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Why Bigger Training Sets Must Yield Diminishing Returns: An Information-Theoretic Speed Limit For AI

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Abstract

It is now well established that AI systems suffer diminishing performance returns as the number of training items is increased. Given that a fundamental limit on the performance of any system is the amount of Shannon information in its training set, we prove that adding one item to a training set of n items cannot provide more than $\Delta H = 0.5 \log_2((n+1)/n)$ bits of additional information. Because ΔH shrinks rapidly as n increases, it is inevitable that the performance of any AI system suffers from diminishing returns as n is increased.

1 Introduction

All modern AI systems rely on large amounts of training data. This remains true of all AI systems that classify inputs into different classes, as well as AI systems that generate images or words (e.g. chatGPT). For the sake of simplicity, we assume the AI systems considered here classify images.

An AI system trained to classify images into cats and dogs (for example) needs a large training set, which typically consists of millions of images. As expected, classification performance increases with the number n of training images. However, it is becoming apparent that there are diminishing returns on the size of training sets(Radford et al., 2019; Sun, Shrivastava, Singh, & Gupta, 2017; Kaplan et al., 2020).

Accordingly, a key question is: Is there a fundamental 'speed limit' for the rate at which the performance of AI systems can increase with n?

2 How much information does each extra training item provide?

Any system which attempts to classify data into different classes must be able to estimate the distribution associated with each class. If each distribution is Gaussian then all of the information implicit in each class is captured by its mean and standard deviation. The uncertainty with which the mean is measured can be defined as the standard deviation in the estimated mean. For example, a system which classifies data into different classes must choose where to place its estimate of the border between classes. If we choose to define this border as (possibly a multiple of) the standard deviation of the mean then the uncertainty of this border decreases with the standard deviation in the estimated mean.

For the present, we assume the training set has a distribution with mean μ and variance σ^2. Given n training items $\{x_1, \ldots, x_n\}$, the estimate of the mean μ is,

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i. \tag{1}$$

And the standard deviation of the estimated mean is

$$\hat{\sigma}(n) \quad \leq \quad \sigma/\sqrt{n}, \tag{2}$$

with equality if the distribution of means is Gaussian. Note that the central limit theorem ensures that, if n is sufficiently large then the distribution of sample means is approximately Gaussian. Consequently, because n is usually large for AI systems, Equation 2 can be expressed as an approximation,

$$\hat{\sigma}(n) \quad \approx \quad \sigma/\sqrt{n}. \tag{3}$$

The uncertainty in the estimated mean can be expressed in terms of Shannon's information theory(Shannon & Weaver, 1949; Stone, 2022), as follows. The differential entropy of a Gaussian distribution with standard deviation $\hat{\sigma}$ is

$$\begin{aligned}
H(n) &= 0.5 \log_2 2\pi e \hat{\sigma}^2(n) && (4)\\
&= 0.5 \log_2 2\pi e + 0.5 \log_2 \hat{\sigma}^2(n) && (5)\\
&= 0.5 \log_2 2\pi e + \log_2 \hat{\sigma}(n) && (6)\\
&= K + \log_2 \hat{\sigma}(n) \text{ bits}, && (7)
\end{aligned}$$

where the constant $K = 0.5 \log_2 2\pi e = 2.0471$ bits. Substituting Equation 3,

$$H(n) \approx K + \log_2 \sigma/\sqrt{n} \tag{8}$$

$$= K + \log_2 \sigma + \log_2 1/\sqrt{n} \tag{9}$$

$$= C + \log_2 1/\sqrt{n} \text{ bits,} \tag{10}$$

$$= C + 0.5 \log_2 1/n \text{ bits,} \tag{11}$$

where the constant $C = K + \log_2 \sigma$. Because the Gaussian is a maximum entropy distribution, any distribution with standard deviation $\hat{\sigma}$ must have an entropy that is less than (or equal to) the entropy of a Gaussian distribution with standard deviation $\hat{\sigma}(n)$. Consequently, the decrease in entropy that results from adding one extra data training item is,

$$\Delta H \approx H(n+1) - H(n) \text{ bits.} \tag{12}$$

Using Equation 11,

$$\Delta H \approx [C + 0.5 \log_2(1/n + 1)] - [C + 0.5 \log_2(1/n)] \tag{13}$$

$$= 0.5 \log_2 (n+1)/n \text{ bits.} \tag{14}$$

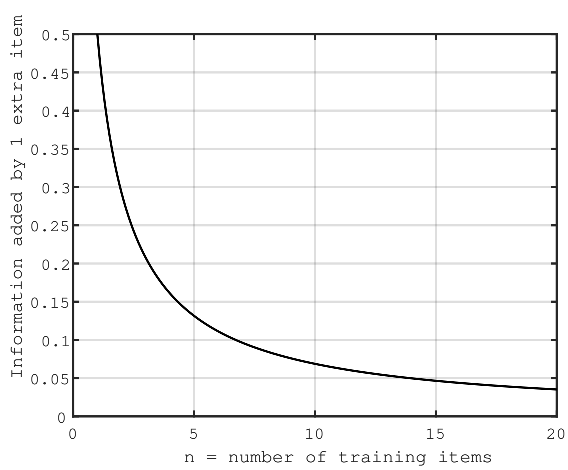

Figure 1: How the maximum amount of Shannon information ΔH (in bits) provided by one additional training item decreases as the number n of training items increases (using Equation 14).

This implies that the rate at which information is gained by adding extra training items does not depend on the standard deviation σ of the training items (even though the absolute *amount* of information gained is proportional to $\log_2 \sigma$, see Equation 8). The rate at which extra information is acquired for each additional training item (Equation 14) is plotted in Figure 1.

3 Discussion

Irrespective of the nature of the task being learned, the number of parameters in a model, the particular training algorithm used, or the speed of the processors used to implement that algorithm, Shannon's information-theoretic data processing inequality places a fundamental limit on the rate at which performance can increase as the number n of training items increases. Given that each extra training item cannot increase the amount of Shannon information gained by more than $\Delta H = 0.5 \log_2((n+1)/n)$ bits, it is inevitable that any system which relies on training data must suffer from diminishing returns on performance as n increases.

Finally, it might be thought that reinforcement learning systems(Sutton & Barto, 2018) do not suffer from diminishing returns because their learning relies on feedback from the environment. However, if that environment is constant then the analysis above implies that even reinforcement learning systems must suffer diminishing returns in each feedback signal (learning trial) they receive as the number of feedback signals is allowed to increase.

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