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Research Paper

Optimal reimbursement schemes in contests☆

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ABSTRACT

Many contests, such as innovation races or sport competitions, often involve reimbursement of expenses. This study examines optimal reimbursement schemes in two-player Tullock contests, analyzing six reimbursement structures: external versus internal funding source each targeting the contest winner, the loser, or both of them. We assess the implications on effort, winning probabilities, and designer payoff under three key conditions: full-reimbursement, neutrality (preserving initial win chances) and viability (positive efforts from players). We find that all the schemes can satisfy viability; and all the schemes except for external reimbursement to the winner can satisfy neutrality. Additionally, all the schemes except internal reimbursement to the winner, and internal or external reimbursement to both players can satisfy full-reimbursement. These findings indicate that optimal reimbursement structures and rates vary depending on the contest structure, and the designer's objectives, such as maximizing effort or maximizing personal payoff.

1. Introduction

Competition is the lifeblood of economic activities, business, sports, war, R&D races, and promotional tournaments. In these situations, participants invest costly resources to secure a prize, which is the essence of a 'contest'. In many such contests, a player may be reimbursed for their expended resources. For instance, the Defense Advanced Research Projects Agency (DARPA) has employed reimbursement strategies in its Grand Challenges, such as the Self-Driving Car Challenge, to encourage bold technological leaps (Goodrich and Olsen, 2003). Similarly, the XPRIZE Foundation has utilized milestone prizes to offset development costs and catalyze innovation in fields like space exploration and environmental sustainability (Diamandis and Kotler, 2012). The U.S. Department of Energy, the NASA and the European Innovation Council have all implemented reimbursement programs to promote innovation in energy efficiency, sustainable space technologies, and blockchain. Beyond R&D, reimbursement schemes are employed in various areas such as defense, or organizational competitions. In military history, war indemnities, where the defeated nation compensates the victor, have served as both a deterrent and a means of reparation (Ferguson, 2001). Reimbursement is often utilized in internal competitions, such as promotional tournaments, to incentivize employee development and enhance organizational performance.

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Reimbursement in contests can be made in several ways that vary in terms of the extent of reimbursement (full or partial), the source of reimbursement (external reimbursement by a third party, or internal reimbursement by one or more of the players), and the outcome-contingent target (the winner and\or the loser). The reimbursement scheme is usually determined by a contest designer who wishes to attain certain goals by affecting the players' incentives. An innovation authority, for example, may select the reimbursement scheme that best promotes the viability of the R&D race, i.e., maximizes the likelihood of a technological breakthrough. The preferred scheme may also be justified by social norms or by guiding and binding principles of fair conduct.

Although reimbursement schemes are inherently important in these contests, there is a scarcity in the literature to systematically analyze and compare various reimbursement schemes. In this study we theoretically focus on six schemes of partial or full-reimbursement. In three schemes, a third party (the designer) reimburses the winner, the loser, or both of them; whereas in the other three schemes one player reimburses the cost to the other, or both of them reimburse each other (reciprocal reimbursement). Moreover, we consider a set of desirable properties such as full-reimbursement, participation constraint, and neutrality (defined in Section 1.1) for each of the schemes and compare their performance in terms of the total effort generated (resources spent), and the designer's payoff.

Reimbursement schemes in contests have been investigated in the literature earlier, although not systematically. Chowdhury and Sheremeta (2011a) constructed a Tullock (1980) contest with effort spillovers where contingent upon winning or losing, the payoff of a player is a linear function of prizes, own effort, and the effort of the rival. This generic structure nests several existing contests in the literature and is used as a special case to analyze models of internal reimbursement. Similar structure of spillover and reimbursement was studied by Baye et al. (2005, 2012), in an all-pay auction set-up, focusing on the designer's payoff and the players' efforts under symmetric prize valuations. Xiao (2018) proved that an all-pay contest with additively separable spillovers has a unique Nash equilibrium. Loser reimbursing the winner has been studied in the literature (Baumann and Friehe, 2012; Carbonara et al., 2015; Luppi and Parisi, 2012; Matros and Armanios, 2009; Yates, 2011) focusing on Tullock contests. Specific models of external reimbursement are also analyzed by Cohen and Sela (2005), Matros (2012), and Thomas and Wang (2017).[1] Matros and Armanios (2009) generalized the external reimbursement by allowing reimbursement of the winner and the loser, jointly or separately, focusing on the designer's payoff under symmetric prize valuations.

1.1. Six schemes of reimbursement

In this study, we introduce a two-player Tullock contest that incorporates the option for effort reimbursement in six mutually exclusive and exhaustive reimbursement schemes. These schemes are categorized based on the source (external or internal) and target (winner and\or loser) of the reimbursement. While each scheme is suitable for particular contexts, contest designers may also face choices between them. Thus, it becomes essential to identify which scheme best fits a given context and whether improvements can be made. Our key questions include which reimbursement scheme is the most desirable, and whether any of them meet multiple desirable criteria simultaneously. A central focus in contest design is determining how designers can achieve their objectives (e.g., designer payoff or total effort) through interventions that influence the contest. In this study we answer these for the first time. Table 1 summarizes the six reimbursement schemes for clarity.

Scheme A: reimbursement of the winner's expenses by a third party (external reimbursement). This reimbursement scheme is common in competitions between employees for a position or job or for promotion to a certain rank. The manager often wishes to motivate the participants by covering the winner's expenses (Thomas & Tung, 1992). Such a reimbursement scheme is also common in R&D races because the contest designer wants to increase the profitability of winning. This scheme has also been studied in contests by Cohen and Sela (2005) and was expanded by Matros and Armanios (2009) and Matros (2012).

Scheme B: reimbursement of the winner's expenses by the loser (internal reimbursement). This reimbursement scheme studied by Baye et al. (2005), in a symmetric all-pay auction set-up. This scheme can be applied in war when victors impose war indemnities on the vanquished side to recoup the costs of warfare and punish the defeated population (Sullo & Wyatt, 2014). It is no wonder that peace treaties between warring nations often resort to legal justifications such as coverage of war expenses. In this scheme, the maximal reimbursement that results in the participation of the players cannot be 100 %.[2] Note that for some non-Tullock contests, full-reimbursement induces an interior equilibrium (e.g., Plott, 1987). Moreover, for Tullock contests with non-linear cost functions or "impact" functions, full-reimbursement also induces an interior equilibrium for some extent of non-linearity (e.g., Carbonara et al., 2015; Farmer and Pecorino, 1999).

Scheme C: reimbursement of the loser's expenses by a third party (external reimbursement). The most common application of this reimbursement is insurance. An insurance company reimburses the loser for their expenses. This reimbursement scheme is plausible in various contexts, such as education, where underperforming students may receive support after failing a test; intra-firm competitions, where designers incentivize participation by offsetting losses; and R&D races aimed at stimulating innovation. Minchuk and Sela (2020) studied this scheme in the context of insurance, where the designer can offer players an insurance option that requires them to pay a premium to cover potential losses. They found that this scheme may be profitable for the designer. This scheme of reimbursement was also studied in Matros and Armanios (2009).

Kovenock and Lu (2020) studied score procurement auctions with all-pay quality bids, where a supplier's score is the difference

[1] Thomas and Wang (2017) studied a Tullock contest with external subsidization regardless of who wins. An important characteristic of this setting is the assumption that there is a limited amount of resource that can be used for subsidies.

[2] The rate of reimbursement is less than 100%; it is equal to the maximal rate that yields an interior equilibrium.

Table 1
Reimbursement schemes.

Schemes		Reimbursement source	
		External (from the designer)	Internal (from the opponent)
Reimbursement target	Winner	Scheme A	Scheme B
	Loser	Scheme C	Scheme D
	Both Winner & Loser	Scheme E	Scheme F (reciprocal reimbursement)

between their quality and price bids. They found that reimbursing the all-pay components of losing or all suppliers increases quality provision and suppliers' payoffs but reduces total surplus and the procurer's payoffs by lowering suppliers' marginal provision costs. Since social marginal quality provision costs remain unchanged and exceed individual costs, the additional quality provision may be overly costly to society, making suppliers' decisions under such subsidies less efficient.

Scheme D: reimbursement of the loser's expenses by the winner (internal reimbursement). This scheme of reimbursement is observed in the aftermath of war when the victor is required to compensate the loser after being convicted of war crimes or when there is an agreement (imposed by the designer or between the parties) on the importance of supporting the weak or needy. A close example without war crime is the Marshall Plan implemented by the USA post-World War II on some of the axis powers in western Europe. This scheme can incentivize weaker players to participate in the competition due to the anticipation that the winner will reimburse the loser for his expenses.

Scheme E: reimbursement of both winner's and loser's expenses by a third party (external reimbursement). This reimbursement scheme is equivalent to a subsidy granted by a third party to all participants in the contest, regardless of the outcome (Glazer and Konrad, 1999). Third parties may provide subsidies to encourage participation and investment in competitions, particularly when there is concern that potential candidates may be deterred by high costs. Examples of such subsidies can be found in various fields, such as government tenders, academic research incentives, and initiatives encouraging manufacturers to invest in green energy (Bai et al., 2019).

Scheme F: reciprocal reimbursement scheme – the winner reimburses the loser, and the loser reimburses the winner (internal reimbursement). This reimbursement scheme is equivalent to expenses subsidy (such as Scheme E), but the source is internal (from the opponents), and with the addition of a lump-sum tax amounting to a certain percentage of the opponent's expenses. While this is a theoretical structure that completes all possible schemes, direct examples are very rare.

1.2. Desirable properties: full-reimbursement, neutrality, and viability

In Tullock's (1980) celebrated contest, the players invest efforts to win a particular prize, and the winning probabilities increase with their efforts (Bevia and Corchon, 2024; Konrad, 2009). Applying this setting, we aim to clarify the preferred scheme of reimbursement for specific objectives of the contest designer, while focusing on three desirable properties: neutrality, viability and full-reimbursement. Below we define and discuss these properties further.

Definition: Neutrality - A reimbursement scheme is considered 'neutral' if it preserves the original ratio (ranking) of players' win probabilities even after the scheme is implemented.

Neutrality (or rank-preservation or plausibility) is defined by a single condition: the ratio of players' winning probabilities in the initial, unaltered contest must not be reversed by any intervention. There are two main reasons for this requirement. First, from a fairness perspective, players might perceive a contest as more legitimate if a subsequent intervention (the reimbursement scheme) does not fundamentally alter their relative chances of success. Hence, from a political standpoint, a policy that drastically alters winning probabilities is likely to be infeasible, as the initially stronger player would have the incentive and means to resist a shift that reduces his status from prospective winner to likely loser. Second, even if such a policy were enacted, the stronger player could choose to withdraw from the contest altogether, effectively nullifying the designer's intentions. To avoid this outcome, the designer refrains from implementing policies that would lead to this reversal. This criterion aligns with Groh et al. (2012), who also characterize it as a neutrality-condition. Note that the setup of some models, including ours, is equivalent to players having the same prize valuation but different linear cost functions. In this case, neutrality means favoring the player who is more cost-efficient.

Definition: Viability – a reimbursement scheme is viable if all players exert positive efforts and receive non-negative payoffs in equilibrium, after the scheme is implemented.

In any contest, particularly in a two-player setting, achieving a viable equilibrium that satisfies players' participation constraints is crucial. Without viability, the contest designer's objectives – which frequently rely on players' efforts – cannot be realized.

Definition: Full-reimbursement - A reimbursement scheme is considered full-reimbursement if a viable pure-strategy equilibrium exists after the scheme is implemented and the target player(s) fully recovers (recover) his (their) expenses by the source player or the designer.

Full-reimbursement incentivizes the player(s) to participate and exert greater efforts compared to a partial refund. However, it is important to note that in some cases, a full refund may prevent the existence of an equilibrium. This can occur if it cancels the source player's incentive to participate in the contest or leads the target player to invest infinite effort. Despite these potential challenges, a full refund also benefits the contest planner by making the contest more attractive and easier to market, ultimately increasing participation and engagement.

1.3. The optimal reimbursement scheme and rate

We aim to provide a theoretical tool with potential applications for finding the optimal reimbursement scheme, including the optimal reimbursement rates, for the following objectives: designer payoff, and total effort. The analysis considers the possibility that the designer is interested, or not interested, in the above three desirable properties. To simplify the illustration of the comparison of the reimbursement schemes, we assume a two-player Tullock contest and a linear cost. These assumptions are common in the literature (e.g. Baye et al., 2005, 2012; Carbonara et al., 2015; Luppi and Parisi, 2012). Reimbursement schemes in contests were earlier analyzed by Cohen et al. (2023). However, they focus on internal funding by the loser, uniform mixed funding, and non-uniform external funding - and demonstrate equivalence in terms of designer profit and player outcomes. But they do not consider viability or neutrality constraints. Additionally, unlike the current analysis, the prize values were symmetric in Cohen et al. (2023).

In patent races, for example, the designer's primary goal is often to maximize innovation efforts that enhance the likelihood of high-quality outputs, such as patents for COVID-19 vaccines. A similar logic applies to sports competitions, where designers recognize that high-effort contests are more appealing to spectators; this drives demand and enhances ticket sales or revenue. Thus, maximizing participant effort aligns naturally with the designer's commercial objectives in these settings (Szymanski, 2003). Scheme E, where the designer provides reimbursement to both the winner and the loser, results in the highest effort than all the other five schemes. Next, Scheme A, where the designer provides reimbursement to the winner, generates the second highest total effort but with a higher designer's payoff than Scheme E. Hence, Scheme E is suitable when outcome quality is prioritized over neutrality. However, Scheme A introduces an imbalance, as the probability of winning for ex-ante weaker players is higher than that of stronger players, which may undermine their intrinsic motivation in the long term.

Schemes E and F maintain both viability and neutrality, but neither allows for full-reimbursement – although the potential rate of reimbursement in Scheme E is higher than in Scheme F. Therefore, the maximal effort in Scheme E is also higher than in Scheme F. However, the maximal designer's payoff in Scheme F (which is equivalent to Scheme B) is higher than the profit in Scheme E (which corresponds to the classic Tullock model and Scheme D).

Scheme C, where the designer reimburses the loser, preserves neutrality and achieves a high, though slightly lower total effort than Scheme A. For cases where full-reimbursement and neutrality are crucial, Scheme C offers a viable compromise that promotes steady effort and intrinsic motivation without disrupting the equilibrium.

Scheme B, in which the loser compensates the winner by reimbursing his expenses, balances the incentives for effort and can achieve neutrality. However, full-reimbursement is not attainable under Scheme B in order to maintain viability; otherwise, competition could be undermined, or political feasibility – as discussed earlier – could be jeopardized. If full-reimbursement for the winner is essential, then the designer might opt for a less balanced Scheme A or a hybrid approach that combines Schemes B and A by reimbursing the losing party partially. In cases without constraints of neutrality or viability, the scheme ranking provided in the next section offers the designer guidance in selecting the most effective reimbursement approach, balancing the competing demands of effort, payoff, and equitable participation in contest environments.

In the next section we first set up the model. We then characterize equilibria in winner reimbursement and in loser reimbursement. The designer's payoff and the total effort are analyzed next, before comparing the six schemes for each criterion. Section 3 concludes by discussing the current results and possibilities of future work.

2. Contests with alternative reimbursement schemes

2.1. Model set-up

To set up the baseline, consider a Tullock (1980) contest with two players ($i = 1, 2$) who compete for a prize that Player i values at $V_i > 0$ by exerting costly effort $x_i \geq 0$. Without loss of generality, let $V_1 \geq V_2$ and assume that players face no fixed cost and unit marginal cost of effort. The probability that Player i wins, (the Contest Success Function, CSF) is: $p_i = \frac{x_i}{x_1+x_2}$ for $(x_1 + x_2) > 0$, and $1/2$ otherwise. The payoff functions in this Tullock contest are: $\pi_i^T = V_i\frac{x_i}{x_1+x_2} - x_i$. The existence and uniqueness of equilibrium follows from Szidarovszky and Okuguchi (1997) and Chowdhury and Sheremeta (2011b). Following standard procedure, in equilibrium, $x_1^{*T} = \frac{(V_1^2 V_2)}{(V_1+V_2)^2}$, $x_2^{*T} = \frac{(V_1 V_2^2)}{(V_1+V_2)^2}$, total effort is $X^{*T} = x_1^{*T} + x_2^{*T} = \frac{(V_1 V_2)}{(V_1+V_2)}$ and the players' winning probabilities are $p_1^{*T} = \frac{V_1}{(V_1+V_2)}$ and $p_2^{*T} = \frac{V_2}{(V_1+V_2)}$, so $\frac{p_1^{*T}}{p_2^{*T}} = \frac{V_1}{V_2} \geq 1$.

Note that we denote the basic Tullock set-up with superscript T. Now, given this set-up, for a reimbursement scheme S $(= A, B, C, D, E, F)$ if we denote the equilibrium variables with superscript S, then viability means: $x_1^{*S} > 0,\ x_2^{*S} > 0$ and neutrality means $p_1^{*S}/p_2^{*S} \geq 1$.[3] Let α denote the fraction of effort reimbursed, sourced either from the designer or the opponent. When the reimbursement comes from the designer, then the payoff of the designer is the difference between the total effort after reimbursement, and some function of the prize-values. Since the prize values are fixed, for all practical purposes, we can ignore that part. The subsequent analysis

[3] Note that the current set-up is equivalent to the players having the same prize valuation ($V_1 = V_2$) but different linear cost functions, $c_i(x_i)$. Assume that $c_1(x_1) < c_2(x_2)$, in this case, neutrality (maintaining the ratio of players' pre-intervention winning probabilities) means favoring the player who is more cost-efficient ($p_1^{*S}/p_2^{*S} \geq 1$).

introduces the reimbursement schemes, evaluating their performance based on viability, neutrality, and the designer's payoff.

2.2. Winner reimbursement: properties

Lemma 1. *Scheme A, i.e., external reimbursement to the winner, is viable but may not be neutral.*

Proof: When the funder is external and the winner is the recipient (Scheme A), then the payoff functions are:

$$\pi_1^A = (V_1 + \alpha x_1)\left[\frac{x_1}{x_1 + x_2}\right] - x_1 \tag{1}$$

$$\pi_2^A = (V_2 + \alpha x_2)\left[\frac{x_2}{x_1 + x_2}\right] - x_2 \tag{2}$$

where the designer reimburses the winner for α proportion of her expenses.

Viability can be drawn directly from Proposition 1 in Liu and Dong (2019) who show interior solutions with positive payoffs. However, as shown by Cohen and Sela (2005), there is an interior equilibrium with non-negative players' payoffs (zero), when $\alpha = 1$, that is, full-reimbursement and viability are satisfied. In such a case, the players' efforts are $x_1^{*A} = V_2$, $x_2^{*A} = V_1$, $p_1^{*A} = \frac{V_2}{V_1+V_2}$, $p_2^{*A} = \frac{V_1}{V_1+V_2}$, so $\frac{p_1^{*A}}{p_2^{*A}} = \frac{V_2}{V_1} \leq 1$. Hence, when the players have asymmetric prize valuations, neutrality is violated. **Q.E.D.**

Note that this result was extended in Matros (2012) by showing that, under $\alpha = 1$, there are corner equilibria in which a player invests more than the prize value of the other player and drives him out of the contest.

Lemma 2. *Scheme B, i.e., internal reimbursement to the winner, cannot be both complete and viable. To attain viability, the rate of reimbursement, α, must satisfy $\alpha \leq \frac{V_2^2}{(V_1^2+V_2^2)} \leq \frac{1}{2}$.*

Proof: When the loser reimburses the winner (Scheme B), then the payoff functions are:

$$\pi_1^B = (V_1 + \alpha x_1)\left[\frac{x_1}{x_1 + x_2}\right] - \alpha x_2\left[1 - \frac{x_1}{x_1 + x_2}\right] - x_1 \tag{3}$$

$$\pi_2^B = (V_2 + \alpha x_2)\left[\frac{x_2}{x_1 + x_2}\right] - \alpha x_1\left[1 - \frac{x_2}{x_1 + x_2}\right] - x_2 \tag{4}$$

where the losing player reimburses the winner for α percent of her expenses. As studied in the literature (e.g., Katz 1988, Farmer and Pecorino 1999), let us show that to obtain a unique interior equilibrium, it is necessary that $\alpha < 1$. In other words, full-reimbursement must be violated.

Notice that (3) and (4) can be presented as:

$$\pi_1^B = V_1\frac{x_1}{x_1 + x_2} - x_1 + \alpha(x_1 - x_2) \tag{5}$$

$$\pi_2^B = V_2\frac{x_2}{x_1 + x_2} - x_2 + \alpha(x_2 - x_1) \tag{6}$$

The first-order equilibrium conditions are:

$$\frac{\partial \pi_1^B}{\partial x_1} = \alpha + \frac{V_1\, x_2}{(x_1 + x_2)^2} - 1 = 0 \tag{7}$$

$$\frac{\partial \pi_2^B}{\partial x_2} = \alpha + \frac{V_2\, x_1}{(x_1 + x_2)^2} - 1 = 0 \tag{8}$$

Given that the second-order conditions are satisfied, since $\frac{\partial^2 \pi_1^B}{\partial x_1^2} = -\frac{2\, V_1\, x_2}{(x_1+x_2)^3} < 0$ *and* $\frac{\partial^2 \pi_2^B}{\partial x_2^2} = -\frac{2\, V_2\, x_1}{(x_1+x_2)^3} < 0$, the players' equilibrium efforts are:

$$x_1^{*B} = \frac{V_1^2\, V_2}{(1-\alpha)(V_1 + V_2)^2} \tag{9}$$

$$x_2^{*B} = \frac{V_1\, V_2^2}{(1-\alpha)(V_1 + V_2)^2} \tag{10}$$

To ensure positive efforts, $\alpha < 1$. Substituting (9) and (10) into (5) and (6), we get:

$$\pi_1^{*\ B} = \frac{V_1\left(V_1^2(1-\alpha) - \alpha V_2^2\right)}{(1-\alpha)(V_1+V_2)^2} \tag{11}$$

$$\pi_2^{*\ B} = \frac{V_2\left(V_2^2(1-\alpha) - \alpha V_1^2\right)}{(1-\alpha)(V_1+V_2)^2} \tag{12}$$

It can be readily verified that these payoffs are not negative,

$$\alpha \le \frac{V_2^2}{\left(V_1^2+V_2^2\right)} \le \frac{1}{2} \tag{13}$$

Hence, viability is satisfied only when (13) is satisfied, that is, full-reimbursement is given up. Note that, for the Tullock CSF, Scheme B with partial reimbursement satisfies the neutrality imperative because $\frac{p_1^{*B}}{p_2^{*B}} = \frac{x_1^{*B}}{x_2^{*B}} = \frac{V_1}{V_2}$. **Q.E.D.**

Note that reimbursement scheme B was studied also by Baye at al. (2005, 2012) in a symmetric all-pay auction set-up. In their settings, interior equilibrium exists also under full-reimbursement ($\alpha = 1$), and not just under partial reimbursement ($\alpha < 1$) as in our setting with a stochastic CSF.

Lemmas 1 and 2 imply the following impossibility rule:

Theorem 1. *No scheme of reimbursement for the winner (Schemes A and B) can satisfy full-reimbursement, neutrality, and viability.*

2.3. Loser reimbursement: properties

In contrast to the Theorem 1, we now show that consistency among the three properties (full-reimbursement, neutrality, and viability) is possible under reimbursement of the loser's costs.

Theorem 2. *Both the internal reimbursement and external reimbursement schemes for the loser (Schemes C and D) can satisfy the full-reimbursement, neutrality, and viability criteria.*

Proof: We will prove this in two parts. First, for external reimbursement (part 1) and then for internal reimbursement (part 2).

Part 1. Suppose that the funder is external, and the loser is the recipient (Scheme C). In this case, the payoff functions are:

$$\pi_1^C = V_1\left[\frac{x_1}{x_1+x_2}\right] + \alpha x_1\left[1 - \frac{x_1}{x_1+x_2}\right] - x_1 \tag{14}$$

$$\pi_2^C = V_2\left[\frac{x_2}{x_1+x_2}\right] + \alpha x_2\left[1 - \frac{x_2}{x_1+x_2}\right] - x_2 \tag{15}$$

Or, alternatively,

$$\pi_1^C = (V_1 - \alpha x_1)\left[\frac{x_1}{x_1+x_2}\right] - x_1(1-\alpha) \tag{14a}$$

$$\pi_2^C = (V_2 - \alpha x_2)\left[\frac{x_2}{x_1+x_2}\right] - x_2(1-\alpha) \tag{15a}$$

where the designer reimburses the loser for α percent of her expenses.

The first-order conditions, in equilibrium, are:

$$\frac{\partial \pi_1^C}{\partial x_1} = \frac{x_2\,(V_1+\alpha x_2)}{(x_1+x_2)^2} - 1 = 0 \tag{16}$$

$$\frac{\partial \pi_2^C}{\partial x_2} = \frac{x_1\,(V_2+\alpha x_1)}{(x_1+x_2)^2} - 1 = 0 \tag{17}$$

Note that the second-order conditions are satisfied, since $\frac{\partial^2 \pi_1^C}{\partial x_1^2} = -\frac{2x_2(V_1+\alpha x_2)}{(x_1+x_2)^3} < 0$ *and* $\frac{\partial^2 \pi_2^C}{\partial x_2^2} = -\frac{2x_1(V_1+\alpha x_1)}{(x_1+x_2)^3} < 0$.

Since $lim_{x_1\to\infty}\pi_1(x_1,x_2) = lim_{x_2\to\infty}\pi_2(x_1,x_2) = -\infty$, players' efforts are bounded from above by some constant $c_i > 0$ making $[0,c_i]$ the set of pure-strategies for each player i ($i = 1,2$). In Appendix A, we show that these upper bounds for players 1 and 2 are V_1/α and V_2/α, respectively. Given that each player's payoff function is concave in its own control variable when the opponent's is fixed and the strategy set is compact, equilibrium existence follows from Theorem 3.1 of Reny (1999), as Scheme C satisfies the Better-Reply-Secure property. Since Scheme C has no corner equilibrium, it must have an interior equilibrium in which $(x_1,x_2) \in (0, V_1/\alpha) \times (0, V_2/\alpha)$ (see Appendix A).

The analytical solution to the Nash equilibrium in efforts under Scheme C, derived from the first-order conditions in Eqs. (16) and (17), is highly complex and results in lengthy third-degree equations, thus, we provide the analytical proof in Appendix A for the existence of an interior equilibrium in efforts for every α (including $\alpha = 1$).

Note that in the symmetric case, where $V_1 = V_2 = V$, we would have obtained :[4]

$$x_1^{*C} = x_2^{*C} = \frac{V}{4-\alpha} \tag{18}$$

Dividing (16) by (17) gives:

$$\frac{x_2}{x_1} = \frac{V_2 + \alpha x_1}{V_1 + \alpha x_2} \tag{19}$$

Since $V_2 < V_1, \frac{V_2+\alpha x_1}{V_1+\alpha x_2} < \frac{V_1+\alpha x_1}{V_1+\alpha x_2}$. Assume that $x_1 < x_2$ and $\frac{x_2}{x_1} > 1$. This results in a contradiction, since $\frac{x_2}{x_1} = \frac{V_2+\alpha x_1}{V_1+\alpha x_2} < \frac{V_1+\alpha x_2}{V_1+\alpha x_2} = 1$, which implies that the inequality $0 < x_2^{*C} < x_1^{*C}$ is necessarily satisfied. Tullock CSF ensures that Player 1′s winning probability remains larger than Player 2′s: $p_2^* < p_1^*$. To sum up, neutrality and viability are satisfied, and full-reimbursement can be satisfied as well.

Part 2: When the winner reimburses the loser (Scheme D). Then the payoff functions are:

$$\pi_1^D = (V_1 - \alpha x_2)\left[\frac{x_1}{x_1 + x_2}\right] + \alpha x_1\left[1 - \frac{x_1}{x_1 + x_2}\right] - x_1 \tag{20}$$

$$\pi_2^D = (V_2 - \alpha x_1)\left[\frac{x_2}{x_1 + x_2}\right] + \alpha x_2\left[1 - \frac{x_2}{x_1 + x_2}\right] - x_2 \tag{21}$$

Since $\frac{x_1}{x_1+x_2} + \frac{x_2}{x_1+x_2} = 1$, Eqs. (20) and (21) can be written as:

$$\pi_1^D = V_1\left[\frac{x_1}{x_1 + x_2}\right] - x_1 = \pi_1^T \tag{20a}$$

$$\pi_2^D = V_2\left[\frac{x_1}{x_1 + x_2}\right] - x_2 = \pi_2^T \tag{21a}$$

That is, the payoff functions of Scheme D are equal to the classic Tullock's benchmark contest, regardless of the rate of reimbursement, α (full-reimbursement can be applied). Moreover, Scheme D satisfies neutrality and viability,[5] as well as the classic Tullock contest does. **Q.E.D.**

To sum up, Theorems 1 and 2 clarify why full-reimbursement of the winner's expenses is unlikely to be realized under certain desirable properties, whereas it is possible when the loser is the recipient. This is because to get a viable contest, there should be some room for competition. Reimbursing the winner may discourage the players, whereas repaying the loser in full provides incentives to "enter" the contest while repaying the winner does not.

2.4. Reimbursement of both winner and loser: properties

The final pair of reimbursement covers both the loser and the winner, whether the reimbursement is internal or external – the outcome remains the same:

Theorem 3. *Both the internal and external reimbursement schemes for both the winner and the loser (Schemes E and F) satisfy the neutrality and viability criteria, but do not satisfy full-reimbursement.*

Proof: Here, we will also prove this in two parts. First, for external reimbursement (part a) and then for internal reimbursement (part b).

Part a. Suppose the reimbursement is provided by an external funder, and both the winner and the loser receive compensation (Scheme E). In this case, the payoff functions are:

$$\pi_1^E = (V_1 + \alpha x_1)\left[\frac{x_1}{x_1 + x_2}\right] + \alpha x_1\left[1 - \frac{x_1}{x_1 + x_2}\right] - x_1 \tag{22}$$

$$\pi_2^E = (V_2 + \alpha x_2)\left[\frac{x_2}{x_1 + x_2}\right] + \alpha x_2\left[1 - \frac{x_2}{x_1 + x_2}\right] - x_2 \tag{23}$$

This scheme corresponds to the classic Tullock benchmark model with a subsidy rate of α:

[4] Matros and Armanios (2009) studied the symmetric version ($V_1 = V_2$) of this reimbursement scheme but allowed reimbursement of both the winner and the loser. They showed that there is an interior equilibrium for any α. They also obtained that under external reimbursement to the loser, the designer's payoff declines in α. Instead, we study situations that allow asymmetry between the prize valuations and show (in Section 2.2) that the optimal α for the designer's payoff decreases as the gap between the asymmetries increases.

[5] Since $x_1^{*D} = \frac{V_1^2 V_2}{V_1+V_2} > 0,\ x_2^{*D} = \frac{V_1^2 V_2}{V_1+V_2} > 0,\ \frac{p_1^{*D}}{p_2^{*D}} = \frac{V_1}{V_2} > 1,\ \pi_1^{*D} = \frac{V_1^3}{(V_1+V_2)^2} > 0$ and $\pi_2^{*D} = \frac{V_2^3}{(V_1+V_2)^2} > 0$.

$$\pi_1^E = V_1\left[\frac{x_1}{x_1+x_2}\right] - x_1(1-\alpha) \tag{22a}$$

$$\pi_2^E = V_2\left[\frac{x_2}{x_1+x_2}\right] - x_2(1-\alpha) \tag{23a}$$

This simple mechanism was studied by Glazer and Konrad (1999). Under this framework, the players' equilibrium efforts and payoffs are positive and equal to: $x_1^{*E} = \frac{V_1^2V_2}{(1-\alpha)(V_1+V_2)^2}$, $x_2^{*E} = \frac{V_1V_2^2}{(1-\alpha)(V_1+V_2)^2}$, $\pi_1^{*E} = \frac{V_1^3}{(V_1+V_2)^2}$ and $\pi_2^{*E} = \frac{V_2^3}{(V_1+V_2)^2}$, and the ratio of players' winning probabilities is given by: $\frac{p_1^{*E}}{p_2^{*E}} = \frac{V_1}{V_2} > 1$. Thus, Scheme E satisfies neutrality and viability. However, full-reimbursement is not possible because when $\alpha = 1$, there is no equilibrium.

Part b. Now, when the funder is internal, and both the winner and the loser is the recipient (Scheme F), namely, reciprocal reimbursement. In this case, the payoff functions are:

$$\pi_1^F = (V_1+\alpha x_1-\alpha x_2)\left[\frac{x_1}{x_1+x_2}\right] + (\alpha x_1-\alpha x_2)\left[1-\frac{x_1}{x_1+x_2}\right] - x_1 \tag{24}$$

$$\pi_2^F = (V_2+\alpha x_2-\alpha x_1)\left[\frac{x_2}{x_1+x_2}\right] + (\alpha x_2-\alpha x_1)\left[1-\frac{x_2}{x_1+x_2}\right] - x_2 \tag{25}$$

This scheme is equivalent to the Tullock classic benchmark model with a subsidy rate α and a lump-sum tax αx_j, where $j \neq i$:

$$\pi_1^F = V_1\left[\frac{x_1}{x_1+x_2}\right] - x_1(1-\alpha) - \alpha x_2 \tag{24a}$$

$$\pi_2^F = V_2\left[\frac{x_2}{x_1+x_2}\right] - x_2(1-\alpha) - \alpha x_1 \tag{25a}$$

Applying the results from Glazer and Konrad (1999), the equilibrium efforts in Scheme F are the same as in Scheme E: $x_2^{*F} = \frac{V_1^2V_2}{(1-\alpha)(V_1+V_2)^2} = x_1^{*E}$, $x_2^{*F} = \frac{V_1V_2^2}{(1-\alpha)(V_1+V_2)^2} = x_2^{*E}$, and thus, the ratio of players' winning probabilities remains: $\frac{p_1^{*F}}{p_2^{*F}} = \frac{V_1}{V_2} > 1$. The players' payoff functions are: $\pi_1^{*F} = \frac{V_1\left(V_1^2-\alpha\left(V_1^2+V_2^2\right)\right)}{(1-\alpha)(V_1+V_2)^2}$ and $\pi_2^{*F} = \frac{V_2\left(V_2^2-\alpha\left(V_1^2+V_2^2\right)\right)}{(1-\alpha)(V_1+V_2)^2}$, which non-negatives only if:

$$\alpha \le \frac{V_2^2}{V_1^2+V_2^2} \tag{26}$$

Similar to Scheme E, Scheme F satisfies neutrality and viability but does not allow full-reimbursement. The key difference between the two schemes lies in the maximum possible reimbursement rate α, which is $\alpha \to 1$ in Scheme E, and $\alpha \le \frac{V_2^2}{V_1^2+V_2^2}$ in Scheme F.

Q.E.D.

2.5. Designer's payoff

Consider a contest designer who wishes to maximize his payoff function ($\pi_d^* = x_1^* + x_2^* - c_d(\alpha,\ x_1^*, x_2^*)$): the difference between the players' total effort and the amount of the reimbursement to one of them in external reimbursement c_d, while upholding both viability and neutrality. Note that in internal reimbursement $c_d(\alpha,\ x_1^*, x_2^*) = 0$. Such an objective is plausible when efforts are transferred to the designer, as often seen in the rent-seeking literature or where the effort is a direct source of revenue for the designer as in sports.

Finding 1. When the prize value asymmetry (V_1/V_2) is low, the optimal reimbursement parameter α for designer payoff maximization in case of internal reimbursement for the loser (Scheme C) is $\alpha = 0$. But when the prize value asymmetry is high, the optimal α is 1.

Finding 2. When the reimbursement is external and the prize value asymmetry is high (low), then reimbursement for the winner (Scheme A) is payoff dominated (dominant) for the designer relative to reimbursement for the loser (Scheme C with optimal α).

Proof and simulations: Note that due to the mathematical-algebraic difficulty in extracting the equilibrium of Scheme C, Findings 1 and 2 are illustrated by simulations.

By applying Reimbursement Scheme A, in interior equilibrium with a full-reimbursement, the designer can secure the following maximal expected net payoff (Cohen and Sela, 2005):

$$\pi_d^{*A} = x_1^{*A} + x_2^{*A} - x_1^{*A}\frac{x_1^{*A}}{x_1^{*A}+x_2^{*A}} - x_2^{*A}\frac{x_2^{*A}}{x_1^{*A}+x_2^{*A}} = \frac{2V_1V_2}{V_1+V_2} = 2X^T \tag{27}$$

Note that Liu and Dong (2019) showed that the maximization of the Designer's profit is obtained through full-reimbursement ($\alpha = 1$). In addition, they showed that the total effort and the designer's payoff in Scheme A are not directly tied to the number of players.

By Theorem 1, Scheme A of external reimbursement of the winner is inconsistent with viability and neutrality. In other words, a designer who insists on upholding these properties cannot apply Scheme A. By Theorem 2, these properties are satisfied when the designer applies Scheme C of external reimbursement to the loser. In this case, their expected net payoff is:

$$\pi_d^{*\ C} = x_1^{*^C} + x_2^{*^C} - \alpha x_1^{*^C}\frac{x_2^{*^C}}{x_1^{*^C}+x_2^{*^C}} - \alpha x_2^{*^C}\frac{x_1^{*^C}}{x_1^{*^C}+x_2^{*^C}} = x_1^{*^C} + x_2^{*^C} - \frac{2\alpha x_1^{*^C}x_2^{*^C}}{x_1^{*^C}+x_2^{*^C}} \tag{28}$$

Given the algebraic difficulty of extracting the players' equilibrium efforts as a function of the prize values from Eq. (18), we use simulation to compare the Designer's payoff margins between Scheme C with other schemes. We start by comparing Scheme C with Scheme A.

Let $V_1 = 100$. Assuming 12 alternative prize valuations V_2 of Player 2 and using the first-order conditions (16) and (17), we obtain by simulation the equilibrium x_1^*, x_2^* and π_d^* of Scheme C, when $0 \le \alpha \le 1$ (but, for convenience, only $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$ are shown) . We also show the equilibrium x_1^*, x_2^*, and π_d^* of Scheme A and Scheme D – which is equal to the classic Tullock model (see Table B1 in Appendix B).

Matros and Armanios (2009) showed that symmetric external reimbursement for the winner is payoff dominant over external reimbursement for the loser. We extend their result by simulation and show in Fig. 1 that when there is high asymmetry in prize valuations, then reimbursing the loser is payoff-dominant for the designer relative to reimbursing the winner. That is, when the value asymmetry (V_1/V_2) is high enough, the result of Matros and Armanios (2009) is reversed: the designer's payoff decreases with α, and the optimal α for maximizing the designer's payoff is 0. As a result, in Scheme C, the optimal α for a designer aiming to maximize profit is $\alpha = 0$ at high asymmetry and $\alpha = 1$ at low asymmetry.

In the simulation, when the asymmetry is high ($V_1/V_2 > 6.786$), the Designer's payoff is larger in Scheme C than in Scheme A, $\pi_d^{*\ C} > \pi_d^{*\ A}$, when $\alpha = 1$. In such a situation, Scheme C is not only consistent with viability and neutrality but gives the designer an advantage (higher payoff) compared to Scheme A. Recall that this advantage exists by Theorem 2, avoiding the reversal of the players' winning probabilities.

Q.E.D

Finding 3. When the reimbursement is internal, reimbursement for the winner (Scheme B) is always more payoff dominant for the designer relative to reimbursement for the loser (Scheme D).

Proof: We now compare the designer's payoff when he applies Schemes B and D. In the former case, the payoff is equal to the total effort, so by (9) and (10),

$$X^{*^B} = x_1^{*^B} + x_2^{*^B} = V_1\ V_2\Big/[(1-\alpha)(V_1+V_2)] \tag{29}$$

Given that $\frac{\partial X^{*B}}{\partial \alpha} > 0$, and since positive efforts and payoffs constrain the reimbursement rate α (it cannot be complete), by (Eq. (13)), the maximal payoff is obtained at:

$$\alpha_{max}^B = V_2^2/\left(V_1^2 + V_2^2\right) \tag{30}$$

which yields the maximal payoff[6]:

$$\pi_d^{*\ B} = \left[V_2\left(V_1^2+V_2^2\right)\right]/[V_1\left(V_1+V_2\right)] \tag{31}$$

Since Scheme D, with a full-reimbursement, is equivalent to no intervention (the benchmark Tullock contest, see Part 2 of the proof of Theorem 2), the maximal payoff is:

$$\pi_d^{*\ D} = V_1V_2/(V_1+V_2) = X^T \tag{32}$$

Hence, Scheme B with $\alpha = \alpha_{max}^B$ is payoff-dominant over Scheme D. That is:

$$\pi_d^{*\ B} = \frac{V_2\left(V_1^2+V_2^2\right)}{V_1\left(V_1+V_2\right)} > \frac{V_2\left(V_1^2\right)}{V_1\left(V_1+V_2\right)} = \frac{V_1V_2}{V_1+V_2} = \pi_d^{*\ D}.\mathbf{Q.E.D}.$$

Finding 4. When reimbursement goes to the winner, external reimbursement (Scheme A) is always payoff-dominant over the most effective internal reimbursement scheme (Scheme B).

Proof: By (27) and (31), and given that $V_1 > V_2$, $\pi_d^{*\ A} = 2V_1V_2/(V_1+V_2)$ is always greater than $\pi_d^{*\ B} = \left[V_2\left(V_1^2+V_2^2\right)\right]/[V_1\left(V_1+V_2\right)]$. **Q.E.D.**

Finding 4 might appear to be counterintuitive – as the designer is willing to reimburse the winner with his own fund rather than allowing the loser to do it. However, note that an internal reimbursement scheme will reduce the incentive to compete as the loser's payoff might become too low. This discouragement is alleviated when the designer reimburses the winner. Similarly, as already studied in the literature (e.g., Cornes and Hartley, 2005), when the prize value is more symmetric, players tend to expend more resources compared to when it is asymmetric.

[6] Note that the maximum payoff the designer can achieve in Scheme B is $\pi_d^{*\ B}$, obtained at α_{max}^B. However, in this situation, α is not optimal, as it does not satisfy viability.

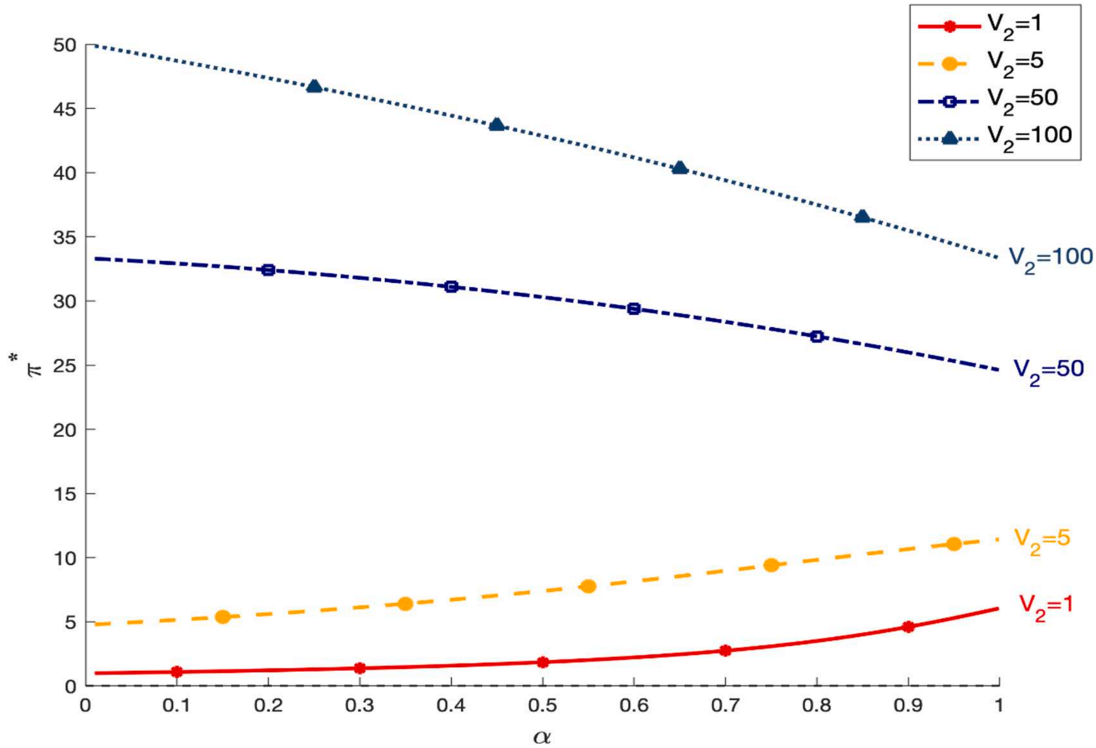

Fig. 1. Designer's payoff in Scheme C, when $V_1 = 100$ and V_2 takes values 1, 5, 50, and 100.

When the designer will have to reimburse it, reimbursement cost outweighs the gain of the extra revenue. Hence, as we see in Finding 5, an internal scheme provides a higher payoff for the designer.

Finding 5. When reimbursement goes to the loser and prize asymmetry is low, then the internal reimbursement (Scheme D) is payoff-dominant over the external reimbursement (Scheme C).

Proof and simulations: Due to the mathematical-algebraic difficulty in extracting the equilibrium of Scheme C, Finding 5 is obtained by simulation. Applying the simulation as in the previous section and noting that $\pi_d^{*\ D} = X^T$, we get Table B1 (see Appendix B), in which $\pi_d^{*\ C}$can be compared with $\pi_d^{*\ D}$. It turns out that only when the prize value asymmetry is high, i.e., $V_1\ /V_2 > 23.421$ (solved by simulations, see also Columns 1, 2, 5, and 8 in Table 2), Scheme C is payoff dominant to Scheme D. **Q.E.D.**

Finding 6. External reimbursement for the winner (Scheme A) is always payoff-dominant relative to internal reimbursement for the loser (Scheme D).

Proof: By (27) and (32), $\pi_d^{*\ D} = X^T$ and $\pi_d^{*\ A} = 2X^T$, which means $\pi_d^{*\ A} > \pi_d^{*\ D}$. **Q.E.D.**

Finding 7. The designer's payoff in an external reimbursement for both the winner and the loser (Scheme E), is the same as in an internal reimbursement for the loser (Scheme D).

Proof: According to Section 2.4, the designer's payoff in Scheme E (total efforts minus the subsidy granted) is the same as the designer's payoff in Scheme D (see Finding 3): $\pi_d^{*\ E} = x_1^{*E} + x_2^{*E} - \alpha x_1^{*E} - \alpha x_2^{*E} = \frac{V_1 V_2}{(V_1+V_2)} = X^T = \pi_d^{*\ D}$ **Q.E.D.**

Finding 8. The designer's payoff in an internal reimbursement for both the winner and the loser (Scheme F), is the same as in an internal reimbursement for the winner (Scheme B).

Proof: According to Section 2.4, The designer's payoff in Scheme F, which equals the total efforts, is: $X^{*F} = \frac{V_1 V_2}{(1-\alpha)(V_1+V_2)}$. Given the maximum value of α obtained in (26), the maximal designer's payoff is: $\pi_d^{*\ F} = \frac{V_2\left(V_1^2+V_2^2\right)}{V_1(V_1+V_2)} = \pi_d^{*\ B}$ **Q.E.D.**

By Findings 2–8, we get:

Result 1. The reimbursement schemes can be ranked in terms of designer's payoff as: $A \succ B \asymp F \succ D \asymp E$. When the prize value asymmetry is sufficiently low, $E \succ C$. When it is high, $C \succ A$.

This result is visualized in Fig. 2. The graphs in Fig. 2 specify the designer's expected net revenue (payoff) when he applies Schemes A, B, C ($\alpha = 0.5$ and $\alpha = 1$), D, E and F.

2.6. Players' efforts

As mentioned in the Introduction, the designer may wish to maximize the players' total effort, that is $X^* = x_1^* + x_2^*$ although he is

Table 2
Comparing the six schemes of reimbursement.

Scheme	Full-reimbursement	Neutral	Viable	Payoff rank		Effort rank	
				Low asymmetry	High asymmetry	Low asymmetry	High asymmetry
A	✓	×	✓	1	2	2	2
B	×	✓	✓	2	3	3	4
C ($\alpha = 1$)	✓	✓	✓	4	1	4	3
C (α : **optimal**)	✓	✓	✓	3	1	4	3
D	✓	✓	✓	5	4	5	5
E	×	✓	✓	5	4	1	1
F	×	✓	✓	2	3	3	4

Note: Scheme D equals to Schemes A-F when $\alpha = 0$. For Scheme B and F full-reimbursement, recall that we assume the maximal partial reimbursement rate that ensures viability: $\alpha = V_2^2/\left(V_1^2 + V_2^2\right)$.

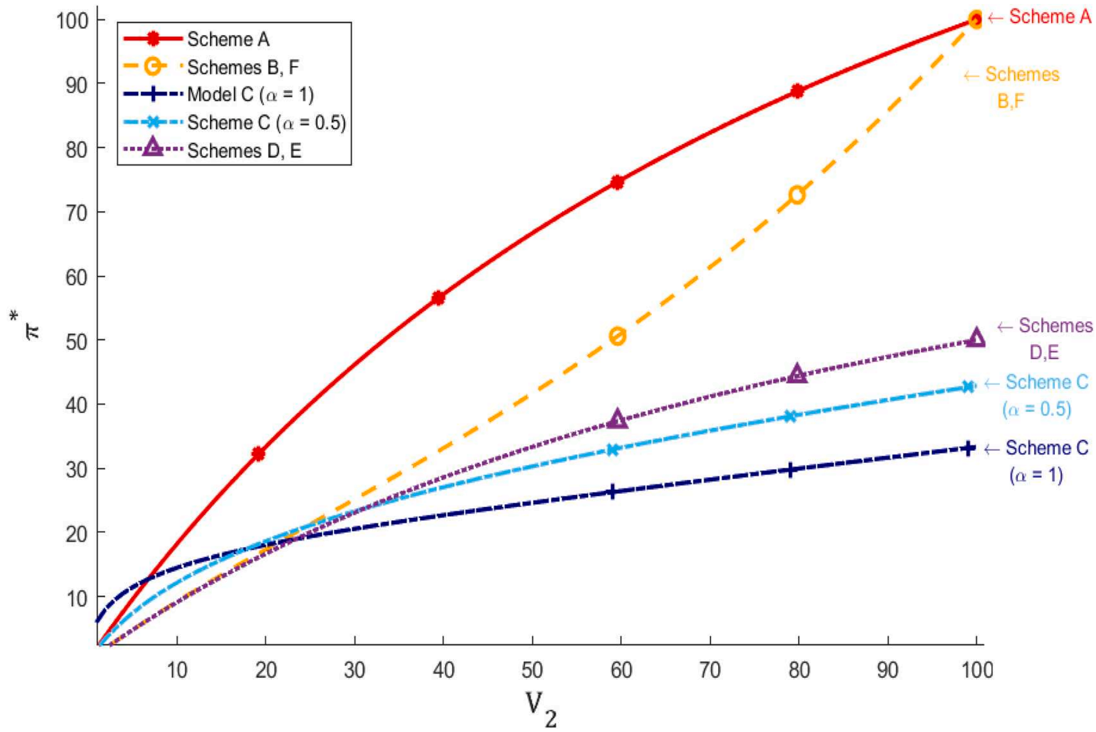

Fig. 2. Comparison of Designer's payoff in Schemes A, B, C, D, E and F when $V_1 = 100$.

reimbursing the players. This objective is common in R&D races in which the quality of the final product, which depends on the effort expended, is the designer's main goal. This is also relevant for the contests with social externalities (such as R&D and education) where more effort benefits society. As shown by Cohen and Sela (2015), the optimal α of Scheme A for the objective of maximization of efforts is $\alpha = 1$.

When the reimbursement is internal (Schemes B, D and F), the total effort is equal to the designer's net payoff (because the designer is not involved in reimbursement), so the optimal α of Schemes B and F in this case is the maximal fraction, $\alpha = \frac{V_2^2}{\left(V_1^2+V_2^2\right)}$ (note, though, that the equilibrium of Scheme D is independent of α).[7,8] By Lemma 1, Finding 1, and Theorem 2, total effort under Schemes A, B, and D is $X^{*A} = V_1 + V_2$, $X^{*B} = X^{*F} = \frac{V_2\left(V_1^2+V_2^2\right)}{V_1\left(V_1+V_2\right)}$, $X^{*D} = \frac{V_1 V_2}{V_1+V_2}$. Finally, in Scheme E, when $\alpha \to 1$, then $X^E \to \infty$. Thus, it is easy to see that:

[7] Chen and Rodrigues-Neto (2023) applied monetary and emotional preferences in Scheme B and found that if the litigants' relative advantages are sufficiently balanced, an increase in either reimbursement expenses or negative relational emotions increases total efforts in equilibrium.
[8] In contrast to our paper, which focuses on symmetric reimbursement rates, Baik and Shogren (1994) found that Scheme C with asymmetric reimbursement rates reduces effort compared to symmetric rates of return.

$X^{*E} \succ X^{*A} \succ X^{*B} \asymp X^{*F} \succ X^{*D}$, which yields:

Result 2. The reimbursement schemes can be ranked in terms of total efforts as: $E \succ A \succ B \asymp F \succ D$. When the prize value asymmetry is sufficiently low, $B \succ C \succ D$, but when it high, $A \succ C \succ B$.

By Table 2 and Fig. 3, $X^{*A} \succ X^{*C} \succ X^{*D}$. The total efforts in Scheme C increases with α, so, here too the optimal α is 1. Our results complement and add to the results of Baye et al. (2012) who studied different types of internal reimbursement in an all-pay auction setting.

Notice that a winner-pay contest corresponds to Scheme C, where the loser receives full external reimbursement ($\alpha = 1$), while the all-pay contest corresponds to Scheme D, which equals a non-intervention Tullock contest. Lagerlöf (2020) studied hybrid winner-pay and all-pay contests and found that the total effort is always lower than in a corresponding all-pay contest with symmetric prizes. This result is consistent with ours. However, in all-pay auctions with variable rewards under incomplete information, where the reward is also a function of the bidder's own bid (a form of reimbursement), there can be paradoxical behavior: reducing rewards or increasing costs may lead to a higher expected sum of bids or, alternatively, a higher expected maximum bid (Kaplan et al., 2002). Yates (2011) shows that in winner-pay contests, a pure-strategy Nash equilibrium exists and is unique under weak assumptions on the CSF.

2.7. *Comparison between reimbursement schemes*

In this study we examine six reimbursement schemes in contests focusing on their impact on total effort and the designer's net payoff, which is calculated as the total effort minus the reimbursement to the winner and/or loser. We find that reimbursing the winner, or both the winner and the loser, cannot satisfy the three criteria of full-reimbursement, neutrality, and viability. Reimbursing the loser is the only scheme that fully satisfies these criteria. Scheme A can be viable but cannot be neutral because it reverses the initial winning probabilities. By sufficiently increasing their investment, the weaker player can come away better off.

Under complete reimbursement, Schemes B, E and F cannot be viable and cannot ensure positive efforts by both rivals. Such a viability requires that the rate of reimbursement does not exceed a certain limit (see Eq. (13)). Any rate above this constraint causes the players to refrain from participating because they will have to pay the loser's expenses if they win, making their expected payoff negative. Complete internal reimbursement for the winner eliminates the incentives to enter the contest while repaying the loser does not. If the prize value asymmetry is high, then Scheme C provides the highest payoff to the designer. When the asymmetry is not particularly high, Scheme A is payoff dominant. However, the optimal total effort is always the largest in Scheme E and the smallest in Scheme D. Table 2 summarizes the findings presented in Section 2.

Table 3 presents the optimal α for each objective function (payoff with various value asymmetry, effort) of the designer in each of the six reimbursement models. If the designer chooses to use Scheme A, the optimal reimbursement rate is $\alpha = 1$ for each of the possible objective functions.[9] While Scheme D is not affected by α at all, the optimal α in Schemes B and F is the maximal $\alpha = V_2^2/(V_1^2 + V_2^2)$. In Scheme C, when the designer is interested in effort or in payoff when prize value asymmetry is low, the optimal alpha is $\alpha = 1$. But when he is concerned with payoff and the value asymmetry is low, the optimal reimbursement rate is $\alpha = 0$. Scheme E is not affected by α for designer's payoff, but for effort maximization, the optimal α is $\alpha \to 1$.

3. Discussion

In this study, we identify the optimal reimbursement schemes in contests for different objectives: incentivizing effort or maximizing designer payoff (total effort, less the designer's reimbursement to the winner/loser). Our analysis also addressed the desirable properties of full-reimbursement, neutrality, and viability.

Our contribution builds upon existing literature on reimbursement and spillovers in Tullock contests (e.g., Chowdhury et al., 2011a, 2011b; Matros and Armanios, 2009). However, this is the very first systematic analysis of six reimbursement schemes (A, B, C, D, E, and F) categorized on the source of funds (internal versus external) and the target of reimbursement (winner, loser, or both), extended to asymmetric valuations. We find that full-reimbursement is generally optimal in Scheme C, even in symmetric value cases, except when maximizing payoff is the sole objective. In Schemes A and C under symmetry we derived the optimal reimbursement rates for objectives that include payoff, effort, and considerations for neutrality and viability. Notably, our findings confirm that Scheme A, with a full-reimbursement rate, is almost the most effective across almost all objectives, though it falls short in neutrality.

In exploring the necessity of the all-pay condition (the classic Tullock contest where each player pays his expenses without getting a reimbursement) in neutral, non-discriminatory contests, we find that the complete elimination of all-pay is achievable by fully reimbursing one player's expenses. Our results show that this full-reimbursement is only viable when the recipient is the losing-party. This effectively supports the weaker player, though this aligns with the designer's interest rather than any ethical motive.

Given that full-reimbursement typically benefits the losing party, we analyzed the designer's preferences between external and internal reimbursement schemes (Schemes C and D). The optimal choice depends on the designer's objectives and the degree of asymmetry in prize valuations. If the designer seeks to reduce disparities in winning probabilities, external Scheme C consistently outperforms internal Scheme D. For objectives focused on maximizing net payoff, Scheme C remains favorable, but only when prize

[9] For a nonlinear reimbursement function, Lie and Dong (2019) showed that if the effort cost function is concave (convex), the optimal reimbursement scheme is (not) to return the full cost to the winner. In addition, Minchuk (2018) showed that if the effort cost function is concave (convex), then reimbursement increases (decreases) designer payoff.

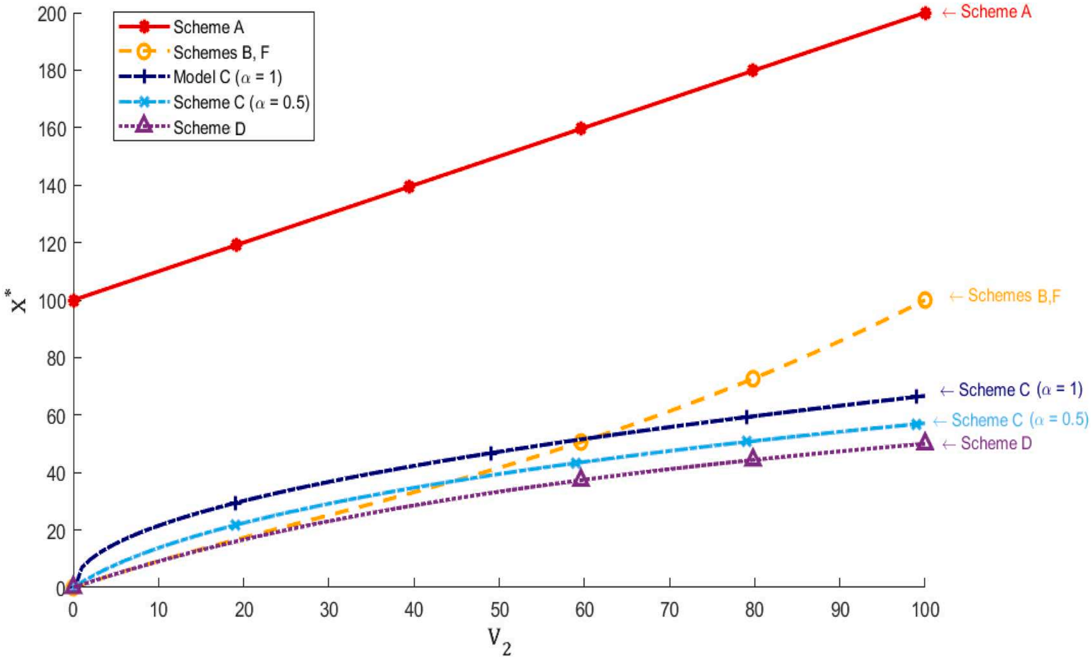

Fig. 3. Comparison of total effort in Schemes A, B, C, D and F when $V_1 = 100$.

Table 3
Optimal α in reimbursement schemes.

Scheme	Target	Source	Designer objective		
			Payoff (Low asymmetry)	Payoff (High asymmetry)	Effort (High\low asymmetry)
A	Winner	External	$\alpha = 1$		
B	Winner	Internal	$\alpha = V_2^2/(V_1^2 + V_2^2)$		
C	Loser	External	$\alpha = 0$	$\alpha = 1$	
D	Loser	Internal	Unaffected by α		
E	Both	External	Unaffected by α	$\alpha \to 1$	
F	Both	Internal	$\alpha = V_2^2/(V_1^2 + V_2^2)$		

valuation asymmetry is substantial. In contexts where total effort is important, such as R&D competitions, Scheme C is again preferred, as external reimbursement of the loser's expenses yields greater overall effort than internal reimbursement through Scheme D.

Our investigation can be extended in various ways. First, the analysis can be extended to more than two players for all six schemes of interest. This will reflect the situations in multi-party competition in R&D. Second, different types of non-linear spillover effects (beyond Baye et al., 2012 or Chowdhury et al., 2011a) can also be introduced. This can broaden the potential applications of the schemes. It will also be possible to show strategic equivalence between different types of reimbursement schemes (Chowdhury and Sheremeta, 2015). Third, both the cost function and the utility function can be generalized with nonlinear curvature. The latter one, specifically, will allow capturing the effects of the reimbursement schemes on risk-averse preferences (Liu and Liu, 2019). Fourth, although affirmative action is a broad area that goes beyond simple reimbursement in contests, it will be possible to analyze reimbursement schemes as a tool of affirmative action and compare those with other relevant tools (See Chowdhury et al., 2023; Mealem and Nitzan, 2016). Finally, there is a scarcity in the experimental literature in analyzing reimbursement schemes and relevant contest design issues. Our study can be used as a theoretical benchmark in experiments to understand behavioral foundations in such contests.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of existence of an interior equilibrium[10]

The in-game payment functions are (14) and (15). Since $lim_{x_1\to\infty}\pi_1(x_1,x_2) = lim_{x_2\to\infty}\pi_2(x_1,x_2) = -\infty$, there exists $c_i > 0$, for player i (i $= 1,2$) such that if a player choose an effort greater then c_i, then her payoff will be negative, while she could have obtained a non-negative payoff by choosing $x_i = 0$. This implies that any equilibrium of the game in which the efforts of the players are bounded from above by c_i and the payoff functions are identical to those of our original game will be also an equilibrium of our game. So, in order to prove the existence of equilibrium in our original game, we may assume that the set of pure strategies of player i is the interval $[0,c_i]$.

Since for i $= 1,2$, the function is π_i is concave in the variable x_i when the other variable is fixed, existence of an equilibrium is guaranteed by Theorem 3.1 of Reny (1999), provided that in addition our game has Reny's better-reply-secure property: if (a) : $\{x^n\}_{n=1}^{\infty}$ is a sequences such that $x = (x_1,x_2) = x^n$ exists and x is not an equilibrium of our game, and (b) : $w_i = lim_{n\to\infty}\pi_i(x^n)$ exists for every i $= 1,2$, then there must be some player i that can secure a payoff greater than w_i at x_i, $i.e$ there exists $0 \le y_i \le c_i$ and an open neighborhood U of x_{-i} such $\pi_i(y_i,\ x'_{-i}) > w_i$ for all $x'_{-i} \in U$.

We show that our game has better-reply-secure property. Let $\{x^n\}_{n=1}^{\infty}$ and (π_1,π_2) as in (a) and (b) above. If $x \in [0,c_1] \times [0,c_2]\backslash\{(0,0)\}$, then the function π_1 and π_2 are continuous at x and hence $w_i = \pi_i(x)$ for i $= 1,2$. Since x is not an equilibrium by assumption, there exists a player i $= 1,2$ which has profitable deviation from her strategy x_i in $x = (x_1,x_2)$. Without loss of generality, if i $= 1$, then there exists $y_1 \in [0,c_1]$ such that $\pi_1(y_1,x_2) > w_1 + \varepsilon$, for some $\varepsilon > 0$. Since π_1 is continuous in (y_1,x_1), $\pi_1(y_1,\ x_2')>\frac{\varepsilon}{2}+w_1$ for every x_2' in some open neighborhood of x_2 and thus 1 can secure at x payoff greater than w_1. Assume now that $x = (0,0)$. Since $lim_{n\to\infty}\left(\frac{x_1^n}{x_1^n+x_2^n} + \frac{x_2^n}{x_1^n+x_2^n}\right) = 1$, we must have either $lim_{n\to\infty}\left(\frac{x_1}{x_1^n+x_2^n}\right) \le \frac{1}{2}$ or $lim_{n\to\infty}\left(\frac{x_2^n}{x_1^n+x_2^n}\right) \le \frac{1}{2}$ (Both limits exist by assumption).

Without loss of generality, assume that $lim_{n\to\infty}\left(\frac{x_1^n}{x_1^n+x_2^n}\right) \le \frac{1}{2}$. Then, $w_1 = lim_{n\to\infty}\pi_1(x_1^n,x_2^n) = lim_{n\to\infty}\left(\frac{x_1^n}{x_1^n+x_2^n}\right)V_1 \le \frac{1}{2}V_1$. Now, for all $(s,t) \in [0,c_1] \times [0,c_2]\backslash\{(0,0)\}$, we have $\pi_1(s,t) = \frac{s}{s+t}(V_1 - \alpha s) - s\,(1\text{-}\alpha)$. Therefore, $lim_{t\to 0}\pi_1(s,t) = V_1 - s$. Thus, if $0 < s < \frac{V_1}{2}$ there exists $\varepsilon > 0$ such that for all $0 < t\langle\varepsilon,\pi_1(s,t)\rangle\frac{1}{2}V_1 \ge w_1$. Which implies that player 1 can secure at $x = (0,0)$ payoff greater than w_1.

This proof aligns precisely with the game conditions under Scheme C, where player i's upper bound c_i may be defined as greater than or equal to their prize divided by the positive constant α, i.e., V_i/α (subject to $\alpha > 0$). Notably, the validity of the proof remains unaffected whether the upper bounds are symmetric across players or vary asymmetrically.

Accordingly, one can establish the existence of an interior equilibrium under Scheme C, $(x_1,x_2) \in (0,V_1/\alpha) \times (0,V_2/\alpha)$. To demonstrate that each player's upper bound can indeed be defined as greater than or equal to V_i/α, consider the following argument. Without loss of generality, fix player 2's effort at some $x_2 \in [0,V_2/\alpha]$ and consider player 1's payoff function (14):

$$\pi_1^C = V_1p_1 + \alpha x_1(1-p_1) - x_1 = (V_1 - \alpha x_1)p_1 - (1-\alpha)x_1 \tag{A1}$$

Where p_1 denotes player 1's probability of success as follows:

$$p_1 = \begin{Bmatrix} \dfrac{x_1}{x_1+x_2}\ \ if\ x_1 + x_2 > 0 \\ 0.5\ \ otehwise \end{Bmatrix}$$

If player 1 choose some $x_1 = (V_1/\alpha) + \epsilon$, where $\epsilon > 0$, then her payoff $\pi_1((V_1/\alpha) + \epsilon,x_2) < 0$. Instead, if she chooses $x_1 = 0$, then she gets payoff $\pi_1(0,x_2) \ge 0$.

We now show that our game does not have any corner equilibrium. Suppose one player chooses zero effort and the other a strictly positive effort. The other has a profitable deviation by marginally reducing their effort to reduce costs while still securing the entire prize. Similarly, in the profile (0,0), each player can profit by deviating to a small $\varepsilon > 0$, winning with certainty at negligible cost. Thus, zero-effort profiles are not equilibria.

Moreover, the strategy profile $(x_1,x_2) = (V_1/\alpha, V_2/\alpha)$ cannot constitute a potential equilibrium. Without loss of generality, fix player 2's effort at $x_2 = V_2/\alpha$ and consider player 1's payoff function expressed in (A1). If $\alpha < 1$, then player 1's choice of $x_1 = V_1/\alpha$ yields payoff $\pi_1(V_1/\alpha, V_2/\alpha) < 0$, which is smaller than her payoff $\pi_1(0, V_2/\alpha) = 0$ under the alternative choice $x_1 = 0$.

Now, if $\alpha = 1$ instead, then player 1's choice of $x_1 = V_1/\alpha$ yields payoff $\pi_1(V_1/\alpha, V_2/\alpha) = 0$, but she can obtain a greater payoff $\pi_1(V_1/\alpha - \epsilon', V_2/\alpha) > 0$ by decreasing her effort to $x_1 = V_1/\alpha - \epsilon'$ for any $\epsilon' \in (0, V_1/\alpha)$. Hence player 1's choice of $x_1 = V_1/\alpha$ is not a best response to player 2's choice of $x_2 = V_2/\alpha$. This rules out $(x_1,x_2) = (V_1/\alpha, V_2/\alpha)$ as a potential equilibrium.

We showed that our game has an equilibrium and there is no corner equilibrium. Therefore, this equilibrium must be an interior one.

Appendix B

Table B1

[10] Special thanks are due to an anonymous reviewer, who helped on the main proof in this appendix.

Table B1
Simulated equilibrium expected net payoff under Schemes A, C, and D.

Value		Scheme A			Scheme C ($\alpha = 0.5$)			Scheme C ($\alpha = 1$)			Scheme D		
V_1	V_2	x_1^{A*}	x_2^{A*}	π_d^{A*}	x_1^{C*}	x_2^{C*}	π_d^{C*}	x_1^{C*}	x_2^{C*}	π_d^{C*}	x_1^{D*}	x_2^{D*}	$\pi_d^{D*} = X^T$
100	**1**	1	100	1	1.855	0.036	1.856	6.470	0.481	6.056	0.980	0.010	0.990
100	**5**	5	100	5	7.350	0.636	7.401	13.030	2.297	11.422	4.535	0.227	4.762
100	**10**	10	100	10	11.890	1.878	12.146	17.070	4.425	14.467	8.264	0.826	9.091
100	**20**	20	100	20	17.540	4.925	18.620	21.750	8.379	18.031	13.889	2.778	16.667
100	**30**	30	100	30	20.980	8.162	23.266	24.690	12.051	20.544	17.751	5.325	23.077
100	**40**	40	100	40	23.300	11.386	27.038	26.810	15.507	22.668	20.408	8.163	28.571
100	**50**	50	100	50	24.940	14.525	30.286	28.450	18.789	24.607	22.222	11.111	33.333
100	**60**	60	100	60	26.150	17.566	33.209	29.760	21.911	26.432	23.438	14.063	37.500
100	**70**	70	100	70	27.030	20.477	35.857	30.860	24.917	28.205	24.221	16.955	41.176
100	**80**	80	100	80	27.690	23.277	38.321	31.810	27.825	29.950	24.691	19.753	44.444
100	**90**	90	100	90	28.200	25.981	40.659	32.620	30.622	31.652	24.931	22.438	47.368
100	**100**	100	100	100	28.571	28.571	42.855	33.333	33.333	33.333	25.000	25.000	50.000

Data availability

No data was used for the research described in the article.

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