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Bidding with Coalitional Externalities: A strategic approach to partition function form games[1]

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Abstract

This paper provides a non-cooperative bargaining approach to analyze games with coalitional externalities. Four well-studied solution concepts for partition function form games are implemented: the externality-free value (Pham Do and Norde (2007), de Clippel and Serrano (2008)), the expected stand-alone value and the consensus value (Ju (2007)) and the extended Shapley value (McQuillin (2009)). We generalize the bidding mechanism introduced in Pérez-Castrillo and Wettstein (2001) to partition function form games. Using these generalizations provides a coherent and structured framework to study strategic aspects of different normative approaches.

Keywords: partition function form games; externalities; implementation; bidding mechanism.

JEL Classification: C71; C72; D62.

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1 Introduction

Economic environments featuring coalitional externalities have been effectively modeled by the game theoretic framework of partition function form games proposed by Thrall and Lucas (1963). This model has been highly useful in studying relevant economic problems such as international trade (e.g., Aghion, Antràs and Helpman (2007)) and environmental agreements (see Chander and Tulkens (1997) and Athanasoglou (2022)). Yet the value theory of this class of games is still under development, despite a rising literature in recent years, of which Kóczy (2018) offers a comprehensive overview.

From the normative point of view, values for such games are studied by, among others, Myerson (1977), Bolger (1989), Feldman (1994), Maskin (2003), Albizuri, Arin and Rubio (2005), Macho-Stadler, Pérez-Castrillo and Wettstein (2007, 2010, 2018), Pham Do and Norde (2007), Ju (2007), de Clippel and Serrano (2008) and McQuillin (2009). Due to the more complicated structure of partition function form games compared to standard transferable utility (TU) games, providing a non-cooperative foundation[1] for values in these environments poses a formidable problem. Only few works, e.g. Maskin (2003), Macho-Stadler, Pérez-Castrillo and Wettstein (2006) and Borm, Ju and Wettstein (2015), have addressed this issue. Maskin (2003) and Borm, Ju and Wettstein (2015) offer alternative perspectives to analyze the strategic aspects emerging in the analysis of partition function form games.[2] The implementation results in Macho-Stadler, Pérez-Castrillo and Wettstein (2006) are applicable only for environments with either purely non-negative externalities or purely non-positive externalities. Thus, providing a non-cooperative basis for axiomatically characterized values for general partition function form games remains an open question.

We want to push further the strategic study of coalitional externalities. The current paper proposes a systematic non-cooperative bargaining approach to partition function form games, which allows for the analysis of general environments with all types of coalitional externalities. Four axiomatically characterized solution concepts for partition function form games will be implemented in this paper. One is the Shapley value for partition function form games introduced by Pham Do and Norde (2007) and extensively studied as the externality-free value by de Clippel and Serrano (2008). The second is the so-called extended Shapley value constructed and analyzed in McQuillin (2009). Both values possess nice axiomatic properties and are well motivated by the underlying rationale of the original

[1]Here we use the phrase of non-cooperative "foundations" just for convenience, as the term is commonly adopted in the literature. We agree with Serrano (2005) that the normative properties of a solution concept are indeed the "foundation" for it. Serrano (2005, 2021) provide an excellent survey for the work of implementations of cooperative solutions.

[2]Similar perspectives appear in Bloch and van den Nouweland (2014) when studying the axiomatic foundations of expectation formation rules.

Shapley value (Shapley (1953)).

Two other solution concepts to be addressed are the expected stand-alone value and the consensus value introduced by Ju (2007). The expected stand-alone value is a generalization of the equal surplus solution in standard TU games to partition function form games, taking into account coalitional externalities. The consensus value is in the same spirit as its TU counterpart (Ju, Borm and Ruys (2007)) that well links and reconciles marginalism and egalitarianism. The underlying procedure to construct the consensus value for partition function form games as well as its axiomatic characterizations show that this value well balances the tradeoff between coalitional effects and externality effects, making it especially appropriate in the context of coalitional externalities.

In this paper, by extending the generalized bidding approach proposed by Ju and Wettstein (2009)[3] to partition function form games, we obtain mechanisms to implement the above four values. This approach does not use the structure of any specific value to generate a specific mechanism tailored for it, but, through the bidding, allows players to consider the payoffs and possible externalities. The emergence of a solution concept, not directly related to the mechanism, serves to highlight intriguing features of the solution concept. The consensus value, for example, emerges as equilibrium outcome when players compete for the right to make a second (i.e., renegotiation) offer rather than arbitrarily assigning it to a particular player. We like to note that the mechanisms introduced in this paper yield the actual values implemented rather than implement them in expected terms.

The results of the paper not only complement those obtained in de Clippel and Serrano (2008), Maskin (2003), Macho-Stadler, Pérez-Castrillo and Wettstein (2006) and Borm, Ju and Wettstein (2015), but also suggest a unified approach to analyze such games. The design of a single basic mechanism to implement several solution concepts provides an opportunity to make direct and critical comparisons among them and highlights the underlying different non-cooperative rationales. We further note that the approach can serve as a toolkit for analyzing partition function form games, both from a non-cooperative point of view (implementation) and a cooperative one (looking for new solution concepts). The mechanisms presented in Borm, Ju and Wettstein (2015), while quite different, are also variants of a basic strategic framework implementing a family of related values for partition function form games.

The next section presents the environment and the values to be implemented. In section 3, we formally describe the bidding mechanisms and show that while using the same bidding stage in all mechanisms, different protocols of renegotiation result in completely different value concepts as equilibrium outcomes. The final section offers further discussions and

[3]The basic multi-bidding mechanism was introduced in Pérez-Castrillo and Wettstein (2001). Variants and further studies of the mechanism can be found in Pérez-Castrillo and Quérou (2012) and Ju (2012).

suggests topics for future research.

2 Partition function form games and the values

We now formally present the model of partition function form games. Let N be the set of players. A coalition S is a subset of N. A *partition* p of N, a so-called *coalition structure*, is a set of mutually disjoint coalitions, $p = \{S_1, ..., S_m\}$, whose union is N. Let $\mathbb{P}(N)$ denote the set of all partitions of N. For any coalition $S \subseteq N$, $\mathbb{P}(S)$ denotes the set of all partitions of S. A typical element of $\mathbb{P}(S)$ is denoted by p_S. Note that two partitions will be considered equal if they differ only by the insertion or deletion of $\emptyset$. That is, $\{\{1,2\},\{3\}\} = \{\{1,2\},\{3\},\emptyset\}$. A pair (S,p) consisting of a coalition S and a partition $p \in \mathbb{P}(N)$ to which S belongs is called an *embedded coalition*, and is nontrivial if $S \neq \emptyset$. Let $\mathbb{E}(N)$ denote the set of embedded coalitions, i.e.

$$\mathbb{E}(N) = \left\{(S,p) \in 2^N \times \mathbb{P}(N) | S \in p\right\}.$$

Similarly, $\mathbb{E}(S)$ denotes the set of embedded coalitions with respect to S. We denote by (N,w) a game in partition function form (or a *partition function form game*) where $w : \mathbb{E}(N) \longrightarrow \mathbb{R}$ is called a *partition function* that assigns a real value, $w(S,p)$, to each embedded coalition (S,p). The value $w(S,p)$ represents the payoff of coalition S, given the coalition structure p forms. By convention, $w(\emptyset,p) = 0$ for all $p \in \mathbb{P}(N)$. The set of partition function form games with player set N is denoted by PG^N.

For a given partition $p = \{S_1, ..., S_m\}$ and a partition function w, let $\bar{w}(S_1, ..., S_m)$ denote the m-vector $(w(S_i,p))_{i=1}^m$. For any $S \subseteq N$ we denote by $[S]$ the partition of S which consists of *singleton coalitions* only, $[S] = \{\{j\} | j \in S\}$, and by $\{S\}$ the partition of S consisting of the coalition S only.

A solution concept on PG^N is a function f, which associates with each game (N,w) in PG^N a vector $f(N,w)$ of individual payoffs in $\mathbb{R}^N$, i.e. $f(N,w) = (f_i(N,w))_{i\in N} \in \mathbb{R}^N$.

We first recall the Shapley value defined by Pham Do and Norde (2007) that is the externality-free value in de Clippel and Serrano (2008). Let $\Pi(N)$ be the set of all bijections $\sigma : \{1, ..., |N|\} \longrightarrow N$. For a given $\sigma \in \Pi(N)$ and $k \in \{1, ..., |N|\}$, we define the partition p_k^σ associated with σ and k, by $p_k^\sigma = \{S_k^\sigma\} \cup [N \backslash S_k^\sigma]$ where $S_k^\sigma := \{\sigma(1), ..., \sigma(k)\}$, and $p_0^\sigma = [N]$. So, in p_k^σ the coalition S_k^σ has already formed, whereas all other players still form singleton coalitions. For a game $w \in PG^N$, define the marginal vector $m^\sigma(w)$ as the vector in $\mathbb{R}^N$ by $m_{\sigma(k)}^\sigma(w) = w(S_k^\sigma, p_k^\sigma) - w(S_{k-1}^\sigma, p_{k-1}^\sigma)$ for all $\sigma \in \Pi(N)$ and $k \in \{1, ..., |N|\}$. The *Shapley value* (Pham Do and Norde (2007)) $\phi(w)$ of the partition function form game (N,w) is defined as the average, over the set $\Pi(N)$ of all bijections, of the marginal vectors, i.e.

$$\phi(w) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} m^{\sigma}(w).$$

It is the unique value satisfying efficiency, additivity, symmetry and the null player property as defined in Pham Do and Norde (2007).

As one can see above, by the formula of $\phi(w)$, the Shapley value value for partition function form games basically abstracts from most of the information provided by the whole partition function form games, as it only considers the worth of a coalition S when other players form singleton coalitions, which is also why it is called the externality-free value in de Clippel and Serrano (2008). As a counterpart to this, one may consider the worth of a coalition S when the rest of the players form the complementary coalition $N \backslash S$ instead of singletons, which is analogous to the core with singleton expectations versus the core with merging expectations for partition function form games (Hafalir (2007)). Indeed this leads to a solution concept called the extended Shapley value by McQuillin (2009). Formally, for a game $w \in PG^N$, define the marginal vector $\bar{m}^{\sigma}(w)$ in $\mathbb{R}^N$ by $\bar{m}^{\sigma}_{\sigma(k)}(w) = w(S^{\sigma}_k, \{S^{\sigma}_k\} \cup \{N \backslash S^{\sigma}_k\}) - w(S^{\sigma}_{k-1}, \{S^{\sigma}_{k-1}\} \cup \{N \backslash S^{\sigma}_{k-1}\})$ for all $\sigma \in \Pi(N)$ and $k \in \{1, ..., |N|\}$. The *extended Shapley value* (McQuillin (2009)) $\phi^{MQ}(w)$ of the partition function form game (N, w) is defined as the average, over the set $\Pi(N)$ of all bijections, of the marginal vectors $\bar{m}^{\sigma}(w)$, i.e.

$$\phi^{MQ}(w) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} \bar{m}^{\sigma}(w).$$

Ju (2007) introduces the consensus value for partition function form games by taking a bilateral perspective and considering both coalitional effects and externality effects when sharing the gains of cooperation. The consensus value is the unique solution that satisfies efficiency, complete symmetry, additivity and the quasi-null player property as defined in Ju (2007). It is shown that the consensus value for partition function form games γ equals the average of the Shapley value (Pham Do and Norde (2007), de Clippel and Serrano (2008)) and the expected stand-alone value. That is, $\gamma(w) = \frac{1}{2}\phi(w) + \frac{1}{2}e(w)$, where $e(w)$ denotes the expected stand-alone value of a partition function form game w and is defined by

$$\begin{aligned} e_i(w) &= \frac{w(N, \{N\})}{|N|} \\ &+ \sum_{S \subseteq N \backslash \{i\} : S \neq \emptyset} \frac{|S|!(|N| - |S| - 1)!}{|N|!} w(\{i\}, \{S\} \cup [N \backslash (S \cup \{i\})] \cup \{\{i\}\}) \\ &- \sum_{j \in N \backslash \{i\}} \sum_{S \subseteq N \backslash \{i,j\}} \frac{|S|!(|N| - |S| - 2)!}{|N|!} w(\{j\}, [N \backslash (S \cup \{i\})] \cup \{S \cup \{i\}\}). \end{aligned}$$

for all $i \in N$. The expected stand-alone value takes players' stand-alone situations as the only input to determine their final payoffs. Hence, it purely measures the externality effects on a player in a partition function form game, compared to the coalitional effects measured by the Shapley value. For a better understanding of the expected stand-alone value, we provide the following explanation. Given a player $i \in N$, she has two choices concerning externalities, either choosing the stand-alone option and receiving the externalities from coalitions consisting of other players or joining some coalitions so as to generate externalities affecting other players standing alone. Thus, the second term in the above expression corresponds to the first choice and can be understood as player i's *expected gain* from the externalities of all possible coalitions not containing i, where the distribution of coalitions is such that any ordering of the players is equally likely. The last term, corresponding to the second choice, is player i's *expected loss* due to joining coalitions, which is taken to be the other players' gains from the externalities of coalitions containing i.

3 The bidding mechanisms and implementation

In this section, we construct bidding mechanisms that implement the above four cooperative solutions for partition function form games. These mechanisms provide a convenient benchmark to evaluate and compare these values from a non-cooperative perspective.

The basic *bidding mechanism* can be described informally as follows: At stage 1 the players bid to choose a proposer. Each player bids by submitting an $(n-1)$-tuple of numbers (positive or negative), one number for each player (excluding herself). The player for whom the net bid (the difference between the sum of bids made by the player and the sum of bids the other players made to her, measuring the player's willingness to become the proposer) is the highest, is chosen as the proposer. Before moving to stage 2, the proposer pays to each player the bid she made. As a reward to the chosen proposer for her effort (represented by her net bid), she is granted with the right to make a scheme how to split the total payoff $w(N, \{N\})$ among all the players at the next stage.

At stage 2 the proposer offers a vector of payments to all other players in exchange for joining her to form the grand coalition. The offer is accepted if all the other players agree. In case of acceptance the grand coalition indeed forms and the proposer receives $w(N, \{N\})$ out of which she pays out the offers made. In case of rejection the proposer "waits" while all the other players go again through a similar game that has the same bargaining protocol with a smaller player set (N minus the rejected proposer).

What are the possible consequences following this rejection? In general, there can be two different scenarios. One is that all the remaining players fail to reach any agreement among themselves again. Then, the hope of forming the grand coalition collapses and the initial proposer will be indeed left alone. The other scenario might be that the remaining players accept the payments made by their chosen proposer, which means that a coalition of all players apart from the initial proposer is formed. In this case, the option of "re-entering" the game for the initial proposer would become realistic. Since now it is a two-party issue, given the potential benefit from cooperation, it seems quite reasonable for both proposers to come back to the table and negotiate again. The following stages will be associated with this renegotiation. That is, in these additional stages the first proposer (in fact, the rejected proposer) and the proposer chosen among the remaining players (when an agreement is reached within themselves) bid and accept further offers.

The first variant implementing the Shapley value for partition function form games (Pham Do and Norde (2007) and de Clippel and Serrano (2008)) has the first proposer (denoted for simplicity by a) make an offer to the proposer chosen among the remaining players (denoted for simplicity by b). The offer is for a to form the grand coalition rather than b. If the offer is accepted the grand coalition forms, a receives $w(N, \{N\})$ and pays the offer, b receives the offer from a and pays all the commitments made by him, and all the other players receive what they were promised.

The second variant implementing the expected stand-alone value has b make an offer to a. If the offer is accepted the grand coalition forms, a receives the offer, b receives $w(N, \{N\})$ and pays the offer to a as well as what he owes to the remaining players.

In the third variant implementing the consensus value, the right to make an offer is endogenously determined through a bidding mechanism between a and b. If a wins, the game proceeds as in the first variant and if b wins, the second variant goes into effect.

To implement the extended Shapley value introduced by McQuillin (2009), we need to consider an alternative bargaining protocol, yet still in the same framework of the bidding mechanism. One can easily appreciate the difficulty of implementing the extended Shapley value: Given that the extended Shapley value considers the worth of a coalition S when the rest of the players form the complementary coalition $N \backslash S$, there could exist good reasons for players to leave the existing coalition and join the "outside" players. A valid implementation mechanism would have to provide no incentive for a proposer to make an offer that will be rejected, but meanwhile, in case of rejection, all the rejected proposers should indeed have the incentive to form a coalition. These two forces seem to work against each other. Finding a mechanism to resolve this conflict and reasonably reconcile the mutually opposite forces is the key to implementing the extended Shapley value.

The mechanism proposed below is inspired by real world practices observed in disputes among political parties as well as business partners, where in case of the breakdown of a political party usually a powerful member quits and forms a new party with his or her followers. Similarly in business, we often see that a leader of a company leaves and founds a rival one, with his or her followers joining from the previous company.

To capture these realistic features we proceed as follows. In case of rejection the proposer a becomes temporarily *inactive*. She is momentarily left out of negotiation to "wait" while all the other players go again through the same procedure starting with a bidding stage. This process continues up to the point where a proposer b's offer is accepted by all other active players. Now all the rejected proposers become active again. They are given the chance to negotiate and form a coalition by themselves. Now a acts as a "big boss" and announces a take-it-or-leave-it offer to every other rejected proposer. If a's offer is rejected the set of rejected proposers forms the singletons partition and the game ends. If the take-it-or-leave-it offer is accepted unanimously, then they form a coalition such that a will receive a payoff of this coalition minus the offers made to all other rejected proposers.

Below we formally describe the bidding mechanisms, which will explicitly explain how these bargaining protocols deal with coalitional externalities.

Mechanism A. If there is only one player $\{i\}$, she simply receives $w(i, \{i\})$. When there are two or more players, the mechanism is defined recursively. Given the rules of the mechanism for at most $n-1$ players, the mechanism for $N = \{1, \ldots, n\}$ proceeds in five stages.

Stages 1 to 3 provide for any set of (active) players S a proposer in S, chosen via a bidding procedure (stage 1), an offer made by the proposer to the rest of the players in S (stage 2), and an acceptance or rejection (stage 3). If stage 3 ends with a rejection, all players in S other than the rejected proposer proceed again through stages 1 to 3 where the set of active players is reduced by excluding the rejected proposer. If stage 3 ends with acceptance, for $S = N$ the game ends; but for a coalition S smaller than N, the game moves to stage 4 and then ends with stage 5. At stage 4 the last rejected proposer makes an offer to the accepted proposer, and at stage 5, the offer is either accepted or rejected and final payoffs are realized.

The mechanism starts with $S = N$.

Stage 1: Each player $i \in S$ makes $s-1$ (where $s = |S|$) bids $b^i_j \in \mathbb{R}$ with $j \neq i$. For each $i \in S$, define the *net bid* of player i by $B^i = \sum_{j \in S\setminus\{i\}} b^i_j - \sum_{j \in S\setminus\{i\}} b^j_i$. Let $i_s \in \arg\max_i(B^i)$ be the proposer, where in case of a non-unique maximizer any of these maximal bidders is chosen as the proposer with equal probability. Once the winner i_s has been chosen, player i_s pays every player $j \in S\setminus\{i_s\}$ her bid $b^{i_s}_j$.

Stage 2: Player i_s makes a vector of offers $x^{i_s}_j \in \mathbb{R}$ to every player $j \in S\setminus\{i_s\}$.

Stage 3: The players other than i_s, sequentially, either accept or reject the offer. If at least one player rejects it, then the offer is rejected. Otherwise, the offer is accepted.

If the offer is rejected, all players in S other than i_s proceed again through the mechanism from stage 1 where the set of active players is $S\backslash\{i_s\}$. Meanwhile, player i_s waits for the negotiation outcome of $S\backslash\{i_s\}$. Dependent upon whether or not the offer made by player i_{s-1} (denoting the proposer of $S\backslash\{i_s\}$) is accepted within $S\backslash\{i_s\}$, player i_s will either be called for renegotiation with i_{s-1} or be left alone. The renegotiation will follow a process as specified below in stages 4 and 5. If being left alone, player i_s will receive a stand-alone payoff.

If the offer is accepted, we have to distinguish between two cases where $S = N$ and $S \neq N$. In the case where $S = N$, which means that all players agree with the proposer on the scheme of sharing $w(N, \{N\})$, the game ENDS. Then each player $j \in N\backslash\{i_n\}$ receives $x_j^{i_n}$ at this stage, and player i_n receives $w(N, \{N\}) - \sum_{j\neq i_n} x_j^{i_n}$. Hence, the final payoff to player $j \neq i_n$ is $x_j^{i_n} + b_j^{i_n}$ while player i_n receives $w(N, \{N\}) - \sum_{j\neq i_n} x_j^{i_n} - \sum_{j\neq i_n} b_j^{i_n}$. In the case where $S \neq N$, stages 4 and 5 are reached.

Stage 4: The rejected proposer preceding i_s, who is denoted by i_{s+1}, makes an offer $\widetilde{x}_{i_s}^{i_{s+1}}$ in $\mathbb{R}$, to player i_s. (The offer is to let i_{s+1} form the coalition $S \cup \{i_{s+1}\}$.)

Stage 5: Player i_s accepts or rejects the offer and the game ENDS.

The payoffs to all the players in $S\backslash\{i_s\}$ are the same independent of whether there was a rejection or an acceptance at stage 5. That is, every player $j \in S\backslash\{i_s\}$ receives $x_j^{i_s}$ and the overall payoff to the player is derived by adding to it all the bids received, which were made by all previously rejected proposers. Hence, the final payoff to player $j \in S\backslash\{i_s\}$ is $x_j^{i_s} + \sum_{t=s}^{n} b_j^{i_t}$.

The payoffs to all other players (i.e., $i_s, i_{s+1}, ..., i_n$) depend on whether or not there was an acceptance at stage 5.

If the offer is accepted then at this stage player i_s receives $\widetilde{x}_{i_s}^{i_{s+1}}$ minus the bids and offer he made to the players in $S\backslash\{i_s\}$, while player i_{s+1} receives $w(S \cup \{i_{s+1}\}, \{S \cup \{i_{s+1}\}\} \cup [N\backslash(S \cup \{i_{s+1}\})]) - \widetilde{x}_{i_s}^{i_{s+1}}$. The overall payoffs to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $\widetilde{x}_{i_s}^{i_{s+1}} - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} - \sum_{j\in S\backslash\{i_s\}} x_j^{i_s} + \sum_{t=s+1}^{n} b_{i_s}^{i_t}$, and the final payoff to player i_{s+1} is $w(S \cup \{i_{s+1}\}, \{S \cup \{i_{s+1}\}\} \cup [N\backslash(S \cup \{i_{s+1}\})]) - \widetilde{x}_{i_s}^{i_{s+1}} - \sum_{j\in S} b_j^{i_{s+1}} + \sum_{t=s+2}^{n} b_{i_{s+1}}^{i_t}$. Moreover, all proposers before i_{s+1} receive their stand-alone payoffs in addition to all the payments received and paid out in the bidding stages they participated in. Hence the final payoff to player i_m for $m > s+1$ is $w(\{i_m\}, \{S \cup \{i_{s+1}\}\} \cup [N\backslash(S \cup \{i_{s+1}\})]) - \sum_{l\in N\backslash(\cup_{k=m}^{n} i_k)} b_l^{i_m} + \sum_{t=m+1}^{n} b_{i_m}^{i_t}$.

If the offer at stage 5 is rejected then at this stage player i_s receives $w(S, \{S\} \cup [N \backslash S])$ minus the bids and the offers she made to the players in S, while player i_{s+1} receives his stand-alone payoff $w(\{i_{s+1}\}, \{S\} \cup [N \backslash S])$. The overall payoff to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $w(S, \{S\} \cup [N \backslash S]) - \sum_{j \in S \backslash \{i_s\}} b_j^{i_s} - \sum_{j \in S \backslash \{i_s\}} x_j^{i_s} + \sum_{t=s+1}^{n} b_{i_s}^{i_t}$, and the final payoff to player i_{s+1} is $w(\{i_{s+1}\}, \{S\} \cup [N \backslash S]) - \sum_{j \in S} b_j^{i_{s+1}} + \sum_{t=s+2}^{n} b_{i_{s+1}}^{i_t}$. Moreover, all proposers before i_{s+1} receive their stand-alone payoffs in addition to all the payments received and paid out in the bidding stages they participated in. Hence the final payoff to player i_m for $m > s+1$ is $w(\{i_m\}, \{S\} \cup [N \backslash S]) - \sum_{l \in N \backslash (\cup_{k=m}^{n} i_k)} b_l^{i_m} + \sum_{t=m+1}^{n} b_{i_m}^{i_t}$.

We note that in the case the mechanism reaches the situation where the set of active players consists of one player only, i.e. $|S| = 1$, the corresponding stages 1 to 3 are redundant and this single player is considered as the proposer for herself whose offer is accepted immediately and the game moves to stages 4 and 5 where she will renegotiate with the previously rejected proposer i_2.

The following theorem shows that for any partition function form game (N, w) satisfying zero-monotonicity[4] , i.e.,

$$w(S, \{S\} \cup p_{N \backslash S}) > w(S \backslash \{i\}, \{S \backslash \{i\}\} \cup_{N \backslash (S \backslash \{i\})}) + w(\{i\}, \{i\} \cup p_{N \backslash \{i\}})$$

for all $S \subseteq N$ and $i \in S$ and all $p_{N \backslash S} \in \mathbb{P}(N \backslash S)$, all $p_{N \backslash (S \backslash \{i\})} \in \mathbb{P}(N \backslash (S \backslash \{i\}))$ and all $p_{N \backslash \{i\}} \in \mathbb{P}(N \backslash \{i\})$, the subgame perfect equilibrium (SPE) outcomes of Mechanism A coincide with the payoff vector $\phi(N, w)$ as prescribed by the Shapley value defined by Pham Do and Norde (2007) and de Clippel and Serrano (2008).

Theorem 3.1 *Mechanism A implements the Shapley value defined by Pham Do and Norde (2007) (i.e., the externality-free value by de Clippel and Serrano (2008)) of a zero-monotonic partition function form game* (N, w) *in SPE.*

PROOF
Let (N, w) be a zero-monotonic partition function form game. The proof proceeds by induction on the number of players n. It is easy to see that the theorem holds for $n = 1$. We

[4]It implies that the size of a coalition is positively correlated with its worth. In games that satisfy this condition, both a coalition and a player outside the coalition are incentivized to admit the player and join the coalition. This type of condition is common in the literature on implementing cooperative solution concepts, e.g., Pérez-Castrillo and Wettstein (2001), Macho-Stadler, Pérez-Castrillo and Wettstein (2006), Vidal-Puga and Bergantiños (2003), and Vidal-Puga (2005).

assume that it holds for all $m \leq n-1$ and show that it is satisfied for n.

First we show that the Shapley value is an SPE outcome. We explicitly construct an SPE that yields the Shapley value as an SPE outcome. Consider the following strategies, which the players would follow in any game they participate in (we describe it for the whole set of players, N, but these are also the strategies followed by any player in a subset S that is called upon to play the game, with S replacing N):

At stage 1, each player $i \in N$, announces $b^i_j = \phi_j(N, w) - \phi_j(N\backslash\{i\}, w|_{N\backslash\{i\}})$,[5] for every $j \in N\backslash\{i\}$.

At stage 2, a proposer, player i_n, offers $x^{i_n}_j = \phi_j(N\backslash\{i_n\}, w|_{N\backslash\{i_n\}})$ to every $j \in N\backslash\{i_n\}$.

At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\phi_j(N\backslash\{i_n\}, w|_{N\backslash\{i_n\}})$ and rejects any offer strictly less than $\phi_j(N\backslash\{i_n\}, w|_{N\backslash\{i_n\}})$.

At stage 4, player i_n makes an offer $\widetilde{x}^{i_n}_{i_{n-1}} = w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ to the selected proposer $i_{n-1} \in N\backslash\{i_n\}$.

At stage 5, player i_{n-1}, the proposer of the set of players $N\backslash\{i_n\}$, accepts any offer greater than or equal to $w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ and rejects any offer strictly less than it.

Clearly these strategies yield the Shapley value for any player who is not the proposer, since the game ends at stage 3 and $b^{i_n}_j + x^{i_n}_j = \phi_j(N, w)$, for all $j \neq i_n$. Moreover, given that following the strategies the offer is accepted by all players, the proposer also obtains her Shapley value.

Note that all net bids equal zero by the balanced contributions property for the Shapley value (Myerson (1980)).

To show that the previous strategies constitute an SPE, note first that the strategies at stages 2, 3, 4, and 5 are best responses: In case of rejection at stage 3 proposer i_n can obtain $w(N, \{N\}) - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ in the end (it pays her to make an offer that is accepted at stage 4, by zero-monotonicity), and all other players play the bidding mechanism with player set $N\backslash\{i_n\}$ and payoff $w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$. By the induction hypothesis, we have the Shapley value as the outcome of this game. That is, each player $j \in N\backslash\{i_n\}$ gets $\phi_j(N\backslash\{i_n\}, w|_{N\backslash\{i_n\}})$. Consider now the strategies at stage 1. If player i_n changes the vector of her bids so that another player becomes the proposer, this will not change her payoff, which would still equal her Shapley value. If she changes the vector of her bids and following it she is still the proposer, it must be that her total

[5] Given a partition function form game (N, w) and a subset $S \subseteq N$, we define the subgame $(S, w|_S)$ by assigning the worth $w|_S(T, p_S) \equiv w(T, p_S \cup [N\backslash S])$ for all $(T, p_S) \in \mathbb{E}(S)$

bid ($\sum_{j\in N\setminus\{i_n\}} b_j^{i_n}$) did not decline, which again means her payoff cannot improve. That is, any deviation of the bidding strategy of player i_n specified at stage 1 cannot improve the payoff of player i_n. Hence, no player has an incentive to change its bid, showing that the given strategy profile is an SPE.

The proof that any SPE yields the Shapley value proceeds similarly to the proof of Theorem 3.1 in Ju and Wettstein (2009) and therefore skipped. □

The key feature of Mechanism A in implementing the Shapley value is that it allows the proposer chosen from the first bidding stage to have the power of making another offer at stage 4 in case she has been rejected at stage 3. One might argue that the right to make a second offer should be awarded to the new proposer who is chosen from the remaining players rather than the original proposer whose offer has been rejected. Such an argument would lead to a new mechanism, which implements the expected stand-alone value.

Mechanism B. Stages $1, 2$ and 3 are the same as in Mechanism A. The difference lies in stages 4 and 5. Again, the mechanism starts with $S = N$.

Stages 1, 2, and 3: Same as in Mechanism A.

If the offer made at stage 3 is rejected, all players in S other than i_s proceed again through stages 1 to 3 where the set of active players is $S\setminus\{i_s\}$. Meanwhile, player i_s waits for the negotiation outcome of $S\setminus\{i_s\}$. Dependent upon whether or not the player i_{s-1}, the proposer of $S\setminus\{i_s\}$, can make his offer be accepted within $S\setminus\{i_s\}$, player i_s will either be called for renegotiation with i_{s-1} or be left alone. The renegotiation will follow the rules as specified below in stages 4 and 5 of the current mechanism.

If the offer is accepted, we have to distinguish between two cases where $S = N$ and $S \neq N$. In the case where $S = N$, the game ends as in Mechanism A. In the case where $S \neq N$, stages 4 and 5 are reached.

Stage 4: Proposer i_s makes an offer $\widetilde{x}_{i_{s+1}}^{i_s}$ in $\mathbb{R}$ to the previously rejected proposer i_{s+1}. (The offer is to pay i_{s+1} this amount for joining in to form the coalition $S \cup \{i_{s+1}\}$.)

Stage 5: Player i_{s+1} accepts or rejects the offer and the game ENDS.

The payoffs to all the players in $S\setminus\{i_s\}$ are the same independent of whether there was a rejection or an acceptance at stage 5, and are identical to the payoffs in Mechanism A.

The payoffs to all other players depend on whether or not there was an acceptance at stage 5.

If the offer is accepted then at this stage player i_s receives $w(S\cup\{i_{s+1}\},\{S\cup\{i_{s+1}\}\}\cup[N\backslash(S\cup\{i_{s+1}\})])-\widetilde{x}^{i_s}_{i_{s+1}}$ minus the bids and offer he made to the players in $S\backslash\{i_s\}$, while player i_{s+1} receives $\widetilde{x}^{i_s}_{i_{s+1}}$. The overall payoffs to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $w(S\cup\{i_{s+1}\},\{S\cup\{i_{s+1}\}\}\cup[N\backslash(S\cup\{i_{s+1}\})])-\widetilde{x}^{i_s}_{i_{s+1}}-\sum_{j\in S\backslash\{i_s\}}b^{i_s}_j-\sum_{j\in S\backslash\{i_s\}}x^{i_s}_j+\sum_{t=s+1}^n b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $\widetilde{x}^{i_s}_{i_{s+1}}-\sum_{j\in S}b^{i_{s+1}}_j+\sum_{t=s+2}^n b^{i_t}_{i_{s+1}}$. Moreover, all proposers before i_{s+1} receive their stand-alone payoffs in addition to all the payments received and paid out in the bidding stages they participated in. Hence the final payoff to player i_m for $m>s+1$ is $w(\{i_m\},\{S\cup\{i_{s+1}\}\}\cup[N\backslash(S\cup\{i_{s+1}\})])-\sum_{l\in N\backslash(\cup_{k=m}^n i_k)}b^{i_m}_l+\sum_{t=m+1}^n b^{i_t}_{i_m}$.

If the offer at stage 5 is rejected then the payoffs of all the players are the same as specified in Mechanism A.

Before stating the main result about Mechanism B, we show the following lemma. Let us first define the subgame $(N\backslash\{i\},w^{-i})$ of (N,w) by

$$w^{-i}(N\backslash\{i\},\{N\backslash\{i\}\})=w(N,\{N\})-w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})$$

and

$$w^{-i}(S,p_{N\backslash\{i\}})=w(S,p_{N\backslash\{i\}}\cup\{\{i\}\})$$

for all $(S,p_{N\backslash\{i\}})\in\mathbb{E}(N\backslash\{i\})\backslash(N\backslash\{i\},N\backslash\{i\})$.

Lemma 3.2 *For any game $w\in PG^N$ we have*

$$\sum_{j\in N\backslash\{i\}}\big(e_j(N,w)-e_j(N\backslash\{i\},w^{-i})\big)-\sum_{j\in N\backslash\{i\}}\big(e_i(N,w)-e_i(N\backslash\{j\},w^{-j})\big)=0$$

for all $i,j\in N$.

Proof

By the definition of the expected stand-alone value, it suffices to show that

$$-|N|e_i(N,w)+w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})+\sum_{j\in N\backslash\{i\}}e_i(N\backslash\{j\},w^{-j})=0$$

for all $i,j\in N$ and $i\neq j$. Obviously,

$$\begin{aligned}|N|e_i(N,w)= &\quad w(N,\{N\})\\ &+\sum_{S\subseteq N\backslash\{i\}:S\neq\emptyset}\frac{|S|!(|N|-|S|-1)!}{(|N|-1)!}w(\{i\},\{S\}\cup[N\backslash(S\cup\{i\})]\cup\{\{i\}\})\\ &-\sum_{k\in N\backslash\{i\}}\sum_{S\subseteq N\backslash\{i,k\}}\frac{|S|!(|N|-|S|-2)!}{(|N|-1)!}w(\{k\},[N\backslash(S\cup\{i\})]\cup\{S\cup\{i\}\}).\end{aligned}$$

and

$$\begin{aligned}
& \sum_{j\in N\setminus\{i\}} e_i(N\setminus\{j\}, w^{-j}) \\
= \quad & w(N,\{N\}) - \sum_{j\in N\setminus\{i\}} \frac{1}{|N|-1} w(\{j\},\{N\setminus\{j\}\}\cup\{\{j\}\}) \\
+ \quad & \sum_{j\in N\setminus\{i\}} \sum_{S\subseteq N\setminus\{i,j\}:S\neq\emptyset} \frac{|S|!(|N|-|S|-2)!}{(|N|-1)!} w(\{i\},\{S\}\cup[N\setminus(S\cup\{i\})]\cup\{\{i\}\}\cup\{\{j\}\}) \\
- \quad & \sum_{j\in N\setminus\{i\}} \sum_{k\in N\setminus\{i,j\}} \sum_{S\subseteq N\setminus\{i,j,k\}} \frac{|S|!(|N|-|S|-3)!}{(|N|-1)!} w(\{k\},[N\setminus(S\cup\{i\})]\cup\{S\cup\{i\}\}\cup\{\{j\}\}).
\end{aligned}$$

Moreover, we know that

$$\begin{aligned}
& \sum_{S\subseteq N\setminus\{i\}:S\neq\emptyset} \frac{|S|!(|N|-|S|-1)!}{(|N|-1)!} w(\{i\},\{S\}\cup[N\setminus(S\cup\{i\})]\cup\{\{i\}\}) \\
= \quad & \sum_{S=N\setminus\{i\}:S\neq\emptyset} \frac{|S|!(|N|-|S|-1)!}{(|N|-1)!} w(\{i\},\{S\}\cup[N\setminus(S\cup\{i\})]\cup\{\{i\}\}) \\
+ \quad & \sum_{S\subsetneqq N\setminus\{i\}:S\neq\emptyset} \frac{|S|!(|N|-|S|-1)!}{(|N|-1)!} w(\{i\},\{S\}\cup[N\setminus(S\cup\{i\})]\cup\{\{i\}\}) \\
= \quad & w(\{i\},\{N\setminus\{i\}\}\cup\{\{i\}\}) \\
+ \quad & \sum_{j\in N\setminus\{i\}} \sum_{S\subseteq N\setminus\{i,j\}:S\neq\emptyset} \frac{|S|!(|N|-|S|-2)!}{(|N|-1)!} w(\{i\},\{S\}\cup[N\setminus(S\cup\{i\})]\cup\{\{i\}\}\cup\{\{j\}\})
\end{aligned}$$

and similarly,

$$\begin{aligned}
& \sum_{j\in N\setminus\{i\}} \sum_{k\in N\setminus\{i,j\}} \sum_{S\subseteq N\setminus\{i,j,k\}} \frac{|S|!(|N|-|S|-3)!}{(|N|-1)!} w(\{k\},[N\setminus(S\cup\{i\})]\cup\{S\cup\{i\}\}\cup\{\{j\}\}) \\
= \quad & \sum_{k\in N\setminus\{i\}} \sum_{S\subsetneqq N\setminus\{i,k\}} \frac{|S|!(|N|-|S|-2)!}{(|N|-1)!} w(\{k\},[N\setminus(S\cup\{i\})]\cup\{S\cup\{i\}\}).
\end{aligned}$$

What remains is clear because

$$\begin{aligned}
& \sum_{S=N\setminus\{i,j\}} \frac{|S|!(|N|-|S|-2)!}{(|N|-1)!} w(\{j\},[N\setminus(S\cup\{i\})]\cup\{S\cup\{i\}\}) \\
= \quad & \frac{1}{|N|-1} w(\{j\},\{N\setminus\{j\}\}\cup\{\{j\}\}).
\end{aligned}$$

□

Theorem 3.3 *Mechanism B implements the expected stand-alone value of a zero-monotonic partition function form game (N,w) in SPE.*

Proof The proof is analogous to that of Theorem 3.1. The differences are in the construction of the SPE strategies and in showing that in any SPE, the final payment received by each of the players coincides with each player's expected stand-alone value. Hence, we first explicitly construct an SPE that yields the expected stand-alone value as an SPE outcome.

To construct an SPE, consider the following strategies.

At stage 1, each player $i \in N$, announces $b^i_j = e_j(N, w) - e_j(N \backslash \{i\}, w^{-i})$, for every $j \in N \backslash \{i\}$.

At stage 2, a proposer, player i_n, offers $x^{i_n}_j = e_j(N \backslash \{i_n\}, w^{-i_n})$ to every $j \in N \backslash \{i_n\}$.

At stage 3, any player $j \in N \backslash \{i_n\}$ accepts any offer which is greater than or equal to $e_j(N \backslash \{i_n\}, w^{-i_n})$ and rejects any offer strictly less than $e_j(N \backslash \{i_n\}, w^{-i_n})$.

At stage 4, a proposer within $N \backslash \{i_n\}$, player i_{n-1}, makes an offer $\widetilde{x}^{i_{n-1}}_{i_n} = w(\{i_n\}, \{N \backslash \{i_n\}\} \cup \{\{i_n\}\})$ to i_n.

At stage 5, player i_n, the "waiting" proposer for the set of players N, accepts any offer greater than or equal to $w(\{i_n\}, \{N \backslash \{i_n\}\} \cup \{\{i_n\}\})$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the expect stand-alone value for any player and constitute an SPE.

Next we show that in any SPE the final payment received by each of the players coincides with each player's expected stand-alone value. We note that if i is the proposer, her final payoff will be $w(N, \{N\}) - (w(N, \{N\}) - w(\{i\}, \{N \backslash \{i\}\} \cup \{\{i\}\})) - \sum_{j \neq i} b^i_j$, whereas if $j \neq i$ is the proposer, i will get final payoff $e_i(N \backslash \{j\}, w^{-j}) + b^j_i$. Hence the sum of the payoffs to player i over all possible choices is (note that all net bids are zero)

$$\begin{aligned} & w(N, \{N\}) - (w(N, \{N\}) - w(\{i\}, \{N \backslash \{i\}\} \cup \{\{i\}\})) - \sum_{j \neq i} b^i_j \\ + \ & \sum_{j \neq i} \left(e_i(N \backslash \{j\}, w^{-j}) + b^j_i \right) \\ = \ & w(\{i\}, \{N \backslash \{i\}\} \cup \{\{i\}\}) + \sum_{j \neq i} e_i(N \backslash \{j\}, w^{-j}), \end{aligned}$$

which, by Lemma 3.2, equals $n \cdot e_i(N, v)$. Since the payoffs are the same regardless of who is the proposer we see that the payoff of each player in any equilibrium must coincide with the expected stand-alone value. □

The result of Theorem 3.3 is intriguing in two aspects. One is that the corresponding Mechanism B also takes coalitional effects into account, but the final equilibrium outcome, i.e., the expected stand-alone value, only involves the externalities on stand-alone coalitions. That is, by shifting the power to make a renegotiation offer from the rejected proposer to the current proposer, we see a striking difference in equilibrium outcomes between Mechanism A and Mechanism B. The underlying key intuition is that Mechanism B allows the proposer who has the power to offer in renegotiation to capture the residual surplus by offering the corresponding stand-alone payoffs to the rejected proposers. The other interesting aspect is that, unlike the result of Theorem 3.2 in Ju and Wettstein (2009), here Mechanism B implements the expected stand-alone value $e(w)$ instead of $E_i(w) = w(\{i\}, [N]) + \frac{1}{|N|}\left(w(N, \{N\}) - \sum_{j \in N} w(\{j\}, [N])\right)$ which is a direct extension of the equal surplus value for TU games. To see this, we offer the following observation. Differing from a classical TU game without externalities, the environment that Mechanism B lies in involves coalitional externalities. Suppose now player i has formed a coalition S containing herself and has the power to make an offer in the renegotiation stage. When she renegotiates with player j, who is the previously rejected proposer, j's reservation value at this moment is a stand-alone payoff $w(\{j\}, \{S\} \cup [N \backslash S])$. This holds true for any such renegotiation stage. Hence, in equilibrium, all such stand-alone payoffs resulting from coalitional externalities will have to be considered.

The above mechanisms A and B take extreme and opposite treatments in case an offer is rejected, which give *a priori* full power to either the rejected proposer or the proposer chosen from the set of remaining players to make a second offer. A less biased option would be giving equal power to the two proposers to make a second offer. That is, let the two compete (by bidding) for the role of being the proposer to make a further offer when they engage in renegotiation. This mechanism as formally described below implements the consensus value for partition function form games.

Mechanism C. The rules of stages 1, 2 and 3 are the same as before. Below we will mainly describe stages 4 and 5 where the difference from mechanisms A and D lies in. The mechanism starts with $S = N$.

Stages 1, 2, and 3: Same as in Mechanism A.

Note that if the offer made at stage 3 is rejected, all players in S other than i_s proceed again through stages 1 to 3 where the set of active players is $S \backslash \{i_s\}$. In the current mechanism, players in $S \backslash \{i_s\}$ actually compete for becoming the proposer i_{s-1} so as to win the right of renegotiating with i_s. The renegotiation between i_{s-1} and i_s is in fact a 2-player bidding game, where both of them simultaneously make bids at stage 4 and the winner will have the right to make a new offer to the other player at stage 5 while the other player accepts or rejects the offer.

Stage 4: Players i_s and i_{s+1} bid for the right to take the role of the proposer. Players i_s and i_{s+1} simultaneously submit bids $\widetilde{b}_{i_{s+1}}^{i_s}$ and $\widetilde{b}_{i_s}^{i_{s+1}}$ in $\mathbb{R}$. The player with the larger net bid pays the bid to the other player and assumes the role of the proposer. In case of identical bids either of the two players is chosen as the proposer with equal probability.

Stage 5: Depending on whether the proposer is i_{s+1} or i_s, the game proceeds as in Mechanism A (when i_{s+1} is the proposer) or Mechanism B (when i_s is the proposer). The payoffs are adjusted by taking into account the bidding at stage 4.

Lemma 3.4 *For any game $w \in PG^N$ we have*

$$\begin{aligned} & |N| e_i(N,w) \\ = \quad & w(\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\}) \\ + \quad & \sum_{j \in N\backslash\{i\}} \frac{w(N,\{N\}) - w(N\backslash\{j\}, \{N\backslash\{j\}\} \cup \{\{j\}\}) - w(\{j\}, \{N\backslash\{j\}\} \cup \{\{j\}\})}{|N|-1} \\ + \quad & \sum_{j \in N\backslash\{i\}} e_i(N\backslash\{j\}, w|_{N\backslash\{j\}}) \end{aligned}$$

for all $i \in N$.

Proof The proof can be constructed along the same line as that for Lemma 3.2. □

Theorem 3.5 *Mechanism C implements the consensus value of a zero-monotonic partition function form game (N,w) in SPE.*

Proof The proof is again similar to that of Theorem 3.1. The differences are once more in the construction of the SPE strategies and in claiming that payoffs must coincide with the consensus value. To explicitly construct an SPE that yields the consensus value, consider the following strategies.

At stage 1, each player $i \in N$ announces $b_j^i = \gamma_j(N,v) - \gamma_j(N\backslash\{i\}, \widehat{w}^{-i})$,[6] for every $j \in N\backslash\{i\}$.

At stage 2, a proposer, player i_n, offers $x_j^{i_n} = \gamma_j(N\backslash\{i_n\}, \widehat{w}^{-i})$ to every $j \in N\backslash\{i_n\}$.

[6]The game $(N\backslash\{i\}, \widehat{w}^{-i})$ is formally defined by $\widehat{w}^{-i}(N\backslash\{i\}, \{N\backslash\{i\}\}) = w(N\backslash\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\}) + \frac{w(N,\{N\}) - w(N\backslash\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\}) - w(\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\})}{2}$ and $\widehat{w}^{-i}(S, p_{N\backslash\{i\}}) = w(S, p_{N\backslash\{i\}} \cup \{\{i\}\})$, for all $(S, p_{N\backslash\{i\}}) \in \mathbb{E}(N\backslash\{i\}) \backslash (N\backslash\{i\}, N\backslash\{i\})$.

At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\gamma_j(N\backslash\{i_n\}, \widehat{w}^{-i_n})$ and rejects any offer strictly less than $\gamma_j(N\backslash\{i_n\}, \widehat{w}^{-i_n})$.

At stage 4, player i_n announces

$$\begin{aligned}\widetilde{b}^{i_n}_{i_{n-1}} = & \quad w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) \\ & + \frac{w(N, \{N\}) - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) - w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})}{2} \\ & - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) \\ = & \quad \frac{w(N, \{N\}) - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) - w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})}{2}\end{aligned}$$

while player i_{n-1} announces

$$\begin{aligned}\widetilde{b}^{i_{n-1}}_{i_n} = & \quad w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) \\ & + \frac{w(N, \{N\}) - w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})}{2} \\ & - w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) \\ = & \quad \frac{w(N, \{N\}) - w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\}) - w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})}{2}.\end{aligned}$$

At stage 5, player i_n makes an offer $\widetilde{x}^{i_n}_{i_{n-1}} = w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ to i_{n-1} and player i_{n-1} makes an offer $\widetilde{x}^{i_{n-1}}_{i_n} = w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ to i_n. Moreover, i_n accepts any offer greater than or equal to $w(\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ and rejects any offer strictly less than it. Similarly, i_{n-1} accepts any offer greater than or equal to $w(N\backslash\{i_n\}, \{N\backslash\{i_n\}\} \cup \{\{i_n\}\})$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the consensus value for partition function form games for any player and constitute an SPE.

To show that in any SPE each player's final payoff coincides with her consensus value, we note that if i is the proposer her final payoff is given by

$$\begin{aligned}& w(N, \{N\}) - w(N\backslash\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\}) \\ - & \frac{w(N, \{N\}) - w(N\backslash\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\}) - w(\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\})}{2} - \sum_{j \neq i} b^i_j\end{aligned}$$

whereas if $j \neq i$ is the proposer, the final payoff of i is $\gamma_i(N\backslash\{j\}, \widehat{w}^{-j}) + b^j_i$.

Hence the sum of payoffs to player i over all possible choices of the proposer is (again note that all net bids are zero)

$$w(N, \{N\}) - w(N\backslash\{i\}, \{N\backslash\{i\}\} \cup \{\{i\}\})$$

$$
\begin{aligned}
- & \quad \frac{w(N,\{N\}) - w(N\backslash\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\}) - w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})}{2} - \sum_{j\neq i} b_j^i \\
+ & \quad \sum_{j\neq i}\left(\gamma_i(N\backslash\{j\},\widehat{w}^{-j}) + b_i^j\right) \\
= & \quad \frac{w(N,\{N\}) - w(N\backslash\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\}) + w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})}{2} \\
+ & \quad \sum_{j\neq i}\left(\frac{1}{2}\phi_i(N\backslash\{j\},\widehat{w}^{-j}) + \frac{1}{2}e_i(N\backslash\{j\},\widehat{w}^{-j})\right) \\
= & \quad \frac{w(N,\{N\}) - w(N\backslash\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\}) + w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})}{2} \\
+ & \quad \frac{1}{2}\sum_{j\neq i}\left(\phi_i(N\backslash\{j\},w|_{N\backslash\{j\}}) + \frac{\frac{w(N,\{N\})-w(N\backslash\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\})-w(\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\})}{2}}{n-1}\right) \\
+ & \quad \frac{1}{2}\sum_{j\neq i}\left(e_i(N\backslash\{j\},w|_{N\backslash\{j\}}) + \frac{\frac{w(N,\{N\})-w(N\backslash\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\})-w(\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\})}{2}}{n-1}\right) \\
= & \quad \frac{1}{2}\left(w(N,\{N\}) - w(N\backslash\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\}) + \sum_{j\neq i}\phi_i(N\backslash\{j\},w|_{N\backslash\{j\}})\right) \\
+ & \quad \frac{1}{2}w(\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\}) \\
+ & \quad \frac{1}{2}\sum_{j\neq i}\left(\frac{w(N,\{N\}) - w(N\backslash\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\}) - w(\{j\},\{N\backslash\{j\}\}\cup\{\{j\}\})}{n-1}\right) \\
+ & \quad \frac{1}{2}\sum_{j\neq i} e_i(N\backslash\{j\},w|_{N\backslash\{j\}}),
\end{aligned}
$$

which, since $w(N,\{N\})-w(N\backslash\{i\},\{N\backslash\{i\}\}\cup\{\{i\}\})+\sum_{j\neq i}\phi_i(N\backslash\{j\},w|_{N\backslash\{j\}}) = n\phi_i(N,w)$ and by Lemma 3.4, equals $n\left(\frac{1}{2}\phi_i(N,w)+\frac{1}{2}e_i(N,w)\right)$, and then yields $n\gamma_i(N,w)$. Since the payoffs are the same regardless of who is the proposer, the payoff of each player in any equilibrium must coincide with the consensus value. □

Mechanism C can be generalized in a natural way by treating the players asymmetrically: bids made by one player are "worth more" than those made by the other. Such a mechanism implements the generalized consensus value of a zero-monotonic partition function form game.

Finally, we present the mechanism to implement the extended Shapley value for partition function form games (McQuillin (2009)). The challenging part in this task is to construct a reasonable bargaining protocol to well balance the two seemingly conflicting forces: the incentive for all players to form the grand coalition N in order to achieve the extended Shapley value payoff versus the temptation posed to the rejected players to form a coalition rather than singletons that is underlying the extended Shapley value. These two forces seem to work in opposite directions since if the former dominates the latter the threatening effect to constrain the payoff of S by $N\backslash S$ would be impossible, whereas if the latter dominates the former the grand coalition would not be formed.

Mechanism D. The rules of stages 1, 2 and 3 are the same as before. We proceed to describe stages 4 and 5.

Stage 4: The first rejected proposer i_n proposes a vector of payments $y_k^{i_n} \in \mathbb{R}$ to any other rejected proposer $k \in \{i_{s+1}, ..., i_{n-1}\}$.

Stage 5: Players $i_{s+1}, ..., i_{n-1}$, sequentially, either accept or reject the proposal made by i_n at stage 4.

If at least one player rejects it, then the proposal is rejected. In this case the game ends with the final partition of N being $\{S\} \cup [N\backslash S]$. That is, all the rejected proposers $i_{s+1}, ..., i_n$ will form singleton coalitions. The payoff to each $j \in S\backslash\{i_s\}$ is given by $\left(\sum_{t=s}^n b_j^{i_t}\right) + x_j^{i_s}$, the payoff to i_s is given by $\left(\sum_{t=s+1}^n b_{i_s}^{i_t}\right) + w(S, \{S\} \cup [N\backslash S]) - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} - \sum_{j\in S\backslash\{i_s\}} x_j^{i_s}$, the payoff to any i_m, $m \in \{s+1, ..., n-1\}$ is $\sum_{t=m+1}^n b_{i_m}^{i_t} + w(\{i_m\}, \{S\} \cup [N\backslash S]) - \sum_{j\in N\backslash\{i_m,...,i_n\}} b_j^{i_m}$, and the payoff to i_n is $w(\{i_n\}, \{S\} \cup [N\backslash S]) - \sum_{j\in N\backslash\{i_n\}} b_j^{i_n}$.

If the proposal is accepted by all these players, then all the rejected proposers $i_{s+1}, ..., i_n$ form a coalition $N\backslash S$ and the game ends with the final partition of N being $\{S\} \cup \{N\backslash S\}$. The payoff to each $j \in S\backslash\{i_s\}$ is the same as above, i.e., $\left(\sum_{t=s}^n b_j^{i_t}\right) + x_j^{i_s}$, the payoff to i_s is given by $\left(\sum_{t=s+1}^n b_{i_s}^{i_t}\right) + w(S, \{S\} \cup \{N\backslash S\}) - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} - \sum_{j\in S\backslash\{i_s\}} x_j^{i_s}$, the payoff to any i_m, $m \in \{s+1, ..., n-1\}$ is $\sum_{t=m+1}^n b_{i_m}^{i_t} + y_{i_m}^{i_n} - \sum_{j\in N\backslash\{i_m,...,i_n\}} b_j^{i_m}$, and the payoff to i_n is $w(N\backslash S, \{S\} \cup \{N\backslash S\}) - \sum_{m=s+1}^{n-1} y_{i_m}^{i_n} - \sum_{j\in N\backslash\{i_n\}} b_j^{i_n}$.

The following theorem shows that for any partition function form game (N, w) satisfying zero-monotonicity, for all $S \subseteq N$ and $i \in S$, the subgame perfect equilibrium (SPE) outcomes of Mechanism D coincide with the payoff vector $\phi(N, w)$ as prescribed by the extended Shapley value defined by McQuillin (2009).

Theorem 3.6 *Mechanism D implements the extended Shapley value defined by McQuillin (2009) of a zero-monotonic partition function form game (N, w) in SPE.*

To better understand how this mechanism would lead to the outcome, it is worth providing an intuitive explanation of the reasoning underlying the behavior of the rejected proposers. Firstly, it is easy to see that stage 5 is essentially an ultimatum game where any rejected proposer i_m, $m \in \{s+1, ..., n-1\}$ will accept an offer if it is no worse than his stand-alone payoff $w(\{i_m\}, \{S\} \cup [N\backslash S])$. Hence, were he rejected, due to the zero-monotonicity condition, there is sufficient incentive for i_n to make an acceptable proposal as such to unite all these rejected proposers to form a coalition $N\backslash S$. This has an impact on i_s when making offers to players in S as she would know that the coalitional payoff of S will indeed be constrained by $N\backslash S$ to be $w(S, \{S\} \cup \{N\backslash S\})$, which is the one underlying the extended Shapley value. Next, we shall observe that, while the threat of $N\backslash S$ to S is credible and therefore affects all the offers, given the zero-monotonicity, any proposer i_m, $m \in \{s+1, ..., n-1\}$, would not have any incentive to make an offer be rejected to end up with a stand-alone payoff. Thus, although the first rejected proposer, i_n, seemingly has a big advantage to potentially gain a lot at the take-it-or-leave-it stage, it is actually impossible for her to exploit it as she would be able to deduce that any subsequent proposer will make an offer to be accepted. Reasoning backwards, the first proposer i_n, if her offer is rejected by $N\backslash\{i_n\}$, will in fact end up with her stand-alone payoff, too. Consequently, in equilibrium, i_n will also make an offer that is accepted by all players in $N\backslash\{i_n\}$.

PROOF (of Theorem 3.6) The proof is an adaption of the above argument, preserving the similar lines as that of Theorem 3.1. Thus, we omit most of it but explicitly construct an SPE that yields the extended Shapley value. Consider the following strategies.

At stage 1, each player $i \in N$ announces $b^i_j = \phi^{MQ}_j(N, v) - \phi^{MQ}_j(N\backslash\{i\}, \tilde{w}^{-i})$,[7] for every $j \in N\backslash\{i\}$.

At stage 2, a proposer, player i_n, offers $x^{i_n}_j = \phi^{MQ}_j(N\backslash\{i_n\}, \tilde{w}^{-i})$ to every $j \in N\backslash\{i_n\}$.

At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\phi^{MQ}_j(N\backslash\{i_n\}, \tilde{w}^{-i_n})$ and rejects any offer strictly less than $\phi^{MQ}_j(N\backslash\{i_n\}, \tilde{w}^{-i_n})$.

At stage 4, after the offer of i_s is accepted by players in S, player i_n announces $y^{i_n}_k = w(\{k\}, \{S\} \cup [N\backslash S])$ to every $k \in (N\backslash S)\backslash\{i_n\}$.

At stage 5, player k, where $k \in (N\backslash S)\backslash\{i_n\}$, accepts any offer that is greater than or equal to $w(\{k\}, \{S\} \cup [N\backslash S])$ and rejects any offer strictly less than it. □

It is helpful to compare[8] Mechanism A and Mechanism D to highlight the strategic difference between the externality-free Shapley value and the extended Shapley value. When

[7] The game $(N\backslash\{i\}, \tilde{w}^{-i})$ is formally defined by $\tilde{w}^{-i}(S, p_{N\backslash\{i\}}) = w(S, \{S\} \cup \{N\backslash S\})$ for all $(S, p_{N\backslash\{i\}}) \in \mathbb{E}(N\backslash\{i\})$.

[8] We thank the anonymous referee for suggesting this comparison.

at least one player rejects i_n's proposal at stage 5 in Mechanism D, i_n's proposal is rejected, and then all the players $i_{s+1}, ..., i_n$ will receive their stand-alone payoffs. The same happens in Mechanism A when i_{s+1}'s proposal is rejected, all the players $i_{s+1}, ..., i_n$ will receive their stand-alone payoffs. However, the two mechanisms differ significantly in the case of acceptance. In Mechanism D, when i_n's proposal is accepted at stage 5, all the players $i_{s+1}, ..., i_{n-1}$ will receive the respective payoffs that are offered by i_n (in equilibrium, these will be their stand-alone payoffs), but player i_n receives a large surplus that is, in equilibrium, $w(N \setminus S, \{S\} \cup \{N \setminus S\}) - \sum_{m=s+1}^{n-1} w(\{i_m\}, \{S\} \cup [N \setminus S])$. In Mechanism A, when a player, say, i_{s+1}'s offer is accepted by i_s at stage 5, this player will receive $v(S \cup \{i_{s+1}\}, \{S \cup \{i_{s+1}\}\} \cup [N \setminus (S \cup \{i_{s+1}\})]) - \tilde{x}_{i_s}^{i_{s+1}}$, which, in equilibrium, will be equal to his marginal contribution to S. Moreover, acceptance at stage 5 in Mechanism D implies coalition $N \setminus S$ is formed, whereas in Mechanism A, the acceptance at stage 5 means coalition $S \cup \{i_{s+1}\}$ is formed.

Obviously, to have the externality-free Shapley value would require the coalition structure of the non-coalitional players to be singletons, i.e., $[N \setminus S]$. On the contrary, the extended Shapley value requires the coalition structure of the non-coalitional players to be $\{N \setminus S\}$. Thus, these two values lie at the two extremes of coalition structures that may be formed by the non-coalitional players. Naturally, one may ask whether we can capture a situation in between.[9] Since Mechanism A and Mechanism D differ in the roles assumed by the rejected proposers in their further negotiations, we cannot follow the logic leading to Mechanism C, which implemented a middle-ground value given by the consensus value. However, one can construct a hybrid mechanism with an exogenous "flip a coin" stage after stage 3 so that either Mechanism A or D is equally likely to follow stage 3. One can readily check that this hybrid mechanism will implement the average of the externality-free Shapley value and the extended Shapley value in subgame perfect equilibria.

4 Discussion and future research

By using a class of bidding mechanisms that differ in the power awarded to the proposer chosen through a bidding process, this paper provided a strategic approach to several well-studied axiomatic solution concepts for partition function form games. It highlights the different non-cooperative rationales of the normative standards applied to environments with externalities. The fact that the equilibria yield outcomes coinciding with these values exactly shows the power of the strategic approach, which provides additional insights underlying the axiomatic values.

The only condition we imposed thus far on the partition function form games is a

[9]We thank the anonymous referee for raising this question.

zero-monotonicity formulation that works for all the implementation mechanisms we have constructed. Technically, this condition can be slightly weakened by specifying a specific coalition structure for $\{N\backslash S\}$. Considering the externality-free value, since the worth of a coalition is defined by having all the other players form a singletons coalition, we can introduce a weaker zero-monotonicity condition using only singleton coalitions, that is, $w(S, \{S\} \cup [N\backslash\ S]) > w(S\backslash\{i\}, \{S\backslash\{i\}\} \cup [N\backslash\ (S\backslash\{i\})]) + w(\{i\}, \{S\backslash\{i\}\} \cup [N\backslash\ (S\backslash\{i\})])$ for all $S \subseteq N$ and $i \in S$, with which Mechanism A still implements the externality-free value. By contrast, for the extended Shapley value, the weakened zero-monotonicity would have $\{N\backslash S\}$ replacing $[N\backslash S]$. Given that this paper aims to provide a general framework for implementing and comparing different solution concepts, we used a single zero-monotonicity condition rather than tailor-made variants for various solutions.

Even though zero-monotonicity only excludes a relatively small number of games, i.e., those where a player joining a coalition may not generate higher payoffs than keeping the two parties apart, it is still worth investigating how to deal with arbitrary cooperative environments. A possible way of handling this problem may follow the spirit of Pérez-Castrillo and Wettstein (2001). Consider implementing the externality-free Shapley value for partition function form games, for example. We now require a proposer to make a proposal consisting of two components: a coalition structure and a vector of offers. Then, if the proposal is accepted, all players will form the coalition structure (rather than only a coalition as in Mechanism A) as specified by the proposer, and the proposer will pay every player the promised offer. The proposer will receive all the payoffs generated by each coalition in the coalition structure. Such a modified mechanism generates an efficient coalition structure, and implements the Shapley value of the superadditive cover of the game.

As we see, introducing the option of renegotiation can result in different equilibrium outcomes and therefore implement various values. Throughout the paper we require a renegotiation between a coalition S and the outside players to happen only when S has already reached an agreement. We do not allow these outside players to play the same bidding mechanism applied to S as that would usually lead to a cycle where no equilibrium may exist. However, this restriction can be weakened to a certain degree by imposing alternative rules (except for the completely laissez-faire case) on renegotiation. Proceeding in this manner might lead to alternative equilibrium outcomes, hence new values for partition function form games. Further interesting directions of research indeed lie in deriving alternative equilibrium outcomes by reasonably modifying the bargaining protocols.

As an example, Borm, Ju and Wettstein (2015) successfully adopts such a bid-offer-renegotiation approach to construct alternative mechanisms that give rise to new solution concepts for partition function form games, namely the rational belief Shapley values.

In contrast to the current paper, where the four solution concepts implemented rely on concrete and fixed coalition structures (e.g., singleton coalition structure for the externality-free Shapley value) when defining the worth of a coalition, the rational belief Shapley values do not necessarily have a fixed coalition structure. Thus, in equilibrium, the corresponding mechanisms induce the relevant players to form "rational" coalition structures appropriate to various solution concepts. In the current paper, a coalition structure is not arbitrarily chosen by any player but has been modeled into the negotiation protocols and will emerge through bargaining.

While renegotiation opens up different ways of modeling the strategic bargaining procedures and serves as a standard building block to study all four solution concepts, we acknowledge that the description of the related extensive form games can be cumbersome. Developing new ideas to construct alternative implementation mechanisms could be very relevant and meaningful. They may not necessarily share the same construct so long as the distinct bargaining protocols can well shed light on the underlying strategic elements of the value concepts and, ideally, be more succinct and straightforward.

Finally, we note that the results obtained in this paper can shed light on many concrete economic and political situations characterized by externalities and cooperative interests and conflicts, such as reaching environmental agreements, coordinating market behavior of firms, the provision of public goods, cost sharing[10] and resolving compensation disputes.

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[10]Cost sharing is a prominent example here, with abundant studies in the literature, from both the axiomatic (e.g., Sprumont (2005)) and strategic (Yeh, Hu and Tsay (2012, 2018)) perspectives. One can generalize the problem to allow for externalities and strategic analysis, for which the current paper may offer a useful approach.

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