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Risk Assessment of the UK Electricity Supply Network: A Preferencebased Decision Support Method

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Abstract

The resilience and reliability of essential infrastructures, such as power grids, are critical for the smooth functioning of societies. With the rapid diffusion of electric vehicles (EVs), reliance on a stable and reliable electric power supply has significantly increased. This necessitates a comprehensive risk analysis framework to understand the reliability of electric power supply systems. Identifying crucial macro-level risks involves a certain degree of uncertainty and requires expert preference elicitation. It is also prominent for a reliable preference elicitation model to appropriately handle the subjective judgments of decision makers (DMs). In this study, a multi-criteria decision analysis (MCDA) perspective is adopted by integrating a spanning trees enumeration (STE) method with the best-worst method (BWM) to capture the hesitancy and uncertainty of DMs in identifying the most crucial risks in the UK electricity supply network system. This approach considers the existence of more than one possible best (i.e., the most favorable) or worst (i.e., the least favorable) criterion in the model. To validate the proposed STE-BWM model, a set of Monte Carlo simulations and a real-world application are implemented coupled with comparative and sensitivity analyses. The simulations are conducted under various defined numerical experiments, and the results indicate a satisfactory success rate of STE (i.e., 65.80%) in identifying the unique best or worst criterion in various experiments. The applicability of the proposed STE-BWM is shown in a case study of the UK electricity supply network risk assessment.

Keywords: Best-worst method, decision analysis, electricity, energy, risk, spanning trees enumeration, uncertainty

1. Introduction

The UK has a long tradition of being on the front end of the NetZero transition. An example of this approach is the intention to gradually eliminate the sale of new petrol- and diesel-powered cars and vans by 2035¹. Additionally, starting in 2035, all newly sold cars and vans will be required to produce zero emissions from their exhaust systems (UK Department for Transport, 2021). These ambitions along with technological advances in manufacturing

¹ Under the former Prime Minister Theresa May's government, the original target date was 2040. During Boris Johnson's tenure the date was brought forward to 2030, to be later delayed to 2035 by Rishi Sunak's government.

processes have led to a significant reduction in the average price of electric vehicles (EVs), causing rapid growth in EV sales and a global increase in lithium-ion battery (LIB) production capacity (Lusty et al., 2022). These evolutions put additional pressure on the electric power supply and vehicle-to-grid (V2G) infrastructure to stay secure, reliable, and resilient. The UK electricity system is passive and complex, with previous research highlighting uncertainty over policy and lack of long-term vision as key sources of stakeholder risk (Connor et al., 2018). Part of the risk analysis requires expert judgments and preference elicitation, which are of an uncertain and subjective nature. In recent years, preference elicitation has gained the attention of researchers from various perspectives including information gathering (Voorberg et al. 2021), modeling preferences (Baak, Goerigk, and Hartisch 2024; Bozóki and Tsyganok 2019), and evaluation and use (Nikou, Mezei, and Sarlin 2015; Boxebeld, Mouter, and Van Exel 2024). However, real-world decision-making processes require dealing with subjective human judgments and circumstances in the decision-making context. Preference elicitation has been commonly applied in methods such as analytic hierarchy process (AHP) (Saaty 1980) and bestworst method (BWM) (Rezaei 2015) to capture preferences of decision makers (DMs). Take an example of BWM as a recent multi-criteria decision analysis (MCDA) method, in which a decision maker (DM) is asked to indicate which decision-making criterion he/she considers as the best and the other as the worst (Q. Wu et al. 2024). In the real world, applying the original BWM with subjective human judgements, choosing only one criterion as either the best or the worst without any level of hesitancy is not always straightforward for DMs. In other words, there might be a set of best and a set of worst criteria instead of just one single best or worst criterion.

In this study, to deal with uncertainty, we propose a hybrid preference elicitation model combining the spanning trees enumeration (STE) method with BWM, namely STE-BWM. In the hybrid model, the STE offers an opportunity for DMs to indicate a range of the best and worst criteria. The STE part of the proposed approach concludes which criterion is the best or worst among the given set of potential best or worst criteria. The STE analysis is based on data already provided in the form of pairwise comparisons by DMs. The result of STE is then fed into the BWM to calculate the final weights and order of criteria.

This study contributes to the literature by proposing a decision-support model that deals with uncertain preferences and empirically applies it to a real-world case study: the UK electricity supply network system risk assessment. To validate the performance of the proposed hybrid approach, a set of Monte Carlo simulations under various defined numerical experiments are

conducted. The simulation analysis is carried out by running numerical analyses in Python. The Python code is available via open access and provides a free decision support tool. This tool can be used by researchers and analysts to apply the STE analysis part of the hybrid approach to their problem, regardless of the scale of the decision-making model being analyzed (see Section 4). The results indicate a satisfactory success rate of the STE in identifying the unique best or worst criterion in various experiments out of a set of potential best or worst criteria. In addition to the numerical analysis, the applicability of the proposed STE-BWM in a real-world application is demonstrated. In this case study of the UK electricity supply network, it has been identified that Natural Disasters (ND), Climate Change (CC), Industrial Action (IA), Affordability (AF), Political Instability (PI), and Sabotage/Terrorism (ST) are the most crucial risks (Vafadarnikjoo et al. 2022; Vafadarnikjoo 2020). These identified risks are prioritized using the STE-BWM in this study. Our research contributes to the multiple criteria decision-making body of knowledge by achieving the following aims of the study:

- (I) To propose a preference elicitation STE-BWM to handle uncertainty in cases where decision makers might struggle to pick only one criterion due to uncertainty, hesitancy, or lack of information.
- (II) To empirically apply the hybrid STE-BWM in a real-world case of an electricity supply network risk prioritization model to show the applicability of the proposed method and to verify the most critical risks to the UK energy supply chain.
- (III) To conduct Monte Carlo simulations under various defined numerical experiments and provide a Python-based decision-support tool to facilitate STE analysis.

In the rest of this paper, background and context will be discussed in Section 2, the proposed STE-BWM are described in Section 3. In Section 4, a Monte Carlo simulation analysis is conducted. To demonstrate the applicability of the method, the STE-BWM is applied in the UK electricity supply network risk assessment, and the empirical findings are presented in Section 5. Finally, Section 6 summarizes the paper.

2. Background and context

In this section, a background on preference elicitation models is provided. It is followed by a literature review on electricity supply network risks and their definition for the purpose of this study.

2.1. Preference elicitation models in multi-criteria decision analysis

This section reviews the relevant literature on preference elicitation models focusing on direct methods using rating scales in the MCDA area. Tversky and Kahneman (1991) presented a

reference-dependent theory of consumer choice which discusses the concept of loss aversion and analyzes its role in shaping preferences for choices. Sarin and Winkler (1992) proposed a preference model to deal with ambiguity of DMs by considering the probability of relevant decision-making events. Keeney and Raiffa (1993) offered a thorough overview of the decision-making process in the face of multiple incommensurable objectives. Recent research has changed our understanding of this problem. Ji et al. (2024) proposed a dependence assessment method based on BWM and cloud model and applied the social network trust graph to determine experts' weights. De Almeida et al. (2024) studied how holistic evaluations (HE) information combines with information from the decomposition approach and what their implications are applying Flexible and Interactive Tradeoff (FITradeoff) method. De Morais Correia et al. (2022) ranked workstations for ergonomic interventions applying Strategic Options Development and Analysis (SODA), Value Focused-thinking (VFT), and FITradeoff method. A few researchers also integrated uncertain non-probabilistic theories with BWM to capture the uncertainties of DMs, such as spherical fuzzy sets (Haseli et al. 2024), calibrated fuzzy BWM (Lopez et al. 2023), and interval-valued intuitionistic fuzzy sets (Dong and Wan 2024).

2.2. Electricity supply network risks

Electricity supply networks, as key critical infrastructures, are indispensable for our economy and society; however, they remain susceptible to various vulnerabilities and risks (Gong et al., 2025). In 1990, the UK electricity industry was privatized and thus unbundled into generation, transmission, distribution, and retailing (Vlahos et al., 1998). We define the electricity supply network as the entire chain of electric power supply from upstream to downstream (Vafadarnikjoo et al., 2020). In previous studies, Kjolle et al. (2012) implemented a cross-sector risk analysis to investigate the cascading impact of failure in the electric power supply in other infrastructures. Hammond and Waldron (2008) identified and ranked major risks in the UK electricity sector by considering various stakeholder groups and quantifying risks by multiplying the likelihood of each risk and its consequences. Silvast (2017) studied the electricity infrastructure and interruptions from a social science perspective and tried to answer how people and organizations react to these interruptions. Staid and Guikema (2015) provided an overview of the risks encountered by an offshore wind farm in the US.

In the context of energy and MCDA, Cinelli et al. (2022) discussed the proper and improper use of MCDA methods in energy system analysis by proposing a decision-support system. Lin et al. (2018) identified risk elements of the New Energy Power System (NEPS) in China and

analyzed their internal influence relations based on D numbers and DEMATEL. Wu et al. (2018) evaluated renewable energy power sources in China applying a fuzzy AHP method and a cumulative prospect theory. Okoro and Kolios (2018) developed and applied a multiple criteria risk assessment framework in a complex oil and gas support structure. Bolsover (2015) employed Bayesian Network (BN) to monitor risks in real-time which would lead to a more efficient decision making in an offshore drilling rig. Chou and Ongkowijoyo (2014) proposed a risk-based approach to compare alternative renewable energy schemes. They applied a hybrid graphical matrix approach with a Monte Carlo simulation. Maxim (2014) prioritized 13 power generation technologies by considering 10 criteria using a weighted sum multi-attribute utility approach. Hunt et al. (2013) proposed a decision support framework tool based on MCDA for complex prediction of decision-making processes in the UK energy sources. Aplak and Sogut (2013) used game theory to evaluate the decision-making process of the industry and environment as two players based on the scope of energy management. The strategies were analyzed using MCDM methods to calculate the performance efficiency values. Ren et al. (2009) studied the causal interrelationships between risk elements in offshore installation operations using a Fuzzy Bayesian Network (FBN). Vafadarnikjoo et al. (2022) reviewed the risks in the UK electric power supply chains and identified the below risks as the most crucial ones by applying neutrosophic revised decision-making trial and evaluation laboratory (NR-DEMATEL) method:

2.2.1. Natural Disasters (ND)

Natural Disasters (ND) are calamitous events of atmospheric, geological, or hydrological origin. These include storms, hurricanes, floods, earthquakes, droughts, tsunamis, landslides, volcanic eruptions, and wildfires. Their impact can be rapid or slow and can disrupt the supply chain or the operation of power generation units. In previous studies, methods other than MCDA have been applied to identify and assess the impact of disasters on power grids, such as social network analysis (He et al., 2025), mathematical modeling (Aldarajee et al., 2020), and probabilistic modeling (Opabola and Galasso, 2024).

2.2.2. Climate Change (CC)

Climate Change (CC) is a long-term alteration of the climate, mainly driven by manmade greenhouse gas (GHG) emissions. Changes in precipitation, cloud coverage, and wind patterns can impact hydropower generation as well as solar and wind power productivity. This can also threaten the capability of cooling thermal power stations to disrupt their operation. The

transformation and transportation of electricity can also be affected by the increased occurrence of extreme weather events (discussed in Section 2.2.1). Salam et al. (2024) reviewed the impact of climate change challenges on the electric power system and explored the role of microgrids in climate change mitigation and adaptation strategies.

2.2.3. Industrial Action (IA)

Industrial Action (IA) is regarded as a major cause of disruptions in energy supply and electricity generation. The electricity sector, as a state-controlled legacy, has connections with powerful labor unions. These unions may be regarded as the main barriers to the liberalization and privatization of the power sector, which is underway in many countries. Hence, the threat of coordinated industrial action often exists. It should be noted that disruptions caused by industrial action are considered short- or medium-term shocks (depending on the definition) (Chalvatzis 2012). As an example, in the oil market, the Venezuelan industrial action in 2002-3 resulted in a gross peak supply loss of 2.6 mb/d (million barrels per day) and is regarded as one of the five most important disruptions of the past decades (Löschel, Moslener, and Rübbelke 2010).

2.2.4. Affordability (AF)

Affordability (AF) refers to the price of energy and capacity of households and business users to afford it. This demonstrates that the availability of energy is insufficient if it is available at very high prices. In business terms, it can lead to activity reduction or even elimination and loss of competitiveness in businesses that are not exposed to high prices. It is also related to vulnerable consumers, who may not be able to meet their basic energy needs, leading to what is known as energy poverty (Gonzalez-Eguino, 2015).

2.2.5. Political Instability (PI)

Political Instability (PI) refers to social unrest or geopolitical changes that affect the security of the energy supply chain and cause disruptions. Political instability can impact all aspects of the electricity supply system, including the supply, network, and demand. In addition to national politics and potential unrest, this risk can be linked to geopolitical changes with trade embargoes and resource nationalism.

2.2.6. Sabotage and Terrorism (ST)

Sabotage and Terrorism (ST) make the electricity supply network confront the serious challenge of providing more security without compromising the inbuilt productivity benefits in highly complicated and interconnected power networks. Disruption of electricity supplies can have a catastrophic impact on national security. Power systems can never be completely safeguarded against a determined attack because the assets are widely dispersed. Nowadays, the increased usage of EVs can potentially pose risks on the electric grid via electric vehicle charging stations (Moghadasi et al., 2022).

3. Proposed STE-BWM method

The implementation steps for the hybrid use of STE and BWM (STE-BWM) are presented in Figure 1. The STE can be accomplished by either Enumerating All Spanning Trees (EAST) (Siraj et al., 2012) or Geometric Mean of All Spanning Trees (GMAST) (Lundy et al., 2017). Siraj et al. (2012) introduced the EAST method to obtain prioritization weights of criteria in pair-wise comparisons. The EAST procedure is explained in the following steps:

Step 1: Obtain the criteria set

$$C = \{F_1, F_2, \dots, F_n\}$$
(1)

Step 2: Acquire the pair-wise comparison matrix of criteria

The obtained pair-wise comparisons can be either complete (without missing values) or incomplete (with missing values).

$$A = \begin{bmatrix} a_{ij} \end{bmatrix} \quad i, j = 1, \dots, n \tag{2}$$

Step 3: Produce the corresponding graph of the pair-wise comparison matrix

The graph can be produced by taking each criterion as a vertex then each non-empty, nondiagonal element of the pair-wise comparison matrix reveals that there is an edge between the two related vertices as in Equation (3); (i, j) represents an edge between vertex i and j.

$$(i,j) = \begin{cases} exists, & a_{ij} \notin \emptyset \\ does not exist, & a_{ij} \in \emptyset \end{cases} \quad i \neq j$$
(3)

Step 4: Generate all spanning trees

The total number of possible spanning trees (η) can be calculated using Kirchhoff's matrixtree theorem (Theorem A.3 in Appendix A). Subsequently, a Gray code algorithm (Appendix B) is used to generate all spanning trees.

Step 5: Compute the weights of criteria from each spanning tree

Knowing that each obtained spanning tree has (n - 1) edges. The weight of i^{th} criterion in k^{th} spanning tree $(w_i^{(k)})$ can be computed by solving a system of n linear equations. For any spanning tree, the (n - 1) equations out of n are constructed based on Equation (4) and the last one indicates the sum of weights must be equal to 1 as shown in Equation (5)

$$w_i^{(k)} = a_{ij} w_i^{(k)} \quad \forall k = 1, \dots, \eta \quad i, j = 1, \dots, n \quad i \neq j$$
⁽⁴⁾

$$\sum_{i=1}^{n} w_i^{(k)} = 1 \qquad \forall k = 1, \dots, \eta$$
⁽⁵⁾

Step 6: Calculate the average of all weights and prioritize criteria

Assuming η is the total number of generated spanning trees then the final weights of criteria (w_i) can be obtained based on the Equation (6)

$$w_{i} = \frac{\sum_{k=1}^{\eta} w_{i}^{(k)}}{\eta} \quad \forall i = 1, ..., n$$
(6)

In the proposed approach, the following steps should be followed:



Figure 1. Implementation steps of the STE-BWM

Step 1: Identify the set of decision-making criteria. The identified criteria can be signified as shown in Equation (7).

$$N = \{C_1, C_2, \dots, C_n\}$$
(7)

Step 2.1: Determine the set of best criteria (i.e., the most critical, most favorable, or most important group of criteria), and the set of worst criteria (i.e., the least critical; least favorable or least important group of criteria). The sets of the best and worst criteria are denoted by Θ and Γ which are identical subsets of *N* as represented in Equations (8) and (9), respectively.

$$\Theta = \{M_1, M_2, \dots, M_m\} \qquad \Theta \subset N, \Theta \neq \Gamma$$
⁽⁸⁾

$$\Gamma = \{L_1, L_2, \dots, L_{n-m}\} \qquad \Gamma \subset N, \Gamma \neq \Theta$$
⁽⁹⁾

Step 2.2: Apply STE to obtain the best or/and worst criterion

In this step, by applying STE (EAST or GMAST), the weights of each combination of the best and the worst criteria are calculated and the maximum weight in Θ determines the best criterion and the minimum weight in Γ determines the worst criterion. The maximum number of calculations equals to $m \times (n - m)$ because of $|\Theta| = m$ and $|\Gamma| = n - m$. For instance, if $|\Theta| = 2$, and $|\Gamma| = 3$, then the STE calculations should be carried out 6 times (i.e., 2 × 3).

Then, the rest of the analysis should be followed from Step 3 to 5 in the Linear BWM (L-BWM) (Rezaei, 2016) as explained below:

Step 3: Establishing the Best-to-Others (BO) preference vector using a 9-point scale In this stage, experts use the linguistic 1-9 rating scale (1 representing equally important to 9 representing extremely more important) to construct a preference vector for the most critical risk (i.e., best) over other risks. A rating scale of 1 means equal preference, and 9 means extreme preference. The resulting BO vector can be represented as $A_B = (a_{B1}, a_{B2}, ..., a_{Bn})$. The notation a_{B1} denotes the preference of the most critical (i.e., the best) risk *B* compared to risk 1, and obviously, the value of a_{BB} will be 1.

Step 4: Establishing the Others-to-Worst (OW) preference vector using a 9-point scale In this stage, experts use the linguistic 1-9 rating scale (1 representing equally important to 9 representing extremely more important) to construct a preference vector of others to the worst (i.e., the least critical) risk. The OW vector can be represented as $A_W = (a_{1W}, a_{2W}, ..., a_{nW})^T$. In the OW vector, the notation a_{1W} denotes the value of a verbal scale for a risk 1 over the worst (i.e., the least critical) risk W, and, naturally, the value of a_{WW} is equal to 1.

Step 5: Finding the optimal weights of identified risks $(w_1^*, w_2^*, ..., w_n^*)$

In this step, the optimized weight of each risk is calculated by minimizing the maximum absolute differences, as shown in the objective function of Model (1).

min
$$max_j \{ |w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W| \}$$
(1)
subject to
$$\sum_j w_j = 1$$
 $w_j \ge 0 \text{ for all } j$

Model (1) is converted into a linear programming problem, which can be represented as Model (2):

$$\operatorname{Min} \xi^L \tag{2}$$

subject to

$$\begin{split} |w_B - a_{Bj} w_j| &\leq \xi^L \text{ for all } j \\ |w_j - a_{jW} w_W| &\leq \xi^L \text{ for all } j \\ \sum_j w_j &= 1 \\ w_j &\geq 0 \text{ for all } j \end{split}$$

In Section 4, a Monte Carlo simulation is implemented to test the validity of the proposed STE-BWM under various numerical experiments.

4. Monte Carlo simulation analysis

Simulation analysis was performed by running numerical analyses in Python. The Python code is available via open-access and provides a free decision support tool¹. Monte Carlo simulation iterations are 10, 50, 250, and 1000. The simulation algorithm, experimental design, and simulation results are presented in the following sections.

4.1. Simulation algorithm

The proposed simulation algorithm has six steps. The simulation algorithm begins by initiating the parameters. Next, it randomly chooses the best and worst criteria based on the given values, then generates initial BO and OW vectors using randomly selected values from a 9-point scale. After that, it calculates input-based consistency to determine if consistency is achieved. If consistent, the algorithm applies the GMAST method to obtain weights for the criteria. Finally, it checks if the maximum number of iterations has been reached; if not, the process repeats, but if the maximum cycle is reached, it aggregates all weights. These steps are elaborated below and all the steps in the algorithm are illustrated in Figure 2.

¹ The codes are accessible at this link: https://github.com/AminVafadar/STE-BWM



Figure 2. The simulation process

Step 1: Initiate the parameters

The parameters include number of criteria (n), number of best criteria ($|\theta|$), number of worst criteria ($|\Gamma|$), and number of cycles (MCN).

Step 2: Randomly choose the best and worst criteria given the values

The best and worst criteria are chosen randomly considering predefined n, $|\theta|$ and $|\Gamma|$.

Step 3: Generate initial BO and OW vectors using randomly selected values from 9-point scale In this section, the generation of initial BO and OW vectors is explained. In this example, there are six criteria (n = 6), three best criteria ($|\theta| = 3$), and one worst criterion ($|\Gamma| = 1$). Thus, there are three BO vectors and one OW vector.

I. Randomly choose the three best criteria and one worst criterion $N = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ thus n = 6. $\theta = \{C_2, C_4, C_5\}$ thus $|\theta| = 3$ and $\Gamma = \{C_3\}$ thus $|\Gamma| = 1$.

Note that the criterion that belongs to the best set cannot belong to the worst set and vice versa.

II. Each best criterion is equally preferred to themselves

With reference to the importance 1-9 rating scale in Rezaei (2016), we use values of 1 as shown below:

		C_1	<i>C</i> ₂	<i>C</i> ₃	C_4	<i>C</i> ₅	С ₆
BO vector 1	<i>C</i> ₂		1				
BO vector 2	C_4				1		
BO vector 3	<i>C</i> ₅					1	

III. Generate a value for a_{BW}

Randomly choose a value from 3 to 9 (with reference to the importance 1-9 rating scale in Rezaei (2016)). In this example, assume that $a_{23} = a_{43} = a_{53} = 8$

		<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	C_4	<i>C</i> ₅	<i>C</i> ₆
BO vector 1	<i>C</i> ₂		1	8			
BO vector 2	<i>C</i> ₄			8	1		
BO vector 3	<i>C</i> ₅			8		1	

IV. Best criteria are equally important compared to each other

The criteria C_2, C_4, C_5 are the best criteria as shown in $\theta = \{C_2, C_4, C_5\}$. Thus, they need to be equally important compared to each other.

		<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	C_4	<i>C</i> ₅	<i>C</i> ₆
BO vector 1	<i>C</i> ₂		1	8	1	1	
BO vector 2	<i>C</i> ₄		1	8	1	1	
BO vector 3	C_5		1	8	1	1	

V. Generate the rest of the remaining values in BO vectors

The remaining values are generated using values between 2 and $a_{BW} - 1$. Here, values between 2 and 7 are randomly chosen for the rest of remaining values.

		<i>C</i> ₁	<i>C</i> ₂	<i>C</i> ₃	<i>C</i> ₄	C_5	<i>C</i> ₆
BO vector 1	С2	7	1	8	1	1	5
BO vector 2	<i>C</i> ₄	6	1	8	1	1	3
BO vector 3	<i>C</i> ₅	4	1	8	1	1	6

VI. To generate OW vector(s), use the a_{BW} values

 $\begin{array}{c|ccccc}
 & OW vector \\
\hline
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To compare the best criteria to the worst criterion, the value of $a_{BW} = 8$ is chosen.

VII. Each worst criterion is equally preferred to themselves

The criterion C_3 as the selected worst criterion is equally important to itself and the value 1 must be put it the OW vector.

	OW vector
	<i>C</i> ₃
<i>C</i> ₁	
C_2	8
C_3	1
C_4	8
C_5	8
<i>C</i> ₆	

VIII. Worst criteria are equally important compared to each other

In this example, we just have one worst criterion. Otherwise, their values would have also been one.

IX. Generate the rest of the remaining values in OW vectors

The remaining values are generated using values between 2 and $a_{BW} - 1$. Here, values between 2 and 7 are randomly chosen for the rest of remaining values.

	OW vector
	<i>C</i> ₃
<i>C</i> ₁	5
C_2	8
C_3	1
C_4	8
C_5	8
C_6	6

Step 4: Calculate input-based consistency

Input-based consistency CR^{I} or the global input-based consistency ratio for all criteria is the maximum value of CR_{j}^{I} which are local consistency levels associated with each criterion (C_{j}) (Liang et al. 2020).

This is formulated as
$$CR^{I} = \max_{j} CR^{I}_{j}$$
 where $CR^{I}_{j} = \begin{cases} \frac{|a_{Bj} \times a_{jW} - a_{BW}|}{a_{BW} \times a_{BW} - a_{BW}} & a_{BW} > 1\\ 0 & a_{BW} = 1 \end{cases}$.

Step 5: Apply GMAST and obtain weights

Lundy et al. (2017) explored the quality of the GMAST method and indicated that as EAST fails to adhere to geometric properties, GMAST can outperform EAST in obtaining final weights. In GMAST, the same steps as EAST should be followed as explained in the proposed STE-BWM (Section 3) but for the final step geometric mean is employed using the geometric mean of all weights $w_i = \sqrt[\eta]{\prod_{k=1}^{\eta} w_i^{(k)}}$ $\forall i = 1, ..., n$ and ultimately to prioritize criteria in an descending order of obtained weights.

Step 6: Check if maximum number of iterations is reached

If yes, all the weights are aggregated using the geometric mean presented in $w_i = \sqrt[\eta]{\prod_{k=1}^{\eta} w_i^{(k)}}$ $\forall i = 1, ..., n$.

4.2. Experimental designs

The set of designed experiments considering all possible scenarios for the Monte Carlo simulations are illustrated in Table 1. The process of an experimental instance is depicted in Figure 3.

0				6	9			
n = 9	1	2	3	4	5	6	7	8
	2	1	1	1	1	1	1	1
	3	2	2	2	2	2	2	
	4	3	3	3	3	3		
	5	4	4	4	4			
	6	5	5	5				
	7	6	6					
	8	7						
				$ \theta $				
n = 8	1	2	3	4	5	6	7	
	2	1	1	1	1	1	1	
	3	2	2	2	2	2		
	4	3	3	3	3			
1	5	4	4	4				
	6	5	5					
	7	6						
				θ				
n = 7	1	2	3	4	5	6		
	2	1	1	1	1	1		
	3	2	2	2	2			
Γ	4	3	3	3				
	5	4	4					
	6	5						
m = 6			$ \theta $					
n = 0	1	2	3	4	5			
	2	1	1	1	1			
IFI	3	2	2	2				
1	4	3	3					
	5	4						
n - 5			θ					
<i>n</i> = 5	1	2	3	4				
	2	1	1	1				
Γ	3	2	2					
	4	3						
n - 1		$ \theta $						
n = 4	1	2	3					
IFI	2	1	1					
1	3	2						
n-3		$ \theta $						
n – 5	1	2						
Γ	2	1						

Table 1. Designed experiments for the Monte Carlo simulations



Figure 3. An experimental instance $(n = 6, |\theta| = 3, |\Gamma| = 1)$

4.3. Simulation results

In cases where more than one best or worst criterion was available, Success (i.e., successful or 100% success rate) means that both the unique best and worst criteria could be identified. Failure (i.e., unsuccessful or 0% success rate) means that none of the best or worst criteria can be uniquely identified. Mixed (i.e., 50% success rate) indicates that either the best or worst criterion was uniquely identified. Table 2 provides average success rate in identifying unique best and worst criteria when using aggregated weights of criteria by applying geometric mean in 10 iterations. Table 3 illustrates the same average success rates under 50 iterations.

	. 0				$ \theta $					Average Success Rate (%)
n =	- 9	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	
	2	\checkmark	\checkmark	Х	√/X	\checkmark	\checkmark	Х	_	
	3	\checkmark	Х	√/X	\checkmark	\checkmark	Х	_	_	
	4	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	_	60.0
1	5	\checkmark	\checkmark	\checkmark	Х	-	_	_	_	
	6	\checkmark	\checkmark	Х	—	_	_	_	_	
	7	\checkmark	Х	_	_	-	_	_	_	
	8	Х	_	_	_	-	_	_	_	
n =	- 8	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	56.5
1	2	\checkmark	\checkmark	\checkmark	√/X	√/X	Х	_	_	

Table 2. Average success rate in 10 iterations (\checkmark =success, X=failure, \checkmark/X =mixed)

	3	\checkmark	√/X	\checkmark	\checkmark	Х	_	_	_	
	4	\checkmark	√/X	\checkmark	Х	_	_	_	_	
	5	\checkmark	\checkmark	Х	_	_	_	_	_	
	6	\checkmark	Х	_	_	_	_	_	_	
	7	Х	_	_	_	_	_	_	_	
n =	: 7	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	
	2	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	_	
151	3	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	56.4
	4	\checkmark	√/X	Х	_	_	_	_	_	
	5	\checkmark	Х	_	_	_	_	_	_	
	6	Х	_	_	_	_	_	_	_	
n =	6	1	2	3	4	5	6	7	8	
	1	_	\checkmark	\checkmark	\checkmark	Х	_	_	_	
	2	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	
	3	\checkmark	\checkmark	Х	_	_	_	_	_	53.3
	4	\checkmark	Х	_	_	_	_	_	_	
	5	Х	_	_	_	_	_	_	_	
n =	: 5	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	Х	_	_	_	_	
	2	\checkmark	\checkmark	Х	_	-	-	-	_	45.8
11	3	\checkmark	Х	_	_	-	-	-	_	
	4	Х	-	_	_	_	_	—	_	
n =	: 4	1	2	3	4	5	6	7	8	
	1	-	\checkmark	Х	_	—	—	_	—	22.2
Γ	2	\checkmark	Х	_	_	-	-	-	_	33.3
	3	Х	-	_	_	_	_	—	_	
n =	: 3	1	2	3	4	5	6	7	8	
IFI	1	—	Х	-	_	-	_	-	-	0.0
	2	Х	_	_	_	_	_	_	_	

As can be seen in Table 2, the average success rate is 60% under n = 9. This value is calculated by taking the average of success rates under various parameter values for $|\Theta|$ and $|\Gamma|$ when n =9. For example, under the parameter settings of $|\Gamma| = 1$ and n = 9 where $|\Theta| = 2, ... 8$, the average success rate is 85.71% (i.e., 6 successes out of 7 cases). The average success rate for $|\Gamma| = 2, n = 9$, and $|\Theta| = 1, ... 7$ is 64.29%. By taking the other average success rates under n = 9 and various possible values for $|\Theta|$ and $|\Gamma|$, the average value of 60% can be computed which is the average of 85.71%, 64.29%, 58.33%, 80%, 75%, 66.67%, 50% and 0%. For the sake of simplicity, merely the final average success rates are presented in Tables 2 and 3.

Table 3. Average success rate in 50 iterations (\checkmark =success, X=failure, \checkmark/X =mixed)

n = 0				(9	Average Success Rate (%)			
n = 9	1	2	3	4	5	6	7	8	65.8
Γ 1	—	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	03.8

	2	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	
	3	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	
	4	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	_	
	5	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	
	6	\checkmark	\checkmark	Х	_	_	_	_	_	
	7	\checkmark	Х	_	_	_	_	_	_	
	8	Х	_	_	_	_	_	_	_	
<i>n</i> =	- 8	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	
	2	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	
	3	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	_	
$ \Gamma $	4	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	62.6
	5	\checkmark	\checkmark	Х	_	_	_	_	_	
	6	\checkmark	Х	_	_	_	_	_	_	
	7	Х	_	_	_	_	_	_	_	
<i>n</i> =	= 7	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	
	2	\checkmark	\checkmark	\checkmark	\checkmark	Х	_	_	_	
1.51	3	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	58.6
1	4	\checkmark	\checkmark	Х	_	_	_	_	_	
	5	\checkmark	Х	_	_	_	_	_	_	
	6	Х	_	_	_	_	_	_	_	
<i>n</i> =	= 6	1	2	3	4	5	6	7	8	
	1	Ι	\checkmark	\checkmark	\checkmark	Х	_	_	_	
	2	\checkmark	\checkmark	\checkmark	Х	_	_	_	_	52.2
$ \Gamma $	3	\checkmark	\checkmark	Х	_	_	_	_	_	55.5
	4	\checkmark	Х	_	_	_	_	_	_	
	5	Х	_	_	_	_	_	_	_	
<i>n</i> =	= 5	1	2	3	4	5	6	7	8	
	1	-	\checkmark	\checkmark	Х	-	-	—	-	
	2	\checkmark	\checkmark	Х	—	—	—	—	—	45.8
111	3	\checkmark	Х	—	—	—	_	—	—	
	4	Х	-	_	_	-	_	_	_	
<i>n</i> =	= 4	1	2	3	4	5	6	7	8	
	1	-	\checkmark	Х	_	_	-	-	-	33.3
$ \Gamma $	2	\checkmark	Х	-	-	-	-	-	-	55.5
	3	Х	-	—	—	—	_	-	_	
<i>n</i> =	= 3	1	2	3	4	5	6	7	8	
111	1	-	Х	_	_	—	_	—	_	0.0
111	2	Х	_	_	_	_	—	_	_	

Figure 4 and Figure 5 show the average success rates under various number of criteria (n) and number of best criteria $(|\Theta|)$ in 10 and 50 iterations, respectively.



Figure 4. Average success rate for varying criteria and best criteria numbers (10 iterations)



Figure 5. Average success rate for varying criteria and best criteria numbers (50 iterations)

Figure 6 and Figure 7 show the average success rates under various number of criteria (n) and number of worst criteria $(|\Gamma|)$ in 10 and 50 iterations, respectively.



Figure 6. Average success rate for varying criteria and worst criteria numbers (10 iterations)



Figure 7. Average success rate for varying criteria and worst criteria numbers (50 iterations)

The results show that (1) STE is unable to identify the best or worst criterion when all the criteria are either the best or worst. This means that no free criterion (i.e., neither the best nor worst) exists. However, in real-world decision-making environment, it barely happens that there would be no free criterion. (2) STE is fully effective in other cases with free criteria. (3) With an increase in the number of iterations, the success rate in identifying the best and worst criteria grows regardless of the number of criteria. This is evident from 10 to 50 iterations. (4) The optimum number of iterations is 50, as the average success rate stabilizes and does not change by increasing to 250 or 1000. (5) The maximum success rate is 65.80%, including non-free cases, and the success rate is 100%, excluding non-free cases (50 iterations, n=9). For baseline analysis, the traditional BWM is considered but since it needs only one best and one worst criterion, it will not allow selecting multiple best or worst criteria in the simulations. However, we applied Fuzzy BWM as the baseline comparison method in our case study analysis (Section 5.4). The analysis of the application of the proposed STE-BWM in the case study of the UK electricity supply network system is discussed in Section 5.

5. Empirical findings in the UK electricity supply network system

In a vulnerability assessment of the UK electric power supply (Vafadarnikjoo et al., 2022), six prominent risks are identified: affordability, natural disasters, industrial action, climate change,

political instability, sabotage, and terrorism. Table 4 presents the most and least important risks identified by five experts. To collect data, 16 experts with rich knowledge and expertise in the UK energy domain were contacted, and 5 responses were received (31% response rate). The BO and OW vectors are listed in Tables 5 and 6, respectively. The BO and OW vectors are based on the preferences of the experts.

	Identified as most important by experts	Identified as least important by experts
AF: Affordability	1	
ND: Natural Disasters	4	
IA: Industrial Action		1, 5
CC: Climate Change	2, 3, 5	
ST: Sabotage/Terrorism	4	3
PI: Political Instability		2, 4

Table 4. Most and least important risks determined by experts

The values of consistency ratio are all within an acceptable threshold lower than 0.1 (Liang et al. 2020).

Experts	The most critical risk	PI	ND	IA	CC	ST	AF
1	AF	1	3	5	2	4	1
2	CC	9	8	3	1	9	5
3	CC	5	3	3	1	4	7
4	ND	7	1	4	3	1	5
4	ST	7	1	4	3	1	5
5	CC	6	2	8	1	2	3

Table 5. Best-to-others (BO) vectors

Table 6. Others-to-worst (OW) vectors

Experts	1	2	3	4	5
The least critical risk	IA	PI	ST	PI	IA
ND	2	6	4	7	7
CC	3	9	9	5	9
ST	1	5	1	7	7
AF	5	7	5	4	5
PI	4	1	3	1	3
IA	1	5	5	3	1

As can be seen in Table 4; expert 4, hesitated to choose only one best criterion (i.e., the most critical risk) between Natural Disasters (ND) and Sabotage and Terrorism (ST); that is why

both were selected. Thus, following the proposed steps of the STE-BWM explained in Section 3, the best criterion for expert 4 can be realized.

Step 1: The identified set of risks are $N = \{AF, ND, IA, CC, ST, PI\}$

Step 2: The set of best and worst risks based on expert 4 are $\Theta = \{ND, ST\}$ (i.e., $|\Theta| = 2$) and $\Gamma = \{PI\}$ (i.e., $|\Gamma| = 1$), respectively. Thus, the STE calculations must be performed twice (i.e., $|\Theta| \times |\Gamma| = 2$). One time for ND and PI and the second for ST and PI. These analyses are presented in the following sections (sections 5.1, 5.2 and 5.3).

5.1. The STE analysis for natural disasters (ND) and political instability (PI)

The STE analyses for ND and PI based on EAST and GMAST are explained in this section. Step 1: The identified set of risks are $C = \{AF, ND, IA, CC, ST, PI\}$

Step 2: Based on the pair-wise comparison vectors provided by expert 4 for ND (i.e., the best risk), and PI (i.e., the worst risk) as shown in Tables 5 and 6, the incomplete pair-wise comparison matrix *A* can be obtained (Table 7). The scale used is from 1-9 in the BWM (Rezaei, 2016).

		1	2	3	4	5	6
		AF	ND	IA	CC	ST	PI
1	AF	1	0.20				4
2	ND	5	1	4	3	1	7
3	IA		0.25	1			3
4	CC		0.33		1		5
5	ST		1.00			1	7
6	PI	0.25	0.14	0.33	0.20	0.14	1

Table 7. The incomplete pair-wise comparison matrix A by expert 4 (ND and PI)

Step 3: The corresponding graph G of the pair-wise comparison matrix A (Table 7) is produced as shown in Figure 8.



Figure 8. The undirected (a), and directed (b) graph G of the matrix A (ND and PI)

Step 4: The Kirchhoff's matrix-tree theorem (Theorem A.3 in Appendix A) is used to obtain the total number of spanning trees. It is known that for each tree, n - 1 = 6 - 1 = 5 edges are needed and as can be seen in Figure 8, the obtained graphs have 9 edges. The total number of spanning trees can be obtained as $\eta = 48$ using the Kirchhoff's matrix-tree theorem. Furthermore, the degree matrix and adjacency matrix of graph *G* are shown in Equation (10) and (11), respectively.

$$D(G) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$
(10)

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$
(11)

Then, the Laplacian matrix of graph G is obtained as represented in Equation (12).

$$L(G) = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 5 & -1 & -1 & -1 & -1 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & -1 \\ -1 & -1 & -1 & -1 & -1 & 5 \end{bmatrix}$$
(12)

 $L^*(G)$ can be attained by omitting any row and the corresponding column of the Laplacian matrix (for instance, by removing row 1 and column 1 or row 2 and column 2 and so on). Then, $|L^*(G)| = 48$ which is the total number of spanning trees for the graph *G* (Figure 8) of the incomplete pairwise comparison matrix *A* (Table 7). Ultimately, the Gray code algorithm is used to generate all 48 spanning trees (Appendix B).

Step 5: the weights of six risks in each of the 48 spanning trees are calculated. The weight of i^{th} risk (i = 1, ..., 6) in k^{th} spanning tree (k = 1, ..., 48) is denoted as $w_i^{(k)}$ and computed based on Equations (4) and (5). All weights are shown in Table 8.

			11.02	0		
Arcs in spanning trees	1: AF	2: ND	3: IA	4: CC	5: ST	6: PI
a21, a26, a36, a25, a24	0.0644	0.3221	0.1380	0.1074	0.3221	0.0460
$a_{21}, a_{26}, a_{25}, a_{24}, a_{23}$	0.0683	0.3417	0.0854	0.1139	0.3417	0.0488
$a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$	0.0698	0.3488	0.0872	0.1163	0.3488	0.0291
$a_{21}, a_{26}, a_{36}, a_{24}, a_{56}$	0.0644	0.3221	0.1380	0.1074	0.3221	0.0460
$a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$	0.0644	0.3221	0.1380	0.1074	0.3221	0.0460
$a_{21}, a_{26}, a_{36}, a_{25}, a_{46}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
$a_{21}, a_{36}, a_{25}, a_{24}, a_{46}$	0.0714	0.3571	0.0714	0.1190	0.3571	0.0238
$a_{21}, a_{36}, a_{25}, a_{24}, a_{16}$	0.0732	0.3659	0.0549	0.1220	0.3659	0.0183
$a_{26}, a_{36}, a_{25}, a_{24}, a_{16}$	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
$a_{21}, a_{36}, a_{24}, a_{56}, a_{23}$	0.0816	0.4082	0.1020	0.1361	0.2381	0.0340
$a_{21}, a_{25}, a_{24}, a_{56}, a_{23}$	0.0683	0.3417	0.0854	0.1139	0.3417	0.0488
$a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$	0.0683	0.3417	0.0854	0.1139	0.3417	0.0488
$a_{21}, a_{24}, a_{46}, a_{56}, a_{23}$	0.0863	0.4317	0.1079	0.1439	0.2014	0.0288
$a_{21}, a_{25}, a_{46}, a_{56}, a_{23}$	0.0605	0.3024	0.0756	0.2160	0.3024	0.0432
$a_{21}, a_{36}, a_{46}, a_{56}, a_{23}$	0.0789	0.3947	0.0987	0.1645	0.2303	0.0329
	Arcs in spanning trees a_{21} , a_{26} , a_{36} , a_{25} , a_{24} a_{21} , a_{26} , a_{25} , a_{24} , a_{23} a_{21} , a_{36} , a_{25} , a_{24} , a_{23} a_{21} , a_{36} , a_{25} , a_{24} , a_{23} a_{21} , a_{36} , a_{25} , a_{24} , a_{56} a_{21} , a_{36} , a_{25} , a_{24} , a_{56} a_{21} , a_{36} , a_{25} , a_{24} , a_{56} a_{21} , a_{36} , a_{25} , a_{24} , a_{46} a_{21} , a_{36} , a_{25} , a_{24} , a_{16} a_{21} , a_{36} , a_{25} , a_{24} , a_{16} a_{21} , a_{36} , a_{25} , a_{24} , a_{16} a_{21} , a_{36} , a_{25} , a_{24} , a_{16} a_{21} , a_{25} , a_{24} , a_{56} , a_{23} a_{21} , a_{26} , a_{24} , a_{56} , a_{23} a_{21} , a_{26} , a_{24} , a_{56} , a_{23} a_{21} , a_{25} , a_{46} , a_{56} , a_{23} a_{21} , a_{25} , a_{46} , a_{56} , a_{23} a_{21} , a_{25} , a_{46} , a_{56} , a_{23} a_{21} , a_{25} , a_{46} , a_{56} , a_{23} a_{21} , a_{36} , a_{46} , a_{56} , a_{23}	Arcs in spanning trees1: AF a_{21} , a_{26} , a_{36} , a_{25} , a_{24} 0.0644 a_{21} , a_{26} , a_{25} , a_{24} , a_{23} 0.0683 a_{21} , a_{36} , a_{25} , a_{24} , a_{23} 0.0698 a_{21} , a_{36} , a_{25} , a_{24} , a_{23} 0.0698 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0574 a_{21} , a_{36} , a_{25} , a_{24} , a_{46} 0.0714 a_{21} , a_{36} , a_{25} , a_{24} , a_{16} 0.0732 a_{26} , a_{36} , a_{25} , a_{24} , a_{16} 0.1644 a_{21} , a_{36} , a_{24} , a_{56} , a_{23} 0.0816 a_{21} , a_{25} , a_{24} , a_{56} , a_{23} 0.0683 a_{21} , a_{26} , a_{24} , a_{56} , a_{23} 0.0683 a_{21} , a_{25} , a_{46} , a_{56} , a_{23} 0.0863 a_{21} , a_{25} , a_{46} , a_{56} , a_{23} 0.0605 a_{21} , a_{36} , a_{46} , a_{56} , a_{23} 0.0789	Arcs in spanning trees1: AF2: ND a_{21} , a_{26} , a_{36} , a_{25} , a_{24} 0.0644 0.3221 a_{21} , a_{26} , a_{25} , a_{24} , a_{23} 0.0683 0.3417 a_{21} , a_{36} , a_{25} , a_{24} , a_{23} 0.0698 0.3488 a_{21} , a_{36} , a_{25} , a_{24} , a_{23} 0.0698 0.3488 a_{21} , a_{26} , a_{36} , a_{24} , a_{56} 0.0644 0.3221 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 0.3221 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 0.3221 a_{21} , a_{36} , a_{25} , a_{24} , a_{56} 0.0644 0.3221 a_{21} , a_{36} , a_{25} , a_{24} , a_{46} 0.0774 0.3271 a_{21} , a_{36} , a_{25} , a_{24} , a_{16} 0.0774 0.3571 a_{21} , a_{36} , a_{25} , a_{24} , a_{16} 0.0732 0.3659 a_{26} , a_{36} , a_{25} , a_{24} , a_{16} 0.1644 0.2877 a_{21} , a_{36} , a_{24} , a_{56} , a_{23} 0.0816 0.4082 a_{21} , a_{25} , a_{24} , a_{56} , a_{23} 0.0683 0.3417 a_{21} , a_{26} , a_{24} , a_{56} , a_{23} 0.0683 0.3417 a_{21} , a_{26} , a_{24} , a_{56} , a_{23} 0.0683 0.4317 a_{21} , a_{25} , a_{46} , a_{56} , a_{23} 0.0605 0.3024 a_{21} , a_{36} , a_{46} , a_{56} , a_{23} 0.0789 0.3947	Arcs in spanning trees1: AF2: ND3: IA $a_{21}, a_{26}, a_{36}, a_{25}, a_{24}$ 0.0644 0.3221 0.1380 $a_{21}, a_{26}, a_{25}, a_{24}, a_{23}$ 0.0683 0.3417 0.0854 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0698 0.3488 0.0872 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0698 0.3488 0.0872 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 $a_{21}, a_{36}, a_{25}, a_{24}, a_{46}$ 0.0714 0.3571 0.0714 $a_{21}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.0732 0.3659 0.0549 $a_{26}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.1644 0.2877 0.1233 $a_{21}, a_{36}, a_{24}, a_{56}, a_{23}$ 0.0816 0.4082 0.1020 $a_{21}, a_{25}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 $a_{21}, a_{25}, a_{46}, a_{56}, a_{23}$ 0.0605 0.3024 0.0756 $a_{21}, a_{36}, a_{46}, a_{56}, a_{23}$ 0.0605 0.3947 0.0987	Arcs in spanning trees1: AF2: ND3: IA4: CC $a_{21}, a_{26}, a_{36}, a_{25}, a_{24}$ 0.0644 0.3221 0.1380 0.1074 $a_{21}, a_{26}, a_{25}, a_{24}, a_{23}$ 0.06644 0.3221 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0698 0.3488 0.0872 0.1163 $a_{21}, a_{26}, a_{36}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{46}$ 0.0774 0.3271 0.1380 0.1074 $a_{21}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.0732 0.3659 0.0549 0.1220 $a_{26}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.1644 0.2877 0.1233 0.0959 $a_{21}, a_{36}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 $a_{21}, a_{26}, a_{46}, a_{56}, a_{23}$ </td <td>Arcs in spanning trees1: AF2: ND3: IA4: CC5: ST$a_{21}, a_{26}, a_{36}, a_{25}, a_{24}$$0.0644$$0.3221$$0.1380$$0.1074$$0.3221$$a_{21}, a_{26}, a_{25}, a_{24}, a_{23}$$0.0683$$0.3417$$0.0854$$0.1139$$0.3417$$a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$$0.0698$$0.3488$$0.0872$$0.1163$$0.3488$$a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$$0.0698$$0.3488$$0.0872$$0.1163$$0.3488$$a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$$0.0644$$0.3221$$0.1380$$0.1074$$0.3221$$a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$$0.0644$$0.3221$$0.1380$$0.1074$$0.3221$$a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$$0.0644$$0.3221$$0.1380$$0.1074$$0.3221$$a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$$0.0644$$0.3221$$0.1380$$0.1074$$0.3221$$a_{21}, a_{36}, a_{25}, a_{24}, a_{46}$$0.0574$$0.2869$$0.1230$$0.2049$$0.2869$$a_{21}, a_{36}, a_{25}, a_{24}, a_{16}$$0.0732$$0.3659$$0.0549$$0.1220$$0.3659$$a_{26}, a_{36}, a_{25}, a_{24}, a_{16}$$0.1644$$0.2877$$0.1233$$0.0959$$0.2877$$a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$$0.0683$$0.3417$$0.0854$$0.1139$$0.3417$$a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$$0.0683$$0.3417$$0.0854$$0.1139$$0.3417$$a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$$0.0683$</td>	Arcs in spanning trees1: AF2: ND3: IA4: CC5: ST $a_{21}, a_{26}, a_{36}, a_{25}, a_{24}$ 0.0644 0.3221 0.1380 0.1074 0.3221 $a_{21}, a_{26}, a_{25}, a_{24}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 0.3417 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0698 0.3488 0.0872 0.1163 0.3488 $a_{21}, a_{36}, a_{25}, a_{24}, a_{23}$ 0.0698 0.3488 0.0872 0.1163 0.3488 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 0.3221 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 0.3221 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 0.3221 $a_{21}, a_{36}, a_{25}, a_{24}, a_{56}$ 0.0644 0.3221 0.1380 0.1074 0.3221 $a_{21}, a_{36}, a_{25}, a_{24}, a_{46}$ 0.0574 0.2869 0.1230 0.2049 0.2869 $a_{21}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.0732 0.3659 0.0549 0.1220 0.3659 $a_{26}, a_{36}, a_{25}, a_{24}, a_{16}$ 0.1644 0.2877 0.1233 0.0959 0.2877 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 0.3417 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683 0.3417 0.0854 0.1139 0.3417 $a_{21}, a_{26}, a_{24}, a_{56}, a_{23}$ 0.0683

Table 8. Weights of risks in all spanning trees (ND and PI)

weights

16	a21, a26, a46, a56, a23	0.0605	0.3024	0.0756	0.2160	0.3024	0.0432
17	$a_{24}, a_{16}, a_{46}, a_{56}, a_{23}$	0.1119	0.4196	0.1049	0.1399	0.1958	0.0280
18	a25, a16, a46, a56, a23	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
19	a36, a16, a46, a56, a23	0.1250	0.3750	0.0938	0.1563	0.2188	0.0313
20	a26, a16, a46, a56, a23	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
21	$a_{21}, a_{16}, a_{46}, a_{56}, a_{23}$	0.0952	0.4762	0.1190	0.1190	0.1667	0.0238
22	$a_{21}, a_{36}, a_{25}, a_{46}, a_{23}$	0.0678	0.3390	0.0847	0.1412	0.3390	0.0282
23	$a_{21}, a_{25}, a_{24}, a_{46}, a_{23}$	0.0702	0.3509	0.0877	0.1170	0.3509	0.0234
24	$a_{21}, a_{26}, a_{25}, a_{46}, a_{23}$	0.0605	0.3024	0.0756	0.2160	0.3024	0.0432
25	$a_{26}, a_{25}, a_{24}, a_{16}, a_{23}$	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
26	$a_{36}, a_{25}, a_{24}, a_{16}, a_{23}$	0.1111	0.3333	0.0833	0.1111	0.3333	0.0278
27	$a_{21}, a_{25}, a_{24}, a_{16}, a_{23}$	0.0706	0.3529	0.0882	0.1176	0.3529	0.0176
28	$a_{21}, a_{36}, a_{25}, a_{46}, a_{56}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
29	$a_{21}, a_{36}, a_{24}, a_{46}, a_{56}$	0.0882	0.4412	0.0882	0.1471	0.2059	0.0294
30	$a_{21}, a_{26}, a_{36}, a_{46}, a_{56}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
31	$a_{26}, a_{36}, a_{24}, a_{16}, a_{56}$	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
32	a36, a25, a24, a16, a56	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
33	$a_{21}, a_{36}, a_{24}, a_{16}, a_{56}$	0.0960	0.4800	0.0720	0.1600	0.1680	0.0240
34	$a_{26}, a_{36}, a_{25}, a_{16}, a_{46}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
35	$a_{36}, a_{25}, a_{24}, a_{16}, a_{46}$	0.0930	0.3488	0.0698	0.1163	0.3488	0.0233
36	$a_{21}, a_{36}, a_{25}, a_{16}, a_{46}$	0.0755	0.3774	0.0566	0.0943	0.3774	0.0189
37	$a_{25}, a_{24}, a_{16}, a_{56}, a_{23}$	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
38	$a_{36}, a_{24}, a_{16}, a_{56}, a_{23}$	0.1290	0.3871	0.0968	0.1290	0.2258	0.0323
39	$a_{26}, a_{24}, a_{16}, a_{56}, a_{23}$	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
40	$a_{21}, a_{24}, a_{16}, a_{56}, a_{23}$	0.0916	0.4580	0.1145	0.1527	0.1603	0.0229
41	$a_{25}, a_{24}, a_{16}, a_{46}, a_{23}$	0.0914	0.3429	0.0857	0.1143	0.3429	0.0229
42	$a_{36}, a_{25}, a_{16}, a_{46}, a_{23}$	0.1081	0.3243	0.0811	0.1351	0.3243	0.0270
43	$a_{26}, a_{25}, a_{16}, a_{46}, a_{23}$	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
44	$a_{21}, a_{25}, a_{16}, a_{46}, a_{23}$	0.0727	0.3636	0.0909	0.0909	0.3636	0.0182
45	$a_{36}, a_{24}, a_{16}, a_{46}, a_{56}$	0.1143	0.4286	0.0857	0.1429	0.2000	0.0286
46	$a_{36}, a_{25}, a_{16}, a_{46}, a_{56}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
47	$a_{26}, a_{36}, a_{16}, a_{46}, a_{56}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
48	$a_{21}, a_{36}, a_{16}, a_{46}, a_{56}$	0.1000	0.5000	0.0750	0.1250	0.1750	0.0250

Step 6: Finally, by obtaining the arithmetic average of all weights for each risk (i.e., EAST) or geometric average (i.e., GMAST), the final weight of each risk can be obtained as shown in Table 9.

_	AF	ND	IA	CC	ST	PI
EAST	0.1010	0.3444	0.0938	0.1410	0.2858	0.0341
Ranking	4	1	5	3	2	6
GMAST	0.0943	0.3392	0.0913	0.1360	0.2788	0.0327
Ranking	4	1	5	3	2	6

Table 9. Average weights of all spanning trees and rankings of risks (ND and PI)

5.2. The STE analysis for sabotage/terrorism (ST) and political instability (PI)

The STE analyses for ST and PI based on EAST and GMAST are explained in this section.

Step 1: The identified set of risks are $C = \{AF, ND, IA, CC, ST, PI\}$

Step 2: The incomplete pair-wise comparison matrix *A* can be obtained as shown in Table 10. It is constructed based on provided pair-wise comparison vectors by expert 4 for ST (i.e., the best risk), and PI (i.e., the worst risk) as shown in Table 5 and Table 6.

		1	2	3	4	5	6
		AF	ND	IA	CC	ST	PI
1	AF	1				0.20	4
2	ND		1			1.00	7
3	IA			1		0.25	3
4	CC				1	0.33	5
5	ST	5	1	4	3	1	7
6	PI	0.25	0.14	0.33	0.20	0.14	1

Table 10. The incomplete pair-wise comparison matrix A by expert 4 (ST and PI)

Step 3: The corresponding graph G of the pair-wise comparison matrix A (Table 10) is produced as shown in Figure 9.



Figure 9. The undirected (a) and directed (b) graph G of the matrix A (ST and PI)

Step 4: The Kirchhoff's matrix-tree theorem (Theorem A.3 in Appendix A) is used to obtain the total number of spanning trees as $\eta = 48$. According to the Kirchhoff's matrix-tree theorem, the degree matrix and adjacency matrix of graph *G* are shown in Equation (13) and (14), respectively.

$$D(G) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$
(13)
$$A(G) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$
(14)

Then, the Laplacian matrix of graph G is obtained as represented in Equation (15).

$$L(G) = \begin{bmatrix} 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & 0 & -1 & -1 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ -1 & -1 & -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & -1 & -1 & 5 \end{bmatrix}$$
(15)

 $L^*(G)$ can be obtained by omitting any row and the corresponding column of the Laplacian matrix. As a result, $|L^*(G)| = 48$ which is the total number of spanning trees for the graph *G* (Figure 9) of the incomplete pairwise comparison matrix *A* (Table 10). Finally, a Gray code algorithm is used to generate all the 48 spanning trees (Appendix B).

Step 5: The weights of six risks in each of the 48 spanning trees are calculated. The weight of i^{th} risk (i = 1, ..., 6) in k^{th} spanning tree (k = 1, ..., 48) is denoted as $w_i^{(k)}$ and computed based on Equations (4) and (5). All weights are shown in Table 11.

				wei	ghts		
No.	Arcs in spanning trees	1: AF	2: ND	3: IA	4: CC	5: ST	6: PI
1	$a_{16}, a_{26}, a_{36}, a_{46}, a_{56}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
2	a16, a26, a36, a46, a54	0.1143	0.2000	0.0857	0.1429	0.4286	0.0286
3	a16, a26, a36, a56, a54	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
4	$a_{16}, a_{26}, a_{36}, a_{46}, a_{53}$	0.1250	0.2188	0.0938	0.1563	0.3750	0.0313
5	a16, a26, a46, a56, a53	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
6	$a_{16}, a_{26}, a_{36}, a_{46}, a_{52}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
7	$a_{16}, a_{36}, a_{46}, a_{56}, a_{52}$	0.1481	0.2593	0.1111	0.1852	0.2593	0.0370
8	$a_{16}, a_{26}, a_{36}, a_{46}, a_{51}$	0.1000	0.1750	0.0750	0.1250	0.5000	0.0250
9	$a_{26}, a_{36}, a_{46}, a_{56}, a_{51}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
10	$a_{16}, a_{26}, a_{56}, a_{53}, a_{54}$	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
11	$a_{16}, a_{26}, a_{46}, a_{53}, a_{54}$	0.1119	0.1958	0.1049	0.1399	0.4196	0.0280
12	$a_{16}, a_{26}, a_{36}, a_{53}, a_{54}$	0.1290	0.2258	0.0968	0.1290	0.3871	0.0323
13	$a_{16}, a_{56}, a_{52}, a_{53}, a_{54}$	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
14	$a_{16}, a_{46}, a_{52}, a_{53}, a_{54}$	0.0914	0.3429	0.0857	0.1143	0.3429	0.0229
15	a16, a36, a52, a53, a54	0.1111	0.3333	0.0833	0.1111	0.3333	0.0278
16	a16, a26, a52, a53, a54	0.1733	0.3032	0.0758	0.1011	0.3032	0.0433
17	a56, a51, a52, a53, a54	0.0683	0.3417	0.0854	0.1139	0.3417	0.0488
18	a46, a51, a52, a53, a54	0.0702	0.3509	0.0877	0.1170	0.3509	0.0234
19	a36, a51, a52, a53, a54	0.0698	0.3488	0.0872	0.1163	0.3488	0.0291
20	a26, a51, a52, a53, a54	0.0683	0.3417	0.0854	0.1139	0.3417	0.0488
21	al6, a51, a52, a53, a54	0.0706	0.3529	0.0882	0.1176	0.3529	0.0176
22	ale, ase, ase, ase, ase, ase	0.0930	0.3488	0.0698	0.1163	0.3488	0.0233
23	a16, a36, a56, a52, a54	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
24	ale, ape, age, app, app, app	0.1644	0.2877	0.1233	0.0959	0.2877	0.0411
25	are, aze, ase, ase, ase, ase	0.0882	0.2059	0.0882	0.1471	0.4411	0.0294
26	$a_{26}, a_{36}, a_{46}, a_{51}, a_{54}$	0.0644	0.3221	0.1380	0.1074	0.3221	0.0460
27	$a_{20}, a_{30}, a_{30}, a_{31}, a_{34}$	0.0960	0 1680	0.0720	0 1600	0.4800	0.0240
28	$a_{10}, a_{20}, a_{30}, a_{51}, a_{54}$	0 1081	0 3243	0.0811	0.1351	0 3243	0.0270
29	alle,	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
30	ale, a26, a46, a52, a53	0.1553	0.2718	0.0680	0.1942	0.2718	0.0388
31	$a_{10}, a_{20}, a_{40}, a_{52}, a_{53}$	0.0789	0.2303	0.0987	0 1645	0 3947	0.0329
32	$a_{20}, a_{50}, a_{40}, a_{51}, a_{53}$	0.0605	0 3024	0.0756	0.2160	0 3024	0.0432
33	alle,	0.0952	0.1667	0.1190	0.1190	0.4762	0.0238
34	$a_{10}, a_{20}, a_{40}, a_{51}, a_{55}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
35	$a_{20}, a_{50}, a_{40}, a_{51}, a_{52}$	0.0574	0.2869	0.1230	0.2049	0.2869	0.0410
36	$a_{30}, a_{40}, a_{30}, a_{51}, a_{52}$	0.0755	0.3774	0.0566	0.0943	0.3774	0.0189
37	$a_{10}, a_{50}, a_{40}, a_{51}, a_{52}$	0.0683	0 3417	0.0854	0 1139	0 3417	0.0488
38	$a_{20}, a_{50}, a_{51}, a_{53}, a_{54}$	0.0863	0.2014	0 1079	0 1439	0.4317	0.0288
39	$a_{20}, a_{40}, a_{51}, a_{53}, a_{54}$	0.0816	0.2381	0.1020	0.1361	0.4082	0.0340
40	$a_{20}, a_{30}, a_{51}, a_{53}, a_{54}$	0.0016	0.1603	0.1020	0.1501	0.4580	0.0229
41	$a_{10}, a_{20}, a_{51}, a_{55}, a_{54}$	0.0510	0.1005	0.1145	0.1074	0.3221	0.022)
42	$a_{20}, a_{20}, a_{21}, a_{22}, a_{24}$	0.00++	0.3221 0.3571	0.0714	0 1100	0.3221 0.3571	0.0738
43	$a_{50}, a_{40}, a_{51}, a_{52}, a_{54}$	0.0714	0 3221	0 1380	0 1074	0 3221	0.0250
<u>4</u> 4	$a_{20}, a_{30}, a_{31}, a_{32}, a_{34}$	0.00	0.3650	0.05/0	0 1220	0.3221	0.0183
45 45	$a_{10}, a_{30}, a_{31}, a_{32}, a_{34}$	0.0752	0.3037	0.0349	0.1220	0.3037	0.0103
т <i>э</i> 46	$a_{40}, a_{50}, a_{51}, a_{52}, a_{53}$	0.0003	0.3024	0.0750	0.1412	0.3024	0.0+32 0.0282
47	$a_{30}, a_{40}, a_{51}, a_{52}, a_{53}$	0.0078	0.3024	0.00-7	0.1412 0.2160	0.3320	0.0202
/ /8	$a_{26}, a_{46}, a_{51}, a_{52}, a_{53}$	0.0003	0.3624	0.0750	0.2100	0.3024	0.0+32 0.0182
40	a16, a46, a51, a52, a53	0.0121	0.5050	0.0707	0.0909	0.5050	0.0102

Table 11. Weights of risks in all spanning trees (ST and PI)

Step 6: The final weight of each risk can be obtained via EAST and GMAST as shown in Table 12.

	AF	ND	IA	CC	ST	PI
EAST	0.1010	0.2858	0.0938	0.1410	0.3444	0.0341
Ranking	4	2	5	3	1	6
GMAST	0.0943	0.2788	0.0913	0.1360	0.3392	0.0327
Ranking	4	2	5	3	1	6

Table 12. Average weights of all spanning trees and rankings of risks (ST and PI)

5.3. STE results

The obtained results and rankings from the STE analysis for Natural Disasters (ND) and Political Instability (PI) (Table 9), and for Sabotage and Terrorism (ST) and Political Instability (PI) (Table 12), are incorporated to reach a conclusion that which one of Natural Disasters (ND) or Sabotage and Terrorism (ST) should be the best risk based on the data obtained from expert 4. The aggregated weights and final rankings obtained from STE (i.e., EAST and GMAST methods) are presented in Table 13.

		EA	ST		
		ND and PI	ST and PI	Average	Ranking
AF	W_1	0.1010065335	0.1010065335	0.1010065335	4
ND	W_2	0.3443855204	0.2857569085	0.3150712145	1
IA	W_3	0.0937656585	0.0937656585	0.0937656585	5
CC	W_4	0.1409772290	0.1409772290	0.1409772290	3
ST	W_5	0.2857569085	0.3443848954	0.3150709020	2
PI	W_6	0.0341081492	0.0341081492	0.0341081492	6
		GM	AST		
		ND and PI	ST and PI	Average	Ranking
AF	<i>W</i> ₁	ND and PI 0.0942570944	0.0942570944	Average 0.0942570944	Ranking 4
AF ND	W ₁ W ₂	ND and PI 0.0942570944 0.3391580310	ST and PI 0.0942570944 0.2788162447	Average 0.0942570944 0.3089871379	Ranking 4 1
AF ND IA	W ₁ W ₂ W ₃	ND and PI 0.0942570944 0.3391580310 0.0912640382	51 and PI 0.0942570944 0.2788162447 0.0912640382	Average 0.0942570944 0.3089871379 0.0912640382	Ranking 4 1 5
AF ND IA CC	W ₁ W ₂ W ₃ W ₄	ND and PI 0.0942570944 0.3391580310 0.0912640382 0.1360483960	ST and PI 0.0942570944 0.2788162447 0.0912640382 0.1360483960	Average 0.0942570944 0.3089871379 0.0912640382 0.1360483960	Ranking 4 1 5 3
AF ND IA CC ST	W ₁ W ₂ W ₃ W ₄ W ₅	ND and PI 0.0942570944 0.3391580310 0.0912640382 0.1360483960 0.2788162447	ST and PI 0.0942570944 0.2788162447 0.0912640382 0.1360483960 0.3391575505	Average 0.0942570944 0.3089871379 0.0912640382 0.1360483960 0.3089868976	Ranking 4 1 5 3 2

Table 13. Aggregated weights and final rankings from STE analysis

As it is shown in Table 13, Natural Disasters (ND) has a bit higher weight compared to the weight of Sabotage and Terrorism (ST) in both EAST and GMAST methods. Thus, in the BWM analysis the Natural Disasters (ND) should be chosen as the most important (best) risk suggested by expert 4.

5.4. Baseline comparison with Fuzzy BWM

In this section, the Fuzzy BWM (F-BWM) (Dong et al., 2021) is utilized to determine the most important criterion based on expert four's input, as presented in Table 14. We used the below scale to convert 1-9 scale in BWM to the Trapezoidal Fuzzy Number (TFN) in F-BWM proposed by Dong et al. (2021).

Numerical scale	Linguistic scale for the	TFN scale
	importance	
1	Equally important (EI)	(1,1,1)
2-3	Weakly important (WI)	(2/3,1,3/2)
4-5	Fairly important (FI)	(3/2,2,5/2)
6-7	Very important (VI)	(5/2,3,7/2)
8-9	Absolutely important (AI)	(7/2,4,9/2)

Table 14. Original BWM scale fuzzy linguistic equivalent (adapted from Roy and Shaw, 2022)

The results indicate that F- BWM could not distinguish a single best criterion and assigns equal ranks to both ND and ST (see Table 15). In this case, getting the average of weights would not be useful to identify the best criterion in F-BWM. This analysis and comparison with the STE analysis as presented in Table 13 shows the suitability of the proposed approach in dealing with situations where there are multiple best or worst criteria and methods like F-BWM are not capable of distinguishing this difference.

	ND (best) and PI (worst)		ST (best) and PI (_	
Risks	Fuzzy weights	Crisp weights	Fuzzy weights	Crisp weights	Ranking
AF	(0.0,0.1267,0.1193)	0.1043	(0.0,0.1267,0.1193)	0.1043	3
ND	(0.0,0.2424,0.2595)	0.2049	(0.0,0.2424,0.2595)	0.2049	1
IA	(0.0,0.1021,0.1193)	0.0879	(0.0,0.1021,0.1193)	0.0879	4
CC	(0.0,0.2038,0.1964)	0.1686	(0.0,0.2038,0.1964)	0.1686	2
ST	(0.0,0.2424,0.2595)	0.2049	(0.0,0.2424,0.2595)	0.2049	1
PI	(0.0,0.0826,0.0631)	0.0656	(0.0,0.0826,0.0631)	0.0656	5

Table 15. Results of the baseline F-BWM method

5.5. Comparison of results and sensitivity analysis

In this section, we apply various BWM approaches including the linear BWM (Rezaei, 2016) and STE (STE-L-BWM), non-linear BWM (Rezaei, 2015) and STE (STE-NL-BWM), the Neutrosophic-enhanced BWM (Vafadarnikjoo et al., 2020) and STE (STE-NE-BWM), F-BWM (Dong et al., 2021) and STE (STE-F-BWM) as well as STE-GMAST and STE-EAST methods using the data provided in Table 5 and Table 6. We provide a comparative analysis of the aggregated weights from five experts. For various BWM implementations, we used the outcome of the STE analysis using EAST and GMAST methods (Table 13). This analysis presumes that the participating experts possess comparable levels of knowledge and professional insight. Consequently, each expert is assigned equivalent significance in the research methodology. The obtained weights from the applied methods and the final ranking of the risks are aggregated and presented in Table 16 and Figure 10.

Risk categories	STE- EAST	STE- GMAST	STE-L- BWM	STE- NL- BWM	STE- NE- BWM	STE-F- BWM	Average	Final ranks
AF	0.1611	0.1521	0.1447	0.1794	0.1824	0.1325	0.1587	3
ND	0.1757	0.1703	0.1810	0.1752	0.1726	0.1594	0.1724	2
IA	0.0999	0.0953	0.1189	0.1319	0.0843	0.1218	0.1087	5
CC	0.3179	0.3102	0.3023	0.2455	0.2889	0.1949	0.2766	1
ST	0.1425	0.1386	0.1467	0.1570	0.1586	0.1257	0.1448	4
PI	0.1030	0.0999	0.1064	0.1110	0.1131	0.1033	0.1061	6

Table 16. Risk significance evaluation and consolidated overall ranking



Figure 10. Risk scores across methods and categories with averages and final ranks

We conduct sensitivity analyses to better understand the impacts of our findings. Our approach attributes various weights to the six methods under 100 simulated scenarios. For each weight scenario, a new aggregated risk score is calculated considering $w \in [0,1]$ and $\sum_{i=1}^{6} w_i = 1$ as shown in Equation (16). A Rank Stability matrix is generated, and it shows frequency of each rank per risk. For example, for the IA risk, it ranks 5th 58 times and 6th 42 times (see Table 17). A Spearman's rank correlation coefficient is used to assess how well the order of one set of ranking matches another (King and Eckersley, 2019). Then, the average of all Spearman's rank correlation coefficients across the 100 simulations is computed.

$$New \ score_i = w_1 \cdot STE - EAST_i + w_2 \cdot STE - GMAST_i + \dots + w_6 \cdot STE - F - BWM_i$$
(16)

As presented in Table 17, the proposed model produces robust and stable risk rankings under variations in method weights. The average Spearman's correlation of 0.975 indicates that the rankings are highly consistent across scenarios. This shows overall robustness in the ranking structure.

Risks	Frequency of each rank (out of 100)			
AF	[0, 1, 99, 0, 0, 0]			
ND	[0, 99, 1, 0, 0, 0]			
IA	[0, 0, 0, 0, 58, 42]			
CC	[100, 0, 0, 0, 0, 0]			
ST	[0, 0, 0, 100, 0, 0]			
PI	[0, 0, 0, 0, 42, 58]			
Average Spearman's correlation: 0.975				

Table 17. Rank stability matrix for sensitivity analysis of methods' weights

We also conduct a second sensitivity analysis to examine the effect of the experts' weights variation. For each method, 100 simulations are run by generating random non-negative weights for the experts (summing to 1). New average scores as the weighted sum of expert scores are computed and new rankings are obtained. Table 18 summarizes this analysis for STE-EAST, STE-GMAST and STE-L-BWM.

Table 18. Rank stability matrix for sensitivity analysis of experts' weights

STE-EAST	STE-GMAST	STE-L-BWM

Dialra	Frequency of each rank	Frequency of each rank	Frequency of each rank	
KISKS	(out of 100)	(out of 100)	(out of 100)	
AF	[1, 33, 22, 42, 2, 0]	[2, 42, 22, 29, 5, 0]	[1, 22, 15, 41, 21, 0]	
ND	[3, 61, 25, 11, 0, 0]	[3, 54, 23, 20, 0, 0]	[8, 61, 12, 17, 2, 0]	
IA	[0, 2, 2, 12, 42, 42]	[0, 0, 5, 15, 39, 41]	[0, 8, 12, 15, 39, 26]	
CC	[96, 2, 2, 0, 0, 0]	[95, 1, 4, 0, 0, 0]	[91, 4, 5, 0, 0, 0]	
ST	[0, 2, 41, 24, 23, 10]	[0, 3, 26, 25, 25, 21]	[0, 5, 42, 21, 16, 16]	
PI	[0, 0, 8, 11, 33, 48]	[0, 0, 20, 11, 31, 38]	[0, 0, 14, 6, 22, 58]	
Average Spearman's correlation	0.854	0.817	0.763	

The sensitivity analysis shows high frequencies at the original ranks (i.e., CC at rank 1 and ND at rank 2).

5.6 Discussion

The final aggregated ranking shows that Climate Change (CC) has the highest importance followed by Natural Disasters (ND) which are the top priority risks in the UK electricity networks. Incorporating estimates until the end of the century, energy network firms use the Met Office UK Climate Projection (UKCP18) tool for long-term climate change adaptation planning since infrastructure usually lasts 30-80 years. The Energy Networks Association (ENA) asked the Met Office in 2020 to assess current research and UKCP18 data to get further knowledge of the effects of climate change on energy infrastructure. This report has led future mitigation plans and shaped present risk evaluations. The ENA group identified climate hazards and natural disasters as the two highest risks to energy network assets: continuous rainfall generating floods, very high temperatures, and cycles of severe rainfall followed by drought times (Energy Networks Association, 2021).

To enhance the robustness of our study's outcomes, we have cross-referenced our findings with industry risk registers maintained by the UK electricity network operators and regulators (e.g., Ofgem and the ENA) to validate our findings. Table 19 compares our findings in terms of ranking of the identified risks with equivalent risk categories as well as priorities inferred from sources such as Ofgem's RIIO-T2 framework, National Grid's climate adaptation reports and ENA publications.

Risk	Study ranking	Industry risk register equivalent	Industry priority	Notes	
CC (Climate Change)	1	Extreme weather	High (rank 1-2)	Strongalignment.Climatehazardsareidentifiedasposingthe highest risk to energy networkassets(EnergyNetworksAssociation, 2021; National GridElectricity Distribution, 2024)	
ND (Natural Disasters)	2	Flooding, Storms	High (rank 1-2)	Strong alignment. Precipitation and Sea level rise are listed as high risks (National Grid Electricity Distribution, 2024)	
AF (Affordability)	3	Cost pressures	Medium (rank 3-4)	Moderate alignment. National Grid emphasizes on affordable energy (National Grid, 2024). Ofgem discussed the impacts of affordability and debt issues on the market reflecting on stakeholders' comments (Ofgem, 2024).	
ST (Sabotage and Terrorism)	4	Physical security threats	Medium- Low (rank 4-5)	Partial alignment. Security is heightened using Critical National Infrastructure (CNI) level 2 to 3 classifications and compliance with the National Protective Security Authority (NPSA) guidance (Ofgem, 2025).	
IA (Industrial Action)	5	Strikes	Low (rank 5-6)	Good alignment. Strikes are not assessed as a priority due to measures that are already in place (Ofgem, 2024).	

Table 19. Comparison of study risk rankings with industry risk register priorities in the UK electricity networks

				Good	alignment.	Ofgem
		Coopolitical		emphasiz	ed a stable	RIIO-2
PI (Political	6	Regulatory	Low (rank	framework and managed external		
Instability)	0	instability	5-6)	risks as g	eopolitical risks	s are less
		Instability		immediat	e (RIIO-2 Fra	umework
				Report, 20	025)	

We observe that the emphasis on climate resilience and extreme weather in industry studies, such as the National Grid's Climate Change Adaptation Report 2024 (National Grid Electricity Distribution, 2024) or Ofgem's RIIO-T2 framework, is consistent with the high priority given to Climate Change (CC) and Natural Disasters (ND) in our rankings. In terms of impact and likelihood, climate change is a top risk, and long-term planning and business operations incorporate high-priority mitigation and adaptation measures. Research literature findings also emphasize the importance of natural disasters impact on the resilience of the UK energy supply network (Espinoza et al., 2016; Wang et al., 2015). More recently, Souto et al. (2024) and Manning et al. (2025) single out a range of extreme weather events as the main drivers of power outages in the UK.

6. Conclusion

As suggested, our study makes two contributions to the literature. First, we proposed and applied the STE-BWM to analyze various risks in the UK electricity network. Applying STE offers an opportunity for DMs to suggest more than one best or worst criterion. The proposed method is capable of calculating which criteria are the best and worst based on pair-wise comparisons already provided by DMs. In the BWM, a DM selects one decision-making criterion as the best criterion (i.e., the most favorable) and another decision-making criterion as the worst one (least favorable). In the real world, it would not always be straightforward for DMs applying BWM to confidently identify only one criterion as either the best or worst criteria with some level of hesitancy. This is likely due to DMs encountering uncertainty, hesitancy, or a lack of information. In this situation, we applied the hybrid decision support method in both theoretical and simulated experiments and practical settings in the UK electricity supply network risk assessment. Second, we applied the method to a real case. This application enables this study to identify climate change and natural disasters as the main risk

categories that can hinder the reliability of the UK electricity network system. The findings are summarized as follows:

- The study's results emphasized climate change and natural disasters as the most significant concern for the reliability of the UK's electricity system.
- The proposed STE-BWM can help integrate uncertainty into the decision-making process. Identification of the best and worst criteria if the experts involved are not fully confident in choosing only one best and/or one worst. The Monte Carlo Simulation validates the ability of the STE with a success rate of 100% when there is at least one free criterion.

However, similar to other studies, there are some limitations to our study. First, as with the other MCDA methods, we collected the views of a limited number of experts. This can be explained by the constraints associated with identifying and recruiting experts from multidisciplinary fields, such as the risks in energy supply network management. Therefore, there was a preference for a small pool of experts rather than a larger pool of respondents without the necessary expertise. Future research can incorporate Artificial Intelligence (AI) and Machine Learning (ML) approaches within decision support systems to extrapolate inputs from a small number of experts to broader contexts, ensuring robustness in the analysis. Furthermore, future research can investigate the application of this method in specific new technologies, such as smart microgrids, to determine how the results can be affected by the size of the network and situational awareness.

Appendix A: Preliminaries-Graph Theory

Definition A.1. cycle (Hein 2001). A cycle is a path with equal beginning and ending vertices, where no edges occur more than once.

Definition A.2. connected graph (Hein 2001). If there is a path between every pair of vertices, then the graph is named a connected graph.

Definition A.3. subgraphs (Benjamin, Chartrand, and Zhang 2015). A graph H is named a subgraph of a graph G if every vertex and edge of H is a vertex and edge of G.

Definition A.4. spanning subgraphs (Benjamin, Chartrand, and Zhang 2015). If the subgraph H of a graph G, has the same vertices as G, then H is a spanning subgraph of G.

Definition A.5. trees (Benjamin, Chartrand, and Zhang 2015). A tree is a connected graph that contains no cycles. It is common to signify a tree by *T*.

Theorem A.1. A graph G is a tree if and only if every two vertices of G are connected by only one path (Benjamin, Chartrand, and Zhang 2015).

Definition A.6. spanning trees (B. Y. Wu and Chao 2004). A spanning tree of a graph G is a subgraph of G which is a tree and includes all the vertices in G.

Definition A.7. a branch and a chord (Chakraborty et al. 2019). Let G be a connected graph then an edge in a spanning tree T of G is named a branch and an edge of G which is absent in the given spanning tree T is named chord.

Definition A.8. directed graphs or digraphs (Bang-Jensen and Gutin 2006). A digraph D that is often written as D = (V, A) includes a non-empty finite set V(D) of elements (vertices) and a finite set A(D) of ordered pairs of distinct vertices (arcs). V(D) and A(D) named vertex set and arc set respectively. In Figure A.1., a digraph D is depicted as an example. The V(D) and A(D) in this example are as follows:



Figure A.1. A digraph D

In digraphs, for an arc like (y, z) the first vertex y is called *tail* and the second vertex is named *head* (i.e., z). It is also said that y dominates z or z is dominated by y. An arc (y, z) is often signified as yz (Bang-Jensen and Gutin 2018). In this paper, the arc (y, z) is shown as a_{yz} .

Theorem A.2. Cayley's tree formula. Cayley (1889) introduced the formula n^{n-2} for counting the number of spanning trees in a complete graph with order n (K_n). For instance, for a K_4 graph, $4^{4-2} = 16$ spanning trees can be obtained as shown in Figure A.2.



Figure A.2. All spanning trees of a complete graph K_4

Definition A.9. degree matrix (Chartrand, Lesniak, and Zhang 2011). Let G be a graph with $V(G) = \{v_1, v_2, ..., v_n\}$, then the degree matrix $D(G) = [d_{ij}]$ is a diagonal $n \times n$ matrix with diagonal values as are shown in Equation A.1.

$$d_{ij} = \begin{cases} degv_i, & if \ i = j \\ 0, & if \ i \neq j \end{cases}$$
(A.1)

Definition A.10. adjacency matrix (Siraj, Mikhailov, and Keane 2012). Let *G* be a graph with $V(G) = \{v_1, v_2, ..., v_n\}$, then the adjacency matrix $A(G) = [c_{ij}]$ where each element c_{ij} represents the number of edges from vertex v_i to vertex v_j .

Theorem A.3. Kirchhoff's matrix-tree theorem (Chartrand, Lesniak, and Zhang 2011). Let G be a labelled graph with adjacency matrix A(G) and degree matrix D(G), then the absolute

value of any cofactor of the Laplacian matrix D(G) - A(G) results in the number of distinct spanning trees of G. The Kirchhoff's matrix-tree theorem helps determine the number of distinct spanning trees of labelled graphs in general and not only in complete graphs.

Appendix B: Gray code algorithm

There are several algorithms in the literature for generating all possible spanning trees in undirected graphs, as reviewed by Chakraborty et al. (2019). In this research, we used the Gray code algorithm developed by Naskar et al. (2009) using Gray codes. First, an initial tree T_0 must be generated by any method such as Breadth-First Traversal (Hein 2001). The T_0 is comprised of n - 1 branches and m - (n - 1) chords. Then, $2^{m-(n-1)}$ binary representations are produced each of length m - (n - 1) namely Gray codes. Subsequently, combination of n - 1 branches and m - (n - 1) chords are calculated for each Gray code in a way that output will contain (n - 1) edges. Finally, each combination should be checked to determine if there is no cycle, and it is a spanning tree.

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Additional information

For the purpose of open access, the author has applied a Creative Commons Attribution (CC BY) licence to any Author Accepted Manuscript version arising from this submission.

Compliance with Ethical Standards

Ethical approval

All procedures performed in this study involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards.

Informed consent

Informed consent was obtained from all the participants.

Disclosure of interest

All authors report that there are no competing interests to declare.

References

- Aldarajee, Ammar H.M., Seyed H. Hosseinian, Behrooz Vahidi, and Shahab Dehghan. 2020.
 'A coordinated planner-disaster-risk-averse-planner investment model for enhancing the resilience of integrated electric power and natural gas networks'. *International Journal of Electrical Power & Energy Systems* 119. https://doi.org/10.1016/j.ijepes.2020.105948
- Aplak, H. Soner, and M. Ziya Sogut. 2013. 'Game Theory Approach in Decisional Process of Energy Management for Industrial Sector'. *Energy Conversion and Management* 74 (October):70–80. https://doi.org/10.1016/j.enconman.2013.03.027.
- Baak, Werner, Marc Goerigk, and Michael Hartisch. 2024. 'A Preference Elicitation Approach for the Ordered Weighted Averaging Criterion Using Solution Choice Observations'. *European Journal of Operational Research* 314 (3): 1098–1110. https://doi.org/10.1016/j.ejor.2023.11.020.
- Bang-Jensen, Jørgen, and Gregory Gutin. 2006. *Digraphs: Theory, Algorithms and Applications*. 2. print. London: Springer.

. 2018. *Classes of Directed Graphs*. New York, NY: Springer Berlin Heidelberg.

- Benjamin, Arthur, Gary Chartrand, and Ping Zhang. 2015. *The Fascinating World of Graph Theory*. Princeton: Princeton University Press.
- Bolsover A. Real-time risk assessment and decision support. Process Saf. Prog. 2015;34(2):183–190. doi:10.1002/prs.11702
- Boxebeld, Sander, Niek Mouter, and Job Van Exel. 2024. 'Participatory Value Evaluation (PVE): A New Preference-Elicitation Method for Decision Making in Healthcare'. *Applied Health Economics and Health Policy* 22 (2): 145–54. https://doi.org/10.1007/s40258-023-00859-9.
- Bozóki, Sándor, and Vitaliy Tsyganok. 2019. 'The (Logarithmic) Least Squares Optimality of the Arithmetic (Geometric) Mean of Weight Vectors Calculated from All Spanning Trees for Incomplete Additive (Multiplicative) Pairwise Comparison Matrices'. *International Journal of General Systems* 48 (4): 362–81. https://doi.org/10.1080/03081079.2019.1585432.
- Cayley, A. 1889. 'A Theorem on Trees'. The Quarterly Journal of Mathematics 23:376–78.
- Chakraborty, Maumita, Sumon Chowdhury, Joymallya Chakraborty, Ranjan Mehera, and Rajat Kumar Pal. 2019. 'Algorithms for Generating All Possible Spanning Trees of a Simple Undirected Connected Graph: An Extensive Review'. *Complex & Intelligent Systems* 5 (3): 265–81. https://doi.org/10.1007/s40747-018-0079-7.
- Chalvatzis, Konstantinos J. 2012. 'Analysis of the Evaluation of Electricity Supply Security : Case Studies for Electricity Sector Incumbents and Policy Makers in Greece and Poland'. PhD Thesis, University of East Anglia. https://ethos.bl.uk/OrderDetails.do?uin=uk.bl.ethos.577681.
- Chartrand, Gary, Linda Lesniak, and Ping Zhang. 2011. *Graphs & Digraphs*. 5th ed. Boca Raton, FL: Chapman and Hall/CRC.
- Chou, Jui-Sheng, and Citra S. Ongkowijoyo. 2014. 'Risk-Based Group Decision Making Regarding Renewable Energy Schemes Using a Stochastic Graphical Matrix Model'. *Automation in Construction* 37 (January):98–109. https://doi.org/10.1016/j.autcon.2013.10.010.
- Cinelli, Marco, Peter Burgherr, Milosz Kadzinski, and Roman Slowinski. 2022. 'Proper and improper uses of MCDA methods in energy systems analysis'. *Decision Support Systems* 163. https://doi.org/10.1016/j.dss.2022.113848

- Connor, P.M., C.J. Axon, D. Xenias, N. Balta-Ozkan. 2018. 'Sources of risk and uncertainty in UK smart grid deployment: an expert stakeholder analysis.' *Energy*, 161:1-9. https://doi.org/10.1016/j.energy.2018.07.115
- De Almeida, Jônatas Araújo, Eduarda Asfora Frej, Lucia Reis Peixoto Roselli, and Adiel Teixeira De Almeida. 2024. 'Analytical Aspects of Combining Holistic Evaluation and Decomposition Elicitation for Preference Modeling in the FITradeoff Method'. *International Transactions in Operational Research*, April, itor.13470. https://doi.org/10.1111/itor.13470.
- De Morais Correia, Lucas Miguel Alencar, Jonhatan Magno Norte Da Silva, Wilza Karla Dos Santos Leite, Ruan Eduardo Carneiro Lucas, and Geraldo Alves Colaço. 2022. 'A Multicriteria Decision Model to Rank Workstations in a Footwear Industry Based on a FITradeoff-Ranking Method for Ergonomics Interventions'. *Operational Research* 22 (4): 3335–71. https://doi.org/10.1007/s12351-021-00671-0.
- Dong, Jiu-Ying, and Shu-Ping Wan. 2024. 'Interval-Valued Intuitionistic Fuzzy Best-Worst Method with Additive Consistency'. *Expert Systems with Applications* 236 (February):121213. https://doi.org/10.1016/j.eswa.2023.121213.
- Dong, Jiu-Ying, Shu-Ping Wan, Shyi-Ming Chen. 2021. 'Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making'. *Information Sciences*, 547, 1080-1104. https://doi.org/10.1016/j.ins.2020.09.014
- Energy Networks Association (ENA). 2021. 'Adaptation to Climate Change Task Group: Gas &Electricity Transmission and Distribution Network Companies'. 3rd Round Climate Change Adaptation Report. https://www.energynetworks.org/assets/images/CCRA3%20report%20v1.0%20final. pdf
- Espinoza, Sebastian, Mathaios Panteli, Pierluigi Mancarella, Hugh Rudnick. 2016. 'Multiphase assessment and adaptation of power systems resilience to natural hazards'. *Electric Power Systems Research* 136. 352-361. https://doi.org/10.1016/j.epsr.2016.03.019
- Gong, Shitao, Linyan Chen, Qianqian Zhou, Xin Gao, Feng Shen. 2025. 'Vulnerability evolution of critical infrastructures: A multidimensional environment-integrated system dynamics analysis'. *Reliability Engineering & System Safety* 256. https://doi.org/10.1016/j.ress.2024.110719
- Gonzalez-Eguino, Mikel. 2015. 'Energy poverty: an overview'. *Renewable and Sustainable Energy Reviews* 47: 377-385. https://doi.org/10.1016/j.rser.2015.03.013
- Hammond, G P, and R Waldron. 2008. 'Risk Assessment of UK Electricity Supply in a Rapidly Evolving Energy Sector'. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy* 222 (7): 623–42. https://doi.org/10.1243/09576509JPE543.
- Haseli, Gholamreza, Reza Sheikh, Saeid Jafarzadeh Ghoushchi, Mostafa Hajiaghaei-Keshteli, Sarbast Moslem, Muhammet Deveci, and Seifedine Kadry. 2024. 'An Extension of the Best–Worst Method Based on the Spherical Fuzzy Sets for Multi-Criteria Decision-Making'. *Granular Computing* 9 (2): 40. https://doi.org/10.1007/s41066-024-00462w.
- He, Futong, Liyan Ji, Cunbin Li. 2025. 'Grid disaster risk identification based on social network analysis'. *Reliability Engineering & System Safety* 256. https://doi.org/10.1016/j.ress.2024.110756
- Hein, James L. 2001. *Discrete Structures, Logic, and Computability*. 2nd ed. Boston: Jones and Bartlett Publishers.

- Hunt, Julian David, René Bañares-Alcántara, and David Hanbury. 2013. 'A New Integrated Tool for Complex Decision Making: Application to the UK Energy Sector'. *Decision Support Systems* 54 (3): 1427–41. https://doi.org/10.1016/j.dss.2012.12.010.
- Ji, Changcheng, Fei Gao, and Wenjiang Liu. 2024. 'Dependence assessment in human reliability analysis based on cloud model and best-worst method'. *Reliability Engineering & System Safety* 242. https://doi.org/10.1016/j.ress.2023.109770
- Keeney, Ralph L., and Howard Raiffa. 1993. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Transferred to digital print. Cambridge New York Melbourne: Cambridge University Press.
- King, Andrew P., and Robert J. Eckersley. 2019. Chapter 2—Descriptive Statistics II: Bivariate and Multivariate Statistics. Statistics for Biomedical Engineers and Scientists; King, AP, Eckersley, RJ, Eds, 23-56. https://doi.org/10.1016/B978-0-08-102939-8.00011-6
- Kjolle, G.H., I.B. Utne, O. Gjerde. 2012. 'Risk analysis of critical infrastructures emphasizing electricity supply and interdependencies'. *Reliability Engineering & System Safety* 105, 80-89. https://doi.org/10.1016/j.ress.2012.02.006
- Liang, Fuqi, Matteo Brunelli, and Jafar Rezaei. 2020. 'Consistency Issues in the Best Worst Method: Measurements and Thresholds'. *Omega* 96 (October):102175. https://doi.org/10.1016/j.omega.2019.102175.
- Lin, Shuaishuai, Cunbin Li, Fangqiu Xu, Ding Liu, and Jicheng Liu. 2018. 'Risk Identification and Analysis for New Energy Power System in China Based on D Numbers and Decision-Making Trial and Evaluation Laboratory (DEMATEL)'. *Journal of Cleaner Production* 180 (April):81–96. https://doi.org/10.1016/j.jclepro.2018.01.153.
- Lopez, Cristina, Alessio Ishizaka, Muhammet Gul, Melih Yucesan, Daniela Valencia. 2023. 'A calibrated fuzzy best-worst-method to reinforce supply chain resilience during the COVID 19 pandemic'. *Journal of the Operational Research Society* 74(9): 1968-1991. https://doi.org/10.1080/01605682.2022.2122739
- Löschel, Andreas, Ulf Moslener, and Dirk T.G. Rübbelke. 2010. 'Indicators of Energy Security in Industrialised Countries'. *Energy Policy* 38 (4): 1665–71. https://doi.org/10.1016/j.enpol.2009.03.061.
- Lundy, Michele, Sajid Siraj, and Salvatore Greco. 2017. 'The Mathematical Equivalence of the "Spanning Tree" and Row Geometric Mean Preference Vectors and Its Implications for Preference Analysis'. *European Journal of Operational Research* 257 (1): 197–208. https://doi.org/10.1016/j.ejor.2016.07.042.
- Lusty, Peter, Pierre Josso, Federick Price, Nitesh Singh, Andrew Gunn, Richard Shaw, Eimear Deady, Hannah Grant, and Nicholas Idoine. 2022. 'Study on future UK demand and supply of lithium, nickel, cobalt, manganese and graphite for electric vehicle batteries'. British Geological Survey Commissioned Report, CR/22/079.
- Manning, Colin, Sean Wilkinson, Hayley J. Fowler, Elizabeth J. Kendon. 2025. 'Antecedent rainfall, wind direction and seasonal effects may amplify the risk of wind-driven power outages in the UK'. *Communications Earth & Environment*, 6, 217. https://doi.org/10.1038/s43247-025-02176-6
- Maxim, Alexandru. 2014. 'Sustainability Assessment of Electricity Generation Technologies Using Weighted Multi-Criteria Decision Analysis'. *Energy Policy* 65 (February):284– 97. https://doi.org/10.1016/j.enpol.2013.09.059.
- Moghadasi, Negin, Zachary A. Collier, Andrew Koch, David L. Slutzky, Thomas L. Polmateer, Mark C. Manasco, and James H. Lambert. 2022. 'Trust and security of electric vehicleto-grid systems and hardware supply chains'. *Reliability Engineering & System Safety* 225. https://doi.org/10.1016/j.ress.2022.108565
- Naskar, Saptarshi, Krishnendu Basuli, and Samar Sen Sarma. 2009. 'Generation of All Spanning Trees'. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.1433035.

- National Grid Electricity Distribution. 2024. Climate change adaptation report: Round 4
(ARP4). [Accessed on 13 May 2025]. Link:
https://www.nationalgrid.co.uk/downloads-view-reciteme/687761
- National Grid. 2024. 'Electricity Transmission-Our Performance 2023/24'. [Accessed on 15 May 2025]. Link: https://www.nationalgrid.com/document/356396/download
- Nikou, Shahrokh, József Mezei, and Peter Sarlin. 2015. 'A Process View to Evaluate and Understand Preference Elicitation'. *Journal of Multi-Criteria Decision Analysis* 22 (5–6): 305–29. https://doi.org/10.1002/mcda.1544.
- Ofgem. 2024. 'Affordability and debt in the domestic retail market-Response document'. [Accessed on 15 May 2025]. Link: https://www.ofgem.gov.uk/sites/default/files/2024-12/Affordability_and_debt_in_the_domestic_retail_market-Response_document.pdf
- Ofgem. 2025. 'Physical security re-opener final determinations'. [Accessed on 15 May 2025]. Link: https://www.ofgem.gov.uk/sites/default/files/2025-04/Physical-Security-Final-Determinations-2025-final.pdf
- Okoro U., Kolios A. Multicriteria risk assessment framework for components' risk ranking: Case study of a complex oil and gas support structure. J. Multi-Criteria Decis. Anal. 2018;25(5–6):113–129. doi:10.1002/mcda.1651.
- Opabola, Eyitayo A., Carmine Galasso. 2024. 'A probabilistic framework for post-disaster recovery modeling of buildings and electric power networks in developing countries'. *Reliability Engineering & System Safety* 242. https://doi.org/10.1016/j.ress.2023.109679
- Ren, J., I. Jenkinson, J. Wang, D. L. Xu, and J. B. Yang. 2009. 'An Offshore Risk Analysis Method Using Fuzzy Bayesian Network'. *Journal of Offshore Mechanics and Arctic Engineering* 131 (4): 041101. https://doi.org/10.1115/1.3124123.
- Rezaei, Jafar. 2015. 'Best-Worst Multi-Criteria Decision-Making Method'. *Omega* 53 (June):49–57. https://doi.org/10.1016/j.omega.2014.11.009.
- 2016. 'Best-Worst Multi-Criteria Decision-Making Method: Some Properties and a Linear Model'. *Omega* 64 (October):126–30. https://doi.org/10.1016/j.omega.2015.12.001.
- RIIO-2 Framework Report. 2025. RIIO-2 Electricity Transmission Annual Report: 2023-24. [Accessed on 15 May 2025]. Link: https://www.ofgem.gov.uk/publications/riio-2electricity-transmission-annual-report-2023-2024
- Roy, Pranith Kumar, Krishnendu Shaw. 2022. 'Developing a multi-criteria sustainable credit score system using fuzzy BWM and fuzzy TOPSIS'. *Environment, Development and Sustainability*, 24(4), 5368-5399. https://doi.org/10.1007/s10668-021-01662-z
- Saaty, Thomas L. 1980. 'The analytic hierarchy process planning, priority setting, resource allocation'. McGraw-Hill International Book Co.
- Salam, Izhar Us, Muhammad Yousif, Muhammad Numan, Moatasim Billah. 2024. 'Addressing the challenge of climate change: the role of microgrids in fostering a sustainable futurea comprehensive review'. *Renewable Energy Focus* 48. https://doi.org/10.1016/j.ref.2024.100538
- Sarin, Rakesh K., Robert L. Winkler. 1992. 'Ambiguity and decision modeling: A preferencebased approach'. *Journal of Risk and Uncertainty* 5, 389-407. https://doi.org/10.1007/BF00122577
- Silvast, Antti. 2017. Making Electricity Resilient: Risk and Security in a Liberalized Infrastructure. London; New York: Routledge, Taylor & Francis Group.
- Siraj, S., L. Mikhailov, and J.A. Keane. 2012. 'Enumerating All Spanning Trees for Pairwise Comparisons'. *Computers & Operations Research* 39 (2): 191–99. https://doi.org/10.1016/j.cor.2011.03.010.

- Souto, Laiz, Robert Neal, James O. Pope, Paula L. M. Gonzalez, Jonathan Wilkinson, Philip C. Taylor. 2024. 'Identification of weather patterns and transitions likely to cause power outages in the United Kingdom'. *Communications Earth & Environment* 5 (49). https://doi.org/10.1038/s43247-024-01217-w
- Staid, Andrea, and Seth D. Guikema. 2015. 'Risk Analysis for U.S. Offshore Wind Farms: The Need for an Integrated Approach'. *Risk Analysis* 35 (4): 587–93. https://doi.org/10.1111/risa.12324.
- Tversky, A., and D. Kahneman. 1991. 'Loss Aversion in Riskless Choice: A Reference-Dependent Model'. *The Quarterly Journal of Economics* 106 (4): 1039–61. https://doi.org/10.2307/2937956.
- UK Department for Transport. 2021. Decarbonising Transport: A Better, Greener Britain. Accessed January 5, 2025. https://assets.publishing.service.gov.uk/media/610d63ffe90e0706d92fa282/decarboni sing-transport-a-better-greener-britain.pdf.
- Vafadarnikjoo, Amin. 2020. 'Decision Analysis in the UK Energy Supply Chain Risk Management: Tools Development and Application'. https://ueaeprints.uea.ac.uk/id/eprint/77909.
- Vafadarnikjoo, Amin, Madjid Tavana, Konstantinos Chalvatzis, and Tiago Botelho. 2022. 'A Socio-Economic and Environmental Vulnerability Assessment Model with Causal Relationships in Electric Power Supply Chains'. *Socio-Economic Planning Sciences* 80 (March):101156. https://doi.org/10.1016/j.seps.2021.101156.
- Vafadarnikjoo, Amin, Madjid Tavana, Tiago Botelho, and Konstantinos Chalvatzis 2020. 'A neutrosophic enhanced best–worst method for considering decision-makers' confidence in the best and worst criteria'. *Annals of Operations Research*, 289(2), 391-418. https://doi.org/10.1007/s10479-020-03603-x
- Vlahos, K., P. Ninios, and D. Bunn. 1998. 'An integrative modelling approach for understanding competitive electricity markets'. *Journal of the Operational Research Society* 49(3), 187-199. https://doi.org/10.1057/palgrave.jors.2600473
- Voorberg, S., R. Eshuis, W. Van Jaarsveld, and G.J. Van Houtum. 2021. 'Decisions for Information or Information for Decisions? Optimizing Information Gathering in Decision-Intensive Processes'. *Decision Support Systems* 151 (December):113632. https://doi.org/10.1016/j.dss.2021.113632.
- Wang, Y., C. Chen, J. Wang, R. Baldick. 2015. 'Research on resilience of power systems under natural disasters-a review'. *IEEE Transactions on Power Systems*, 31(2). 1604-1613. https://doi.org/10.1109/TPWRS.2015.2429656
- Wu, Bang Ye, and Kun-Mao Chao. 2004. *Spanning Trees and Optimization Problems*. Boca Raton: Chapman & Hall/CRC.
- Wu, Qun, Xinwang Liu, Ligang Zhou, Jindong Qin, and Jafar Rezaei. 2024. 'An Analytical Framework for the Best–Worst Method'. Omega 123 (February):102974. https://doi.org/10.1016/j.omega.2023.102974.
- Wu, Yunna, Chuanbo Xu, and Ting Zhang. 2018. 'Evaluation of Renewable Power Sources Using a Fuzzy MCDM Based on Cumulative Prospect Theory: A Case in China'. *Energy* 147 (March):1227–39. https://doi.org/10.1016/j.energy.2018.01.115.