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Interacting with Yielding Vehicles: A Perceptually Plausible Model for Pedestrian Road Crossing Decisions

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***Abstract*—As autonomous driving technology advances, automated vehicles (AVs) will increasingly share road space with pedestrians, creating significant challenges for AV systems. Effective interaction between AVs and pedestrians is one of the key hurdles. Pedestrian simulation tools offer the potential to expedite the evaluation and refinement of these interactive capabilities. However, existing research lacks efforts to model pedestrian behavior in vehicle-yielding scenarios, resulting in distorted modeling results. This paper proposes a perceptually plausible road-crossing decision model that creates temporal-dynamic crossing decisions across a range of vehicle-yielding scenarios. Specifically, a proposed hybrid perception strategy explains how pedestrians may apply psychophysical cues to make crossing decisions. Discrete choice models based on the hybrid perception strategy combined with a crossing initiation model reproduce the details of crossing decisions: the decision and its timing. An empirical dataset collected in a pedestrian simulator is applied to validate the model. Additionally, the latest crossing decision models, i.e., the evidence accumulation model and the artificial neural networks approach, are employed as comparisons. The results show that the proposed model accurately reproduces crossing decision patterns affected by diverse vehicle kinematics in vehicle-yielding scenarios in a perceptually plausible manner. Our results strengthen the notion that there is a perceptual threshold for pedestrians to control their decision-making strategy. The proposed theory and approach bring insights into the computational pedestrian road-crossing behavior and have practical implications in traffic simulation and AV development.**

***Index Terms*—Pedestrian-vehicle interaction, Road crossing decision, Perceptually plausible, Vehicle-yielding scenario, Simulation.**

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I. INTRODUCTION

ROAD crossing, where pedestrians walk towards the opposite sidewalk while avoiding potential collisions with approaching vehicles, is one of the most important road behaviors in traffic. Simulating pedestrian crossing behavior has critical implications for traffic safety, policy-making, management, and infrastructure development. In recent years, with the development of AVs and the great expectations for highly AVs, more research has been drawn to this area [1], [2]. The emerging concern is that extending the deployment of AVs from a few confined areas, which pose a lower risk for pedestrians, to a variety of operational design domains could inevitably heighten conflicts with pedestrians [3]. The failure of AVs to comprehend pedestrian behavior and interact appropriately with them may not enhance traffic efficiency and safety as anticipated but rather exacerbate traffic dilemmas and introduce additional issues [4]. Therefore, for the needs of AV simulation testing, it becomes even more urgent to accurately simulate pedestrian crossing behavior. However, current research is still lacking in how to realistically and explainably model pedestrian road-crossing decisions (PRDs). Below we review in detail the observation and modeling studies related to pedestrian crossing behavior to summarize the current limitations in the literature.

A. Pedestrian behavior simulations

The lack of pedestrian simulation technology, particularly for vehicle-pedestrian interaction simulation, could limit the development of AVs. This has given rise to extensive modeling research in this area. In microscopic scenarios involving pedestrians and vehicles, researchers have developed expert models and data-driven models to simulate pedestrian behavior. Expert models, such as the Social Force model (SFM) [5] and Cellular Automata model (CAM) [6], rely on empirical motion rules. Teknomo [7] applied SFM to simulate and analyze pedestrian walking behavior during crossings. Meanwhile, Zhang et al. [8] developed a CAM, simulating vehicle-pedestrian conflicts through probabilistic functions. On the other hand, data-driven models, like Artificial Neural Networks (ANNs) and Reinforcement learning (RL) approaches, learn pedestrian road behavior from naturalistic datasets or within predefined environments. For instance, Ma et al. [9] utilized ANNs to learn pedestrian walking behavior, considering relative spatial and motion relationships between pedestrians and other objects in videos. Martinez et al. [10] employed multiple agents RL to learn pedestrian walking behavior within a social force-based environment. Nevertheless, these models primarily concentrate on simulating pedestrian motion, disregarding pedestrian decision-making. Although, in certain cases, like within a controlled single corridor, pedestrian movement embodies a series of time-continuous decisions, in pedestrian-vehicle interaction scenarios, pedestrians' intentions are affected and frequently change due to the highly dynamic time-varying nature of traffic. Often, pedestrians' intentions have changed, but they have not yet been reflected in behavioral changes, such as crossing gap rejections. Hence, it is necessary to separate PRDs simulation from pedestrian behavior simulation into a specific study.

B. Observation studies on pedestrian road-crossing behavior

Pedestrian road-crossing behavior has been a focus of extensive research for several decades. Recent advancements in virtual reality and sensor technology have propelled this field to new heights [11]–[13]. Relevant observational studies were conducted in simulators, test tracks, and naturalistic road environments. In simulator studies, researchers primarily utilize two types of virtual reality devices: CAVE (Cave Automatic Virtual Environment) systems and head-mounted displays (HMDs) [14], [15]. CAVE systems comprise large screens surrounding participants, enabling them to perceive high-resolution computer-generated images. HMDs, on the other hand, provide 3D goggles that deliver high-quality images and enhance immersion by isolating the viewer from the real world. In naturalistic observation studies, researchers use various sensors, such as drones and traffic cameras, to record pedestrians' and vehicles' behaviors [16]. Those experiments yield data that reflect actual pedestrian behavior and decision-making patterns in diverse real-world situations. Test track experiments, conducted at dedicated sites, represent a compromise between simulator studies and naturalistic observations, balancing control and realism [12].

Pedestrian behavior in virtual environments is influenced by the display interface, world model, and the way participants interact with virtual environments. Despite current advanced VR devices offering high-fidelity displays, challenges persist. Inaccurate environment modeling could lead to imprecise estimation of spatiotemporal information by pedestrians [17]. Moreover, HMDs may reduce the field of view and compress distance perception [18]. In contrast, naturalistic observations are real and natural but also difficult to control. In such settings, pedestrians may approach crosswalks and observe approaching vehicles in various ways, involving numerous variables that complicate the attribution of specific behavior patterns [16], [19]. Test track environments offer a compromise between virtual and naturalistic observations, addressing the limitations of both methods. However, this approach sacrifices some freedom and realism; for instance, pedestrians are generally not permitted to perform actual crossings for safety reasons [12], [18]. Given the different characteristics of these three experimental methods, they are usually suitable for studying different aspects of pedestrian crossing behavior. Simulator studies are mostly used to explain the relationship between specific crossing patterns and several factors, while naturalistic observations are primarily used to investigate the actual crossing behavior of pedestrians under real conditions from a relatively macro perspective. For instance, some studies found that pedestrians usually initiate their crossing decisions from non-stationary states and this dynamic process can be distinguished into different stages: approach, appraising, and crossing [16], while others observed that pedestrians approach and interact with vehicles from multiple directions [19], [20]. Although test tracks can also study pedestrian crossing patterns under controlled conditions, they are more suitable for situations requiring accurate perception, such as investigating pedestrians' perceptions of external human-machine interfaces [21], [22].

Using the aforementioned experimental methods, existing studies have identified numerous factors affecting PRDs, including vehicle kinematics, pedestrian heterogeneity, and environmental conditions. Regarding vehicle kinematics, strong evidence suggests that vehicle speed, spatial and temporal distance influence PRDs [2], [23]. Several studies have demonstrated that pedestrians can perceive vehicle deceleration behavior and adjust their decisions accordingly [24], [25]. Concerning pedestrian heterogeneity, factors such as age, gender, and distractions have received considerable attention. Researchers agree that older adults and children make riskier decisions when crossing the street than middle-aged and young adults [26], [27]. Due to deterioration in movement and perceptual abilities, older adults' PRDs are characterized by lower walking speeds, longer decision times, and acceptance of larger gaps [11], [16], [28], [29]. Child pedestrians have a poor ability to perceive approaching vehicle speeds and primarily rely on spatial distance when judging safe crossing gaps [30], [31]. Risk-taking and non-checking traffic behaviors are common among child pedestrians of all ages [17], [27], [32]. Gender differences in PRDs have also been observed. Males generally wait less time to cross the street than females, are more likely to accept smaller crossing gaps, and are exposed to greater crossing risks [27], [33], [34]. Concerning distractions, numerous studies have shown that visual-manual distractions, such as mobile phone use, occupy pedestrians' visual perception and cognitive resources, resulting in their inability to accurately assess traffic conditions [35]. Additionally, different types of distractions (e.g., auditory and visual-manual) have varying effects on pedestrians [36]. Overall, current research has revealed numerous behavioral patterns and characteristics of PRDs. The challenge of modeling these behaviors to establish high-fidelity vehicle-pedestrian interaction simulation models has attracted significant research attention. The following section introduces PRD-related modeling research.

C. Models for pedestrian road-crossing decisions

Modeling PRDs holds critical implications for simulating traffic interactions. Existing computational models for PRDs have been developed based on various theories and hypotheses, such as rule-based choice models [37], data-driven approaches [38], game theoretical models [39], and cognitive models

[40]. Regarding rule-based choice models, since pedestrians usually make crossing decisions by evaluating gaps between approaching vehicles (also known as gap acceptance behavior), this concept has led to the development of gap acceptance models. For example, HCM2000 [41] proposed a gap acceptance model, wherein the gap acceptance threshold is a constant influenced by road width and pedestrian walking speed. Himanen et al. [42] assumed that the gap acceptance threshold followed binomial distribution and proposed a binary logit gap acceptance model. Zeng et al. [43] combined SFM with a binary logit gap acceptance model to simulate pedestrian crossing behavior. For data-driven approaches, [44] and [37] applied ANNs and Support Vector Machine as solvers for gap acceptance models, respectively. Wang et al. [45] established a deep RL model, assuming PRDs were made based on the pedestrians' noisy sensory information of vehicle kinematics. For game theoretical models, PRDs were often modeled as sequential games, such as the Stackelberg game and the Chicken game [1], [46], where vehicle and pedestrian decisions were determined by solving the Nash equilibrium of a given payoff matrix. Furthermore, one type of model, namely evidence accumulation models (EAM) [25], [40], was based on cognitive theories and assumed that PRDs were the result of an evidence accumulation process and were determined after the accumulated evidence reached a given threshold. All the above-mentioned models characterize PRDs from different perspectives, but these methods either fully or partially have not bridged the research gaps, as identified and discussed below.

D. Research Gaps in pedestrian road-crossing decision modeling

Very few models specifically solved PRDs in vehicle-yielding scenarios and struggled to handle several critical issues. Firstly, most existing models merely relied on the gap acceptance assumption [5]. However, vehicle-yielding scenarios are complex, requiring pedestrians to continuously observe vehicles' status and update their decisions until their expectations are satisfied [47]. The gap acceptance assumption ignores pedestrians' estimation of vehicle-yielding behavior. Therefore, existing models have not sufficiently considered the impacts of vehicle kinematics on pedestrians. For example, some models neglected the impacts of vehicle deceleration behavior [48], while other models assumed that vehicle speed has a monotonic impact on PRDs [1]. Yet, previous observations indicated that vehicle deceleration behavior has impacts on pedestrians [49] and the impact of vehicle speed is non-monotonic [24]. Furthermore, existing models fail to capture the temporal-dynamic nature of PRDs in vehicle-yielding scenarios, i.e., the delay between decision and action, known as crossing initiation time. For example, in [50], [51], the decision-making threshold for PRDs was a deterministic linear model. However, previous observations found that the distribution of PRDs along time in vehicle-yielding scenarios tends to follow bimodal patterns [40], as shown in Fig. 1. The bimodal distribution pattern suggested that pedestrians prefer to cross the road when traffic gaps are sufficiently

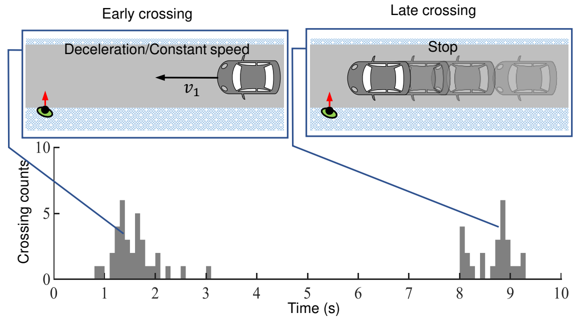

Fig. 1. The bimodal distribution pattern of PRDs in vehicle-yielding scenarios

large or when vehicles are about to stop. In between these two situations, few pedestrians cross the road. Accordingly, in order to simulate PRDs in vehicle-yielding scenarios, it is crucial to reproduce temporal-dynamic decisions.

Regarding the existing data-driven methods, for example, in [44] and [37], they are also based on gap acceptance assumption. Not only do they have the problems mentioned above, but they are also less interpretable, making it difficult to explain the modeling results. Regarding game theoretical models, these models typically assumed strong interactions between pedestrians and vehicles, with one actor constantly responding to the actions of another actor tactically [1], [46]. However, according to naturalistic observations, such strong interactions only account for a small part of all interactions, and most interactions are implicit and can be completed with just a glance [52], [53]. Additionally, most payoff matrices of games were manually designed models, lacking support from behavioral theories and exhibiting limited simulation capabilities [54].

Generally, existing approaches for PRD simulation are rarely based on specific behavioral or psychological theories and do not describe perceived information from the pedestrian perspective. Instead, external physical factors that may not be directly available to pedestrians, like vehicle distance, are often used [55], [56]. As we discussed in Section I-B, most vehicle kinematics, age, and distraction effects are attributed to the impacts on the pedestrians' perception system. Hence, to understand PRDs, we need to investigate the involved perceptual cues and identify their functions. When crossing the road, pedestrians rely on their visual perception of the space around them. Vision cues have been demonstrated as the main source of information used by pedestrians [57]. Specifically, the well-established perception theory indicates that as an object moves close to the observer, its increasing image on the observer's retina can cause the observer to perceive it as an approaching object [58]. Evidence has shown that psychophysical perceptual cues, such as visual looming and τ, are closely related to PRDs [59]. Hence, simulating PRDs based on human perception mechanisms may provide anthropomorphic and perceptually plausible results.

Finally, it is worth noting that the EAM, guided by cognitive theories, provides a powerful explanation tool for PRDs, which could solve some of the above issues. However, those models focused on simulating cognitive processes instead of crossing behavior, resulting in relatively high computational resource consumption. Furthermore, due to limitations in cognitive model complexity, it is unclear whether the EAM could effectively incorporate perceptually plausible cues into the model. Since EAM is currently the most suitable model for solving PRDs in vehicle-yielding scenarios, this study uses EAM as a baseline model for comparison.

E. Research contributions

To address these research gaps mentioned above, we develop a perceptually plausible model for PRDs in vehicle-yielding scenarios, i.e., the PT-PRD model. As a basis, the proposed hybrid perception strategy draws on an established framework of visual space perception, in which pedestrians adopt different visual cues to evaluate risks during the vehicle approach [24], [60]. We extend this framework by showing how pedestrians selectively use visual cues to finalize their crossing decisions in vehicle-yielding scenarios. An empirical dataset collected in a highly immersive CAVE-based simulated environment is applied to test the model (For details on the dataset, please see [61]). These results show that the proposed model captures well the rather subtle patterns seen in the empirical data in terms of both crossing decisions and crossing initiation times. The simulating decisions align well with the observations and show comparable or better modeling performance than the EAM and the ANNs approach, supporting the good performance of the PT-PRD model.

The rest of this paper is organized as follows. Section 2 presents the details of the PT-PRD, EAM, and ANNs approach. The empirical dataset, calibration, and evaluation methods are introduced in Section 3. In Section 4, the simulation results and key features of the proposed model are discussed. Finally, Section 5 presents the discussion and conclusions.

II. Methodology

PRDs may involve several cognitive stages, such as perception, comprehension and decision making, and response execution [62]. Consequently, the development of a computational model for PRDs involves addressing two pivotal questions. The first question pertains to the perceptual information utilized by pedestrians. A psychophysical representation is established for two visual cues that pedestrians may perceive. However, merely identifying these visual cues is insufficient for reproducing PRDs. It is equally essential to understand how pedestrians employ these visual cues in making crossing decisions. To address this, a hybrid perception strategy is proposed, delineating when and how pedestrians utilize specific cues. Subsequently, discrete choice and initiation models, grounded in the hybrid perception strategy, are introduced to characterize PRDs. Finally, a brief introduction to EAM and data-driven models as a comparison is provided.

A. Psychophysical representations of visual information

Change rate of visual angle. When an object approaches an observer, its enlarged image on the observer's retina allows

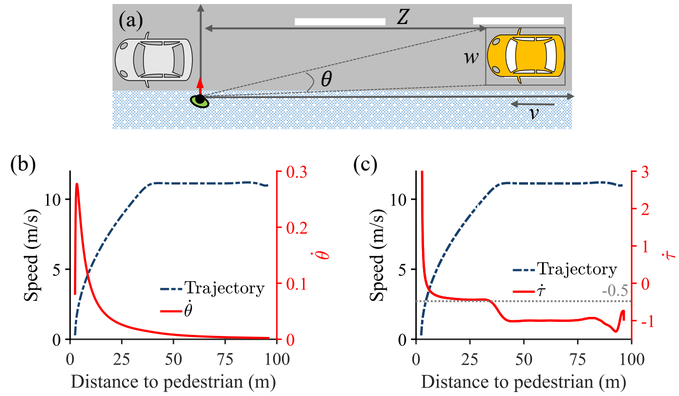

Fig. 2. (a) Visual cues in road-crossing scenarios. (b) and (c) show curves of $\dot{\theta}$ and $\dot{\tau}$ in a specific scenario, where the vehicle drives at 25 mph (11.18 m/s), brakes at 38.5 m from the pedestrian with a constant rate of deceleration, -1.73 $\mathrm{m/s^2}$, and stops 2.5 m from the pedestrian. The corresponding $\dot{\theta}$, $\dot{\tau}$ values and vehicle speed are shown in (b) and (c). For detailed information on traffic scenarios considered in this study, please refer to the empirical data in Section III-A

the observer to perceive it as an approaching object [58]. The expansion rate of the image is correlated to the sensation of collision threat [58], [63], generally quantified as the change rate of the visual angle subtended by the approaching object at the observer's pupil [64]. Suppose a road-crossing scenario, as shown in Fig. 2a, a vehicle approaching a pedestrian at speed v. The visual angle subtended by the car is specified by θ, and its first temporal derivative is given by:

$$\theta = 2\tan^{-1}(\frac{w}{2Z}) \Rightarrow \dot{\theta} = \frac{wv}{(Z)^2 + w^2/4} = \frac{w}{T^2 v + w^2/4v} \tag{1}$$

where $\dot{\theta}$ refers to the change rate of visual angle. Z and w denote the vehicle distance from the pedestrian and its width. According to the above equation, $\dot{\theta}$ is positively correlated with vehicle speed and negatively correlated with vehicle distance, suggesting that pedestrians perceive a higher risk of collision as vehicle speed increases (when distance is constant) or distance decreases (when speed is constant). Moreover, since the vehicle distance from the pedestrian equals the product of the time gap and vehicle speed, replacing the distance with the time gap, T, and the speed reveals that the time gap and the speed have a negative impact on $\dot{\theta}$, suggesting pedestrians could perceive lower collision risk as vehicle speed decreases (when time gap is constant) or time gap decreases (when speed is constant).

Change rate of τ. To avoid potential collision events, humans require both the spatial and temporal properties of objects. However, $\dot{\theta}$ does not provide veridical information on the Time to Collision (TTC) of an approaching car [60]. For instance, as shown in Fig. 2b, it can be found that although the car slows down significantly for a while, $\dot{\theta}$ still increases and then dramatically decreases just a short while before the car comes to a full stop. Hence, $\dot{\theta}$ does not seem to be a very informative and reliable cue for identifying deceleration. In addition to $\dot{\theta}$, it would seem like pedestrians could benefit from using some visual cues corresponding to the TTC of

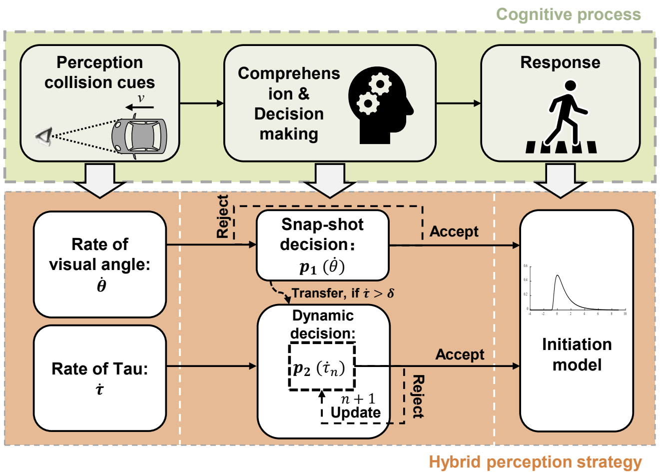

Fig. 3. PRD modeling framework based on hybrid perception strategy.

the approaching vehicle. Prior studies have demonstrated that there is one visual cue that specifies the TTC, i.e., τ, the ratio of visual angle to the change rate of visual angle [64], [65], and its first temporal derivative is given by:

$$\tau = \frac{\theta}{\dot{\theta}} \Rightarrow \dot{\tau} = \frac{Zd}{v^2} - 1 \tag{2}$$

where d is the deceleration rate of the vehicle. $\dot{\tau}$ has been found to be relevant for detecting whether a collision will occur [64]. Suppose that at the t time point, a vehicle begins to brake with a constant d. According to a simple kinematics relationship, d is adequate to stop a vehicle safely in front of the pedestrian only if the following equation satisfies:

$$\frac{v^2}{2d} \leq Z \Leftrightarrow \frac{1}{2} \leq \frac{Zd}{v^2} \tag{3}$$

which means the distance the vehicle will take to stop should be less than, or equal, its current distance from the pedestrian. Afterwards, combing (2) and (3), then we can get:

$$\frac{1}{2} \leq \dot{\tau} + 1 \Leftrightarrow \dot{\tau} \geq -\frac{1}{2} \tag{4}$$

Therefore, it has been mathematically proven that a value of $\dot{\tau} \geq -0.5$ represents that the current deceleration is adequate, and the collision events can be avoided. Further, look at Fig. 2c, $\dot{\tau}$ equals -1 when the vehicle maintains a constant speed. As the car slows down with a constant deceleration rate and stops in front of the pedestrian, $\dot{\tau}$ rapidly exceeds -0.5 and then increases approximately exponentially. Therefore, the psychophysical cue, $\dot{\tau}$, could be the information used to characterize the yielding behavior of the vehicle and judge if the collision events can be avoided.

B. Hybrid perception strategy

Suppose a vehicle first travels at a constant speed or brakes very lightly and then slows down significantly at a distance from the pedestrian. Initially, the car maintains a constant speed, which means there may not be enough visual cues for pedestrians to detect vehicle deceleration behavior, i.e., $\dot{\tau} \leq -0.5$. In these situations, pedestrians may rely on $\dot{\theta}$, which are easy to acquire and process [66]. Consider another situation where the distance between pedestrians and vehicles is too great for pedestrians to tell if the vehicle is giving way to them. Hence, pedestrians in this situation may still tend to use $\dot{\theta}$ to judge if the collision is imminent rather than estimating the behavior of the vehicle. In contrast, when the vehicle drives close to pedestrians and decelerates significantly, i.e., $\dot{\tau} \geq -0.5$, pedestrians then tend to rely on $\dot{\tau}$ to judge if the vehicle can slow down or stop in front of them. $\dot{\tau}$ provides veridical and reliable information about the approaching vehicle at the time [24]. Moreover, in previous observations [59], [61], it has been found that many pedestrians quickly finalized their crossing decisions based on the collision risk before they detected the yielding behavior of the approaching vehicle. These findings suggest that pedestrians may prioritize $\dot{\theta}$ visual cues for crossing decision-making.

Consequently, the above discussion posits that pedestrians can flexibly adjust their crossing decisions based on a hybrid perception strategy, as illustrated in Fig. 3. Specifically, pedestrians tend to prioritize $\dot{\theta}$ for crossing decision-making. Only when $\dot{\tau} \geq \delta$, pedestrians instead use $\dot{\tau}$ as the main cue to their crossing decision. δ is a threshold indicating that pedestrians detect the deceleration behavior of the vehicle. According to (4), it is initially determined to be -0.5. We will verify this hypothesis based on data later.

C. Formulations of the perceptually plausible PRDs model

Two succinct road-crossing decision models are formulated based on the proposed hybrid perception strategy:

Snap-shot decision model: Considering the hybrid perception strategy, pedestrians prioritize $\dot{\theta}$ to assess the collision risk from an approaching vehicle. If no additional cues indicate a decrease in the vehicle's speed, making a decision to cross the road again after a prior rejection would be irrational, as $\dot{\theta}$ continues to increase, signifying a persistent rise in the collision risk. Therefore, pedestrians' rational behavior in such traffic situations involves either waiting for another crossing opportunity (allowing the vehicle to pass first) or waiting for the vehicle to yield. In this context, pedestrians should make their decisions relatively swiftly to avoid missing opportunities (Fig. 3). According to the assumption, the snap-shot decision based on $\dot{\theta}$ is expressed by:

$$p_1(\dot{\theta} \mid \beta_0, \beta_1) = \frac{1}{1 + \exp\left(-\beta_1 \ln(\dot{\theta}) - \beta_0\right)} \tag{5}$$

where $p_1(\dot{\theta})$ denotes pedestrian binary logit crossing probability for the approaching vehicle with a $\dot{\theta}$ value, which $\dot{\theta}$ only refers to the change rate of visual angle at the time point when a traffic gap is available, or pedestrians first observe the approaching vehicle. In is the natural logarithmic transformation [14]. β_0 and β_1 are the model parameters that need to be estimated based on the data using the Maximum Likelihood Estimation method (Please refer to III-C Section).

Dynamic decision model. However, when the snap-shot decision is rejected by pedestrians and the deceleration behavior of the vehicle becomes evident, i.e., $\dot{\tau} \geq \delta$, pedestrians then shift to make their decision based on the yielding behavior of the approaching vehicle. It is assumed that pedestrians dynamically assess the crossing opportunity based on $\dot{\tau}$ until they finally decide to cross the road (Fig 3). Consequently, the decision model based on $\dot{\tau}$ is dynamic, re-running after each rejection, and is expressed as:

$$\begin{aligned} &P_t(\dot{\tau}_t) = p_2\left(\dot{\tau}_t \mid \beta_2, \beta_3\right) \cdot \left(1 - P_{t-1}(\dot{\tau}_{t-1})\right) \\ &\text{where} \quad p_2\left(\dot{\tau}_t \mid \beta_2, \beta_3\right) = \beta_3 \dot{\tau}_t + \beta_2; \quad p_2 \in [0, 1] \end{aligned} \tag{6}$$

In (6), $P_t(\dot{\tau}_t)$ denotes the recursive crossing probability at the t-th step. The term $p_2(\dot{\tau}_t)$ represents the crossing probability derived from a linear regression based on the $\dot{\tau}_t$ and is constrained within the range [0, 1]. The model parameters, β_2 and β_3, are estimated from empirical data using the Maximum Likelihood Estimation method (Please refer to III-C Section). The following example illustrates how to calculate the recursive crossing probability P_t. Initially, P_0 equals to $p_1(\dot{\theta})$, reflecting a snapshot decision made previously according to (5). Then, using (6), the second step calculates $p_2\left(\dot{\tau}_1\right)$ and $P_1(\dot{\tau}_1)$. This step is repeated for T recursion steps until the vehicle stops. Finally, when the vehicle stops, $p_2\left(\dot{\tau}_t\right)$ equals 1 and the recursion ends.

Initiation model. As shown in Fig. 3, the third part of the PT-PRD model is the initiation model, accounting for the temporal information of crossing decisions. In this study, initiation time refers to the time duration between when the rear end of the previous car passes the pedestrian position and when pedestrians start crossing. We have previously demonstrated that the distribution of pedestrian crossing initiation time, t_{int}, could be represented using the Shifted-Wald model [67], also known as the Inverse Gaussian model, given by:

$$\begin{aligned} &SW(t_{int} \mid a, \alpha, \gamma) \\ \Rightarrow &\frac{a}{\sqrt{2\pi (t_{int} - \gamma)^3}} \exp\left(\frac{-[a - \alpha(t_{int} - \gamma)]^2}{2(t_{int} - \gamma)}\right) \end{aligned} \tag{7}$$

where the Shifted-Wald model is controlled by three parameters, namely a, α, and γ. a affects the deviation of the distribution around the mode. α influences the magnitude of the tail. γ represents the shift of the distribution [68]. We further establish two Shifted-Wald models, $SW_1\left(a_1, \alpha_1, \gamma_1\right)$ and $SW_2\left(a_2, \alpha_2, \gamma_2\right)$, that are responsible for snap-shot decisions and dynamic decisions, respectively. a_1, a_2, α_1, α_2, and γ_1 are parameters that need to be estimated based on the data using the Maximum Likelihood Estimation method (Please refer to III-C Section). γ_2 represents the time point corresponding to the dynamic decision steps, t.

Finally, combing (5), (6), and (7), the density function of the PT-PRD model is as follows:

$$f(t_{int} \mid \dot{\theta}, \dot{\tau}_t) = p_1(\dot{\theta}) \cdot SW_1(t_{int}) + \sum_{t=1}^{T} \left(P_t(\dot{\tau}_t) \cdot SW_2(t_{int})\right) \tag{8}$$

where $f(t_{int} \mid \dot{\theta}, \dot{\tau}_t)$ indicates that, given the model parameter set and variables, $\dot{\theta}$ and $\dot{\tau}_t$, the dynamic pedestrian crossing probability density function of the vehicle approach process is the joint function of snap-shot decision function and all dynamic decision functions. The simulation process of the PT-PRD model is illustrated in Algorithm. 1.

Algorithm 1 The simulation process of the PT-PRD model

Output: Decisions: $\boldsymbol{u}$

Crossing initiation time: $\boldsymbol{t}_{int}$

$N_r = N$ // Remaining pedestrians number N_r and total pedestrians number N
for t time step in time T **do**
  for i pedestrian in N_r **do**
    $u_i \leftarrow Binomial(1, p_1(\dot{\theta}))$ // Sampling: crossing decision
    if $u_i == 1$ **then**
      $t_{int,i} \leftarrow SW_1$ // Sampling: crossing initiation time
    else if $\dot{\tau}_t \geq \delta$ **then**
      $u_i \leftarrow Binomial(1, p_2(\dot{\tau}_t))$ // Sampling: crossing decision
      if $u_i == 1$ **then**
        $t_{int,i} \leftarrow SW_2$ // Sampling: crossing initiation time
      end if
    end if
  end for
  $N_r = N_r - \text{length}(\boldsymbol{u}_i)$ // Update remaining pedestrians
end for

D. Evidence accumulation model

To demonstrate the performance of the PT-PRD model, EAM is chosen as a comparison model [40], defining a psychological term called generalized TTC, given as follows:

$$\begin{aligned} \bar{\tau} = \tau & \quad \text{(TTA effect)} \\ + \beta_Z\left(Z/v' - \tau\right) & \quad \text{(Distance effect)} \\ + \beta_{\dot{\tau}}(\dot{\tau} + 1) & \quad \text{(Vehicle yielding effect)} \\ + \beta_H H & \quad \text{(eHMI effect)} \\ + \infty \text{ if } \tau < \tau_p & \quad \text{(two vehicles)} \end{aligned} \tag{9}$$

where the different $\beta_?$ above are coefficients. The generalized TTC comprises τ, $\dot{\tau}$, Z, v', and τ_p, and H. v' denotes the prior speed, which is the typical speed a pedestrian assumes the vehicle was traveling before seeing the vehicle. τ_p indicates the TTC at which pedestrians judge that the vehicle has passed. H means if the vehicle turns on the external human-machine interface (eHMI). Based on the generalized TTC, EAM is not purely based on perceptual cues but incorporates real vehicle kinematics. The defined generalized TTC was applied as the psychological cue to feed into an evidence accumulation model, given by:

$$\begin{aligned} &\frac{dA}{dt} = -\lambda A(t) + s(t) + \epsilon(t) \\ &t' = \min(t) \text{ s.t. } A(t) > A' \end{aligned} \tag{10}$$

where $s(t) = \sigma(m(\bar{\tau} - \bar{\tau}'))$, showing that the generalized TTC is transferred through the Sigmoid function, where m and $\bar{\tau}'$

are a scaling factor and a threshold. The accumulation unit, $A(t)$, adds $s(t)$ and white noise, $\epsilon(t)$, every timestep, t, and forgets part of the information with ratio λ. Finally, a crossing decision is made at time t' when the evidence threshold A' is passed. σ, $\beta_?$, τ_p, λ, m, $\bar{\tau}'$, and A' are coefficients that need to be estimated. For detailed information on the EAM, please refer to [40].

E. Data-driven approach: time-dynamic artificial neural networks for PRD modeling

Besides the EAM, we are also interested in comparing the PT-PRD model with a data-driven approach, to see its uniqueness. However, to the authors' knowledge, existing data-driven approaches have not shown a line on modeling time dynamic PRDs in vehicle-yielding scenarios, instead simulating discrete gap acceptance decisions [37], [44]. The modeling granularity of these discrete gap acceptance decisions is much lower than that of time-dynamic modeling methods, resulting in results that cannot be fairly compared. Therefore, we proposed a time-dynamic PRD model using vanilla ANNs for comparison purposes. Specifically, sampled pedestrian decisions and vehicle kinematic cues were arranged in time. Ensure that each sampling point has the corresponding cues and decisions, as follows:

$$\begin{aligned} & I=(1,2,3,...,i,...,z)\,; \\ & \mathbf{VK}=\{VK_1,VK_2,VK_3,...,VK_i\}\,; i\in I \\ & \mathbf{u}=\{u_1,u_2,u_3,...,u_i\}\,; i\in I \end{aligned} \tag{11}$$

where I is a set of sampling points with size z. VK_i is a vehicle kinematic cue at sampling point i, including vehicle speed, acceleration rate, and distance. u_i is pedestrian's binary crossing decision at sampling point i. Here, the sampling frequency of the data is 10 Hz. Based on these data, a time-dynamic PRD model is established using a vanilla ANNs. Initially, the $\mathbf{VK}$ vector is used as input to a Hidden layer, followed by a BatchNorm (BN) layer and ReLu activation, ϕ, as follows:

$$\begin{array}{rc} \text{Input layer :} & \mathbf{X}=(\mathbf{VK},\mathbf{u}) \\ \text{Hidden layers :} & H_n=\phi\left(\text{BN}\left(\mathbf{W}_n\mathbf{X}+\mathbf{b}_n\right)\right) \\ \text{Output layer :} & \mathbf{P}=f_{\text{sigmoid}}\left(H_n\right)) \end{array} \tag{12}$$

where $\mathbf{W}_n$ and $\mathbf{b}_n$ are parameters of ANNs. Since pedestrians' crossing decisions are in binary format, the output layer is a Sigmoid layer, f_{sigmoid}, and the loss function is a binary Cross Entropy function accordingly:

$$\ell(VK_i,u_i)=-\left[u_i\log(p(VK_i))+(1-u_i)\log(1-p(VK_i))\right] \tag{13}$$

where $p(VK_i)$ is the crossing probability obtained by the ANNs with VK_i as input.

III. Calibration and validation

A. Empirical data

The empirical data used in this study was originally collected and applied to compare the impacts of different external human-machine interfaces on pedestrian road-crossing behavior by [61] (Ethics approval number: LTTRAN-107).

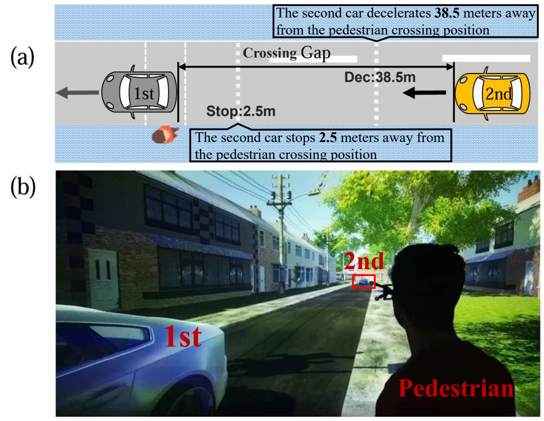

Fig. 4. Diagrams of experiment and simulation vehicle-yielding scenario. (a) Schematic top view of the road-crossing scenario. (b) A photo shows the traffic scenario in the HIKER simulator from a participant's perspective, where the first car is about to pass the participant and the second car comes into view [61].

For detailed information on the experiment, please refer to that study. Here, a summary of parts of the data we used is provided. The dataset was collected using a CAVE-based pedestrian simulator at the University of Leeds. Sixty participants (36 male and 24 female, aged 19 to 36) were recruited via the driving simulator database. Their ages ranged from 19 to 34, with an average of about 28.

Apparatus and experiment design. The cave-based pedestrian simulator includes three wall projections and a floor projection. Eight 4K projectors project the images at 120 Hz. The walking environment in the simulator is 9 meters long and 4 meters wide, providing participants with ample walking space. Ten cameras track the tracking glasses on the participant's head to adjust the images to fit the participant's perspective. Regarding the design of the experiment, A residential block scenario with a 3.5m wide one-lane road and an uncontrolled intersection was generated in the simulator using Unity (Fig. 4b). A row of trees was included on one side of the road to indicate the starting position for the pedestrian. For the traffic scenario, there were two vehicles, 1.95 m wide and 4.95 m long, driving in the center of the road. The first car started 96 m away from the pedestrian, and the second car kept one of the four time gap sizes behind the first car, i.e., 2, 3, 4, or 5 s. In the beginning, both vehicles drove at one of the three constant speeds, i.e., 25, 30, or 35 mph. The first car always maintained a constant speed. However, the second car started decelerating at a constant rate when it arrived at 38.5 m from the participant and came to a stop at a distance of 2.5 meters from the participant (Fig. 4a). Accordingly, the deceleration rates for 25, 30, and 35 mph were 1.73, 2.50, and 3.40 m/s^2, respectively.

Procedures. This experiment applied stationary crossing initiation, wherein participants initially stood at the curb and pressed a button to initiate the scenario. Two vehicles then appeared on the road, creating a crossing opportunity when the first vehicle passed the participant (Fig. 4a). Participants were instructed to cross the road between two vehicles when they felt comfortable and safe to do so, either after the first

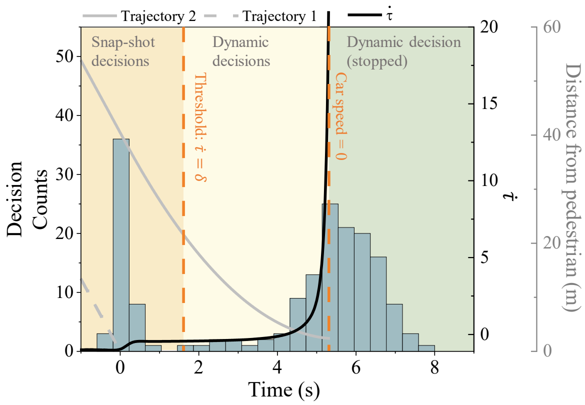

Fig. 5. Data partitioning example. The histogram of crossing initiation time overlaps with the $\dot{\tau}$ curve and the distance curves of the approaching vehicles (grey, dashed curve for the first car and solid curve for the second car). The data are divided into three parts: snap-shot decision, dynamic decision (deceleration), and dynamic decision (stopped), based on the criteria: $\dot{\tau} = \delta$ and the speed of the second car is zero. Moreover, the data in the dynamic decision (deceleration) are further split into several subgroups based on consecutive $\dot{\tau}$ intervals.

vehicle or when the second vehicle slowed down or stopped. A trial ended when they reached the opposite pavement. The experiment had twelve traffic scenarios with three initial speeds and four initial time gaps. Each participant experienced 36 trials in a random order, resulting in 2160 trials in total. The experiment also included other scenarios, but the present data only came from the scenarios where the approaching vehicle decelerated without external human-machine interfaces.

B. Data processing

Before fitting the model to the data and analyzing the results, the data needed to be properly reduced and processed to meet modeling requirements. The crossing decisions fell into three categories: snap-shot decisions, dynamic decisions when the vehicle decelerated, and dynamic decisions after the vehicle stopped. Fig. 5 shows an example of data partitioning where data are grouped in terms of $\dot{\tau}$ value and the speed of the approaching vehicle. Specifically, those pedestrians who crossed the road when $\dot{\tau}$ was smaller than δ, were grouped into the snap-shot decisions. Those who crossed the road when $\dot{\tau}$ was bigger than δ and before the car fully stopped were grouped into the dynamic decision (deceleration). The others who crossed the road after the car had come to a complete stop were sorted into the dynamic decision (stop) group. According to (6), we assume that in the dynamic decision (deceleration), pedestrians could recurrently evaluate the crossing opportunity until they feel comfortable crossing. Therefore, the data in the dynamic decision (deceleration) group were further sorted into a few subgroups to represent the above process. The division method is that the $\dot{\tau}$ curve of the approaching vehicle (ranging from δ to 20) was divided into 43 intervals. The length of intervals increased in increments according to the equation, i.e., $2\mathrm{e} - 8 \times i^5 + 0.003$, where i denotes the number of the interval between 1 and 42. Finally, Data belonging to the same $\dot{\tau}$ interval were classified as one subgroup.

Two main metrics were measured for each trial of data: the crossing initiation time and binary crossing decision. The crossing initiation time is the time point when pedestrians start to cross the road. The base point of crossing initiation time was set to the time point when the first vehicle passed the pedestrian (Fig. 4 and 5). Moreover, the following criteria were used to identify the onset of PRDs: (a) The pedestrian's longitudinal position should exceed the pavement edge. (b) The change in longitudinal position should be more than 0.003 m over a simulated time step of 120 Hz. (c) To rule out incomplete crossings, where pedestrians start to cross but then return to the pavement, participants must step out one meter from the pavement edge one second after meeting the first two conditions. The binary crossing decision indicates whether pedestrians cross the road in a certain group or subgroup.

C. Model calibration

For the PT-PRD model, the decision model and initiation model were fit to different types of data. Specifically, the snap-shot decision model, (5), was fitted to binary crossing decision data, while the dynamic decision model, (6), was applied to road crossing probability data. For the initiation models, crossing initiation time data for snap-shot decision groups and dynamic decision groups varied in scale and thus required normalization to a single scale. In the snapshot decision group, initiation times were normalized based on the time when the rear end of the first vehicle passed the pedestrian. For the dynamic decision groups, initiation times were normalized using time points corresponding to the lower bounds of the relevant $\dot{\theta}$ intervals. Subsequently, two Shifted-Wald models, i.e., SW_1 and SW_2, were fitted to the normalized initiation time data for the snapshot and dynamic decision groups, respectively.

Parameter estimation was performed by identifying the optimal parameter set that maximizes the model's likelihood using the Maximum Likelihood Estimation method. This maximization is equivalent to minimizing the negative log-likelihood function. The log-likelihood functions for the decision models, (5), (6), and (7), and Wald models were established following:

$$\min_{\Theta} \left(-\ln \mathcal{L}_i(\Theta)\right)$$
$$\mathcal{L}_i(\Theta) = \begin{cases} \mathcal{L}_1\left(\beta_0, \beta_1\right) = \prod_{m=1}^{12} p_1\left(\dot{\theta}_{\mathrm{m}}\right) \\ \mathcal{L}_2\left(\beta_2, \beta_3\right) = \prod_{t=1}^{T} p_2\left(\dot{\tau}_t\right) \\ \mathcal{L}_3(a, \alpha, \gamma) = \prod_{s=1}^{S} SW\left(t_{int,s}\right) \end{cases} \tag{14}$$

where m denotes the number of snapshot decisions, which corresponds to the twelve condition groups of experimental data. t represents the number of dynamic decision steps, and s is the size of the crossing initiation time. The crossing initiation time functions of snapshot and dynamic decisions, $SW_1\left(a_1, \alpha_1, \gamma_1\right)$ and $SW_2\left(a_2, \alpha_2\right)$, were estimated separately. To minimize these functions, MATLAB's 'fminunc' algorithm was utilized [69]. All parameters are shown in Table I

For the EAM, parameters were estimated using the same dataset; therefore, we adopted the estimates from [40]. It is

TABLE I
ESTIMATED PARAMETERS OF MODELS

PT-PRD model								
β_0	β_1	β_2	β_3	a_1	α_1	γ_1	a_2	α_2
-10.34	-2.25	0.01	0.01	8.09	4.50	1.47	2.40	2.23
EAM								
First car	σ	λ	m	$\bar{\tau}'$	A'	τ_p	β_Z	$\beta_{\dot{\tau}}$
	0.6	1.8	0.6	∞	0.8	-0.1	0.8	0.6
Second car	σ	λ	m	$\bar{\tau}'$	A'	τ_p	β_Z	$\beta_{\dot{\tau}}$
	0.6	1.8	0.6	1.6	0.8	$-\infty$	0.8	0.6

important to note that EAM accounted for data with the eHMI effect, which was not considered in our model. All parameters are shown in Table I. The ANNs approach consists of an input layer with 4 features, three hidden layers with decreasing numbers of units (64, 32, 16), and a total of 3169 parameters ($[64\times4, 64\times1]+[64\times1, 64\times1]+[32\times64, 32\times1]+[32\times1, 32\times1]+[16\times32, 16\times1]+[16\times1, 16\times1]+[1\times16, 1\times1]$).The training process utilizes the Adam optimizer and is configured to run for a maximum of 50 epochs with mini-batches of 5000 samples. The initial learning rate is set to 0.001.

D. Validation of the perception threshold theory

Although the collision perception threshold, δ, has been mathematically proven to be -0.5 according to (4), it is necessary to validate the plausibility of the theoretical results with data. Therefore, an exhaustive grid search method over δ was carried out by finding the minimum Root Mean Square Error (RMSE) of predicted road-crossing probability, given by:

$$\text{RSME} = \sqrt{\frac{1}{N}\sum\left(p_o - p_{pre}\right)^2} \tag{15}$$

where p_0 and p_{pre} are the observed and predicted road-crossing probability of all groups. $N = 12 + T$ is the number of all decision groups. The δ range from -0.8 to 1 was uniformly divided. Hence, the tendency of modeling bias could be visualized as the function of δ.

E. Model evaluation

Moreover, to evaluate the similarity between simulated results and actual data. The Kolmogorov-Smirnov (KS) test was applied as follows [70]:

$$D_{n,m} = \sup\left|\boldsymbol{F}_n(x) - \boldsymbol{F}_m(x)\right| \tag{16}$$

where sup denotes the supremum function. $\boldsymbol{F}_n(x)$ and $\boldsymbol{F}_m(x)$ are the empirical cumulative distribution functions (ECDF) of the observed data and simulated result. n and m represent the size of the samples. The K-S test rejects the null hypothesis (i.e., two samples are drawn from the same probability distribution), if $D_{n,m}$ is bigger than the threshold given by the selected significance level (0.05 significance level was chosen). The range of the $D_{n,m}$ is from 0 to 1, measuring the maximum vertical distance between the ECDF of a sample and the ECDF of a reference distribution. 0 means that the sample exactly conforms to the reference distribution, and 1 means that the sample does not resemble the reference distribution at all. In the study, the data were applied as the reference distributions. The simulated results were obtained using the Mart-Carlo sampling method based on the PT-PRD model, and the number of samples is 200 to match the experiment data size. In addition to the KS test, RSME and relative root square mean error (rRSME) are also applied to evaluate the simulation performance, given by the following equation:

$$\text{rRMSE} = \sqrt{\frac{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(\hat{y}_i\right)^2}} \tag{17}$$

where y and $\hat{y}$ are simulations and empirical results. n is the size of the sample. rRMSE is the RMSE normalized by the root square value and can be used to compare different types of measurements. An rRMSE value between 0-0.1 indicates excellent model accuracy [71].

IV. Results and Analysis

In this section, the validation of the perception threshold, δ, is first presented. Then, the PT-PRD model is compared with the EAM and the ANNs approach to evaluate its modeling performance. Afterward, we look in detail to investigate if the PT-PRD model captures the vital behavioral patterns of PRD influenced by diverse vehicle kinematics.

A. Collision perception limitation in vehicle yielding scenarios

As shown in Fig. 6, the modeling errors are graphed against the variable δ. The most favorable alignment between the model and the data occurs at approximately -0.5, especially when the car's collision cues slightly exceed -0.5, for instance, -0.44. Deviating from this range of δ values results in an increasing trend in RMSE values for both snapshot and dynamic decisions. Consequently, empirical data support the threshold hypothesis in pedestrian hybrid perception strategy in vehicle yield scenarios, as illustrated in (4). When δ is around -0.5, slightly surpassing this value, pedestrians could perceive the

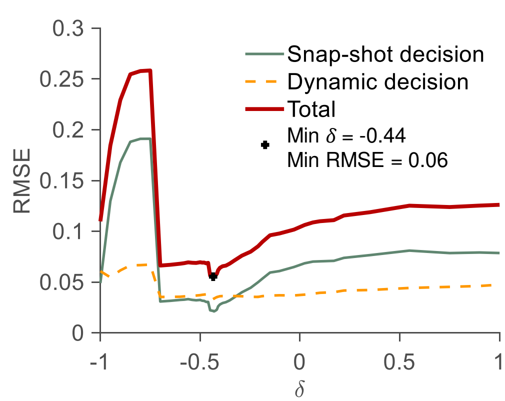

Fig. 6. The modeling RMSE is plotted as a function of δ. The RMSE curves for snap-shot decisions, dynamic decisions, and all decisions are presented, respectively.

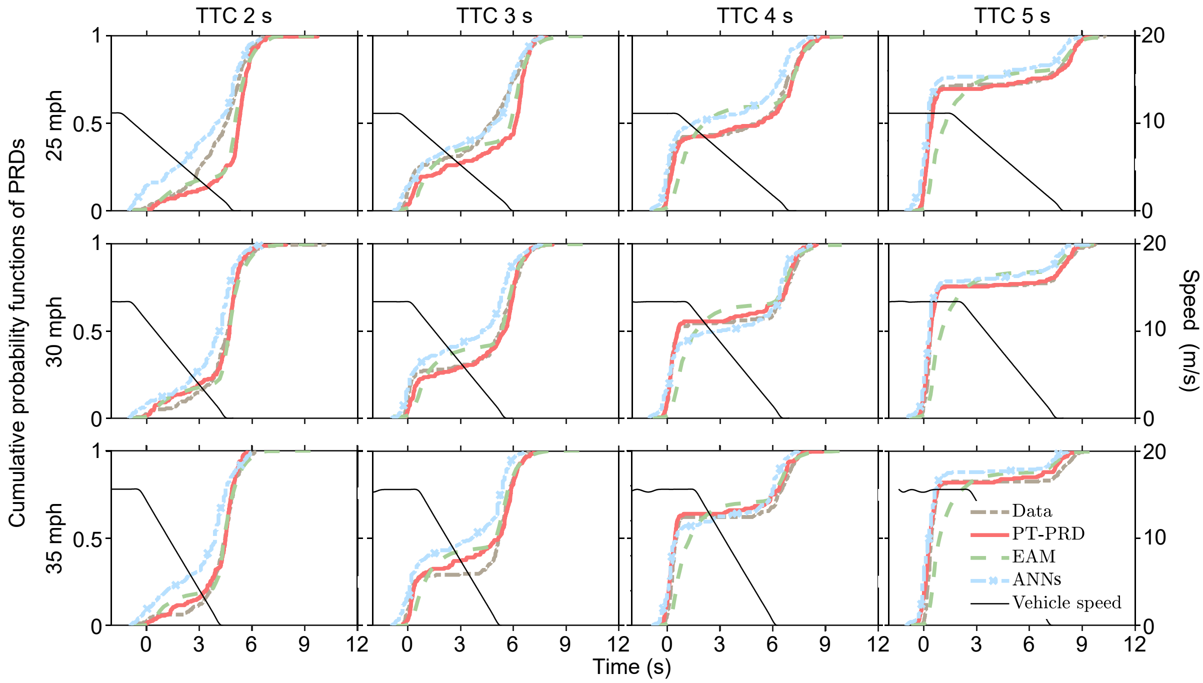

Fig. 7. Comparison of modeling PRDs. Twelve panels present cumulative probability functions for PRDs produced by empirical data, PT-PRD model, EAM, and ANNs approach in twelve traffic conditions. The speed of the yielding vehicle is also presented.

yielding behavior of deceleration vehicles, which leads to a change in the decision-making strategy. In the subsequent validations, a fixed δ value of -0.44 is employed.

B. Model comparison

PRDs. As shown in Fig. 7, the simulated cumulative probability functions (CDFs) for PRDs provided by the PT-PRD model, EAM, and ANNs approach are compared to the empirical data across all traffic conditions. First of all, all models simulate the bimodal pattern of PRDs, where most pedestrians tend to cross the road when the vehicle is relatively far from them or about to stop. These simulation results align with empirical observations [25], [61]. However, as shown in Table. II, under most traffic conditions, the KS test rejects that EAM's and ANNs approaches' simulations and empirical data share the same distributions at a 0.05 significance level. For example, in the 25 mph speed and 4 s TTC condition, EAM indicates that quite a few pedestrians cross the road when the vehicle decelerates but still travels at a relatively high speed (approximately between 1 s and 3 s). In contrast, the CDF of the empirical data is almost horizontal during this duration, indicating very few pedestrians cross in this situation. Such discrepancy between the EAM or ANNs approach and empirical data is consistent across the other 8 traffic conditions. Compared to the EAM and ANNs approach, the PT-PRD model's simulation performance is significantly better. The KS test accepts simulated results in 10 out of 12 conditions, suggesting that the PT-PRD model establishes distributions similar to the empirical data. Generally, the PT-PRD model exhibits superior capabilities than the EAM and ANNs approach based on the selected traffic scenario and dataset.

Crossing initiation time. The mean crossing initiation time of simulated results is shown in Fig 8. Although the EAM and ANNs approach predict mean crossing initiation time across a range of experimental conditions at a good level, RSME = 0.49 and 0.69, the PT-PRD model performs relatively better with RSME = 0.35. it can be found that the EAM and ANNs approach both tend to underestimate the mean crossing initiation time, as most results are lower than the identity line. Combining with PRDs results in Fig. 7, it can be inferred that this is because the EAM and ANNs approach predict more

TABLE II
KS TEST RESULTS

Model	Speed and TTC conditions (mph,s)											
	25, 2	25, 3	25, 4	25, 5	30, 2	30, 3	30, 4	30, 5	35, 2	30, 3	30, 4	30, 5
PT-PRD	0.28*	0.24*	0.11	0.10	0.13	0.13	0.06	0.12	0.09	0.11	0.16	0.12
EAM	0.27*	0.22*	0.27*	0.39*	0.13	0.19*	0.31*	0.37*	0.10	0.18*	0.31*	0.37*
ANNs	0.18*	0.08	0.18*	0.21*	0.24*	0.21*	0.14	0.23*	0.30*	0.25*	0.17*	0.33*

$*$: the KS test rejects the null hypothesis at a 0.05 significance level.

pedestrians to cross the road before the vehicle stops, and the mean crossing initiation time is, therefore, smaller. Again, the overall performance of the PT-PRD model on mean crossing initiation time indicates good agreement between observed and predicted results, suggesting that the simulation power of the proposed model is acceptable.

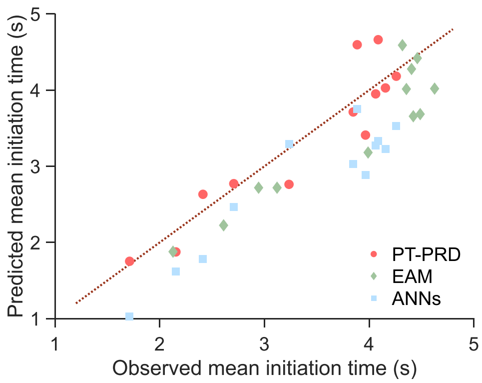

Fig. 8. Comparison of observed and simulated mean crossing initiation time. The identity line is denoted as a dotted line. The calculated RSMEs for the PT-PRD model, EAM, and ANNs approach are 0.29, 0.49, and 0.69, separately.

The above analysis demonstrates that the PT-PRD model can better simulate PRDs in vehicle-yielding scenarios than the EAM and established ANNs approach. In the following sections, the power of the PT-PRD model on reproducing vehicle kinematics impacts on PRDs will be analyzed in more detail to measure its simulation capabilities accurately.

C. Simulation performance

Impacts of vehicle kinematics on PRDs. As illustrated in Fig. 9a and b, PRDs in three decision groups are plotted as a function of TTC and vehicle speed, respectively. In Fig. 9a, the PT-PRD model faithfully replicates all TTC impacts (rRMSE = 0.04), demonstrating that, with the increase in TTC, more pedestrians are inclined to make snap-shot decisions and fewer pedestrians cross the road when the vehicle decelerates or comes to stops. This behavior trend aligns with findings from prior studies [40], suggesting that pedestrians are more prone to rely on TTC cues for their crossing decisions than on vehicle-yielding cues. Moreover, under the 5-second TTC condition, there is a notable decrease in the number of pedestrians crossing when the vehicle decelerates. This could be attributed to the extended temporal distance impeding pedestrians' perception of vehicle behavior, as discussed in [24].

In Fig. 9b, the PT-PRD model effectively captures the majority of speed-related impacts on PRDs (rRMSE = 0.11). With increasing vehicle speed, there is a noticeable trend of more pedestrians making snap-shot decisions. Previous studies have indicated that pedestrians are more likely to cross the road in conditions of higher vehicle speed when TTC is controlled [72]. Our previous study [14] demonstrated that this unsafe behavior pattern may be attributed to limitations in human visual systems. The PT-PRD model accurately reflects this decision pattern using psychophysical visual cues, once again underscoring the efficacy of psychophysical cues in decision modeling. Moreover, the PT-PRD model suggests that, as vehicle speed increases, fewer pedestrians cross the road when the vehicle decelerates. This observation could also be attributed to the extended temporal distance impeding pedestrians' perception of vehicle behavior. In the specific scenario under consideration, we controlled TTC, resulting in a longer spatial distance as the vehicle speed increased. It is noteworthy that the model incorrectly describes the impact of vehicle speed on PRDs when the vehicle comes to a stop. This discrepancy may be attributed to higher modeling biases in lower-speed conditions, as shown in Fig. 7

Impacts of vehicle kinematics on crossing initiation time. In Fig. 9c and d, pedestrians' crossing initiation time is presented as a function of TTC and vehicle speed, respectively. In Fig. 9c, the PT-PRD model effectively represents all TTC impacts on crossing initiation time with deemed acceptable performance (rRMSE = 0.15), illustrating that pedestrians who tend to cross the street during vehicle deceleration and stopping phases exhibit delayed crossing as TTC increases. Moving to Fig. 9d, the influence of vehicle speed on crossing initiation time is portrayed. Once again, the simulation performance of the PT-PRD model is decent (rRMSE = 0.07). The model captures the notable trend that the crossing initiation time of PRD (stop) decreases as vehicle speed increases. These observed behavioral patterns are attributed to the vehicle's kinematics in our specific traffic scenarios; as shown in Fig. 7, with an increase in TTC or vehicle speed, the time for deceleration either increases or decreases. Consequently, these factors affect pedestrians' crossing initiation time.

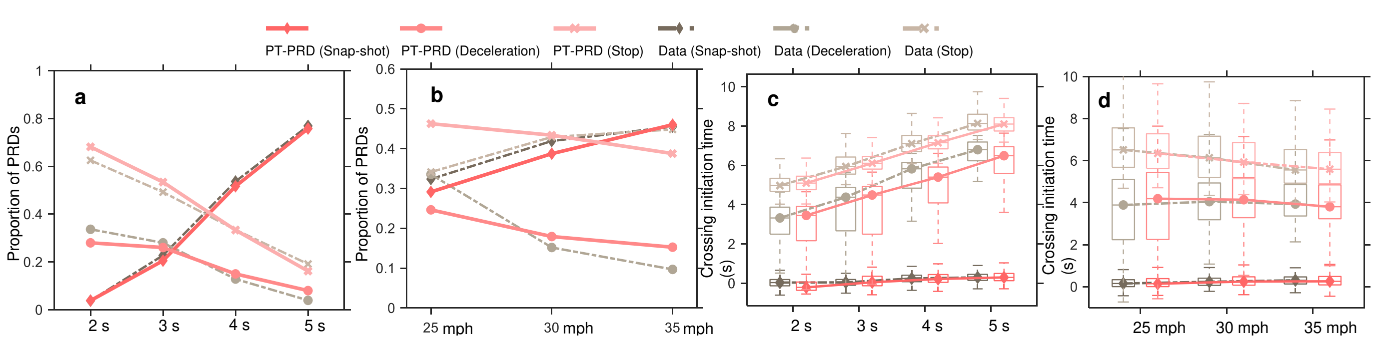

Fig. 9. Simulation results of the PT-PRD model as a function of vehicle speed, TTC, and decision group. Each panel displays simulation results and data using distinct line colors. Gradient color changes and varied symbols designate results for different decision groups.

V. Discussion and conclusion

This study demonstrates that PRDs in complicated vehicle-yielding scenarios can be perceptually plausibly described using a sequence of discrete models grounded in a hybrid perception strategy formalized by the PT-PRD model. PRDs in vehicle-yielding scenarios present heightened complexity. This complexity arises not only due to the bimodal distribution of PRDs in the presence of yielding vehicles but also because PRDs are dynamically influenced by real-time changes in vehicle kinematics [59], [73]. Unlike conventional models, in which pedestrians base crossing decisions on the gap acceptance assumption and ground truth vehicle kinematic cues, the PT-PRD model extends the crossing decision-making mechanism guided by pedestrian visual perception. The results indicate that the proposed model adeptly reproduces PRDs across a range of vehicle-yielding scenarios. A mechanistic explanation is provided for PRDs in such scenarios: pedestrians make crossing decisions employing snap-shot and dynamic decision-making strategies based on different available psychophysical visual cues. A thorough discussion follows, elucidating the novel insights that our study brings to the simulation of pedestrian crossing behavior.

A. Hybrid perception strategy for PRDs

In road-crossing scenarios, where vehicles could adjust their speed in response to pedestrians, such as driving at a constant speed or decelerating, pedestrians require multiple kinematical cues from the vehicles to accurately assess the situation. However, owing to the limitations of the visual system, previous studies have shown that pedestrians struggle to judge actual kinematic cues effectively [72], [74]. Therefore, to simulate human-like pedestrian behavior, it is necessary to explore how pedestrians use clever strategies to compensate for functional limitations. This study demonstrates an alternative approach by employing psychophysical visual cues to model PRDs. The model incorporates potential correlations between psychophysical visual cues and decisions, simulating crossing behavior in various vehicle-yielding scenarios. Specifically, when the deceleration behavior of the approaching vehicle is absent or not clearly evident, pedestrians tend to make snap-shot decisions and rely on the simple optical expansion cues, i.e., $\dot{\theta}$. However, when the visual cues, $\dot{\tau}$, for detecting vehicle-yielding behavior become available, pedestrians transition to a dynamic decision strategy, updating their decisions based on the time-varying $\dot{\tau}$. Our modeling results support that there may be a collision perception threshold, δ, accounting for the transition process between two cross-decision strategies. In a word, one of the key notions highlighted by our results is that pedestrian behavior is simulated based on the visual affordance of vehicle kinematics.

Additionally, the proposed model is compared to the latest PRD model, namely the EAM and ANNs approach [40]. EAM characterizes PRDs through a drift-diffusion process grounded in time-varying cues, emphasizing the simulation of the accumulation process of evidence within the cognitive system rather than perceptually plausibly characterizing crossing behavior. ANN is based on data-driven modeling, directly revealing the patterns behind the data at the expense of interpretability. In contrast, the PT-PRD model delineates PRD using discrete choice models and the hybrid perception strategy, with clearer behavior and perception plausibility, which thereby makes the PT-PRD model different from the latest approach.

B. Temporal-dynamic PRDs

Furthermore, the proposed model surpasses the majority of conventional PRD models, such as gap acceptance models [51], [75], by considering the temporal-dynamic nature of PRDs, i.e., the crossing initiation time [40]. The initiation of PRDs has been demonstrated to have critical implications for achieving realism PRDs [4], [76]. Our proposed model employs a Wald distribution model to account for the timing of PRDs in vehicle-yielding scenarios. The plausibility of the assumptions is supported by simulation results, wherein the PT-PRD model accurately reproduces a range of crossing initiation patterns influenced by diverse vehicle kinematics.

C. Practical Implications

The proposed model holds practical implications for traffic research in various ways. Intuitively, the PT-PRD model can be integrated into traffic simulation tools to generate human-like PRDs for pedestrian agents, thereby enhancing the realism of traffic simulations. The temporal-dynamic PRDs simulation may drive pedestrian simulation to a more fine-grain level. Beyond the context of conventional traffic, our model has significance for the development of AVs. The model can be applied to virtual AV testing platforms to control pedestrian agents in the simulated environment, thereby improving the realism of simulated traffic interactions. In addition, the model avoids directly using actual kinematic cues, such as vehicle distance and speed, and instead formalizes perceptual cues. This treatment could easily model the effects of age or distraction on pedestrian perception, thereby achieving the modeling of pedestrian heterogeneity [77].

D. Limitations

The proposed model, while demonstrating overall effectiveness, presents certain limitations that require further refinement. Notably, the model's performance is suboptimal in specific scenarios, particularly under conditions of 2 s and 25 mph, and 3 s and 25 mph, as illustrated in Fig. 7. In these instances, the model exhibits a more conservative tendency compared to empirical data, suggesting that pedestrians likely consider additional cues to enhance their confidence in crossing decisions, especially in situations involving low speeds and short TTC conditions. One plausible explanation for this discrepancy is that pedestrians may interpret low-speed driving behavior as an indication of yielding intent. Consequently, as visual perception theory advances, there exists potential for further model refinement to address these nuances.

The significance of pedestrian heterogeneity in PRD modeling cannot be overstated. Although the current study did not explicitly account for this factor, future iterations of the model should incorporate variables such as age, gender, and distraction to reflect real-world pedestrian behavior more accurately. This inclusion would enhance the model's applicability and predictive accuracy across diverse pedestrian populations. While this study utilized empirical data collected in a simulator environment for model evaluation, it is important to acknowledge that naturalistic data more closely approximates actual pedestrian behavior. Real-world pedestrian crossings typically involve non-stationary initiation and encompass dynamic phases such as approaching, appraising, and crossing, whereas this study focused primarily on the transition from a static position to crossing. To broaden the applicability of these research findings to real-world scenarios, further investigation and analysis using naturalistic data are essential.

Finally, it is worth noting that increasing the number of parameters and complexity of the neural network may improve the modeling performance, however, excessive number of parameters and worse interpretability will only further strengthen the significance of this study.

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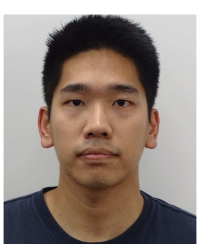

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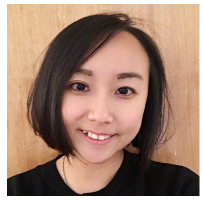

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