Transportation Geotechnics 37 (2022) 100859

Contents lists available at ScienceDirect

Transportation Geotechnics

journal homepage: www.elsevier.com/locate/trgeo

Blind field tests to determine Rayleigh wave velocity on a high-speed railway environment: The reliability of seismic surface waves methods

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ARTICLE INFO

Keywords:
Seismic surface waves
MASW
Dispersion
High-speed railways

ABSTRACT

To ensure track stability and good earthwork performance for high-speed railways the surface wave velocity in the earthworks should exceed the train's speed. Specifications for high-speed rail are now stating minimum Rayleigh wave (V_R) velocities to be checked during construction. This requires suitable reliable geophysical tests, however there are no defined standards or data collection and processing protocols. Additionally, the analysis of these data traditionally relies on the practice and experience of those undertaking the work which can introduce variability in results.

This paper presents the results of a blind comparison trial investigating Multichannel Analysis of Surface Waves test (MASW) for use in high-speed rail earthworks compliance evaluation, concentrating on assessment of Rayleigh wave velocity (V_R). Testing was undertaken by four companies, at the same site on natural ground and a stabilised trial embankment. Defined tests protocols and a test of their own design against a specification to assess V_R were used. The anonymised V_R results show reasonable agreement in the dispersive character of the soil if higher modes are carefully considered when picking a dispersion curve. The V_R results were then investigated against depth using a rule of thumb. Such an approach avoids the inversion processing step (to get more traditional Vs against depth) which potentially introduces variability. This suggests that direct Rayleigh wave data could be used by earthworks engineers to give routine compliance assessment and then if required further investigation undertaken in areas of compliance concern within the overall earthwork.

Introduction

The performance of earthworks in all linear transport infrastructure projects is integral to ensuring good performance of the asset over its life. For this reason, earthwork specification, design, construction and monitoring are crucial. Increasingly more complex requirements of embankment fill material are checked before and during construction to certify that the properties used in the design are met in the field [15].

The new high-speed HS2 railway line in the UK requires a maximum operational speed of 360 km/h, a design life of 120 years, with an intended limited maintenance input over its lifetime to maximise operational availability. This requires specification of detailed target earthwork designs to be met, which must be checked on site during construction. Such compliance verification includes the design of trial embankments and compliance assessment in areas presenting differences in geological conditions along the extensive route of the new railway.

Of the many important earthwork parameters to be assessed, and of relevance in this paper are earthwork layer stiffness, and its consequent relationship with the earthwork's surface wave velocity. If a train's speed approaches the expected Rayleigh wave's speed (V_R) of the ground, dynamic effects may occur [24], which can manifest themselves in transient deformation of the track system which has implications for overall track stability and performance. This train-track and ground-borne vibrations behaviour often termed "critical velocity effects" has been assessed by many authors, especially in very low stiffness "soft" deposits, for example in the high-speed trainline X-2000 in Ledsgard, Sweden [23].

Therefore, for HS2 it is a requirement that suitable geotechnical tests should be undertaken during site investigations and during earthwork construction to ensure a minimum surface wave velocity is reached to a minimum depth to guard against the critical velocity deformations

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https://doi.org/10.1016/j.trgeo.2022.100859
Received 30 May 2022; Received in revised form 8 September 2022; Accepted 10 September 2022
Available online 16 September 2022

described above.

Overall good performance of the railway earthwork and specifically embankments is achieved by the selection and good compaction of appropriate materials, however, a check on ground wave speed via an appropriate testing technique during embankment construction gives further assurance. Such target wave speeds for compliance are often specified as Vs (shear wave velocity), which linked to elastic theory can be related to soil Stiffness $Go = \rho Vs^2$ and hence minimum soil stiffnesses values can also be defined to ensure compliance. However, it is ultimately Rayleigh wave velocity (V_R) that is of interest for critical velocity and the defining of minimum Vs or stiffness values for critical velocity compliance comes from links to V_R via analysis.

There are no standard data collection and processing protocols for testing for Seismic Surface Wave methods (SSWMs). Seismic velocity can be measured through a variety of methods and generally borehole methods are considered highly reliable in conventional geophysical investigations, but present disadvantages in earthworks compliance, such as their high cost, they only provide soil information at one single point, are invasive and time-consuming. Therefore, despite their perceived accuracy they are not really appropriate for embankment construction layer testing, or for routine testing over many miles of a linear infrastructure scheme as part of construction compliance. However, they do form a useful part of initial ground investigation.

SSWMs on the other hand are more suitable for estimating V_R (and hence Vs if required), being non-invasive, relatively easy, cheaper, and proportionately quicker to implement. SSWMs measure the dispersive character of a site to give V_R and then come to a solution through resolving an inverse problem, to give an estimate of the value of Vs and its change with depth. This leads to a non-unique solution, since the inversion is linked to the way the results are obtained, and frequently is reliant on the analyst's experience of looking at such data and in interpreting and manipulating data in post processing, often linked to an initial ground model (either known or assumed from experience).

Therefore, given the nature of seismic surface wave measurement, there are significant areas where variability in the collected and processed data can occur [11], [12]. Different processes can be used to acquire the data and get to a site-specific solution and coupled with the reliance on the experience of those undertaking the work has potential for introducing variability in results. Additionally, on busy construction sites the conditions on the day of the test can have an impact in the data quality (wind noise, rainfall or construction operations etc). Different approaches could lead to different values being obtained and the confidence in that data. This is of relevance when different teams or firms may be engaged in the collection of earthworks compliance data along long sites within a large project like the HS2 scheme. This becomes important during construction if the velocity is measured around the values that may be of concern but is of course less of an issue if compliance values are significantly exceeded.

This paper reports the results of a blind trial to anonymously compare surface seismic wave method results (specifically MASW as it is more widely offered by contractors) obtained from the same site at the same locations, on a trial lime stabilised embankment and the natural subgrade. The tests were performed anonymously by a number of experienced geophysical companies all acquiring data from the same layout of survey lines, and a bespoke survey where the client asked them to design a survey for a specific requirement to test to a depth of 20 m.

The paper concentrates on the dispersive characteristics of the site under test (i.e., the collection of Rayleigh wave data versus frequency via MASW- Multichannel Analysis of Surface Waves). The purpose is to evaluate how reliable these methods are in terms of the data collection and dispersion curve assessment to check the reliability on the Rayleigh wave velocity (V_R) data assessed on site. The purpose of these comparisons will help the industry better understand the issues that could potentially influence the test repeatability and reproducibility and hence reliability of field earthwork surface wave compliance assessment across a large and lengthy scheme.

Given the inversion data processing step is identified as a source of possible variability and as V_R is primarily of interest for critical velocity (and is measured directly) an approach of presenting V_R versus depth results has been proposed to see if inversion to Vs is always necessary to check compliance (hence, please note the inversion results to estimate Vs are not presented herein although are discussed in outline).

The paper is organised as follows. Following the background information above an introduction to seismic surface wave methods is made, including information on data acquisition and analysis. After these details of past work, using geophysical tests in railway earthwork investigations is presented followed by a brief review of past work that has looked at the comparisons and trials between geophysical methods in more conventional site investigations linked to the variability observed. The issues related to inversion of data to get to Vs are also discussed.

The field trial design is then explained, with a description of the soil conditions at the test sites and detail of the testing protocols adopted. The test results are then presented including dispersion curves and variability of results. Finally, the alternative representation of the results of V_R versus depth for earthwork engineers to assess initial compliance is presented.

Seismic surface wave methods

This section explains the basic principles of Seismic surface wave evaluation and data processing relevant for application in earthworks compliance assessment, to specifically assess Vs and V_R.

The evaluation of such waves can be split into invasive down borehole techniques and surface wave evaluation methods. This paper concentrates on surface methods and in particular MASW, borehole methods not being deemed suitable for earthwork compliance. Having explained the methods, issues from past work on comparison trials are presented, however it should be noted that no comparison trials on railway earthworks for critical velocity assessment linked to limiting values of V_R could be found in the literature.

Surface Wave Principles and Methods for earthworks compliance

Surface-wave methods are based on the identification of the dispersive characteristics of a site and on the further estimation of Vs through inversion. Surface waves exist along media with a free surface, such as the earth-air or the earth-water interface. Their amplitude decreases exponentially with depth. In the presence of a homogeneous medium the induced particle motion becomes negligible within around the depth of one wavelength [17,18,19,25] and they show geometrically dispersive behaviour [7] in vertically heterogeneous material. This means that different wavelengths propagate at different depths and therefore for each frequency, the phase velocity measured is dependent on the specific subsurface layer and its properties [7,19].

The surface methods typically used are MASW and CSW (Continuous Surface Waves) and they involve the collection of seismic wave velocity data (typically of V_R versus frequency, note only MASW is considered in this paper). The collected MASW data undergo processing steps including picking the appropriate mode (usually the fundamental) of the dispersion image. However, if the soil stratification is such that there is a stiffer surface layer and a softer layer trapped between stiffer ones, then the fundamental mode may not necessarily be dominant. Then a filtering and inversion process to give Vs is applied Such values are then used in earthworks compliance to then check against derived targets of Vs as a proxy for stiffness or V_R (as defined above).

In MASW typically 24 or 48 geophones are spaced along a line to be tested with similar equipment used as in seismic refraction/ reflection testing. Typically, a hammer impacting a plate, or an accelerated weight drop source is used to generate a test wave and trigger geophone sampling. Shots (hammer impacts) are usually undertaken on both ends of the seismic line, and at the centre and in any other locations along the geophone line that are considered necessary [7]. The spacing of the geophones and impact points is adjusted depending on the purpose and design of the survey as a function of expected ground behaviour and depth to be tested (see Section 2.2), which requires experience to develop.

Data acquisition

In MASW data acquisition, to acquire good quality data from seismic surveys, should have a high signal to noise ratio (S/N) over a wide frequency range, so it should allow modal recognition and separation, noise filtering and uncertainty estimation. To achieve this, it is widely accepted to take multiple recordings (shots) per location and to stack (sum) the data from the shots in the processing step. The main factors that influence the data are related to the spatial sampling of surface waves (i.e., the geophone spacing ΔX) and total array length, since frequently the minimum detectable wavelength λ is equal to ΔX [18]. The number of receivers is also important, such that the uncertainty is reduced when more geophones are used for a given array length, as well as source offset (i.e., the position of the seismic source relative to the geophone array [18]. The receiver array has a great impact on the wavenumber resolution Δk and consequently to mode's separation which is important for dispersion curve picking (discussed below). Generally, there is no limit on the maximum obtained wavelength for a specific array, meaning that wavelengths longer than the array may be observed, as they depend on the site global behaviour and on the frequency, range contained in the signal.

Long arrays contribute better to modal separation and vertical resolution, whereas short arrays produce a better S/N ratio and are less affected by high-frequency attenuation [18,19]. However lateral variations could be detected by the acquisition of shots at the opposite side of the array (forward and reverse shot) and the comparison of the data [7].

Data processing

Shear wave velocity Vs analysis is typically done by deriving the experimental dispersion curve and then using this to get the best estimate of Vs with depth through an inversion process. Wavefield transforms are widely used to analyse the signal in domains where waves are easily identified and their properties are estimated [18], such as f-p (frequency-slowness) or f-c (frequency-phase velocity). The f-c domain is obtained through applying different Fourier transform frequency analysis to produce what is called dispersion images [3]. In these images, the maximum energy is picked [2,3], for what are called dispersion curves.

In the case where the dispersion curves of forward and reverse shots are identical or superimposed, a stacked dispersion image can be obtained for the augmentation of S/N ratio [3] and for identifying the existence of lateral variations [19]. Furthermore, windowing techniques may be applied to identify the existence of lateral variations in the data. These however produce leakage in the frequency-wavenumber (f-k) spectrum that creates ripples in the data which usually prevent higher modes from being identified [19]. Pasquet & Bodet [16] have suggested that generally, the best compromise between resolution and lateral variations should be applied through trial-and-error tests as there may be no perfect criterion for data processing. This is supported by the results of interviews with practitioners on the commercial application of geophysics data processing [12], which shows the interpretation and experience of the analysis is important and can lead to variability in this trial-and-error process.

V_R is also measured through the MASW method, and the data analysis is more straightforward than the one required for Vs estimation, since it includes just the identification and picking of maximum energy in the dispersion image in the phase velocity- frequency domain (thus avoiding the inversion step). Also, Wavelength (λ) and Rayleigh wave velocity (V_R) are related to each other with the frequency as:

$$\lambda = 360/(\text{phase difference}),\ V_R = f\lambda \tag{1}$$

Typically, the depth of ground (Z) which has a significant influence on the wave velocity is therefore assumed to be approximately one third of the wavelength in the presence of soils with continuously increasing Vs with depth, and therefore needs to be treated as an approximation in analysis [9,14]:

$$Z = \frac{\lambda}{3} \tag{2}$$

Past work on comparison of seismic test methods and data processing

Much work has been undertaken in the past to compare the data collected from various seismic techniques be they borehole or surface methods. Additionally, work has been undertaken to look at the impact of data analysis methods or how to avoid the inversion process. Some of these are discussed below. Typically, these projects have been undertaken for site characterisation purposes. None have been found that form part of the evaluation of tests for the construction assessment or for use in compliance assessment for Critical velocity issues for High-Speed railways.

The InterPacific project investigated three sites with variable materials in Italy and France with a variety of geophysical techniques. In total 14 teams of experts processed the collected data, which gave reasonable agreement between the separately collected and produced dispersion data. However, some variability was observed regarding Vs profiles, mostly in stratigraphic features. This was attributed to the non-uniqueness limitations of the inversion process, in its ability to identify interfaces and it was believed that if a priori information from borehole data were given about the soil model, reliability would be improved. Furthermore, data resolution deteriorated at depth [8].

Many studies have been undertaken evaluating the accuracy of Vs estimates compared with Vs data from invasive borehole methods in various locations. The results reported have shown a difference in Vs varying from 11 to 20 % [13,26,5,22,4].

To avoid uncertainties introduced through the inversion process, various researchers have studied different algorithms to process data. For example, the accuracy of the WAVe method was checked in determining average $V_{s,30}$ in five case studies and the results have been quantitatively compared to more rigorous inversion approaches, all providing similar results [1]. Furthermore, a synthetic case was used to prove that the error of estimated and true velocity values compared from applying the linear relationship of weighted average Vs at various depths and surface wave phase velocity at various wavelengths, when known for some models, without data inversion, was of the same order [20]. A simple method suitable in industry applications for avoiding inversion was also proposed, focusing on the direct estimation of Vs time-average models based on the data for one-way time for a specific datum plan depth. If a 1D Vs model is known along the seismic line, with the relevant dispersion curve, a relationship of wavelength and depth on the time-average velocity model can be estimated. This approach provided models with less than 10 % uncertainties on both field and synthetic data [21]. These approaches work well when the experience and skill on the analyst using the method is appropriate and they are consistently applied. However, in practical terms this is perhaps hard to achieve when lots of data is being assessed by many teams.

Notwithstanding, while inversion can be performed routinely and repeatably, this should be considered in the context of the purpose of the data collected and for fast routine and extensive tests to check compliance figures may lead to extra sources of error between testing teams (it should also be noted, although not presented here, the Vs-depth profiles obtained through inversion in this trial presented significant differences, and examples are included in [11]).

Geophysics use on railway earthworks

Limited railway earthworks testing has been published, in the UK and existing "classic network" railway earthworks (made of complex and laterally varying geomaterials of varying quality and typically 150 years old) were tested for stiffness variations through a combined survey of MASW and CSW. MASW was used to locate voids and CSW to identify the passage from fine to coarse material. This showed that these methods, in combination, mapped stiffness changes in complex materials [10]. In Ireland, MASW testing was attempted to map the steeply sloping bedrock under a settled rail embankment [6]. However, again these methods have been used for material characterization in existing railways and not to check Rayleigh wave velocity during the construction phase.

The blind field trial

Therefore, to evaluate the nature of differences in field measured V_R (and by analysis values such as Vs or $\mathbf{G}_0$) and to compare data for different analysis teams, an anonymous blind field trial was designed to compare MASW readings from the same site at the same location, to give an assessment of differences in MASW data collection for compliance testing purposes. This section describes the field trial design, data collection and processing methods used.

Four of the major companies that apply MASW in the UK participated in the blind trial. Their teams consisted of experienced and qualified geophysicists. The same strictly designed survey protocols were given to all companies and then the firms also had the opportunity to design their own survey aiming to map the subsurface down to 20 m depth. The tests were undertaken over a single week and each firm was allocated a day to test.

The tests were located at Long Itchington Wood in Warwickshire, UK adjacent to the HS2 main works. Two test sites (described henceforth as pads) where chosen and were adjacent to each other. One pad was a treated trial stabilised embankment (Pad S). The second was an area of natural ground (Pad N) from which the topsoil had been removed. Pad N was located at the top of a recently excavated cutting slope. Prior to the trial as part of project ground investigation some previous MASW testing had been undertaken and ground borehole logs were available near the site but no a priori information was provided to the trial firms at this initial stage, but some limited information was provided to help subsequent modelling.

Stabilised pad (S)

The stabilised trial pad was built of 10 layers, 0.3 m thick, of Mercia mudstone won on site, stabilised by mixing with 1.5 % lime and addition of water to achieve optimum material compaction. Its dimensions were 80 m long, 8 m wide and 3 m high. The pad was founded on the natural ground of weathered Mercia Mudstone with the topsoil removed.

Natural ground pad (N)

This pad consisted of the natural ground material, stripped of topsoil and comprised interbedded weathered Mercia mudstone and clays of the Sidmouth Mudstone Formation, similar to the material under the stabilised pad. Due to site constraints Pad N was slightly shorter than Pad S at approximately 60 m long.

Survey design

The tests undertaken per pad were seismic profiles, one set strictly specified and followed by all contractors. The second was an open design created by each company with the requirement of reaching 20 m depth. The lines all had the same longitudinal run and the same centreline, The test lines that were strictly specified are shown in Fig. 1 and the own design ones in Table 1.

The aim of assessing a few similar test protocols (with no target depth specification requirement) was to see the level of repeatability and variability of collected and processed data when receivers are placed at exactly the same points, using similar equipment on the same ground. The purpose of requesting an individual survey design requiring a specific depth of assessment was to understand in a more realistic scenario what differences the client could expect to perhaps see in the obtained response from a variety of teams and methods.

In the case of the strictly specified tests, all used 24 spiked geophones of 4.5 Hz resonance frequency. As Pad N was slightly shorter than Pad S there was no space to deploy the 2 m geophone spacing/ 10 m offset line and only the other two surveys shown in Fig. 1 were undertaken on Pad N.

Following data collection each firm was asked to analyse the data using their standard approaches. Each company used a commercially available software of its choice, and the dispersion curves were picked by the geophysicist's experience based on past work with similar datasets. The software used, and the processing approaches used are summarized in Table 2 where details are available.

Results

The results obtained by each company for each pad (S then N) and test configuration are presented below. Firstly, for the specified tests then the open "own" design tests. The picked dispersion curves are presented, and the resulting curves then compared directly in graphs. Comparisons are made in terms of the survey configuration used, the

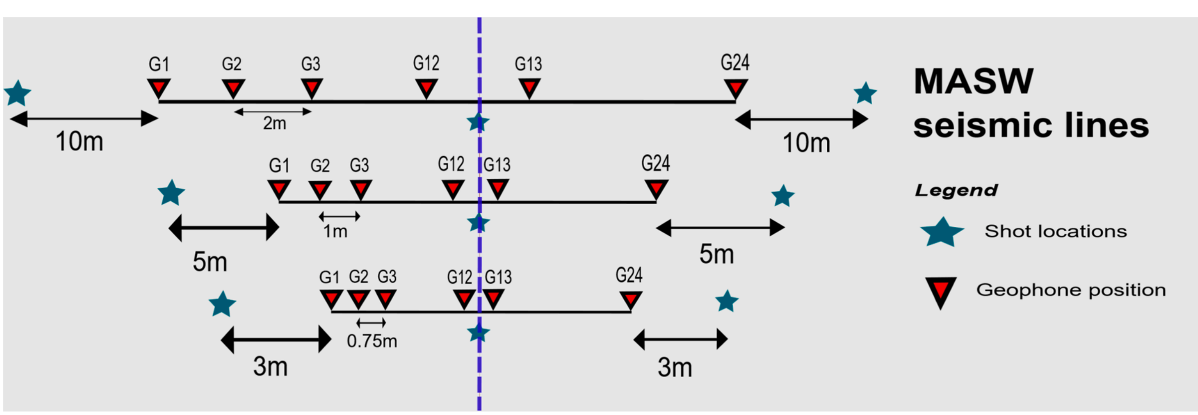

Fig. 1. The design of the surveys, given to participants to be applied to both test areas (S and N- the dotted line corresponds to the common centre point for all surveys, between Geophones No 12 and 13 (also note the 10 m offset line was not assessed on Pad N due to space constraints).

Table 1
Test approaches designed by the contractors to assess the ground to 20 m depth with MASW.

Pads	Configurations Designed by Each Company			
	A	B	C	D
S	Land streamer with 48 No. 4.5 Hz vertical geophones at 1 m spacing. Variable offset shots (12, 9, 6 and 3 m at both off-ends) with a hammer	48 No. 4.5 Hz fixed geophones at 1 m spacing. Variable offset shots (10 m at both off-ends and then every 1 m) with a hammer	48 No. 4.5 Hz fixed geophones at 1 m spacing. Offset shot at 10 m and then every 4th geophone with a hammer	24 No. 4.5 Hz fixed geophones at 2 m spacing. Variable offsets (10, 8, 6, 4, 2 m) with a PEG-40 wt drop
N	Land streamer with 24 No. 4.5 Hz vertical geophones at 1 m spacing. Data from multiple spreads was acquired with the spread moved along the survey line at 3 m intervals. Variable offset (6, 3 and 0 m) shots with a hammer	48 No. 4.5 Hz fixed geophones at 1 m spacing. Variable offset shots (1 m from first geophone and 2 m from last) and shots every 1 m with a hammer	40 No. 4.5 Hz fixed geophones at 1 m spacing. Offset shot at 10 m and then every 4th geophone with a hammer	23 No. 4.5 Hz fixed geophones at 2 m spacing. Variable offsets (6, 4, 2 m) with a hammer

Table 2
The software and processing approach chosen by each analyst.

Company	A	B	C	D
Software	SurfSeis 6	ParkSeis	Geogiga Surface Plus	SurfSeis 6
Dispersion analysis approach	Muting and filtering	Muting (setting to zero), to optimise the dispersion images/ no muting for data arriving before the surface wave signals	Precise analysis details were not provided	Suppression of the first mode using an FK filter adversely impacted the fundamental mode. Combined advanced and HRLRT processing in some cases separated the modes sufficiently.

acquisition parameters, the commercial software used, and the field equipment selected (according to Table 1), depending on the specific survey objective.

It is worth noting that only the dispersion images and the relevant dispersion curves picked are included, since the focus of the survey and this paper was to check the reliability of the Rayleigh wave velocity (V_R) data.

The firms did subsequently undertake inversion analysis, and this is presented elsewhere [11], as space precludes detailed discussion in this paper. A typical example of the inversion results is shown in Fig. 2. This clearly shows the significant variability in processed Vs versus depth values between companies. These are attributed to a series of parameters, such as the "noise" from an active construction site, the analyst's experience, the values selected in the inversion starting model etc. (all factors which have been identified by various authors- see Section 2).

Stabilised pad (S)

Figs. 3–5 show the dispersion images in the phase velocity- frequency domain per company and per survey configuration for Pad S. The derived summary dispersion lines are presented by a different colour in the comparison plots in Fig. 6. To help anonymise the participating companies, each dispersion curve has been adjusted to greyscale, as each firm uses their own house colours and styles in their plots. Tables 3 and 4 present the summary velocity data with depth and variability calculated as the percentage difference between V_R max and min against V_R min.

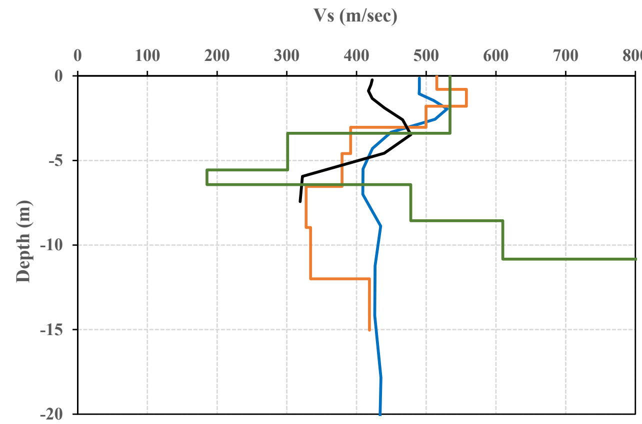

Fig. 2. Typical example comparing Vs-depth models following inversion from companies showing variability in data (Pad S, 1 m geophone spacing test).

In terms of dispersion images, there are clearly noticeable differences between the companies in respect of the frequency ranges presented which appears to come down to the experience of the analyst, and their confidence in presenting information at high and low frequencies based on factors that may have influenced site data collection. It should be noted that high frequency data can be more subject to the assessment of the fundamental mode in the response, especially when there are a number of closely spaced modes which seemed to be apparent in these data (potential examples are labelled in Fig. 3). Low frequency data tend to have a weaker signal (because greater energy is needed to generate strong waves at depth).

For the longest line of 2 m spacing, most geophysicists picked Rayleigh wave dispersion estimates at frequency range 20–60 Hz. It should be noted that the maximum depth of penetration that can be achieved by an MASW survey is somewhat dependent on the ability to generate and record a low frequency signal. To achieve this, a relatively long survey line is required so that low frequency signals can disperse from higher frequency ones. Quiet site conditions are required so that low frequency signals can be recorded at sufficient distance from the shot point without being swamped by higher amplitude signals generated by nearby plant. This was not the case at this site, with many construction machines being operated near the test location, and at times testing had to wait for such "noise" to subside.

In terms of the highest frequency, that was obtained by Company A at almost 100 Hz, but this was picked from experiential judgement since the fundamental mode (M0) maximum energy was clear only up to 60 Hz, (and picked to this level by the other companies). However, Company A chose to pick this since in the presence of higher order modes, M0 was seen to continue below the first higher order as a faint but distinct signal (see Fig. 3).

In terms of minimum frequency, it seems difficult for the companies to get reliable data and only companies A and B picked from 5 to 25 Hz. This is because it is difficult for the analysts to separate different modes (due to mode superposition of M0 with higher ones) which may mean that higher modes (with higher velocities) get picked. This reduces confidence in these values since the phase velocity changes sharply with frequency (or depth). Even though these velocities are higher than the target ones specified by HS2 and it is unlikely to be of engineering concern, attention should be paid since misidentification of secondary or tertiary harmonics of the fundamental mode can lead to potential over-

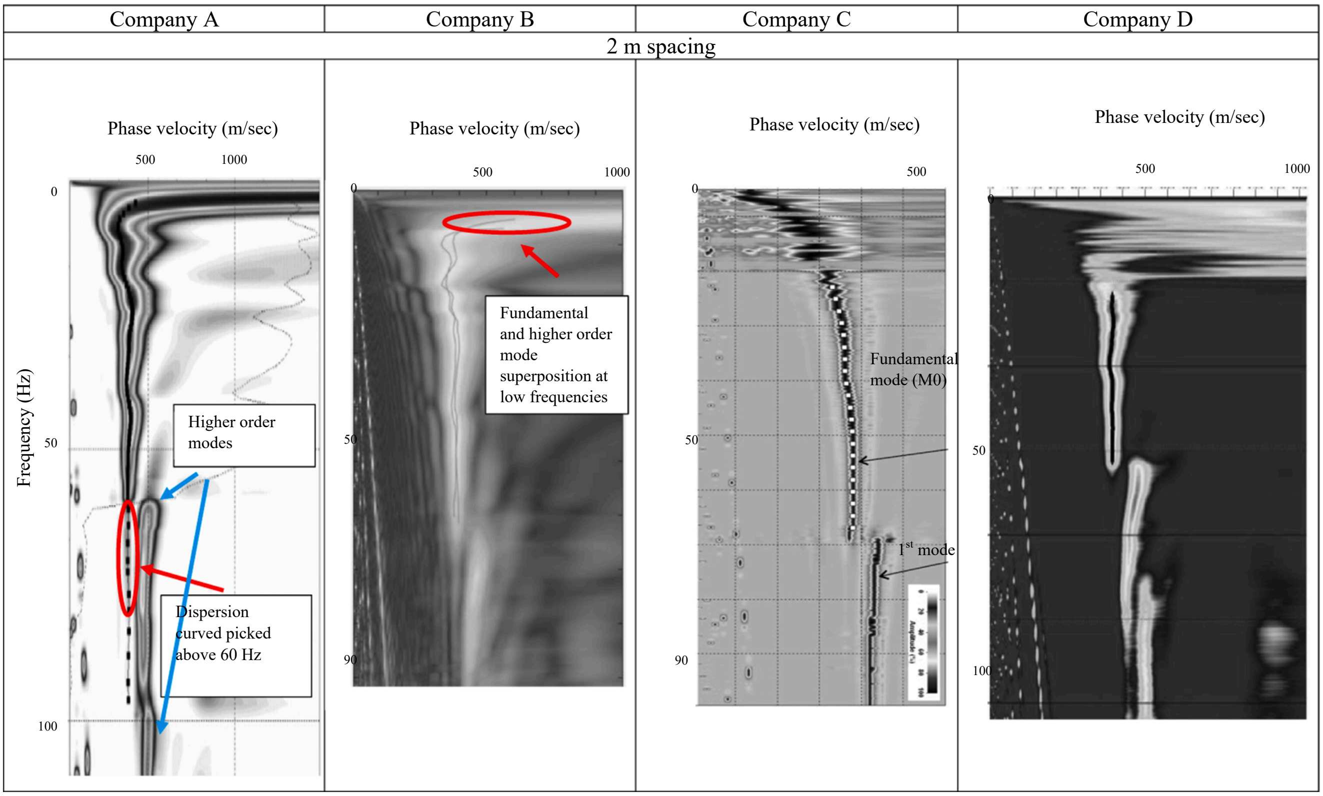

Fig. 3. All dispersion images analysed in the phase velocity-frequency domain per company (all lines) at Pad S for 2 m geophone spacing.

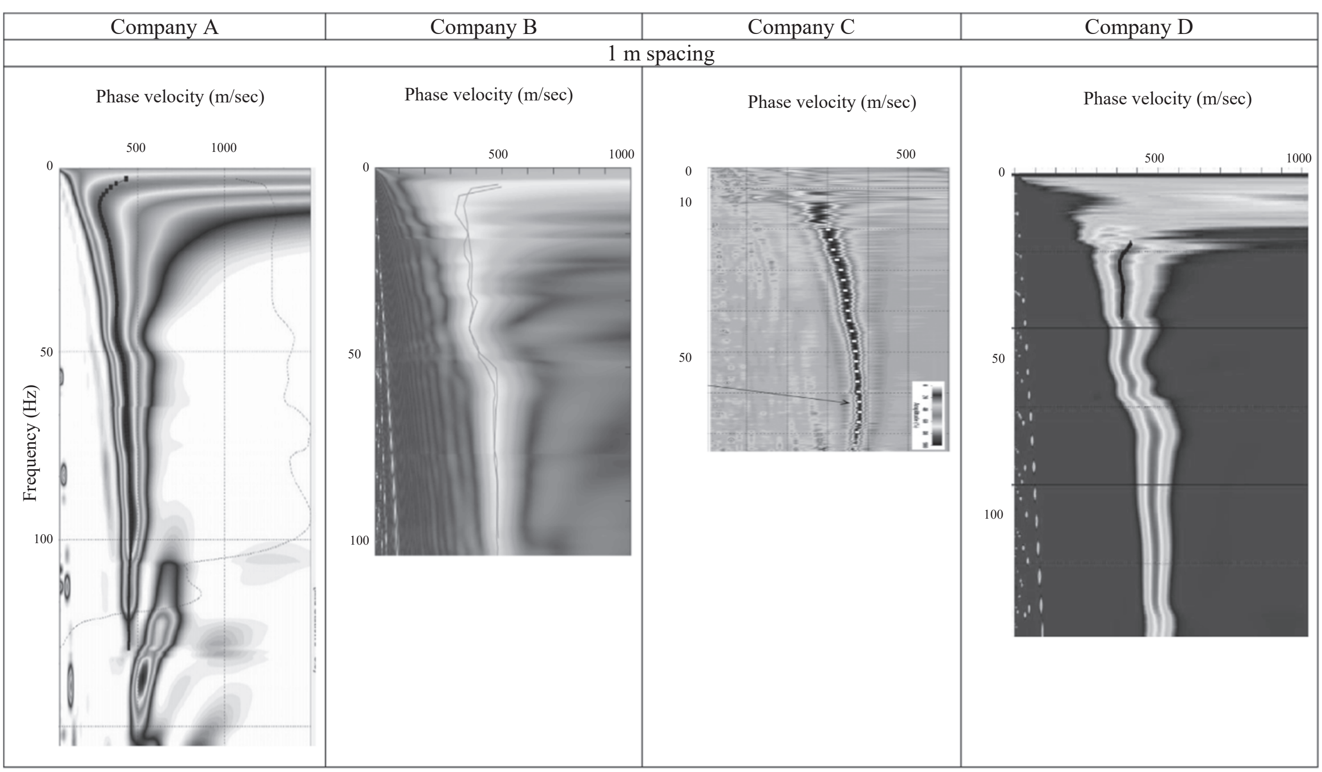

Fig. 4. All dispersion images analysed in the phase velocity-frequency domain per company (all lines) at Pad S for 1 m geophone spacing.

estimation of Rayleigh wave velocity and accordingly to possible over optimistic assessments of engineered earthwork performance.

From equation (1), velocity contrasts for frequencies higher than around 25 Hz, correspond to wavelengths (λ) between 12 and 16 m. Equation (2) implies the depth of ground influencing this change is at approximately 4.5 m, larger than the actual change at 3 m at the base of

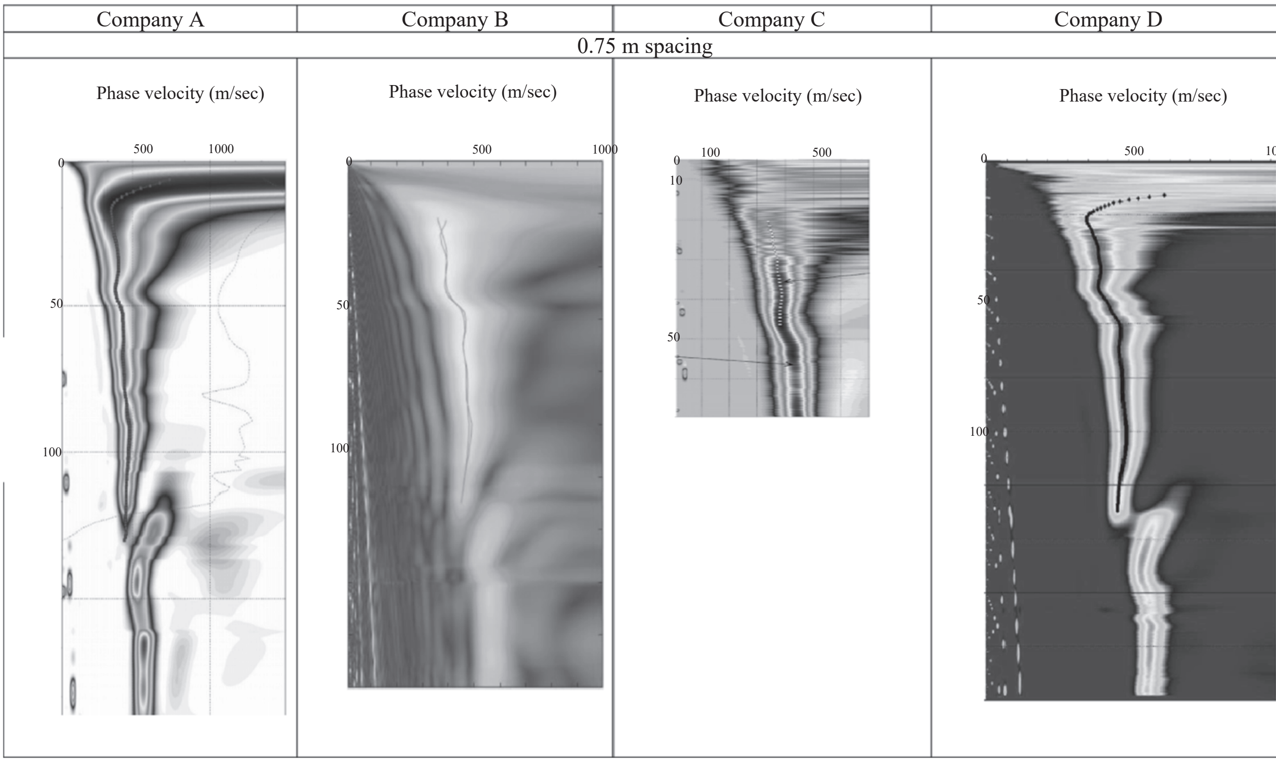

Fig. 5. All dispersion images analysed in the phase velocity-frequency domain per company (all lines) at Pad S for 0.75 m geophone spacing.

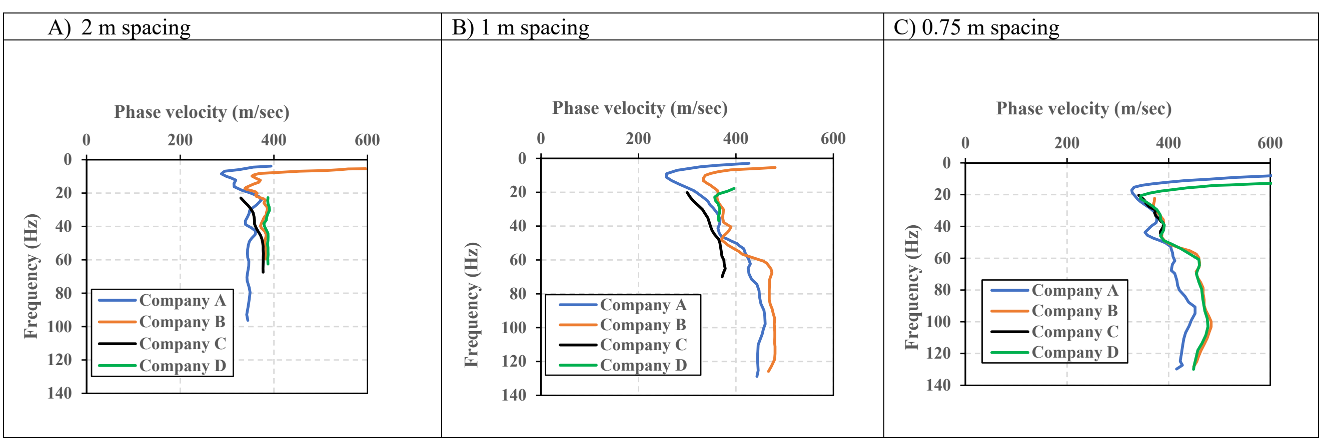

Fig. 6. Comparison of the dispersion curves for all seismic line lengths for Pad S.

Table 3
The % V_R difference for all profiles of Pad S, examined at frequencies of 10–50 Hz with an increment of 10 Hz.

Survey configuration	2 m spacing	1 m spacing	0.75 m spacing
Difference in V_R between the companies			
10 Hz	18.2 %	31.4 %	53.9 %
20 Hz	17.6 %	21.7 %	11.4 %
30 Hz	12.1 %	13 %	1.7 %
40 Hz	8 %	12.1 %	6.9 %
50 Hz	11.2 %	9.6 %	3.3 %

Table 4
The difference between maximum and minimum velocities picked for participant's own design in Pad S.

Frequency (Hz)	V_R min	V_R max	Comments	V_R difference (%)
10	326.31	373.44	only companies. A, B,D	14.4
20	299.98	376.84		25.6
30	331.25	391.17		18.1
40	347.81	385.46		10.8
50	343.3	386.98		12.7

the stabilized pad. However, this relationship of depth and wavelength is only an approximation and calculations can be used just as an indication rather than absolute value (further discussed below). Company D did not identify particularly strong velocity contrasts with depth compared to other companies, and like Company C, was only able to present results above 25 Hz.

By reducing the survey length to 1 m spacing, slightly greater variation in results between the companies at frequencies at 10 Hz (greatest depth) is observed (i.e., 31.4 % versus 18.2 % in the case of 2 m spacing), but less variation at higher frequencies. Significantly, the frequency range over which data is presented is increased for all companies, with two presenting data to much greater than 60 Hz and no significant change to the minimum frequencies picked. All companies seem to have mapped velocity contrasts, and a significant change is observed at 60 Hz. This corresponds (using the approach above) to a depth of influence of approximately 2.5 m which is much closer to the known change at 3 m. This shows that reducing the geophone spacing increased the ability of the tests to identify the shallower soils.

By reducing the geophone spacing even more, resolution at shallow depths is further improved, with 3 of the 4 companies presenting results to 120 Hz (Fig. 6-C). The results are similar to the 1 m spacing line, but the change in specification allowing more companies to get high quality data over a large frequency range, especially in shallower depth (higher frequencies), would be of significant benefit if used as construction verification.

Overall, comparing the results on Fig. 6 in terms of difference in survey line length, it is shown that in all cases the results are broadly in line with the expected ground conditions (high velocities at high frequencies in the stabilised materials, a velocity contrast at intermediate frequencies for the weathered mudstone, and a further velocity contrast at low frequencies in the un-weathered mudstone present at depth), with evidence of behaviour of a stiff layer overlying a softer layer below (further discussed below).

The difference between maximum and minimum velocity picked for frequencies 10–50 Hz with an increment of 10 Hz (See Table 3) revealed that this is higher (21.7–53.9 %) at 10–20 Hz for all survey lengths and below 20 Hz the variability becomes smaller. The best match (3.3. % difference) for the most superficial layers (i.e., 50 Hz) was identified in the smallest array of 0.75 m spacing, providing better resolution.

In terms of the companies' own design to reach 20 m depth for Pad S, without a consistent test specification, the MASW results (Fig. 7) show a wider spread than those in Fig. 6. The overall frequency range is similar to the previous tests with the 2 m geophone spacing, but companies C and D were able to present results over a greater range. There is no significant increase in the low frequency content of the results despite these being targeted (for depth). The variability in velocities every 10 Hz was about the same, from 12.7 to 18.1 % and the highest difference was 25.6 %, observed at 20 Hz (Table 4).

Natural Ground pad (N)

Figs. 8 and 9 show the dispersion images in the phase velocity- frequency domain per company and per survey configuration for Pad N and Fig. 10 shows the comparative curves picked for each survey for this pad. Tables 5 and 6 present the summary velocity data with depth and variability calculated as the percentage difference between V_R max and min against V_R min.

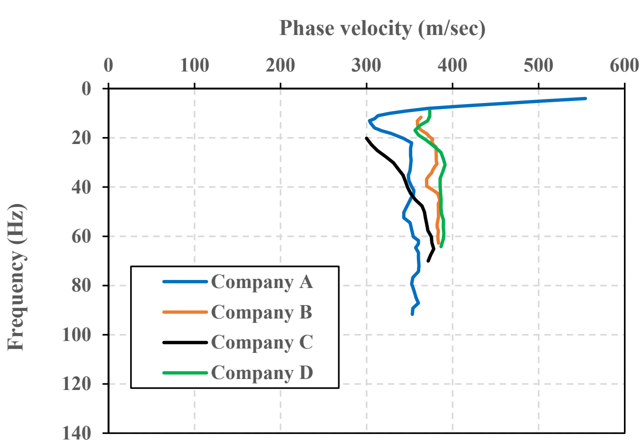

Fig. 7. The dispersion data for companies' own design in Pad S.

Based on the dispersion data of the longest line (1 m spacing, Fig. 8), company B picked the maximum energy at the lowest frequencies, starting from 7 Hz, but velocity variation was significantly high for small frequency changes, perhaps making these higher depth data questionable. S/N ratio of the shot for this company was very high but differed markedly from one another (i.e., forward and reverse shots), possibly indicating significant ground heterogeneity. Over the interval 5–25 Hz the dispersion curves of the stacked curves picked for both off-ends exhibit similar responses, albeit with velocity distortions occurring at different frequencies.

On the other hand, company A picked a very clear fundamental mode of dispersion. For this company, higher order modes in data collected across Pad N were not as prevalent as in data collected with a similar test layout on Pad S, with lower amplitude responses observed. This is attributed to the lack of an artificially stiffened near surface layer which is likely to aid the transmission of higher frequency seismic energy. Additionally, it was noted from depth validity calculations, that the maximum observable frequency for data collected across Pad N was lower than for the equivalent line data on Pad S (Fig. 10-A, 89 Hz and 124 Hz accordingly). This is again attributed to the lack of the stiffened material at the surface which aids the propagation and reduces the attenuation of high frequency signals. Company C picked the dispersion at 20–40 Hz (Fig. 8) but seemed to not have identified any strong velocity contrasts, compared to others. Company D manually picked the dispersion curves on this pad, but these were of mixed quality, with the fundamental mode being less coherent. For the 1 m spacing profile, the fundamental mode is identified between approximately 10 and 40 Hz, with higher modes dominating the spectrum as frequency increases (Fig. 8).

In terms of the shortest line of 0.75 m spacing, only data from companies A, B and D were made available and could be compared. Company A identified the dispersion at the highest frequency ranges, from 11 to 100 Hz (Fig. 10-B). Overall, it is evident that reducing the geophone spacing has enabled each company to extend the high-frequency range of the data by approximately 20 Hz, which gave better resolution in the near surface. For company D it has been unclear the trend of the fundamental mode from around 12–18 Hz, so energy picking at low frequencies could be inexact.

The difference between maximum and minimum velocity picked for frequencies 10–50 Hz with an increment of 10 Hz revealed that this is higher (14.4–18.6 %) at 10 and 40 Hz for the longest survey length (Table 5). Conversely, for the 0.75 m spacing line the smallest velocity difference was observed at 10 Hz (11.8 %). Between the two lines, at 10 Hz the better match was given at 0.75 m spacing line.

It should be noted that the survey area was not immediately adjacent to the engineered embankment, so direct extension of the findings from the other pad to investigate greater depths below the embankment would not be an entirely reliable process similar. However, it is likely that the results obtained from Pad S have some bearing on the likely physical properties of the material found below the embankment, as the underlying ground conditions are similar.

In terms of the companies' own design to reach 20 m depth, it should be noted data from Company C is not available for these comparative plots. The difference between the obtained results and the specified test arrangements are similar to those for Pad S. A notable difference is the very small frequency range over which company D presented the results, possibly due to high ambient noise from the site works (Fig. 11). The variability in results every 10 Hz was significantly high at greater depths (Table 6, 63.5 % at 10 Hz). A very good agreement was observed at more superficial layers (just 1.6 % at 50 Hz) at shallower depth (Table 6).

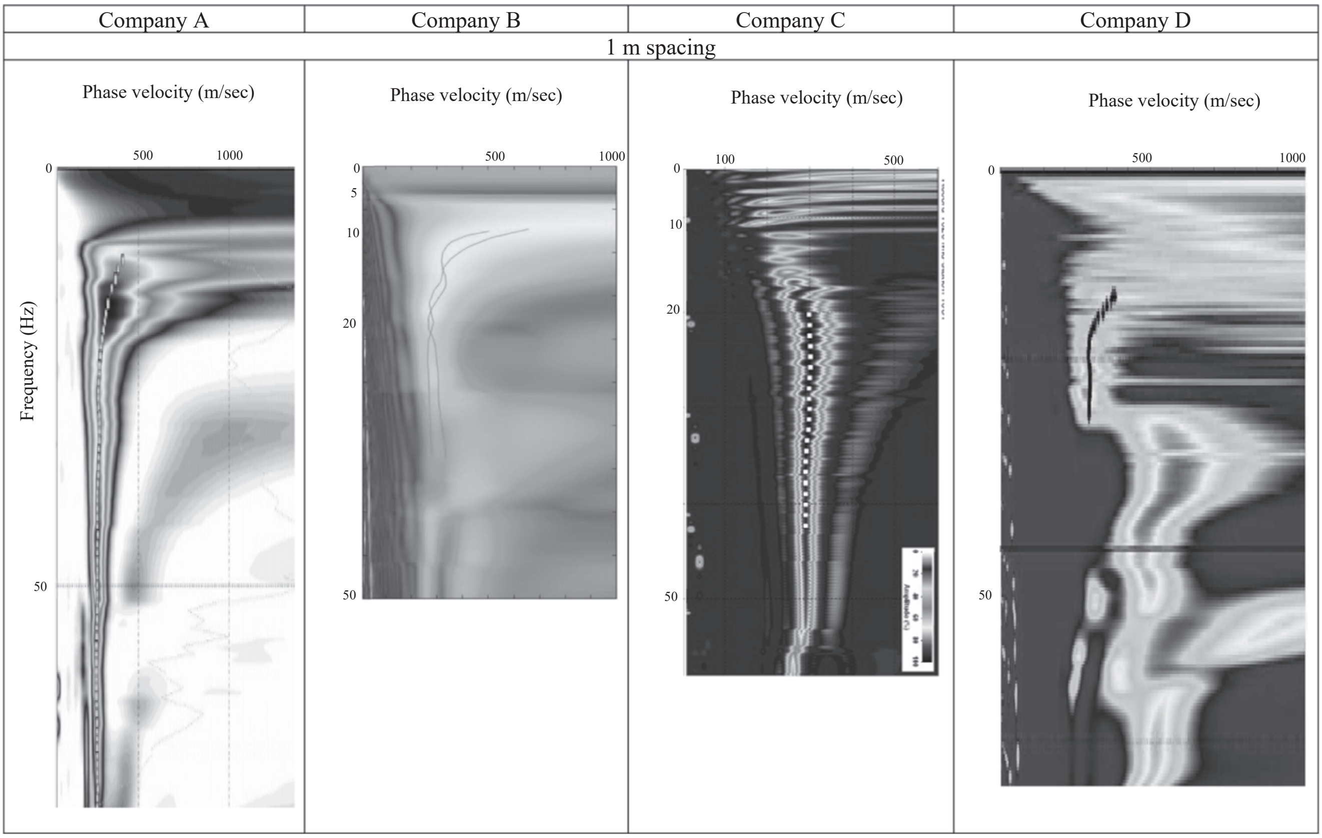

Fig. 8. All dispersion images analysed in the phase velocity-frequency domain per company (all lines) at Pad N for 1 m geophone spacing.

Discussion

Implications of the trial for compliance testing

The outcome of the blind trial showed that in terms of phase velocity, the differences observed between companies were becoming smaller as the frequency increased, meaning that the accuracy and consistency appears reasonable particularly in the near surface. This difference (in most cases no more than 12 % at the surface) among companies corresponds to frequencies 40–50 Hz which, based on equation (2), is translated to depths 2.5–3.2 m below ground surface. At lower frequencies the difference become larger, if the target values are exceeded significantly then this variability seems acceptable. If these values are around compliance target values further data inspection should be undertaken of geotechnical test data in the area to give more confidence and understand the implications or causes. For example, engineers checking geotechnical test results, as part of an overall earthwork assessment, could evaluate the obtained MASW results together with any E_{v2} or CBR/Point load results and when a less stiff zone is potentially identified.

In terms of resolution, as expected, this revealed that smaller length of seismic lines gives more detailed results in the near surface, since most companies evaluated higher frequencies in the case of 0.75 m geophone spacing. An exception was company C whose maximum frequency data on Pad S were 70 Hz for 1 m spacing and 45 Hz when the spacing was further reduced to 0.75 m. Company D as well, did not manage to produce a dispersion image showing clearly the fundamental mode, so could not properly pick the dispersion curve in the case of 1 m spacing on the same pad. These examples are mostly due to the high noise level coming from machines operating on site. This is relevant when trying to design tests to specific depths on live sites and shows again that the MASW data need to perhaps be considered in context of the whole suite of compliance information.

The results obtained when companies applied MASW based on their own design to reach 20 m depth again showed variability between them, but similarly the difference in maximum and minimum velocities was getting smaller as frequency increased. It is difficult to directly compare the two test sets (specified and own design), since they represent slightly different test objectives, and give information on different response relationships of the materials. This again supports the view that the data are part of an overall suite of compliance information that needs to be considered in the round.

In some of the survey lines, the companies reported difficulty in identifying the Rayleigh wave’s fundamental mode from the dispersion images, partly because of significant “ground” noise on site when collecting results. This may explain why some companies identified different extents of frequency ranges in their presented dispersion curves. In cases where the presence of higher order modes was interfering with the fundamental mode at very low frequencies, the phase velocities that were picked by analysts changed sharply with depth. Therefore, this confirms that attention should be paid when interpreting and presenting results in the low frequency range and ideally any phase velocity-frequency plots need to be assessed in conjunction with the dispersion image as they are produced (as reported as undertaken by the companies) to ensure that no mode superposition is taking place, that might lead to false or misinterpreted compliance numbers.

Impact of the stabilized pad on results

From what is known about the ground conditions at the test sites the stiff, stabilized soils in the trial embankment (pad S) would be expected to have shear wave velocities higher than 300 m/s. The natural ground is likely to have shear wave velocities of approximately 200 m/s in the weathered zones, increasing significantly in the weak mudstones to values higher than 300 m/s. Because the thickness of the stabilized embankment and weathered zone are relatively small, the effect on the

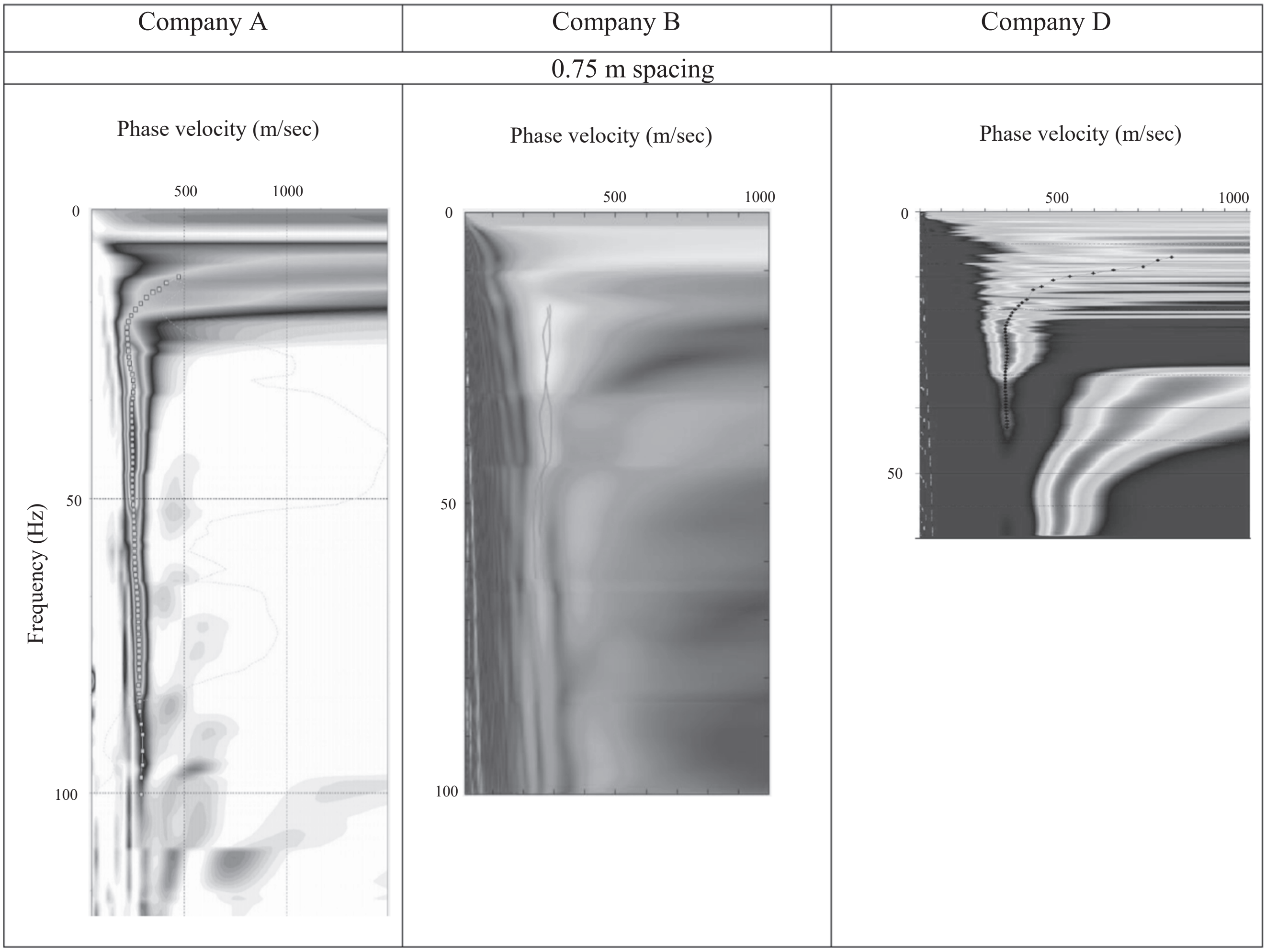

Fig. 9. All dispersion images analysed in the phase velocity-frequency domain per company (all lines) at Pad N for 0.75 m geophone spacing.

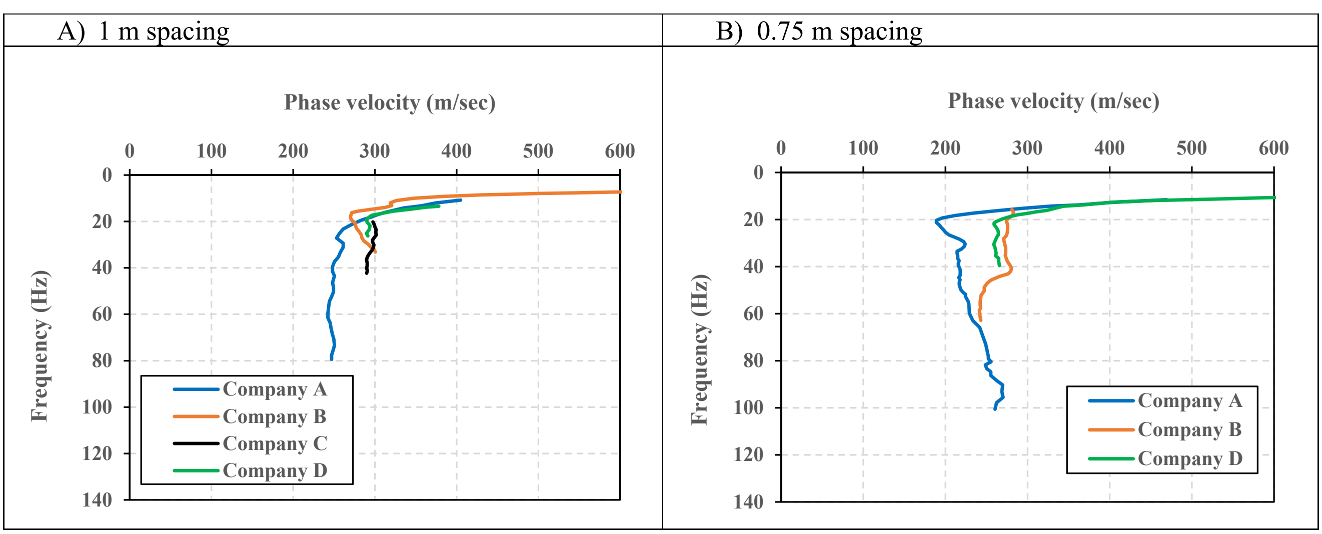

Fig. 10. The dispersion curves for all seismic line lengths for Pad N.

Rayleigh wave velocities over different frequencies may be expected to be complex as waves may not be entirely constrained within a layer and cross layer boundaries. It is important to note for the embankment Pas S, this could be compounded by the fact that the embankment is stiffer than the natural soil so it cannot be assumed in analysis (as is typically the case) that the speed of Rayleigh waves increases as frequency reduces. This atypical situation, when testing on well-engineered embankments, should therefore be considered a possibility within the initial survey design and subsequent data interpretation.

Table 5
The V_R difference for all profiles of Pad N, examined at frequencies of 10–50 Hz with an increment of 10 Hz.

Survey configuration	1 m spacing	0.75 m spacing
Difference in V_R between the companies		
10 Hz	18.6 %	11.8 %
20 Hz	8.1 %	45.6 %
30 Hz	14.4 %	21.2 %
40 Hz	16.8 %	28.2 %
50 Hz	Only company A reached this frequency	13.2 %

Table 6
The difference between maximum and minimum velocities picked for participant's own design in Pad N.

Frequency (Hz)	V_R min	V_R max	Comments	V_R difference (%)
10	286.46	468.36	only companies A, B	63.5
20	257.8	280.5		8.8
30	252.73	289.28		14.5
40	262.66	287.77		9.6
50	241.34	245.24	only companies A, B	1.6

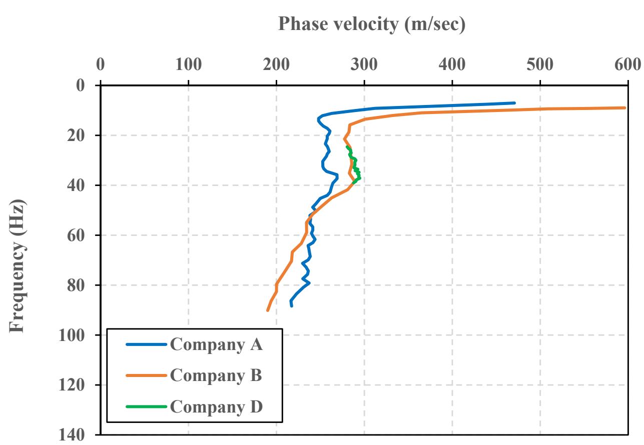

Fig. 11. The dispersion data for companies' own design in Pad N.

Alternative presentation of V_R against frequency with depth contours

The Rayleigh wave frequency-phase velocity domain data presented above show the detailed V_R response of the ground could be used directly to check and verify that the target specified values are met in any earthwork. However, an understanding of the plots in respect to their response with depth below ground surface is also useful should there be any areas of concern around target levels. Therefore, a method of presenting the data in a way that also indicates depth below ground surface is also helpful, especially if used by earthworks engineers who may not be experts or experienced in geophysics. Quickly being able to identify in the dispersion data, the V_R at or near a target value or that looks unusual, geotechnical engineers will be able to know approximately the depth of a weak zone and investigate further or seek further advice perhaps by specific inversion modelling of the layer to get Vs or to undertake further geotechnical tests/ boreholes in selected areas/depths of concern.

Equation (2) enables the depth of ground influencing the Rayleigh wave velocity at a given frequency to be estimated, and from this, contour lines of the "depth to zone of influence" can be plotted on the dispersion curve data. This has been done for both pads in Figs. 12 and 13 (an example of how the calculations is included on Table 7).

In the interest of brevity, the paper only presents the contour lines for the profile of 1 m geophone spacing and 5 m offset for both pads (N and S), but this approach can readily be applied in all dispersion curve data.

These plots are helpful for interpretation of the V_R data, assessing whether the data are confirming anticipated good performance (based on prior knowledge of the ground conditions) and for identifying any anomalies. However, they perhaps should not be used for detailed interpretation of the ground condition, particularly in areas of marginal performance. It should be noted that interpretation can be quite subjective and the depth zones of influence corresponding to these V_R values might not reveal a zone of concern for dynamic issues or further investigation if the wrong (i.e., higher order) mode is chosen. Additionally, equation (2) may lead to misinterpretation because 1/2 or 1/4 wavelength may be more appropriate in a given circumstance to the 1/3 plotted above, depending on the soils tested as this is an empirical rule, so again judgement is needed.

However, the proposed data representation can be a useful tool for quick and easy data interpretation by engineers not having any particular experience with seismic surface waves methods, when looking at wider compliance data.

Conclusions

Seismic surface waves (i.e., Rayleigh waves) generated by the passage of high-speed trains are considered in the design and specification of high-speed rail earthworks as they may cause dynamic effects in the ground if the train's speed approaches the Rayleigh wave velocity of the underlying soil (critical velocity effect).

Measuring the Rayleigh wave velocity of an earthwork during construction can be undertaken to check compliance against minimum specified wave speed target values to guard against the critical velocity effect. This can be achieved through MASW, the most commonly used seismic surface waves technique.

A trial was performed aimed to assess variability in V_R results via comparison of collected data from a series of geophysics tests by several companies using similar techniques of wave speed assessment. Two sites were evaluated, one on natural ground and one on a 3 m high lime-stabilised embankment. The following are the key findings of the work:

- MASW is a relatively straightforward method to implement for collecting seismic surface wave data during earthwork construction mainly as it is non-invasive and widely used. However, some challenges need to be considered, such as the noise on site coming from construction plant operating, people, rain or wind, which can all impact the data quality.
- In some of the survey lines, the companies reported difficulty in identifying the Rayleigh wave's dominant mode from the dispersion images, partly because of significant "ground" noise on site when collecting results. This may explain why some companies identified different extents of frequency ranges in their presented dispersion curves.
- In cases where the presence of higher order modes might interfere with the fundamental mode, (at very low frequencies), attention should be paid when interpreting results in the low frequency range, to ensure that no mode superposition is taking place, that could lead to false or mis-interpreted compliance numbers.
- The dispersion curves obtained from MASW showed a good agreement between the companies and the smallest differences in phase velocity were observed at high frequencies (i.e., shallower depths).
- Shortening the survey line by reducing the geophone spacing increased the data resolution at high frequencies and the frequency range over which companies were able to supply results. This finding was especially noticeable for the stabilised pad. On low-height very stiff embankments there are therefore potentially advantages to using smaller geophone spacings to target specific embankment layers or depths.

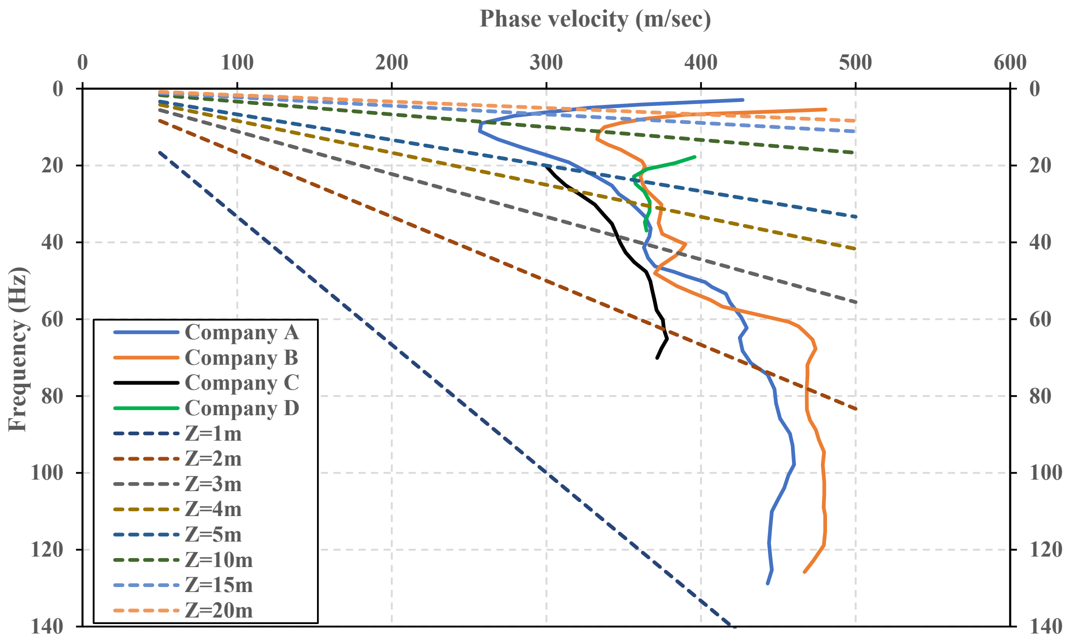

Fig. 12. The dispersion curves and the theoretical depth contour zones for pad S (profile of 1 m spacing/ 5 m offset).

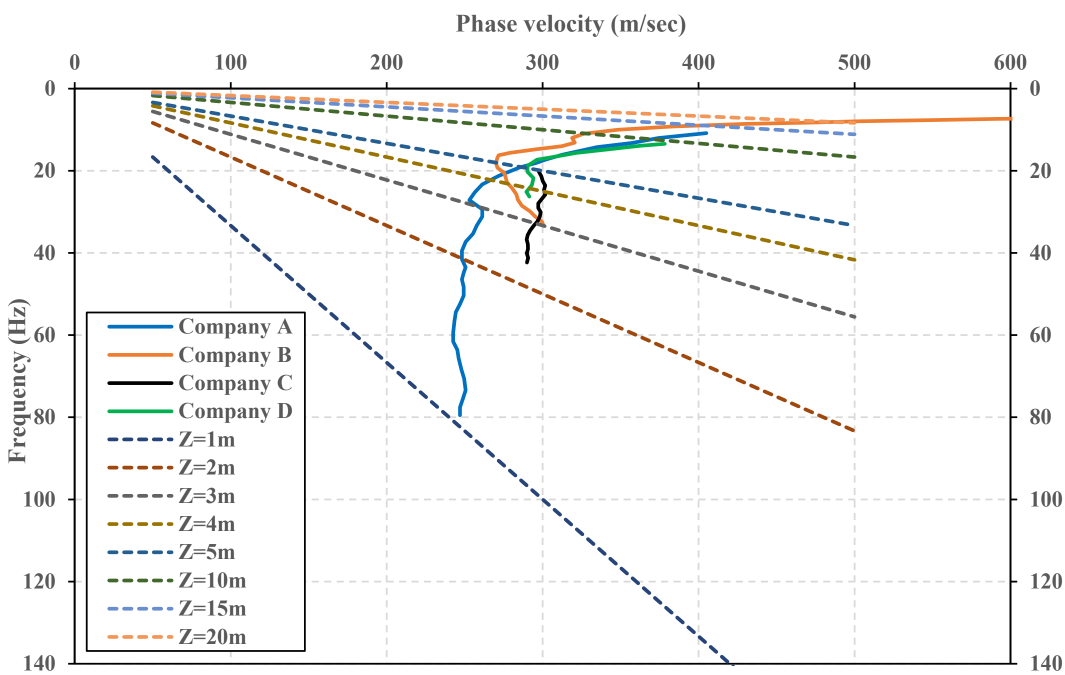

Fig. 13. The dispersion curves and the theoretical depth contour zones for pad N (profile of 1 m spacing/ 5 m offset).

- With the appropriate test set-up, MASW data could identify the different materials (stabilised and underlying ground) at both sites.
- Apart from ensuring that Rayleigh wave velocities on a site are beyond the critical target value, phase velocity over frequency data can be also used for identifying weak zones in an earthwork. Based empirical relationships noted in the literature, engineers can translate frequency values to depth. This can help non-geophysicists rapidly interpret these data to give confidence in the results being presented and to quickly identify any potential areas of concern within an earthwork that may need further interpretation.
- However, attention should be paid when interpreting the data from the depth contour lines, as there might be zones of concern where the Rayleigh wave velocity is picked based on the existence of higher order modes (see above).
- The potential for any highly specified well-built engineered earthwork to be stiffer that the underlying materials need to be considered in analysis when testing though the earthwork, into potentially less stiff underlying materials. As this is atypical or the usual situation.

From a geotechnical earthwork engineering point of view, it is clear there is variability in the data which must be considered in any interpretation around absolute values, targets and limits. In areas of potential concerns and consequent corrective action, the use of MASW results requires careful consideration in the context of the full suite of

Table 7
The values used for the construction of the depth (Z) contours of Figs. 12 and 13.

	λ = 3	λ = 6	λ = 9	λ = 12	λ = 15	λ = 30	λ = 45	λ = 60
	Z = 1 *m*	Z = 2 *m*	Z = 3 *m*	Z = 4 *m*	Z = 5 *m*	Z = 10 *m*	Z = 15 *m*	Z = 20 *m*
λ	3	6	9	12	15	30	45	60
velocity (m/s)	frequency (Hz)							
50	16.7	8.3	5.6	4.2	3.3	1.7	1.1	0.8
100	33.3	16.7	11.1	8.3	6.7	3.3	2.2	1.7
150	50.0	25.0	16.7	12.5	10.0	5.0	3.3	2.5
200	66.7	33.3	22.2	16.7	13.3	6.7	4.4	3.3
300	100.0	50.0	33.3	25.0	20.0	10.0	6.7	5.0
500	166.7	83.3	55.6	41.7	33.3	16.7	11.1	8.3

compliance data. MASW tests have a role to play as part of this compliance test suite. Further research is underway to understand in detail where potential differences in the dispersion curves come from and on the effect of each field set-up and processing approaches on the results.

CRediT authorship contribution statement

Katerina Kyrkou: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Supervision, Project administration. **Matthew Frost:** Conceptualization, Writing – review & editing. **Paul Fleming:** Conceptualization, Writing – review & editing. **Nick Sartain:** Conceptualization, Writing – review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The authors would like to acknowledge Balfour Beatty Vinci for giving access to their site and trial embankment, the staff and companies participating in the field tests and the teams processing the data, (anonymously) as well as HS2 Ltd. for funding this research project.

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