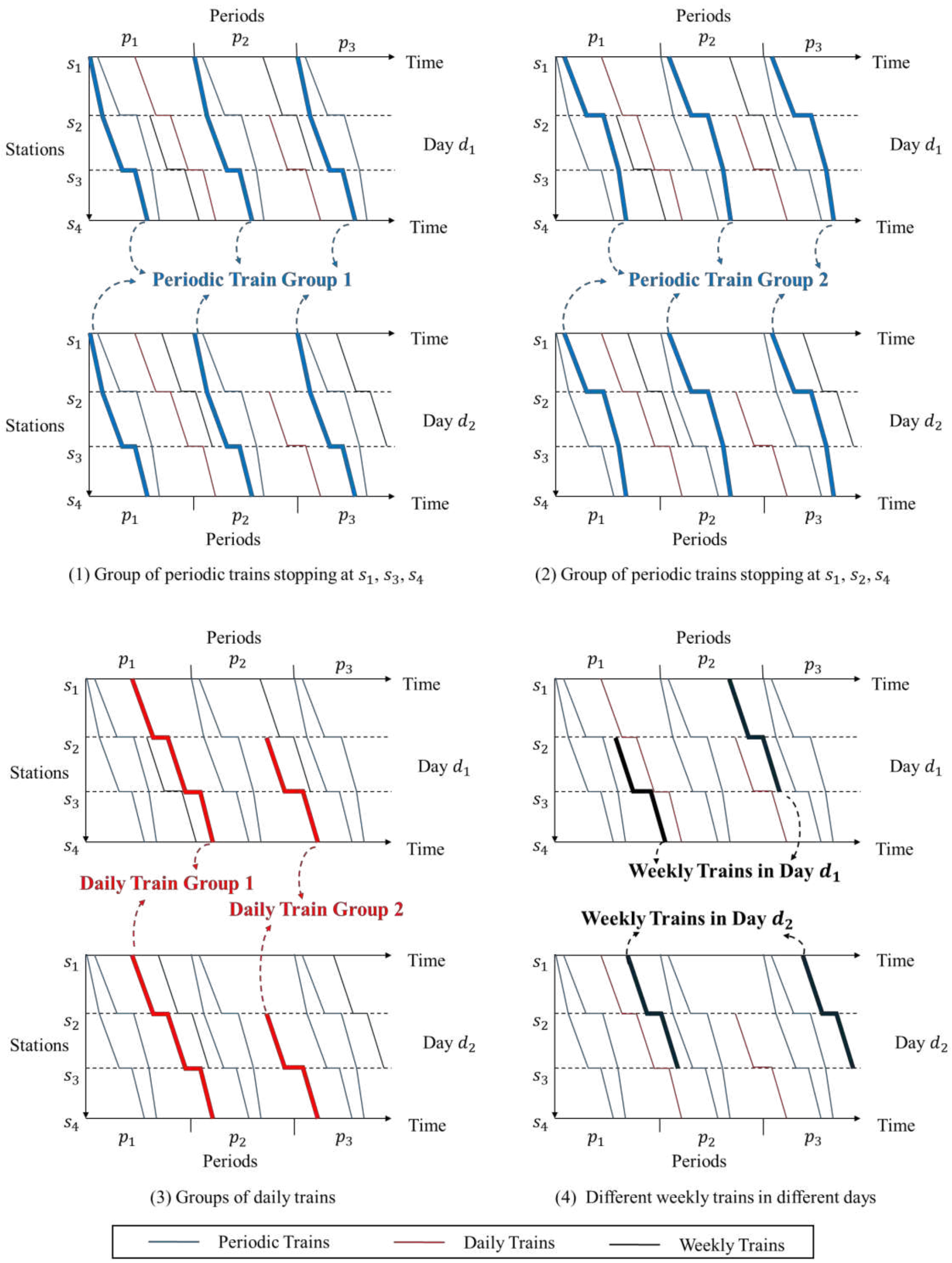

Figure 1 Periodic trains, daily trains, and weekly trains in a weekly timetable

Weekly Trains: These trains introduce flexibility into the timetable by operating only on specific days of the week, allowing better alignment with varying passenger demand. Figure 1 (4) illustrates an example of weekly trains operating on different days in the weekly timetable.

Our previous work (Nie et al., 2022) proposed an efficient approach for generating a weekly line plan, determining the origin, destination, frequency, composition, and weekly mode (periodic, daily, or weekly) of trains in different periods and days. This study focuses on adjusting train stops and schedules, using the weekly line plan from that earlier work as input. The origin, destination, frequency, composition, and weekly mode of all trains specified in the given line plan are treated as fixed inputs and are not subject to optimization in the WTTSP problem.

Based on these classifications, the WTTSP problem can be defined as follows: Given the configuration of the HSR line and its stations; detailed passenger demand data over a week; and the origin, destination, frequency, composition, and weekly mode of all trains, determine the stopping patterns and the departure and arrival times of each train at each station. The objective is to minimize the total travel costs of both passengers and trains, while ensuring consistency in stop patterns and schedules for all trains within the same periodic or daily train group.

2.2 Assumptions

Assumptions 1 through 3 establish the foundational concepts of weekly train operations, consistent with those in Nie et al. (2022), and are briefly reviewed here. Assumption 4 outlines the modeling approach for the passenger preference. Assumptions 5 through 7 simplify practical train operation rules to facilitate the formulation of the WTTSP model and the development of the algorithm.

Assumption 1: The WTTSP problem considers two types of high-speed train compositions: short-composition vehicles with 8 carriages and long-composition vehicles with 16 carriages (or 17 carriages on some Chinese HSR lines). Train composition is used as input for determining feasible passenger routes.

Assumption 2: In the passenger demand data, passengers of the same OD pair and departing in the same period and on the same day, are grouped into a single **passenger group**. It is assumed that all passengers within a group share the same train preferences. This departure period is regarded as their preferred period, reflecting typical passenger travel behaviors. Passengers from different groups are assumed to have distinct demand characteristics, due to differences in OD pairs, departure periods, and travel days, capturing variations in passenger behavior across time and routes.

Assumption 3: To ensure a basic level of service along the HSR line, certain trains are designed as **mandatory trains,** operating at fixed times even during low-demand periods. These are referred to as. In this study, long-distance trains connecting the terminal stations of the HSR line, departing at standard hours such as 8:00, 9:00, or 10:00 are considered mandatory. These trains make limited stops and prioritize high speeds to serve long-distance passengers.

Assumption 4: Passenger travel behavior is influenced by three factors: (1) the difference between their preferred departure period and the boarding train's departure period, (2) the travel speed of the train, which is affected by the number of stops and dwell times, and (3) the transfer times required when changing trains.

Assumption 5: Stations along the HSR line are categorized as **major** or **local** based on characteristics such as station capacity, passenger volumes, and the number of originating/terminating trains. Each station is also assigned a level, and both trains and passengers are classified accordingly, as shown in **Table 1**.

Assumption 6: All trains in the weekly timetable are assumed to have the same technical speed and acceleration. Once a train's stopping pattern is determined, its travel time between any two adjacent stations become fixed.

Assumption 7: Train dwell times at stations are adjustable within a predefined range based on the station's level. Major stations, equipped with more arrival and departure tracks, can accommodate more simultaneous train stops and offer greater dwell time flexibility. Fast trains may also overtake slower trains at major stations if the latter have extended dwell durations.

Assumption 8: Passenger transfers are only permitted at major stations, and transfer times are restricted to a predefined range.

Table 1 Stations, trains, and passenger classifications

	Level 1	Level 2	Level 3
Station	**Major Stations**: Located at key points in the network, usually connecting multiple HSR lines or at large cities with high passenger demand	**Local Stations:** Typically located along HSR lines connecting two major stations. Local stations are further classified into different levels according to the passenger volume.	
Train	**Level 1 Trains**: Fast trains stopping only at level 1 major stations.	**Level 2 Trains**: Relative fast trains stopping at level 1 or level 2 stations	**Level 3 Trains**: Slow trains stopping at any stations
Passenger	**Level 1 Passengers**: Direct passengers between level 1 stations	**Level 2 Passengers:** Passengers between level 2 or level 1 stations.	**Level 3 Passengers:** Passengers between level 3 stations and any other stations.

3 MATHEMATICAL FORMULATIONS

This section presents the formulation of the Weekly Train Operation Network (WTON) and the WTTSP model. Section 3.1 introduces the arcs and nodes in the WTON, which represent the stopping patterns, schedules, and the corresponding passenger flows. Section 3.2 describes the operational rules for periodic and daily trains within the WTON to ensure consistency in their stops and schedules. Section 3.3 defines the model's objective, decision variables, and constraints, all of which are constructed based on the WTON framework.

3.1 Arcs and Nodes in the Weekly Train Operation Network (WTON)

Let $s \in S$ represent the stations along the HSR line, where each major station $s \in S^M$ generates a departure time-space node n_{sdt}^D and an arrival time-space node n_{sdt}^A for every time $t \in T$ on each day $d \in D$ within the WTON. These nodes represent the departure and arrival of both passengers and trains. Local stations, located between adjacent major stations, are not explicitly modeled as individual time-space nodes in the network.

In the WTON, **traveling arcs** connect two adjacent major stations and indicate that trains stop at both major stations as well as selected local stations in between. By selecting traveling arcs with different stopping patterns, various combinations of local stations can be served. Once a stopping pattern is chosen, the departure and arrival times at the corresponding major stations are determined accordingly.

Between the arrival and departure nodes at the same major station, **dwelling arcs** represent the time trains and passengers spend at the station. After alighting at a major station, passengers may transfer to another departing train via a transfer arc, which is formulated similarly to a dwelling arc but is applicable only to passengers.

Figure 2 illustrates an example of the arc formulation in the WTON:

Traveling Arc: A traveling arc, denoted as $a^R_{(ss',d,tt',\lambda)}$, represents a train or passengers departing from major station s at time t on day d, and arriving at major station s' at time t', following a stopping pattern $\lambda \in LP_{ss'}$, where $LP_{ss'}$ is the set of all possible stopping patterns between stations s and s'. For brevity, the arc notation is simplified $a^R_{ss'dt\lambda}$.

According to Assumption 6, once the stopping pattern λ of a traveling arc a is determined, the travel time $(t' - t)$ is recorded as τ_a, which includes three components: the dwelling time at each local station where the arc a stops, the acceleration and deceleration time associated with each intermediate stop, and the travel time between the origin and destination of the arc a.

All traveling arcs include stops at their origin and destination stations. The selected stopping pattern λ determines which intermediate local stations are served. Typically, traveling arcs are defined between adjacent major stations. If a major station lies between the origin and destination, it is not included in the stopping pattern, representing a long-distance direct service between non-adjacent major stations.

For instance, in Figure 2, arcs a_1, a_2, and a_3 all depart from s_1 at time t_1 on day d_1. Both a_2 and a_3 stop at major stations s_3, but differ in their local station stops: a_2 bypasses local station s_2, while a_3 includes a stop at s_2, resulting in different arrival times at s_3. In contrast, a_1 skips s_3 entirely and continues to s_4. All stops along a traveling arc serve as potential origins and destinations for passengers, and seating capacity constraints are applied on each segment between adjacent stops to limit the number of passengers that can board.

Dwelling Arc: A dwelling arc, denoted as $a^D_{(ss,d,tt')}$ and simplified as $a^D_{sdtt'}$, represents a train dwelling at major station s, transitioning from the arrival node at time t on day d to the departure node at time t'. According to Assumption 7, the dwell time at major stations falls within a range T^D, such that $(t' - t) \in T^D$. In Figure 2, dwelling arcs a_5, a_6, and a_7 represent different dwell times for passengers and trains.

Transfer Arc: A transfer arc, denoted as $a^T_{sdtt'}$, represents passengers transferring at major station s from a train arriving at time t on day d to another train departing at time t'. According to Assumption 8, the transfer time is constrained within a range T^T, such that $(t' - t) \in T^T$. In Figure 2 transfer arc a_{21} represents passengers transferring at station s_3.

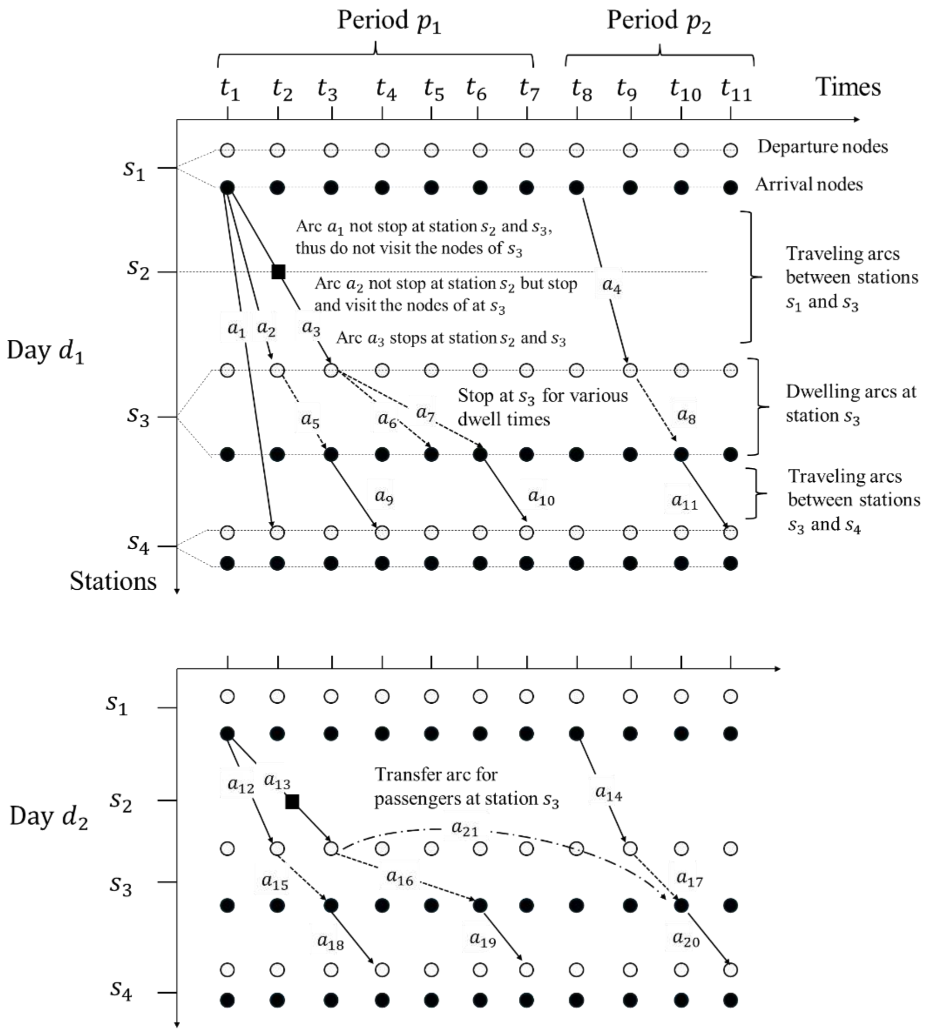

Figure 2 Arcs in WTON

3.2 Periodic and Daily Trains in the WTON

Based on the WTON, the consistency of stops and schedules for trains within the same periodic and daily train groups in the WTTSP problem is formulated through time-space route constraints. Let $p \in P$ represent a period, with the length each of period is denoted as τ^P. Integer k represents the index of a period p in the set P.

The routes of periodic trains in the WTON are formulated as follows: Traveling arcs that connect major stations s and s', follow the same stopping pattern λ, and depart at intervals of integer multiples of τ^P, form a periodic traveling arc group. Suppose the earliest departure in this group occurs at time t, the group can be denoted as:

$$A_{ss't\lambda}^{R(P)} = \{a_{ss'dt'\lambda}^{R} | t' = t + k \cdot \tau^P,\ k \in [0, |P| - 1],\ d \in D\}$$

Similarly, a periodic dwelling arc group between time t_1 and t_2 at station s is denoted as:

$$A_{st_1t_2}^{D(P)} = \left\{a_{sdt_1't_2'}^{D} \middle| t_1' = t_1 + k \cdot \tau^P, t_2' = t_2 + k \cdot \tau^P, k \in [0, |P| - 1],\ d \in D\right\}$$

In this case, for trains l and l' from the same periodic train group, which operate during periods indexed by k and k' on days d and d', respectively, if train l occupies a traveling arc $a^R_{ss'dt\lambda}$ from the periodic group $A^{R(P)}_{ss't_0\lambda}$, where $t = t_0 + k \cdot \tau^P$, train l' must occupy the corresponding traveling arc $a^R_{ss'd't'\lambda}$ from the same group $A^{R(P)}_{ss't_0\lambda}$, where $t' = t_0 + k' \cdot \tau^P$. This ensures that train l' operates in the k'-th period on day d', maintaining the same periodic structure as train l.

Similarly, if train l occupies a dwelling arc $a^D_{sdt_1't_2'}$ from the group $A^{D(P)}_{st_1t_2}$, where $t_1' = t_1 + k \cdot \tau^P$ and $t_2' = t_2 + k \cdot \tau^P$, then train l' must occupy the corresponding dwelling arc $a^D_{sd't_1''t_2''}$ from the same group, where $t_1'' = t_1 + k' \cdot \tau^P$ and $t_2'' = t_2 + k' \cdot \tau^P$.

An example is provided in Figure 2, where traveling arcs $\{a_2; a_4; a_{12}; a_{14}\}$ and $\{a_9; a_{11}; a_{18}; a_{20}\}$ form two periodic groups, while dwelling arcs $\{a_5; a_8; a_{15}; a_{17}\}$ form another periodic group. These groups are also depicted in Figure 3. If a periodic train in period p_1 and day d_1 select arcs a_2, a_5, a_9, then the trains from the same periodic train group but in different periods and days will select $\{a_4; a_8; a_{11}\}$, $\{a_{12}; a_{15}; a_{18}\}$, and $\{a_{14}; a_{17}; a_{20}\}$ to maintain schedule consistency.

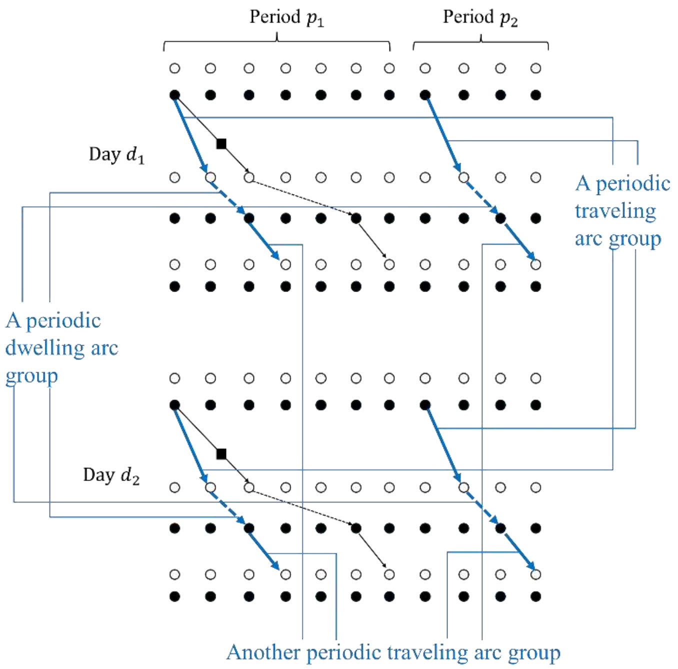

Figure 3 Periodic traveling and dwelling arc groups in the WTON

The routes of daily trains in the WTON are formulated similarly: Traveling arcs that connect major stations s and s', follow the same stopping pattern λ, and depart at time t form a daily traveling group, denoted as:

$$A^{R(D)}_{ss't\lambda} = \{a^R_{ss'dt\lambda} | d \in D\}$$

Similarly, a daily dwelling arc group is denoted as:

$$A_{stt'}^{D(D)} = \{a_{sdtt'}^{D} | d \in D\}$$

In this case, for trains l and l' from the same daily train group, operating on days d and d', respectively, if train l occupies a traveling arc $a_{ss'dt\lambda}^{R}$ in a daily group $A_{ss't\lambda}^{R(D)}$ or a dwelling arc $a_{sdtt'}^{D}$ in a daily group $A_{stt'}^{D(D)}$, then train l' must occupy traveling arc $a_{ss'd't\lambda}^{R}$ and dwelling arc $a_{sd'tt'}^{D}$ from the same daily groups on day d'.

Consider the example in Figure 2, where arcs $\{a_2;\ a_{12}\}$, $\{a_4;\ a_{14}\}$, $\{a_9;\ a_{18}\}$, $\{a_{11};\ a_{20}\}$, $\{a_3;\ a_{13}\}$, and $\{a_{10};\ a_{19}\}$ form different daily traveling arc groups, and arcs $\{a_5;\ a_{15}\}$, $\{a_8;\ a_{17}\}$, and $\{a_6;\ a_{16}\}$ form different daily dwelling arc groups. These groups are also illustrated in Figure 4. If a daily train on day d_1 select arcs a_2, a_5, a_9, then a train from the same daily group on day d_2 will select a_{12}, a_{15}, a_{18}.

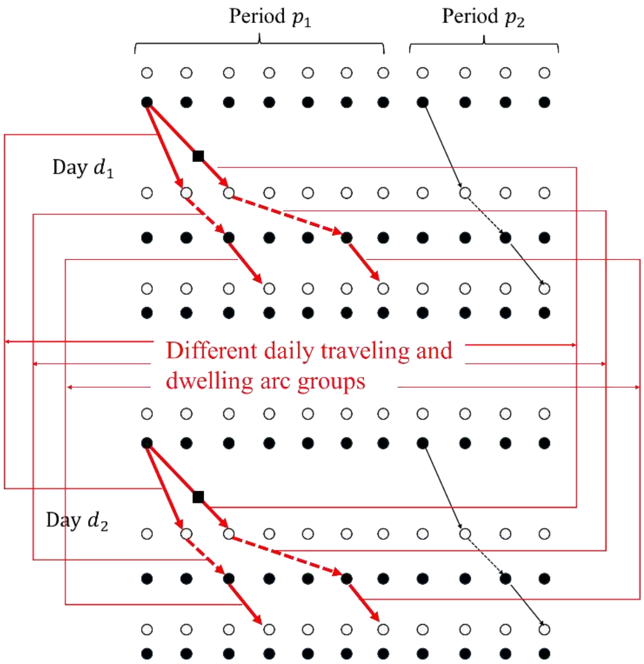

Figure 4 Daily traveling and dwelling arc groups in the WTON

3.3 Weekly Train Timetabling Integrating Stop Planning (WTTSP) Model

Table 2 The meaning of sets and parameters for WTTSP model

Notation	Meaning
U	The set of passenger groups, and u represents an individual passenger group.
U_d	The set of passenger groups on day d.
L	The set of trains in input weekly line plan, and l represents an individual train.
L_d	The set of trains on day d.

PT	A periodic train group, and $\mathcal{PT}$ represents the set of all periodic train groups.
DT	A daily train group, and $\mathcal{DT}$ represents the set of all daily train groups.
A_d	The set of arcs on day d.
A_{n+}	The set of arcs departing from the node n in the WTON.
A_{n-}	The set of arcs arriving at the node n in the WTON.
A_{sdt}^{Dep}, A_{sdt}^{Arr}	The set of traveling arcs departs from or arrives at station s at time t on day d
A_s^R	The traveling arcs that stop at station s.
A_u	The set of traveling, dwelling, and transfer arcs between the origination and destination of passenger group u.
AA	The set of the pair of traveling arcs (a, a'), which are crossed between two adjacent stations. For example, the departing time of a at station s_n is later than a', but the arrival time of a at station s_{n+1} is earlier than a'.
SS_a	The set of the pair of stations (s, s'), which are both stops of the traveling arc a.
$U_{ss'}$	The set of passenger groups whose origination or destination is between stations s and s'. In this case station pair (s, s') can be part of the traveling routes of passengers.
S_u^M	The set of major stations between the origination and destination of passenger group u.
τ_s^{Dep}	The safety departure headway interval between adjacent trains at station s.
τ_s^{Arr}	The safety arrival headway interval between adjacent trains at station s.
d_a	The operating date of arc a
v_u	The passenger volume of passenger group u.
d_u	The departing date of passenger group u.
s_u^{Ori}, s_u^{Des}	The origination station and destination station of passenger group u.
s_l^{Ori}, s_l^{Ter}	The origination station and termination station of train l.
d_l	The operating date of train l.
k_l	The index of the period of train l in the period set P.
ε_l	The seating capacity of train l.

The sets and parameters used in the WTTSP model are listed in Table 2. Decision variables related to the routes of trains are described as follows:

- x_a^l is a binary variable that equals 1 if train l occupies the arc a, and 0 otherwise.
- ϖ_t^l is a binary variable, that equals 1 if train l departs at time t, and 0 otherwise.

- ϱ_t^l is a binary variable, that equals 1 if train l terminates at time t, and 0 otherwise.

When determining routes, passengers select arcs in a manner similar to trains. For each passenger group, consisting of a specific number of individuals, the distribution of passengers across selected arcs can be determined and used to calculate their total travel costs. If an OD pair is not adequately served, some passengers may be unable to find a train. The number of such unserved passengers is accumulated, and a corresponding penalty is added to the objective function. Based on this logic, the decision variables related to passenger groups are defined as:

- y_a^u is a continuous variable representing the volume of passengers from group u traveling through arc a.

- z^u is a continuous variable representing the volume of passengers in group u unable to board any train.

The objective of the WTTSP model is defined in (r1). The first and second terms represent the total travel cost for all trains l and all passenger groups u, respectively. The third term accounts for the number of unserved passengers, acting as a penalty for insufficient service on certain OD pairs. The parameter ω is the weight of passenger travel cost.

For trains, the travel cost includes a basic operation cost c_B^l and an additional travel cost c_a^l for occupying arc a. The values of c_a^l depends on the time consumption of arc a. To ensure that trains depart within the periods specified in the weekly line plan, an additional cost is incurred if the arc a departs from station s_l^{Ori} outside of period p_l.

For passengers, the travel cost includes the cost θ_a^u for occupying the arc a and a penalty cost φ for passengers unable to board any train. Similar to c_a^l, the value of θ_a^u depends on the time consumption for arc a. Additionally, a higher cost is incurred if arc a departs from the passengers' origin station s_u^{Ori} outside of their preferred period p_u.

$$Min{:}\, Z = \omega\left(\sum_{l,a} c_a^l x_a^l + \sum_{l,t} c_B^l \varpi_{\Delta t}^l\right) + \sum_{u,a} \theta_a^u y_a^u + \varphi \sum_u z^u \qquad \text{(r1)}$$

Constraints for train operation are formulated as (r2) through (r11). The **departure and arrival constraints** (r2), along with the **flow balance constraints** (r3), (r4), and (r5) define feasible time-space routes for trains on the WTON. The **safety headway constraints** for **departure and arrival** (r6) and (r7) ensure the intervals between departure and arrival times of adjacent trains at any station. The **crossing constraint** (r8) prevents the time-space routes of any two trains from intersecting between any two adjacent stations. However, based on the findings of Yue et al. (2016), this crossing rarely occurs in Chinese HSR lines while the safety headway constraints are satisfied, thus constraints (r8) are not considered in solving WTTSP model. **Periodic train operation constraints** (r9) and (r10) along with **daily train operation constraints** (r11) and (r12) ensure the consistency of stops and schedules for periodic and daily trains within the same groups.

$$\sum_{t} \varpi_t^l = \sum_{t} \varrho_t^l \qquad \forall l \in L \tag{r2}$$

$$\sum_{a \in A_{n_{sd_l t}^+}} x_a^l = \sum_{a \in A_{n_{sd_l t}^-}} x_a^l \qquad \forall l \in L\,;\ \forall s \in \left(S^M - s_l^{Ori} - s_l^{Ter}\right);\ \forall t \in T \tag{r3}$$

$$\sum_{a \in A_{n_{s_l^{Ter} d_l t}^+}} x_a^l = \sum_{a \in A_{n_{s_l^{Ter} d_l t}^-}} (x_a^l + \varrho_t^l) \qquad \forall l \in L; t \in T \tag{r4}$$

$$\sum_{a \in A_{n_{s_l^{Ori} d_l t}^+}} (x_a^l + \varpi_t^l) = \sum_{a \in A_{n_{s_l^{Ori} d_l t}^-}} x_a^l \qquad \forall l \in L; t \in T \tag{r5}$$

$$\sum_{l \in L_d, t' \in [t, t+\tau_s^{Dep}], a \in A_{sdt}^{Dep}} x_a^l \le 1 \qquad \forall s \in S;\ t \in T; d \in D \tag{r6}$$

$$\sum_{l \in L_d, t' \in [t, t+\tau_s^{Arr}], a \in A_{sdt}^{Arr}} x_a^l \le 1 \qquad \forall s \in S;\ t \in T; d \in D \tag{r7}$$

$$\sum_{l} \left(x_a^l + x_{a'}^l\right) \le 1 \qquad \forall (a, a') \in AA \tag{r8}$$

$$x^l_{a^R_{ss'd_l t_l \lambda}} = x^{l'}_{a^R_{ss'd_{l'} t_{l'} \lambda}} \qquad \forall s, s' \in S^M;\ \lambda \in LP_{ss'};\ \forall t \in T;\ t_l = t + k_l \cdot \tau^T;\ t_{l'} = t + k_{l'} \cdot \tau^T;\ \forall PT \in \mathcal{PT};\ \ \forall\, l, l' \in PT \tag{r9}$$

$$x^l_{a^D_{sd_l t_l t'_l}} = x^{l'}_{a^D_{sd_{l'} t_{l'} t'_{l'}}} \qquad \forall s \in S^M;\ \forall t \in T; t_l = t + k_l \cdot \tau^T;\ t'_l = t' + k_l \cdot \tau^T;\ t_{l'} = t + k_{l'} \cdot \tau^T;\ t'_{l'} = t' + k_{l'} \cdot \tau^T; (t' - t) \in T^D;\ \ \forall PT \in \mathcal{PT};\ \ \forall\, l, l' \in PT \tag{r10}$$

$$x^l_{a^R_{ss'd_l t \lambda}} = x^{l'}_{a^R_{ss'd_{l'} t \lambda}} \qquad \forall s, s' \in S^M;\ \lambda \in LP_{ss'};\ \forall t \in T;\ \ \forall DT \in \mathcal{DT};\ \forall\, l, l' \in DT \tag{r11}$$

$$x^l_{a^D_{sd_l tt'}} = x^{l'}_{a^D_{sd_{l'} tt'}} \qquad \forall s \in S^M;\ t \in T; (t' - t) \in T^D;\ \ \forall DT \in \mathcal{DT};\ \forall\, l, l' \in DT \tag{r12}$$

$$x_a^l \in \{0,1\} \qquad \forall l \in L; a \in A_{d_l} \tag{r13}$$

$$\varpi_t^l \in \{0,1\} \qquad \forall l \in L \tag{r14}$$

$$\varrho_t^l \in \{0,1\} \qquad \forall l \in L \tag{r15}$$

Constraints for passengers are formulated as (p1) through (p5). The **passenger volume constraints** (p1) and (p2) limit the total volume of passengers departing from the origin of each group. Passenger **flow balance constraints (p3),** similar to train flow balance constraints (r3), ensure the proper flow of passengers. The **passenger boarding constraints** (p4) ensure that passengers can only board the trains that stop at their origin or destination stations. S**eating capacity constraints** (p5) link the routes of trains and passengers while limiting the number of passengers traveling through each arc.

$$\sum_{t, a \in (A_{n_{s_u^{Ori} d_u t+}} \cap A^R)} y_a^u + z^u = v_u \qquad \forall u \in U \tag{p1}$$

$$\sum_{t,a\in(A_{n_{s_u^{Des}d_u t-}}\cap A^R)} y_a^u + z^u = v_u \qquad \forall u \in U \tag{p2}$$

$$\sum_{a\in(A_{n_{sd_u t+}}\cap A_u)} y_a^u = \sum_{a\in(A_{n_{sd_u t-}}\cap A_u)} y_a^u \qquad \forall\, u \in U; s \in S_u^M;\ t \in T \tag{p3}$$

$$y_a^u = 0 \qquad \forall u \in U; a \notin A^R_{s_u^{Ori}}\ ;\ a \notin A^R_{s_u^{Des}} \tag{p4}$$

$$\sum_{u\in(U_{ss'}\cap U_{d_a})} y_a^u \le \sum_{l\in L_{d_a}} \varepsilon_l x_a^l \qquad \forall a \in A^R; \forall(s,s') \in SS_a \tag{p5}$$

$$y_a^u \in [0, p_u] \qquad \forall u \in U; a \in A_{d_u} \tag{p6}$$

$$z^u \in [0, p_u] \qquad \forall u \in U \tag{p7}$$

Incorporating a weekly planning horizon and integrating both train timetabling and stop planning, significantly increases the number of decision variables and constraints in the WTTSP model.

First, the number of decision variables x_a^l, ϖ_t^l, and ϱ_t^l, along with constraints (r2)–(r5) depends on the number of trains l. Similarly, the number of variables y_a^u and z^u, and constraints (p1)–(p4), depends on the number of passenger groups u. Compared with traditional single-day planning models, the inclusion of weekly demand data and a larger train set substantially increases model scale.

Second, the number of time-space arcs in WTON affects the scale of decision variables x_a^l and y_a^u and constraint (p5). Meanwhile, the number of time-space nodes determines the size of constraints (r6)–(r8). Since stop planning is integrated, many traveling arcs are created to represent different stopping patterns, further expanding the model compared to conventional timetabling frameworks.

Third, the number of periodic train groups determines the scale of constraints (r9) and (r10), and the number of daily trains determines the scale of constraints (r11) and (r12). These consistency constraints increase the model's structural complexity and further differentiate it from classical approaches.

4 ALGORITHM

To address large-scale practical scenarios, a **Lagrangian relaxation algorithm** is developed to solve the WTTSP model. The safety headway constraints (r6) and (r7), along with the seating capacity constraints (p5), are relaxed and incorporated into the objective function, allowing the original model to be decomposed into more tractable submodels, as detailed in Section 4.1. The operation constraints for periodic and daily trains remain enforced and are satisfied by solving the submodels sequentially, in accordance with the weekly schedule of trains.

In each iteration, a relaxed solution representing a lower bound on the optimal solution, is obtained by independently solving the submodels. Simultaneously, a feasible solution representing an upper bound, is determined using a customized passenger demand-matching strategy, introduced in Section 4.2. After each iteration, the Lagrangian multipliers are updated according to the procedure described in Section 4.3.

4.1 Decomposing the WTTSP model

The large number of trains included in a weekly timetable significantly increases the complexity of finding an optimal solution. To address this, the proposed algorithm decomposes the WTTSP model into submodels for determining the timetables of subgroups of trains. The complete set of train subgroups is denoted as $\mathcal{L}$, while an individual subgroup is denoted as L^{Sub}. A train subgroup may consist of either a periodic train group, a daily train group, or a single weekly train.

The Lagrangian multipliers for relaxing the safety headway constraints (r6) and (r7), and the seating capacity constraints (p5), are denoted by λ_{sdt}, λ'_{sdt}, and $\sigma_{ass'}$, respectively. The relaxed objective is formulated as (s1), where the binary parameter $\epsilon^{u}_{ass'}$, equals 1 if passenger group u belongs to the set $U_{ss'} \cap U_d$. This relaxed objective can be reformulated as (s2), consisting of a constant term $\sum_{s,d,t}(\lambda_{sdt} + \lambda'_{sdt})$, the sum of travel costs and multipliers Z^{Rel}_{l} for all trains l in all subgroups L^{Sub}(formulated as (s3)), and the sum of travel costs and multipliers Z^{Rel}_{u} for all individual passenger groups u (formulated as (s4)).

$$
\begin{aligned}
Min{:}\, Z^{Rel} = Z &+ \sum_{s,d,t} \lambda_{sdt} \left(\sum_{l\in L_d, t'\in\left[t,t+\tau_s^{Dep}\right], a\in\left(A_{n_{sdt'}+}\cap A^R\right)} x_a^l - 1 \right) \\
&+ \sum_{s,d,t} \lambda'_{sdt} \left(\sum_{l\in L_d, t'\in\left[t,t+\tau_s^{Arr}\right], a\in\left(A_{n_{sdt'}-}\cap A^R\right)} x_a^l - 1 \right) \\
&+ \sum_{a\in A^R, (s,s')\in SS_a} \sigma_{ass'} \left(\sum_{u\in\left(U_{ss'}\cap U_{d_a}\right)} y_a^u - \sum_{l\in L_{d_a}} \varepsilon_l x_a^l \right)
\end{aligned}
\tag{s1}
$$

$$
Min{:}\, Z^{Rel} = \sum_{L^{Sub}\in\mathcal{L}} \left(\sum_{l\in L^{Sub}} Z_l^{Rel} \right) + \sum_{u} Z_u^{Rel} - \sum_{s,d,t} (\lambda_{sdt} + \lambda'_{sdt})
\tag{s2}
$$

$$
\begin{aligned}
Z_l^{Rel} = \omega \sum_{a} c_a^l x_a^l + \omega \sum_{t} c_B^l \varpi_t^l &+ \sum_{l\in L_d, t'\in\left[t,t+\tau_s^{Dep}\right], a\in\left(A_{n_{sdt'}+}\cap A^R\right)} \lambda_{sdt} x_a^l \\
&+ \sum_{l\in L_d, t'\in\left[t,t+\tau_s^{Arr}\right], a\in\left(A_{n_{sdt'}-}\cap A^R\right)} \lambda'_{sdt} x_a^l \\
&- \sum_{a\in A^R, (s,s')\in SS_a, l\in L_{d_a}} \sigma_{ass'}\, \varepsilon_l x_a^l
\end{aligned}
\tag{s3}
$$

$$
Z_u^{Rel} = \sum_{a\in A^R, (s,s')\in SS_a} \epsilon_{ass'}^{u} \sigma_{ass'}\, y_a^u + \sum_{a} \theta_a^u y_a^u + \varphi z^u
\tag{s4}
$$

Submodels for routing train subgroups L^{Sub}: The objective of the submodel for train group L^{Sub} is to minimize $\sum_{l\in L^{Sub}} Z^{Rel}_{l}$. The decision variables include x^l_a, ϖ^l_t, and ϱ^l_t for each train $l \in L^{Sub}$, subject to the routing constraints (r2)–(r5) from the WTTSP model. If L^{Sub} is a periodic train group, constraints (r9) and (r10) are included to ensure the consistency of stops and schedules.

If L^{Sub} is a daily train group, constraints (r11) and (r12) apply to enforce daily regularity.

After solving the submodels separately, the resulting objective values $\sum_{l\in L^{Sub}} \overline{Z_l^{Rel}}$ are treated as the Lagrangian relaxation costs of each train subgroup. Subgroups with relatively lower relaxation costs typically attract higher passenger volumes or are less likely to violate safety headway constraints.

Submodels for routing passenger group $\boldsymbol{u}$: The objective of each passenger group's submodel is to minimize Z_u^{Rel}. The only decision variable is y_a^u, subject to constraints (p1)–(p4), which ensure proper routing of passengers. Since these submodels only involve continuous variables, they are computationally efficient and can be solved quickly using commercial solvers. Multiple passenger group submodels can be processed in parallel, improving overall algorithm performance.

4.2 Updating the Upper Bound

The lower bound of the optimal solution is defined as the highest objective value obtained among all relaxed solutions. In each iteration, the relaxed solution provides a lower bound, calculated as the sum of the Lagrangian relaxation costs, $\sum_{l\in L^{Sub}} \overline{Z}_l^{Rel}$ and $\overline{Z}_u^{Rel}$.

Conversely, the upper bound is the smallest objective value among all feasible solutions. A In the current iteration, a feasible solution is generated based on the same relaxation cost values. Following the traditional approach used in integrated train timetable studies (e.g., Yue et al., 2016; Xu et al., 2021), a customized passenger demand-matching strategy for feasible solution generation is proposed.

Let A denote the set of all arcs in the WTON. To generate a feasible solution, once a train subgroup has been routed, all traveling arcs that depart within the safety headway intervals of these trains, conflicting the safety headway constraints, are removed from A. The remaining arcs are denoted as A^{Train}, which contains all feasible traveling and dwelling arcs for routing remaining trains. The set of arcs selected by already-routed trains is denoted as A^{Psg}, and passenger groups search for their routes only within this set. This process consists of the following steps:

Step 1. **Initialization:** Set $A^{Train} = A$ and $A^{Psg} = \emptyset$

Step 2. **Routing Train Subgroups:** Order each train subgroup L^{Sub} by the values of $\frac{\sum_{l\in L^{Sub}} \overline{Z}_l^{Rel}}{|L^{Sub}|}$. Then following process is applied to route the trains in each L^{Sub}.

> **Step 2-1**. **Routing a part of passengers:** Passengers are routed on arcs in A^{Psg}. Since A^{Psg} is significantly smaller than A, all the passenger groups can be routed simultaneously. The set of passengers unable to find feasible routes is denoted as U^{Sub}.
>
> **Step 2-2**. **Finding preferred routes for remaining passengers:** Passenger groups in U^{Sub} are routed on arcs in A^{Train}. The passenger volumes traveling between stations (s, s') along the arc $a \in A^{Train}$ are recorded as $\mathcal{P}_{ass'}$.
>
> **Step 2-3**. **Adjust multipliers:** For each pair of stations (s, s') along the arc a, the multipliers $\sigma_{ass'}$ are adjusted according to the equation (s5), where ξ represents the

adjustment range of $\sigma_{ass'}$

$$\sigma'_{ass'} = \sigma_{ass'} - \xi\mathcal{P}_{ass'} + \frac{1}{|A|}\sum_{a\in A;(s,s')\in SS_a} \xi\mathcal{P}_{ass'} \quad \text{(s5)}$$

Step 2-4. **Solving submodels:** Using the adjusted multiplier $\sigma'_{ass'}$ to replace $\sigma_{ass'}$ in the objective, the submodel for routing trains in L^{Sub} on arcs in A^{Train} is solved. The values of the decision variables are recorded as $\overline{x}_a^l$, $\overline{\varpi}_t^l$, and $\overline{\varrho}_t^l$.

Step 2-5. Updating A^{Train}: For each train $l \in L^{Sub}$, if $\overline{x}^l_{a^R_{ss'dt\lambda}} = 1$, the traveling arcs $a \in A^{Dep}_{sdt'}$ for any time $t' \in [t - \tau_s^{Dep}, t + \tau_s^{Dep}]$ and traveling arcs $a \in A^{Arr}_{s'dt''}$ for any time $t'' \in [t + \tau_\lambda - \tau_{s'}^{Arr}, t + \tau_\lambda + \tau_{s'}^{Arr}]$ are removed from A^{Train} if they are present.

Step 2-6. Updating A^{Psg}: For each train $l \in L^{Sub}$, if $\overline{x}_a^l = 1$, the corresponding traveling or dwelling arc a is added to A^{Psg}. If traveling arcs $a^R_{s'sd(t-\tau_a)\lambda}$ for any s' is included in A^{Psg}, and the $a^R_{ss''dt\lambda}$ for any s'' is also included, passengers can transfer between corresponding trains thus the transfer arc $a^T_{sdtt'}$ is added to A^{Psg}.

Step 3. **Finalizing the upper bound solution:** After all train subgroups L^{Sub} have been routed, the values of $\overline{x}_a^l$, $\overline{\varpi}_t^l$, and $\overline{\varrho}_t^l$ are fixed in the feasible (upper bound) solution. All passenger groups are then routed on arcs in A^{Psg}, and the values of variables y_a^u and z^u are recorded as $\overline{y}_a^u$ and $\overline{z}^u$. The upper bound is calculated as the objective value $\overline{Z}$, using equation (r1) with all decision variables fixed.

The novelty of this customized demand-matching strategy lies in Steps 2-1 to 2-3. In Step 2-2, passengers who cannot be served by already-scheduled trains search for their preferred routes using arcs still available for unscheduled trains. In Step 2-3, the Lagrangian multipliers are adjusted so that unscheduled trains are more likely to select passenger-preferred routes, thus improving service quality.

4.3 Updating the Lagrangian Multipliers

This section outlines the process for updating the Lagrangian multipliers in the $(i+1)$-th iteration, denoted as $\lambda_{sdt}^{(i+1)}$, $\lambda_{sdt}^{\prime(i+1)}$, and $\sigma_{ass'}^{(i+1)}$, based on their corresponding values for the i-th iteration, denoted as $\lambda_{sdt}^{(i)}$, $\lambda_{sdt}^{\prime(i)}$, and $\sigma_{ass'}^{(i)}$.

A one-dimensional vector V is constructed, where each element corresponds to a specific multiplier and represents its updating gradient. The elements for multiplier $\lambda_{sdt}^{(i)}$, $\lambda_{sdt}^{\prime(i)}$, and $\sigma_{ass'}^{(i)}$ are represented as $V\left[\lambda_{sdt}^{(i)}\right]$, $V\left[\lambda_{sdt}^{\prime(i)}\right]$, and $V\left[\sigma_{ass'}^{(i)}\right]$, respectively, and are computed using equations (s6), (s7), and (s8).

The updated multipliers $\lambda_{sdt}^{(i+1)}$, $\lambda'^{(i+1)}_{sdt}$, and $\sigma_{ass'}^{(i+1)}$ are calculated in equations (s9)–(s12), where the step size ψ and γ are determined based on the order of magnitude of the multipliers.

$$V\left[\lambda_{sdt}^{(i)}\right] = \sum_{l\in L_d, t'\in[t,t+\tau_s^{Dep}], a\in A_{sdt}^{Dep}} \bar{x}_a^l - 1 \qquad \forall s\in S;\ t\in T; d\in D \tag{s6}$$

$$V\left[\lambda'^{(i)}_{sdt}\right] = \sum_{l\in L_d, t'\in[t,t+\tau_s^{Arr}], a\in A_{sdt}^{Arr}} \bar{x}_a^l - 1 \qquad \forall s\in S;\ t\in T; d\in D \tag{s7}$$

$$V\left[\sigma_{ass'}^{(i)}\right] = \sum_{u\in(U_{ss'}\cap U_{d_a})} \bar{y}_a^u - \sum_{l\in L_{d_a}} \varepsilon_l \bar{x}_a^l \qquad \forall a\in A^R; \forall(s,s')\in SS_a \tag{s8}$$

$$Z^{Mul} = \frac{\bar{Z} - \sum_{L^{Sub}\in\mathcal{L}}\left(\sum_{l\in L^{Sub}} \bar{Z}_l^{Rel}\right) - \sum_u \bar{Z}_u^{Rel}}{|V|} \tag{s9}$$

$$\lambda_{sdt}^{(i+1)} = max\left\{\lambda_{sdt}^{(i)} + \psi\cdot Z^{Mul}\cdot V\left[\lambda_{sdt}^{(i)}\right],\ 0\right\} \qquad \forall s\in S;\ t\in T; d\in D \tag{s10}$$

$$\lambda'^{(i+1)}_{sdt} = max\left\{\lambda'^{(i)}_{sdt} + \psi\cdot Z^{Mul}\cdot V\left[\lambda'^{(i)}_{sdt}\right],\ 0\right\} \qquad \forall s\in S;\ t\in T; d\in D \tag{s11}$$

$$\sigma_{ass'}^{(i+1)} = max\left\{\sigma_{ass'}^{(i)} + \gamma\cdot Z^{Mul}\cdot V\left[\sigma_{ass'}^{(i)}\right],\ 0\right\} \qquad \forall a\in A^R; \forall(s,s')\in SS_a \tag{s12}$$

5 COMPUTATIONAL EXPERIMENTS

Using Chinese high-speed railway lines as the context, case studies of varying scales are constructed to evaluate both the optimization efficiency of the WTTSP model and the solution performance of the Lagrangian relaxation algorithm. The experiments are conducted on a personal computer equipped with an Intel i5-10500 CPU and 64GB of RAM.

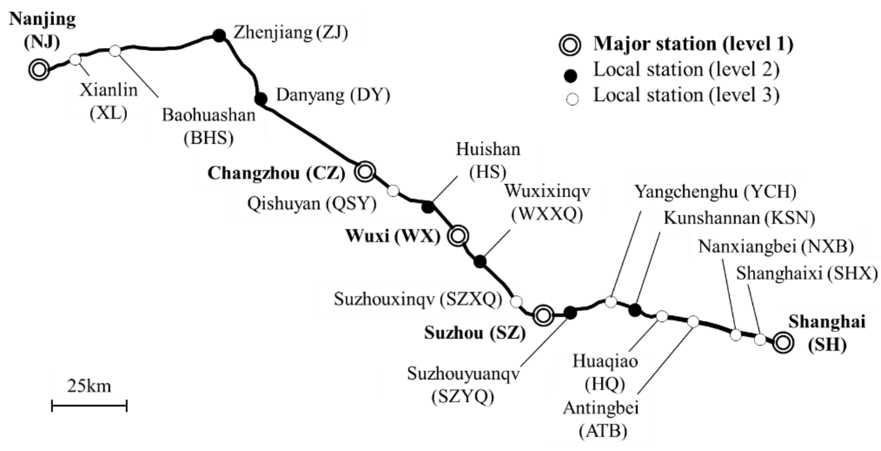

Figure 5 Stations along the Nanjing-Shanghai HSR line

5.1 Solution Performance Analysis based on Small-scale Cases

The parameters for case construction and algorithm execution are detailed in Table 3. The maximum solution time for CPLEX is limited to 7,200 seconds.

The complexity of the cases increases progressively from Case ID 1 to Case ID 5, as indicated by the number of stations and the volume of trains included in the weekly line plan.

Cases ID 1, ID 2, and ID 3 are based on a simplified HSR network with only five stations, which is also used in Nie et al. (2022) for generating small-scale weekly line plans. In contrast, Cases ID 4 and ID 5 are based on the Nanjing-Shanghai HSR line, which spans 311 km and includes 20 stations (as shown in Figure 5), representing a medium-range intercity HSR line connecting two major metropolitan areas.

As the complexity increases, the performance of CPLEX degrades significantly. In Case ID 2, CPLEX fails to reach an optimal solution with a gap of 0%. In Case ID 4, it returns an infeasible timetable where no passengers can board any train, making the solution practically unusable. In Case ID 5, CPLEX cannot even construct the WTTSP model or generate any feasible solution.

Although the runtime per iteration of the Lagrangian relaxation algorithm increases with case complexity, the solution gaps remain relatively stable across all cases. This demonstrates the algorithm's robustness and scalability. For Case ID 4, which reflects a practical HSR line, the best solution obtained from the Lagrangian relaxation algorithm is only one-quarter of the objective value yielded by CPLEX.

Table 3 Parameters for small-scale case studies

Parameter	Value
Maximum speed of trains (km/h).	300
Additional traveling time for one more stop (min).	2
Safety departure and arrival headways $\tau_s^{Dep}, \tau_s^{Arr}$ (min).	3
Range of dwelling times T^D at major stations (level 1), level 2 local stations, and level 3 local stations (min).	[6,12]; [4,10]; [2,8]
Range of passenger transfer time T^T (min).	[20, 30]
Critical parameters ω, φ, and ξ in the model and algorithm.	30; 100; 75
Steps ψ and γ of updating Lagrangian multipliers.	1,000; 1,000

Table 4 Solution performance comparison between CPLEX and Lagrangian relaxation

ID for small-scale cases		1	2	3	4	5
Testing HSR line		Toy line with 5 stations			Nanjing-Shanghai line with 20 stations	
Total number of trains in a week		10	18	36	244	394
CPLEX	Solution times	290s	7,200s	7,200s	7,200s	7,200s
	Objective value of the best solution	18,480	31,675	351,560	4.58×10^7	--
	Gap of the best solution	7.66%	38.60%	64.02%	106.83%	--
Lagrangian relaxation	Average time consumption for an iteration	1s	3s	32s	26min	42min
	Objective value of the best solution	24,080	36,000	187,515	1.22×10^7	7.94×10^6
	Gap of the best solution	28.67%	51.50%	32.51%	42.35%	33.86%

In the small-scale cases, the use of manually defined inputs leads to a mismatch between passenger demand and the origin-destination configurations of the available trains. This inconsistency limits the performance of our strategy, which is designed to be data-driven and aligned with realistic operational patterns. Consequently, the solution gaps reported in Table 4 for these cases are relatively large. Among these, Case ID 2 exhibits the largest gap, and its iteration process is illustrated in Figure 6. As shown by the red line in the figure, the lower bound estimation mechanism in the current implementation is neither fully optimized nor tailored to the specific structure of the problem. This limitation reduces the accuracy of lower bound estimates and slows the convergence of the iterative process, directly contributing to the relatively large observed solution gaps.

Nonetheless, from an optimization perspective, Figure 6 demonstrates a clear and steady improvement in the objective value of the best-found solution across iterations, validating the effectiveness of the proposed algorithm.

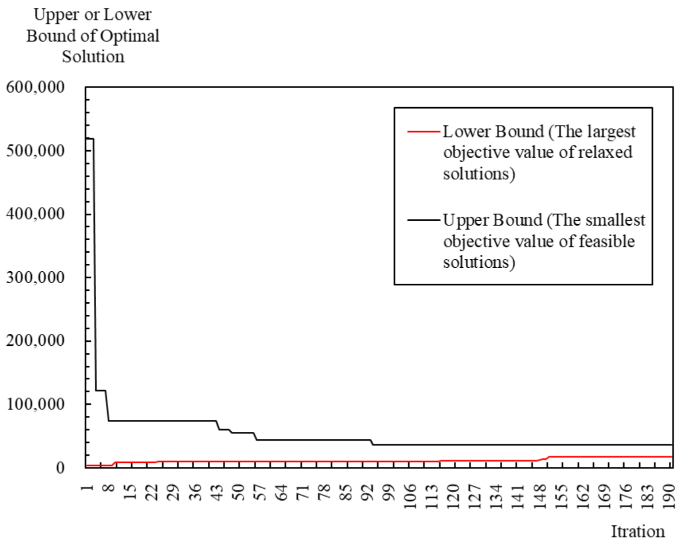

Figure 6 Iteration process of Lagrangian relaxation algorithm

5.2 Solution Performance Analysis based on Nanjing-Shanghai HSR line

In this section, practical-scale cases are constructed based on the Nanjing-Shanghai HSR line, as illustrated in Figure 5. The weekly line plan developed in our previous work (Nie et al., 2022) is used as input to define the origins, destinations, compositions, and operational frequencies of trains for each period and each day.

To reduce computational complexity, a consistent timetable is applied from Monday to Thursday, a separate timetable is used for Friday and Sunday, reflecting their similar demand patterns on this HSR line, and a unique timetable is set for Saturday. In Nie et al. (2022), solutions using common timetables for similar-demand days were compared with solutions employing distinct timetables for all seven days. The results showed that consolidating similar-demand days can help the algorithm achieve better solutions within limited computation time.

The remainder of this section is structured as follows: Section 5.2.1 discusses the values of critical parameters in the proposed WTTSP model and the Lagrangian relaxation algorithm. In Section 5.2.2, solutions with different sets of alternative stopping patterns are analyzed to demonstrate the impact of flexible stop adjustments. Section 5.2.3 highlights the benefits of integrating stop planning by comparing the weekly timetables obtained using the WTTSP model with those generated by a sequential approach. In Section 5.2.4, the optimized weekly timetable is compared with the manually determined timetable used in practice, providing an analysis of the optimization performance of the WTTSP model. Section 5.2.5 compares the weekly timetable with a daily-varying timetable generated using a similar algorithm, illustrating the trade-offs between regularity and demand adaptability.

5.2.1 Discussing the Values of Critical Parameters

Several key parameters significantly influence the solution performance of the WTTSP model:

- **Weight $\boldsymbol{\omega}$ of train travel costs** in the objective function affects the trade-off between improving passenger travel convenience and reducing train operation costs.
- **Adjustment parameter $\boldsymbol{\xi}$** in the feasible solution generation process modulates the amplitude of the Lagrangian multipliers. This helps align passenger and train routes, though it may increase train costs to some degree.
- **Penalty value $\boldsymbol{\varphi}$** for passengers unable to board any train impacts the balance between increasing the number of served passengers and improving the service for passengers who can easily board.

When analyzing the influence of any specific parameter, the values of the other parameters are kept constant. Setting $\xi = 10$ and φ=100, the travel costs for passengers and trains under different values of weight ω is recorded in Figure 7. The data points between $\omega = 1000$ and $\omega = 50$, show an elbow-shaped trend, with the inflection point occurring at $\omega = 100$. For $\omega > 100$, reducing passenger travel costs becomes less significant as train travel costs continue to rise. For $\omega < 50$, a reduction in algorithmic efficiency leads to a comprehensive increase in the travel costs for both passengers and trains. Thus, for practical-scale case studies, **the weight $\boldsymbol{\omega}$ is set as 100.**

Similarly, Figure 8 records the changes in travel costs for passengers and trains with varying values of ξ. Another elbow-shaped trend emerges. When $\xi \in [0,10]$ passenger travel costs decrease notably while train costs increase. Between ξ=10 and ξ=100, the costs for trains and passengers stabilize, with no significant changes. For $\xi \in [100,500]$, the reduction in passenger travel costs becomes negligible, and when $\xi \geq 500$, the solution remains unchanged. Therefore, **$\boldsymbol{\xi}$ is set to 50, a median value within the $\mathbf{[10, 100]}$ range**.

Figure 9 illustrates how the average travel costs of boarding passengers and the proportion of passengers unable to board (un-boarding passengers) change with different values of φ. When $\varphi \in [10,50]$, the proportion of un-boarding passengers decreases significantly. For $\varphi > 20$, the average travel costs for boarding passengers increase, reflecting the cost of planning additional stops to accommodate more passengers. Based on the trends in Figure 9, **$\boldsymbol{\varphi}$ is set to 70, at which point the proportion of un-boarding passengers falls to 2% for the first time and remains nearly constant thereafter.**

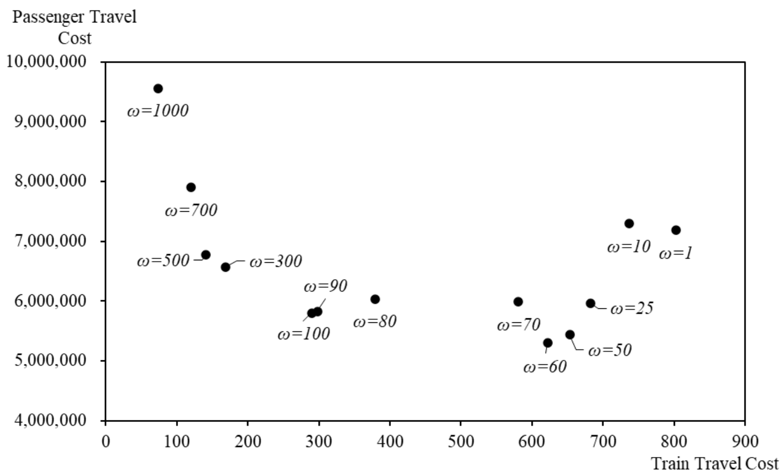

Figure 7 Changes of travel costs of trains and passengers with different values of ω

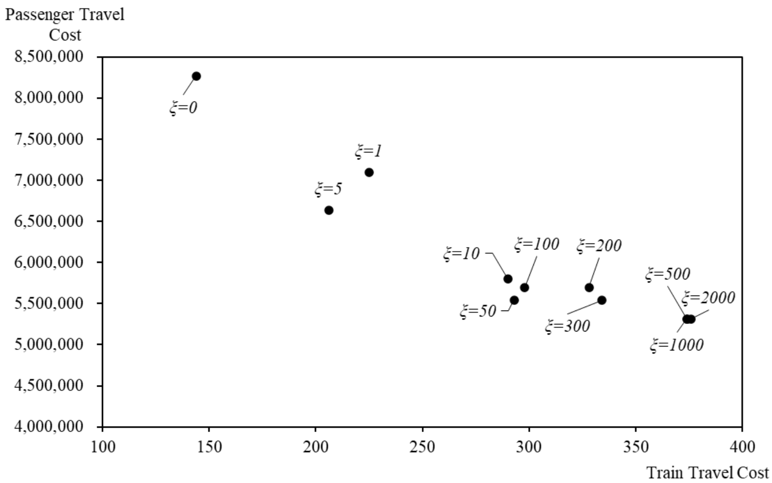

Figure 8 Changes of travel costs of trains and passengers with different values of ξ

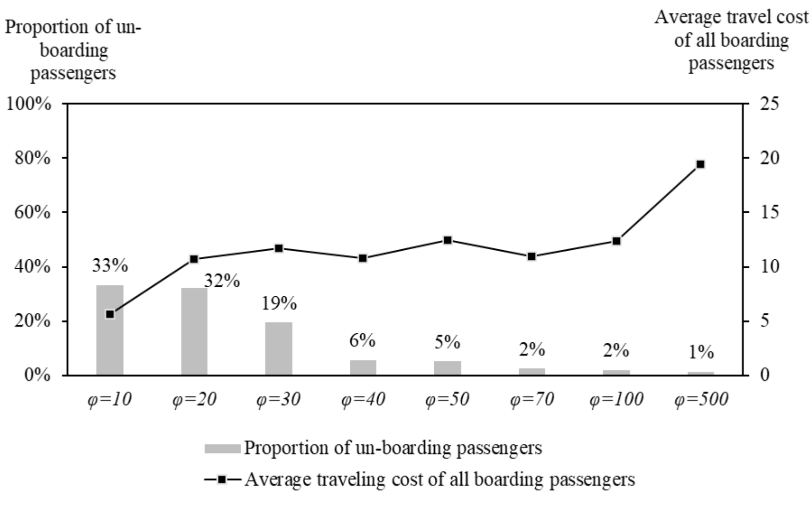

Figure 9 Average travel cost of boarding passengers and the proportion of un-boarding passengers in the solutions with different values of φ

5.2.2 Solution Comparison with Different Sets of Stopping Patterns

The scale of the stopping pattern set determines the flexibility of train stops, which in turn affects solution performance. Seven sets of alternative stopping patterns were constructed following the rules of practical scenarios, each with varying numbers of options.

Set 1 is termed the "Totally all-stopping pattern." In this set, all trains are fixed to stop at every station they pass. This is the smallest set, making it the easiest to solve.

Set 2 is the "Hierarchical all-stopping pattern." Trains are categorized into three levels, as outlined in Table 1. Level 1 trains stop at all level 1 stations they pass, level 2 trains stop at all level 1 and 2 stations, and level 3 trains stop at every station. This pattern is widely used on European and Japanese HSR lines.

Set 3 is the "Flexible hierarchical pattern." It builds on Set 2, but allows level 2 trains to skip one or two level 2 stations, and level 3 trains to skip one or two level 3 stations. This set introduces more flexibility, adapting to HSR lines with more stations and complex passenger demand.

Set 4 is the "Hybrid pattern." It extends Set 3 by allowing both level 1 and level 2 trains to choose stopping patterns for both level 1 and 2 stations, while level 1 trains remain fixed to level 1 stops. Level 3 trains can choose between level 2 and 3 stops. In this set, train stops are more responsive to passenger demand rather than predefined rules. This set aligns with the practical stopping rules of the Nanjing-Shanghai HSR and is applied in section 5.2.1.

For larger-scale models, solving a single iteration may become computationally infeasible, preventing the algorithm from updating bounds and finding better solutions. To explore the limits of the algorithm, **Sets 5, 6, and 7** were constructed. **Set 7**, the "Enumerated pattern," allows level 2 trains to stop or skip at any level 1 or 2 station, and level 3 trains to stop or skip at any level 1, 2, or 3 station, with the condition that they stop at least once at a level 2 station. **Sets 5 and 6** are derived from Set 7 by excluding certain stopping patterns, creating sets of varying sizes.

Table 5 Solutions with different sets of stopping patterns

Set of stopping patterns		1	2	3	4	5	6	7
Number of alternative stopping patterns between major stations	NJ-CZ	1	3	10	10	15	15	15
	CZ-WX	1	3	4	4	4	4	4
	WX-SZ	1	3	4	4	4	4	4
	SZ-SH	1	3	12	12	51	63	98
Average time consumption of each iteration		43min	62min	85min	93min	245min	Out of memory while solving the 213-rd train subgroup	Out of memory while solving the 52-th train subgroup
Gaps		9.31%	8.00%	11.12%	9.53%	10.21%		
Objective value of the best-found solution ($\times 10^6$)		6.50	6.08	5.88	4.97	6.50		
Train travel cost of the best-found solution		1856	347	397	507	465		
Passenger travel cost of the best-found solution ($\times 10^6$)		6.69	6.05	5.84	4.92	6.45		

Table 5 records the solutions obtained using Sets 1 to 7. The algorithm could only solve one-quarter of the train subgroups during the first iteration of solving Set 7 before running out of memory. This suggests that applying practical stop rules to exclude certain patterns instead of considering all possible stops is essential. In Set 6, only five train subgroups remained unsolved, indicating that this set's scale is near the algorithm's capacity.

Comparing the results of Sets 1 through 4, larger sets provide more suitable services and improve the weekly timetable quality. However, the performance of Set 5 shows that overly large stopping pattern sets can reduce solution efficiency. Set 4, which achieves the best objective value, strikes an optimal balance. It provides enough flexibility to improve service quality while minimizing the time consumed per iteration. This allows for more iterations and increases the likelihood of finding high-quality solutions.

5.2.3 Solution Comparison between Integrated and Sequential Optimization.

The proposed model and algorithm integrate stop planning into weekly timetabling. To demonstrate the advantages of this integration, we compare it to a sequential approach, which uses the stops given in Nie et al. (2022) as input and only optimizes the train timetable.

Table 6 compares key indicators of both the sequential and integrated solutions. The most notable improvement from the integrated optimization is the significant increase in the number of passengers departing during their preferred periods. While the sequential approach adjusts trains primarily by canceling or shifting their departure periods, the integrated approach goes further by adjusting train stops, resulting in more tailored service and reduced passenger travel costs. Additionally, passenger travel speed and average load factor show slight improvements, reflecting better overall service quality in the weekly timetable produced by the integrated approach.

Table 6 Comparison of the practical timetable, weekly timetable obtained by sequential method, and weekly timetable obtained by proposed method

	Practical timetable	Weekly timetable			Daily-varying timetable
		Generated using sequential approach	**Generated using our algorithm**	7 timetables for 7 days	
Train travel cost	524	498	507	514	499
Average travel speed of trains (km/h)	142.17	210.71	209.24	210.21	209.35
Passenger travel cost ($\times 10^6$)	6.72	5.21	4.92	5.10	4.87
Average number of intermediate stop of passengers	1.97	0.89	0.92	0.93	0.90
Average travel speed of passengers (km/h)	129.75	174.52	177.77	177.19	178.04
Average load factor of trains	68.23%	71.10%	71.45%	70.42%	73.25%
Average load factor of mandatory trains	95.08%	97.13%	100%	100%	100%
Proportion of passengers departing in preferred periods	70.14%	69.26%	78.46%	78.28%	82.31%
Proportion of passengers departing in preferred periods or the adjacent periods	84.42%	93.13%	94.24%	94.10%	96.22%

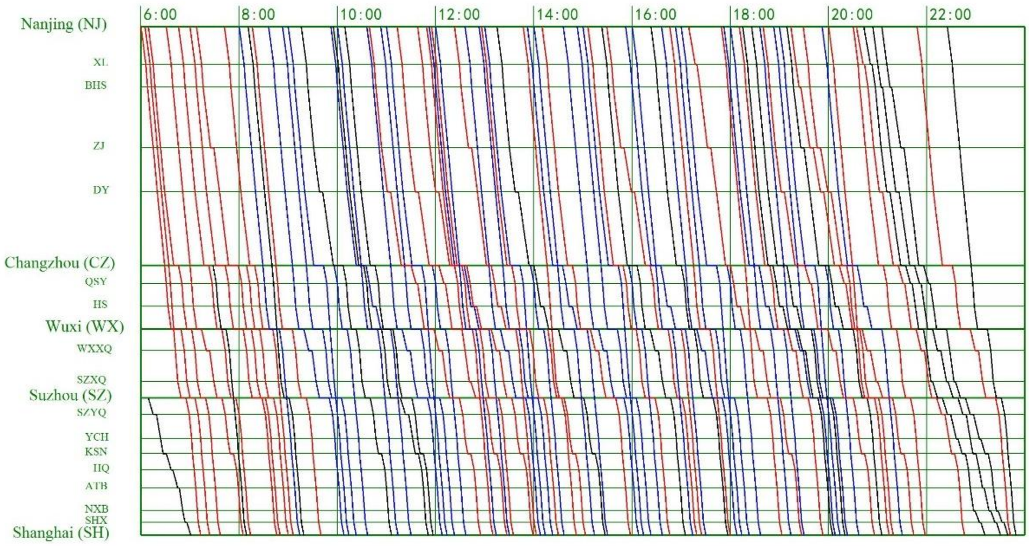

Figure 10 Weekly timetable of Nanjing-Shanghai HSR line (Mon. to Thu.)

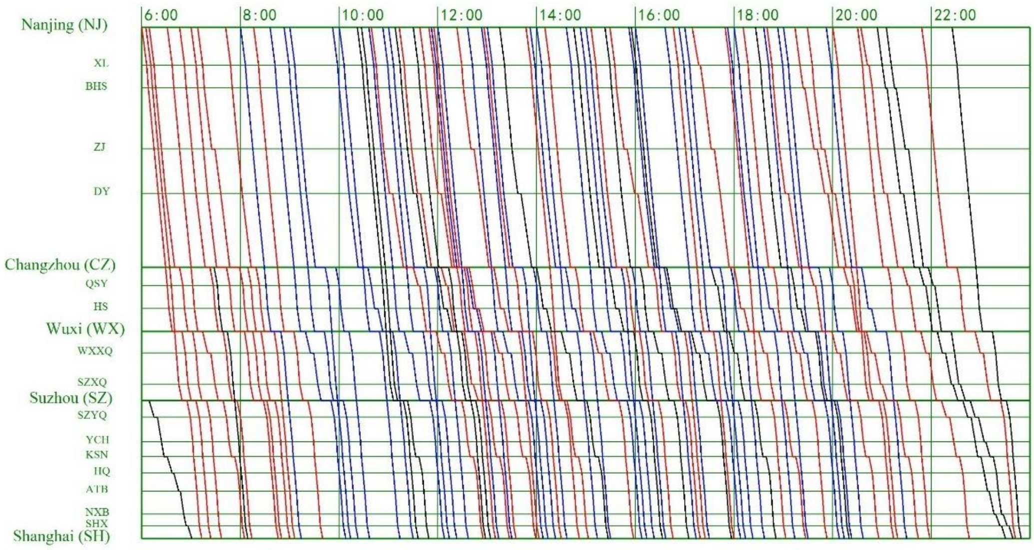

Figure 11 Weekly timetable of Nanjing-Shanghai HSR line (Fri. and Sun.)

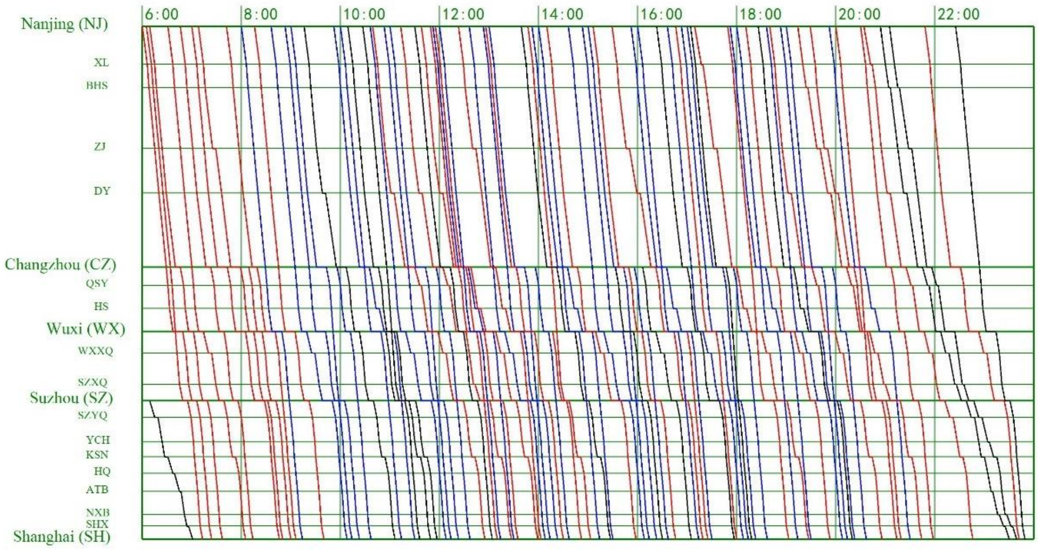

Figure 12 Weekly timetable of Nanjing-Shanghai HSR line (Sat.)

5.2.4 Solution Comparison between Weekly Timetable and Practical Timetable.

Figures 10, 11, and 12 illustrate the timetables for different days generated by the proposed Lagrangian relaxation algorithm. The blue, red, and black lines represent periodic, daily, and weekly trains, respectively. Compared to the manually determined practical timetable (indicators shown in Table 6), the advantages of the weekly timetable obtained using the proposed WTTSP model are analyzed as follows.

The traveling speeds of both trains and passengers have been significantly improved through the optimization of stops and dwelling times. Figure 13 shows the distribution of traveling speeds for passengers and trains in the weekly timetable. In the WTTSP model, train stops are constrained by the three train levels, resulting in three distinct peaks in train numbers at speeds of 250 km/h, 200 km/h, and 140 km/h, which represent different service levels. Similarly, passenger numbers peak around 250 km/h, 200 km/h, and 120 km/h (slightly lower due to transfer time). This distribution reflects a better spatial alignment between passengers and trains in the weekly timetable. Furthermore, the proportion of passengers departing during preferred periods is improved, indicating a better temporal match between passengers and services.

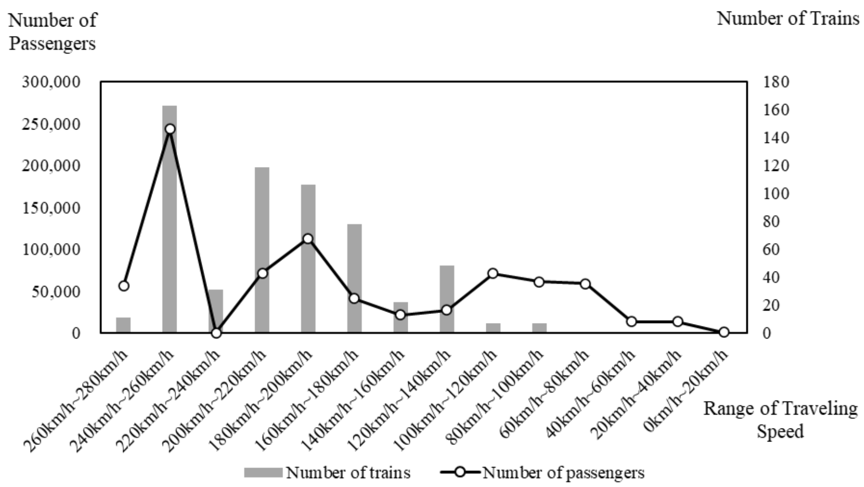

Figure 13 Traveling costs distribution of passengers and trains

Figure 14 compares the transport capacity (measured by measured by the total seatkilometers, calculated as the sum of each train's seat number multiplied by its mileage) and passenger demand (measured by passengerkilometers, calculated as the sum of all passengers' travel distances) for each period of each day in both the weekly and practical timetables. In the practical timetable, some periods, such as 16:00–18:00, exhibit excessive transport capacity. The weekly timetable addresses this mismatch by eliminating unnecessary trains to reduce train operating costs and scheduling trains according to passenger preferences to minimize passenger travel costs.

As a result of the improved spatial and temporal alignment between passengers and trains, the load factors of trains in the weekly timetable have increased, particularly for mandatory trains. These trains achieve speeds of up to 280 km/h with a 100% load factor, offering premier services with enhanced crew service and seating options, thereby attracting more passengers and boosting revenue for HSR companies.

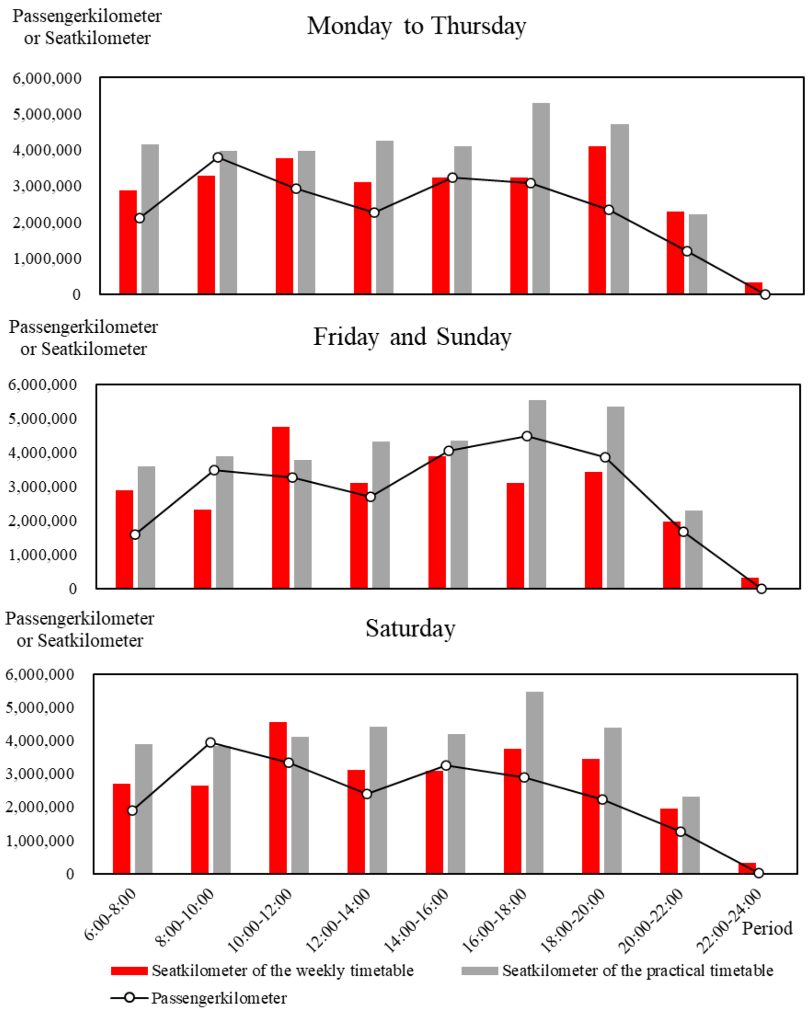

Figure 14 The seatkilometers and passengerkilometers of each period in each day

5.2.5 Solution Comparison between Weekly Timetable and Daily-Varying Timetables.

This section compares two alternative timetable strategies with the weekly timetable proposed in this study. The first alternative is the "7-for-7 weekly timetable," in which each of the seven days is assigned a distinct timetable, including Monday through Thursday. The second is the "daily-varying timetable," where the proportions of periodic and daily trains are set to zero, resulting in completely different timetables for each day. The performance indicators for these strategies are presented in Table 6. As this comparison was also explored in Nie et al. (2022), the same origins, destinations, and train frequencies from that study are adopted for consistency.

Due to their similar generation processes, the proposed weekly timetable and the 7-for-7 weekly timetable produce comparable outcomes in terms of passenger and train travel speeds and train load factors. However, the 7-for-7 timetable achieves slightly lower travel costs for both passengers and trains. This marginal improvement is attributed to the increased flexibility in scheduling, although the higher model complexity may result in suboptimal solutions when computational time is limited. Given sufficient computational resources, using distinct timetables for all seven days, even for days with similar demand, can further improve demand-matching performance and overall solution quality.

In contrast, the daily-varying timetable greatly simplifies the optimization process, as each day is scheduled independently. As shown in Table 6, this approach leads to the lowest passenger travel cost, as it imposes no constraints on periodic or daily train operations. This allows for full scheduling flexibility, enabling trains to be optimally aligned with daily demand patterns and thus achieving a high level of demand matching. However, as discussed in the Introduction, such flexibility leads to highly irregular timetables, which may cause confusion for passengers. For those adjusting their travel across different days or time periods, a more regular timetable offers significant convenience by maintaining consistent departure patterns. Furthermore, the inclusion of periodic and daily trains enables passengers who miss a scheduled train to catch the next service within a predictable time window, or to make reliable transfers between regular services—enhancing both travel experience and service reliability.

5.3 Solution Comparison between Nanjing-Shanghai and Beijing-Shanghai HSR lines

The Beijing-Shanghai HSR line spans 1302 km and has 23 stations, representing long-distance HSR lines that pass through multiple metropolitan areas. Unlike the Nanjing-Shanghai line, the Beijing-Shanghai line operates different timetables on Fridays and Sundays. The weekly timetable for this line is shown in Figures 15 to 18, and the key characteristics of its weekly schedule, compared to the Nanjing-Shanghai line, are summarized as follows:

(1) Frequent Overtaking at Stations. Overtaking is more common due to the larger number of level 2 major stations along the Beijing-Shanghai HSR compared to the Nanjing-Shanghai HSR. This results in a higher volume and greater variety of stopping patterns for level 2 and level 3 trains. To minimize gaps between faster and slower trains, a fast train may overtake multiple slow trains at stations to maintain its speed.

(2) Large Gaps at the Start and End of the Day. The longer distance of the Beijing-Shanghai line leads to longer travel times. Trains departing from one terminal station often arrive much later at the other terminal compared to the Nanjing-Shanghai line. This creates significant gaps in the timetable at the beginning and end of the day at the terminal stations. Short-distance trains, such as those serving commuters, can be scheduled in these gaps. For example, while one daily short-distance train operates between Suzhou and Shanghai on the Nanjing-Shanghai line, at least five short trains run daily on the Beijing-Shanghai line.

(3) Fewer Trains Connecting the Two Terminal Stations. The weekly plan for the Beijing-Shanghai HSR includes many shorter-distance trains between key metropolitan areas like Tianjin, Jinan, Xuzhou, and Nanjing. These shorter trains fill gaps between faster and slower long-distance trains, making the timetable more saturated. In contrast, the Nanjing-Shanghai HSR features fewer intermediate cities with high passenger volumes between Nanjing and Shanghai, such as Suzhou, Wuxi, and Changzhou. The short-distance trains on this line all terminate in Shanghai, where the high volume of terminating trains causes oversaturation, leading to the cancellation of many short-distance trains to ensure service for passengers between Nanjing and Shanghai.

6 CONCLUSION

To better align HSR services with fluctuating passenger demand across different times of the day and various days of the week, this study proposes the WTTSP model, which uses the origin, destination, and operational frequencies of trains for each period and day as inputs and determines train stopping patterns, as well as with departure and arrival times at each station. Trains are categorized into three types based on their weekly operation modes: **periodic trains**, which repeat in each period daily; **daily trains**, which maintain consistent stopping patterns and schedules across all days; and **weekly trains**, which are scheduled flexibly to accommodate demand variations. Passenger routing is also incorporated into the model, with the objective of minimizing the combined travel costs of trains and passengers.

To solve the integrated optimization problem efficiently, the model is reformulated as a route-searching problem for trains and passengers through the construction of WTON. A Lagrangian relaxation algorithm is applied, which relaxes the safety headway and seating capacity constraints and decomposing the model into subproblems for routing subgroups of trains and passengers. In each iteration, these subproblems are solved independently to obtain a relaxed solution and a lower bound of the optimal solution. A feasible solution is obtained through a customized demand-matching strategy to determine the upper bound. Based on these relaxed and feasible solutions, the Lagrangian multipliers are updated for the next iteration.

The effectiveness of the proposed model and algorithm is validated through case studies on Chinese HSR lines. In small-scale scenarios, the algorithm significantly outperforms the commercial solver CPLEX in both solution quality and computation time, demonstrating superior efficiency. In practical-scale experiments based on the Nanjing-Shanghai HSR line, the results reveal the algorithm's computational limitations highlight the benefits of integrated optimization over conventional sequential methods. Notably, the optimized weekly timetable improves passenger travel speeds and departure-time satisfaction. An additional case study on the Beijing-Shanghai HSR line further demonstrates the algorithm's versatility, showing its applicability to various types of HSR lines.

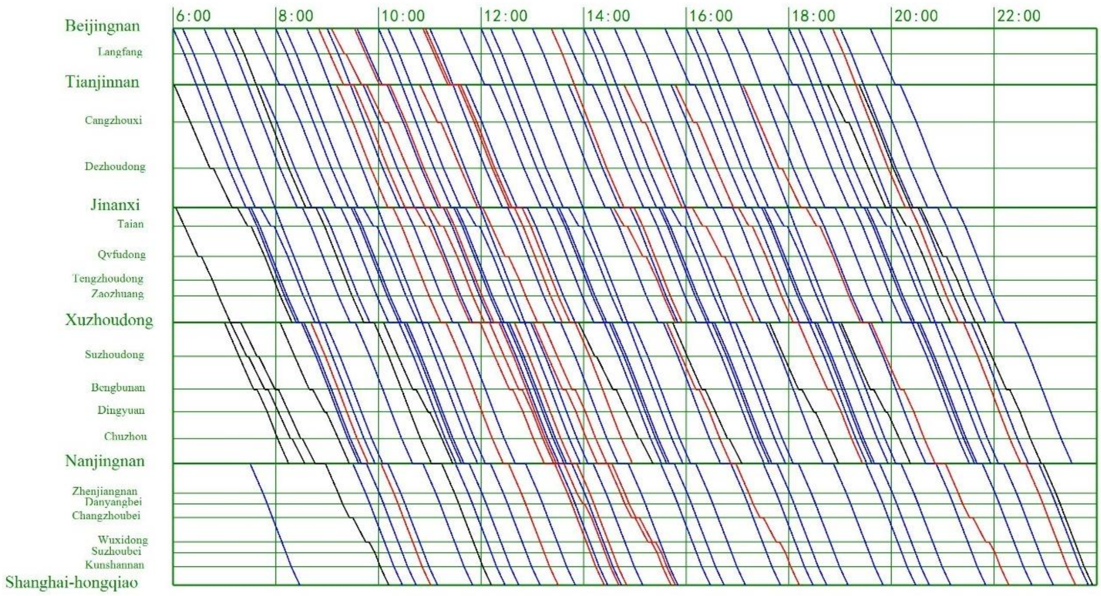

Figure 15 Weekly timetable of Beijing-Shanghai HSR (Mon. to Thu.)

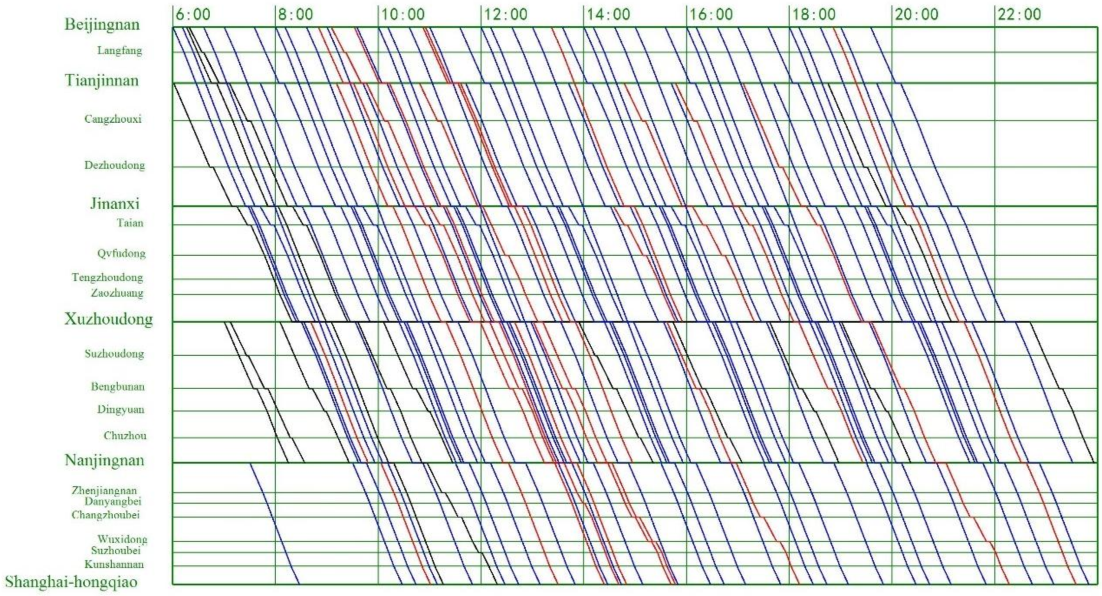

Figure 16 Weekly timetable of Beijing-Shanghai HSR (Fri.)

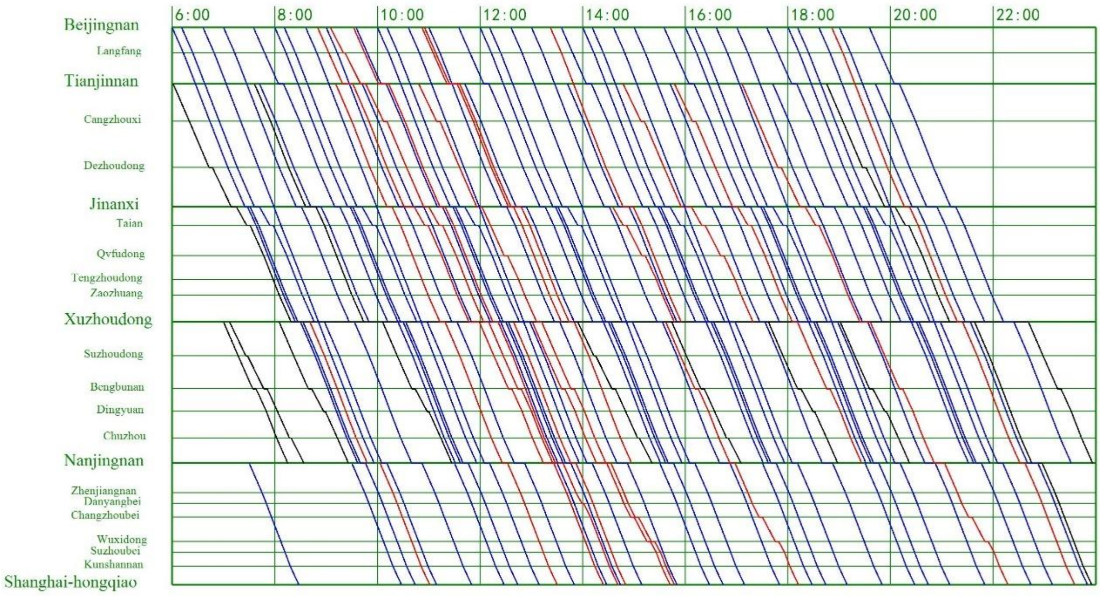

Figure 17 Weekly timetable of Beijing-Shanghai HSR (Sat.)

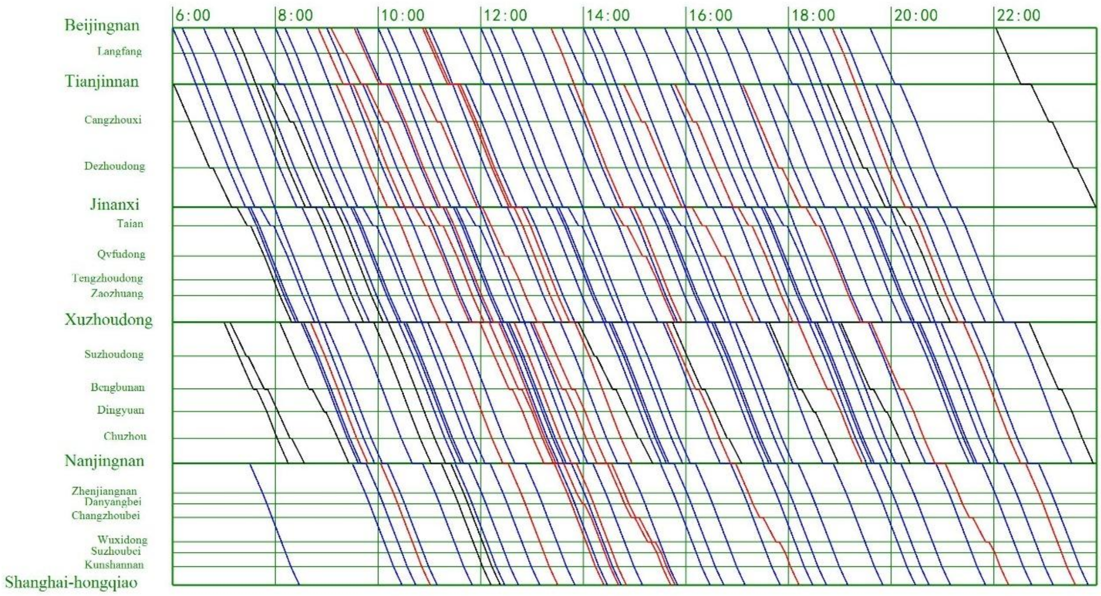

Figure 18 Weekly timetable of Beijing-Shanghai HSR (Sun.)

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