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Graphical Abstract

A review & critique of optimal fail-safe structural design

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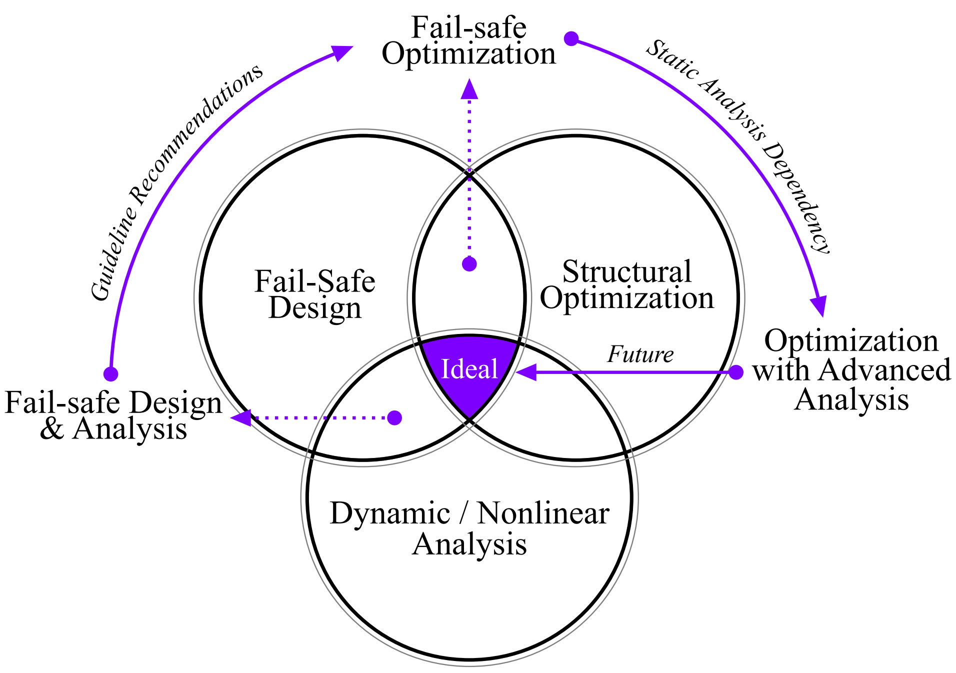

Highlights

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- The areas of structural optimization and fail-safe/robust design have generally been independent research themes, operating separately.

- 'Fail-safe optimization' has sought to reconcile fundamental differences in approach between these two fields.

- A review of fail-safe optimization is presented, critiquing the unification of the two parent themes.

- Fail-safe optimization works are found to miss important considerations well-known in the fail-safe/robust design community.

- To advance the state-of-the-art, structural optimization and fail-safe design must be addressed concurrently, with both aspects implemented in a fair proportion.

A review & critique of optimal fail-safe structural design

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Abstract

Structural material optimization holds great promise for reducing the embodied carbon in future infrastructure. However, optimized solutions typically lack robustness and may be susceptible to disproportionate collapse under damaging events such as blasts, collisions, or material corrosion. Fail-safe optimization offers a solution by balancing material efficiency with the ability to meet design requirements after a defined damage event. This paper reviews the state-of-the-art in fail-safe optimization, identifying current knowledge gaps and evaluating methodologies against existing guidelines for fail-safe design. A brief overview of fail-safe and collapse-resistant design is included to contextualize the comprehensive summary of the current literature. Key findings highlight the need for further investigation into buckling failure, allowing partial collapse, buildability aspects and the role of bending within fail-safe frame optimization. Additionally, a significant gap is identified between the types of analysis recommended in fail-safe design guidelines and those used in optimization tools, with the latter often neglecting non-linear and dynamic effects. To address this disconnect, the paper discusses optimization algorithms and techniques through the lens of incorporating more advanced analysis. This review aims to advance the development of fail-safe optimization, guiding improvements in current tools to enhance structural efficiency and safety against life-threatening events.

Keywords: Fail-safe optimization, Structural optimization, Disproportionate collapse, Robustness, Accidental actions, Review

1. Introduction

1.1. Context

Global challenges such as the climate emergency, the reduction of available raw materials, and the pursuit of economic gain have significantly increased interest in the research area of structural material optimization. This practice seeks to unify design and analysis within an automated process, enabling the identification of the best feasible design more efficiently. By defining the design requirements (e.g., global equilibrium, stress and deflection limits), specifying what constitutes a good structure (e.g., minimum structural volume), and determining the means of evaluating a solution's validity (e.g., analysis type), algorithms can be employed to explore potential design solutions and guide the process toward the optimal structure. Structural optimization can thus be viewed as an approach to engineering design from an inverse problem perspective [1].

However, structural safety remains paramount, and damage-resistant properties are essential for designs to be confidently used in practice. Structures identified through typical optimization formulations often lack these characteristics as minimizing material usage inherently reduces redundancy, making them more susceptible to abnormal loading or damaging events [2]. The design of material-efficient structures that ensure safety after a destructive event is known as fail-safe optimization [3]. This type of structural optimization aims to balance material efficiency and redundancy, finding the minimum volume solution that provides a desired level of structural robustness, typically defined by the extent of damage a structure can withstand.

This literature review investigates current fail-safe optimization research, assessing the tools against contemporary recommended fail-safe design methodologies and identifying current knowledge gaps.

1.2. Definitions

Within the field of fail-safe design, many researchers have commented on the general lack of consensus on terminology [4, 5, 6], with Starossek & Haberland [5] suggesting the lack of agreement has inhibited the development of procedures and design standards. Here, a set of definitions is given based on the curated literature in an attempt to provide clarity for this paper without exacerbating the inconsistency of the current terminology.

Structural Damage: *An unplanned and abnormal variation or change of a structure's properties, which entails weakening and negative consequences. Damage includes impacts on material properties and/or geometry of a structure. This may occur at any point during the structure's life cycle [7, 8, 9]. Note that statistical variation of certain system parameters, e.g. the natural uncertainty of material properties such as yield stress values, are not considered structural damage in this review.*

Redundancy: *The ability of a structural system, post damage, to redistribute forces - which were once taken by the damaged elements - among its undamaged members and connections through alternative load paths or multiple load-transfer mechanisms.* [10, 6]

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Robustness: *The ability of a structure to avoid consequences disproportionate to an event which causes damage. It is a matter of residual capacity to maintain function after a change in the structure or its environment. It, therefore, must be defined on a case-by-case basis, in line with specific design or analysis investigation goals.* [4, 11]

(Note the term 'robustness' in the context of optimization is commonly used to describe a type of optimization problem where the design parameters possess some degree of uncertainty and ensuring the resulting structure is insensitive to such uncertainties [12]. Therefore, in this article, the term 'structural robustness' is used to denote the former to avoid confusion with the latter.)

Disproportionate Collapse: *Refers to an event where damage to a relatively small part of a structure results in damage or collapse over a much larger area. Disproportionate collapse relates to the final size of the damaged region compared to the initial amount of damage without describing the structural behaviour occurring during the event or the triggering circumstances. Whether a resulting end damage is disproportionate is potentially subjective and thus requires careful definition for a given structure.* [4, 11]

Progressive Collapse/Failure: *A description of the response of a structure during a damaging event where local structural damage propagates through a chain reaction mechanism leading to further failure within the structure. Progressive collapse can be a mechanism that creates a disproportionate collapse event; however, the mechanism does not guarantee that a large disproportion of damage will always occur.* [4, 13]

Key Element: *An element which can cause disproportionate collapse if subject to a substantial reduction in capacity due to a structural damage event.*

Critical Element: *An element which, if damaged, will significantly reduce the performance of the entire structure.*

Fail-safe Structure: *A structure which is capable of fulfilling the relevant design requirements after a certain portion of the structure has been subjected to structural damage.* [14, 15]

1.3. Study scope and structure

It is now possible to define the scope of this study. Here, the focus is specifically on the optimization of fail-safe structures. Fail-safe structures offer one strategy to produce robust structures which are not susceptible to disproportionate collapse; the key difference from more general risk-based approaches is 'the acceptance that failures will occur for one reason or another despite all precautions taken against them' [14, p. 363]. For this reason fail-safe design is a popular approach across various industries from aerospace to civil engineering (albeit under varying terminologies). Relatively mature analysis methods and design guidelines are available. However, the rise of optimization-driven approaches has been in large part separated from this body of knowledge. The aim of this paper is therefore to bridge that gap by identifying discrepancies between the two fields, and providing a common base from which work can progress towards more realistic models for fail-safe optimization.

The organisation of this paper is illustrated in Fig. 1. First, in Sect. 2, a summary of modern fail-safe design methodologies are presented to provide context to research on fail-safe optimization. The state-of-the-art in fail-safe optimization is then presented in Sect. 3, incorporating a critique of the work against contemporary design practice and highlighting key knowledge gaps. A significant mismatch between current optimization tools and recommended design guidelines is observed in the analysis methods employed. Hence, in Sect. 4 potential optimization techniques are assessed through the lens of their potential for incorporating the advanced analysis methods demanded by current best practices in fail-safe design. Overall, it is hoped that this work will aid in the development of practical fail-safe optimization design, reducing the material consumption of our structures whilst improving safety.

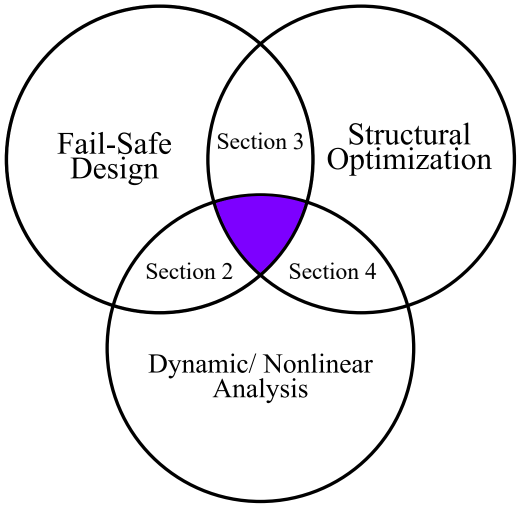


Figure 1: Paper organisation, with the solid purple centre representing the ideal state of optimal fail-safe structural design tools.

2. Fail-Safe Design & Analysis

2.1. Section Motivation & Aims

Throughout a structure's lifespan, it may be subject to some form of damage which negatively influences its performance and ability to provide its design requirement. This damage can come in many forms, including material degradation, fire, earthquake, vehicle collisions, blasts, or natural storms. Due to the rise of climate change and escalation of geopolitical tensions, global infrastructure is becoming increasingly exposed to such severe events [16]. It is imperative that performance after the occurrence of damage can be guaranteed to ensure the safety of the people who occupy and use these structures and to reduce economic loss [4]. The area of fail-safe design and progressive collapse has thus received growing attention over the last few decades.

This section reviews modern aspects of the collapse-resistance design and structure analysis to provide context to fail-safe optimization. Sect. 2.2 provides an overview of principal contemporary methodologies employed in this field. The most relevant approaches and considerations for optimization-driven design are then explored: critical member identification (Sect. 2.3) and dynamic response after a damage event (Sect. 2.4). Here, the focus is on providing the required context for the topic of fail-safe optimization, and readers are referred to Elkandy et al. [16], Adam et al. [4], and Kiakojouri et al. [17] for comprehensive reviews of the current research on the progressive collapse of structures.

2.2. Design & Analysis Methodologies

Like for many design problems, structural engineers often refer to codes and guidelines to aid their design of fail-safe structures (e.g. GSA 2003 [18], UFC 4-023-03 [19], and Eurocode 1 [20]). In literature, Ellingwood [13], Byfield et al. [21], and Russell et al. [22] present the use of design codes from a historical perspective, highlighting the importance of progressive collapse events such as Ronan Point (London, 1968), the A.P. Murrah Federal Building (Oklahoma, 1995) and the World Trade Center (New York, 2001) have had on the development of design codes and guidelines. Byfield et al. [21] highlight the main ways modern codes recommend designing against progressive collapse, including techniques such as member-tying, designing for local resistance of key members, and alternative load path design. The latter methodology will be the primary technique focused on in this review due to its applicability in fail-safe optimization. Fang and Fan [6] identify four main design features to create redundant structures: structural form, material property, member ductility, and continuity. They look to define which parts of the design stages, construction, and maintenance should these main features be addressed. Starossek [5] proposes a framework for designing a collapse-resistant structure, placing design requirements and objectives into different classification levels, leading the designer to consider incorporating different degrees of structural robustness into their structures. Alternatively, López [23] considers the importance of recording cases of disproportionate collapse, outlining a general methodology for establishing a database and taxonomy and suggesting methods in which to accurately describe the damaging event and the mechanism of damage propagation for comprehensive logging which can enable engineers to learn from previous failures.

Many authors have proposed different metrics for quantifying fail-safe structures' redundancy and other desirable qualities. Refs. [24, 25, 26, 8] have looked to define indexes based on different structural characteristics as a means of placing numerical quantification to the impact of a damage case and the redundancy/structural robustness of a structure. Whilst most studies have established indexes for frame and truss-based structures, Kranz et al. [27] established a methodology to measure fail-safe qualities for continuum structures, measuring different load paths through an image processing method.

Others have viewed the problem of fail-safe design from a probability and risk analysis perspective. Pretlove et al. [28] applied the natural variation of material properties into the fail-safe assessment of a simple structure, yielding a probability of failure for a given damage case and system parameter definitions. Frangopol [24] undertook studies on probabilistic measures, establishing a reliability index based on the variability of certain design variables. Likewise, Baker et al. [29] established a framework to assess system structural robustness when subject to structural damage, founding the methodology through risk analysis, allowing for the consideration of indirect consequences of damage. Similarly, Mousavi & Gardoni [30] propose an integrity index based on the difference between the maximum and minimum failure probabilities of the structural components of a system. Ellingwood [13] introduced some basic principles of risk-informed decision-making for assessing and mitigating risks and proposed a mathematical framework for quantifying the probability of disproportionate damage occurring. Beck et al. [31] looked to compare the structural robustness indexes of [29] and [32], assessing them on their suitability for measuring strengthening via a codified alternative load path method and their applicability for describing solutions from risk-based optimization.

McKay et al. [33], and Byfield et al.[21] reviewed the different structural analysis methods from various codes and guidelines for the prevention of disproportionate collapse, with McKay et al. [33] highlighting some of the inconsistencies with the application of dynamic load factors used in static analysis and their general overly conservative nature. Many of these codes and guidelines recommend the use of dynamic analysis for most structural and damage types. The importance of dynamic effects and their consequences on design is discussed in greater detail in Sect. 2.4. Kiakojouri et al. [17] and Elkady et al. [16] reviewed numerical modelling techniques, highlighting the strengths and weaknesses of the finite element method, the applied element method [34], and the discrete element method [35] for their accuracy in simulating different levels and forms of damage. Kiakojouri et al. [17] criticised the use of the finite element method, claiming that it cannot easily predict the collapse mechanism and sequence, although this has occasionally been achieved (e.g. Smith [36, 37]). Supervised machine learning approaches have also been applied for the dynamic assessment of progressive collapse [38].

2.3. Critical member identification

Within the built environment, structures are typically formed of multiple elements – thus introducing multiple potential failure points. As a result, analysing the impact of member damage for every possible damage scenario can become highly laborious and time-consuming. This is particularly true for truss-like structures or in the case of continuum structures as considered in Sect. 3.2, where the effect of damaging any given member is hard to quantify through engineering intuition. For a structure with n number of members and each damage case is defined by the damage of k number of members, the total number of unique damage conditions which can be analysed is given by the binomial coefficient, as shown in equation 1:

$$\text{Total Unique Damage Conditions} = \binom{n}{k} = \frac{n!}{k!(n-k)!}. \quad (1)$$

A 4-bar example is shown in Fig. 2, illustrating all the potential damage cases for $k = 1$ and $k = 2$. Consider a truss

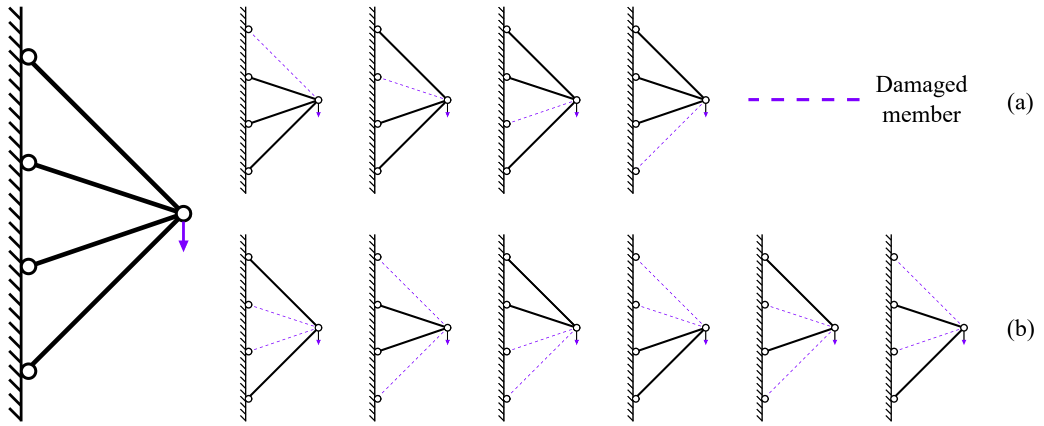


Figure 2: Illustration of all possible damage cases for a 4 bar cantilever truss structure. (a) Single element damage cases. (b) Two element damage cases.

composed of 20 members ($n = 20$), where each damage case is defined by the damage of two members ($k = 2$). There exist 190 unique damage cases that would require analysis. If an engineer employs the recommended non-linear dynamic analysis approach, assessing all possible damage cases would naturally incur significant computational costs and time. In response to this, a significant amount of research has looked into the identification of critical members.

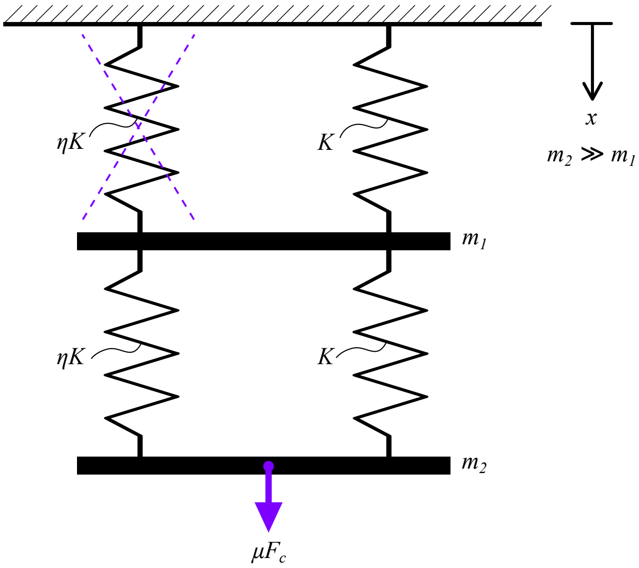
Energy-based methodologies for critical member identification have been a popular approach for many researchers. Smith [36] presented a systematic search procedure, quantifying the criticality of a damage case by comparing the change in strain energies of the remaining members to their rupture energies. Smith develops this methodology in [37], allegorising the collapse of a building to fast fracture in metals to find the sequence of damaged members that requires the least amount of damage effort yet results in structural collapse. Similarly, Feng [39] utilises a strain energy-based methodology by employing a change in total strain energy robustness definition to identify critical members.

Other researchers have approached the problem by looking to establish coefficients of importance for given members based on the structure's load capacity or stiffness information to identify their damage impact. Frangopol and Curley [24] used their limit-state-based redundancy definition to identify critical members. Likewise, the identification of worst-case damage scenarios, measured through the reduction of limit analysis load factors, was formulated as an optimization problem by Kanno [40] and solved as a mixed integer linear programming problem. To find regions within a standard power transmission tower sensitive to member damage, Eslamlou et al. [41] utilised non-linear dynamic analysis to determine impact factors from damage cases assigned at different locations in the structure. Feng et al. [42] presented a methodology centred on information from a system's stiffness, applying an element importance coefficient by comparing the determinant of the damaged and undamaged tangent stiffness matrices. Similarly, Fang [43] presents a plastic importance coefficient based on the change in the ultimate plastic bearing capacity of the damaged structure and the elastic bearing capacity of the undamaged structure. Furthermore, quantifying the importance of members for a given loading event based on failure probabilities has also been investigated. By considering a systematic reliability-based methodology, Felipe et al. [44] ranked elements based on their combined probability of initiating failure progression (structural importance) and their probability of being damaged (vulnerability).

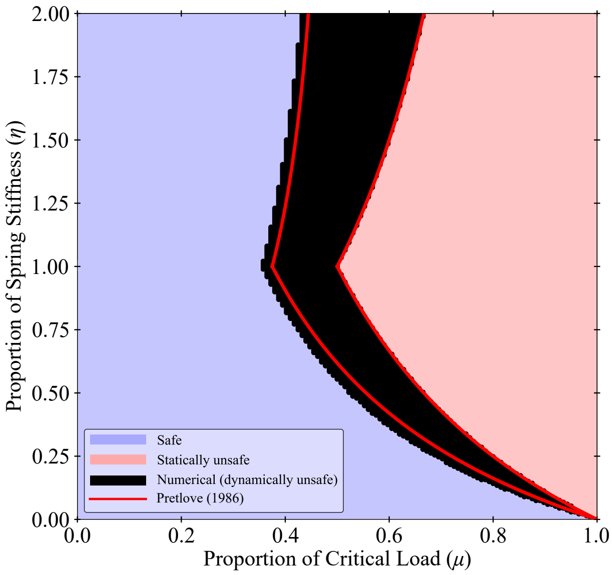
2.4. Dynamic Action & Effects After Damaging Events

There is a consensus among researchers and guidelines that using dynamic analysis for measuring the effects of sudden damage events is recommended over any static analysis. The short study by Pretlove [45] acts as a good demonstration of the importance of dynamic action. Pretlove [45] remarks that a structure will undergo a complex transient dynamic response after sudden damage as the system's kinetic energy is gradually removed through damping and thus approaches a new static equilibrium. However, during the dynamic phase, the system's displacement will overshoot concerning its final static position, thus converting its kinetic energy to strain energy, leading to heightened element stresses. By using this energy conversion and considering a simple 1-D parallel spring system (see Fig. 3a), Pretlove [45] demonstrated analytically the non-conservative nature of static analysis. It was shown that when certain conditions are met, the dynamic analysis will infer that a progressive collapse mechanism will be triggered whilst a static analysis does not, as shown by the black region in Fig. 3b. These results were later verified numerically by Whiteside et al. [46]. However, the extent of the dynamic influence is generally case-dependent and entails complex behaviours which provide challenges to model. The rest of this subsection focuses on the dynamic action of trusses due to the typologies prevalent in fail-safe optimization.

Many researchers have looked to undertake physical testing on truss structures subject to sudden damage to measure the dynamic effects and overall collapse-resistant mechanisms [28, 47, 48, 49, 50]. However, due to the difficulties of such experiments and the limitations of the number of damage cases



(a)



(b)

Figure 3: Pretlove [45] static/dynamic comparison study. (a) 1-D parallel spring system that experiences an instantaneous spring removal. (b) A plot of system parameters and the result of the structure's safety based on the type of analysis used. The black zone is the numerical replication by Whiteside et al. [46].

which can be applied, numerical modelling, most commonly employing the finite element method, is used to perform fail-safe analysis [51, 41, 48, 52, 53, 50, 46].

Rapid damage events induce other complex dynamic events in the structure, such as acoustic stress wave propagation. Several studies have modelled stress wave propagation through truss structures [54, 55, 56]. However, its consideration in fail-safe analysis has been sparse. Jiang & Chen [51] and Goto et al. [57] claim that the considerations of stress waves can be neglected, as their magnitude will generally be less than the stress developed by the form redistribution; however, additional investigation into the importance of stress wave propagation is required across a broader range of structure types.

The rupture time of an element during a damage event can significantly affect the dynamic effects experienced by a structure. Studies by Mozos & Aparicio [58] and Whiteside et al. [46] have investigated this, with their findings indicating that an increase in removal time decreases the dynamic response, the magnitude of which being linked to the modal time periods of the damaged structure. This characteristic is illustrated in Fig. 4.

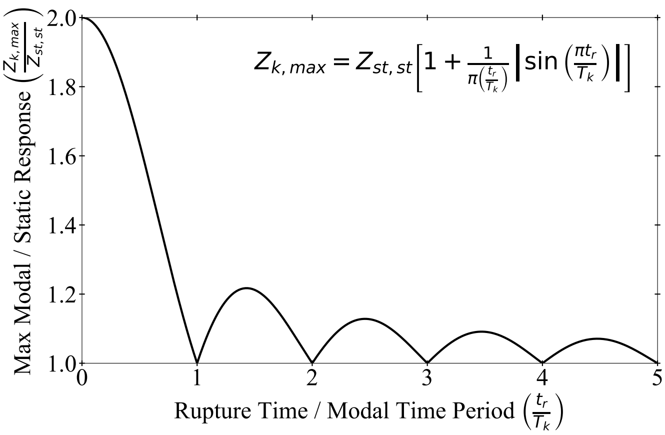


Figure 4: Plot of the generalised coordinate ratio of the maximum modal response to the static response over the ratio of element rupture time to the modal time period, showing the general influence of the rupture time on the dynamic response of a structure. Findings from Mozos & Aparicio [58].

Due to some of the challenges in performing dynamic analysis for the process of fail-safe design, it is common to utilise a static analysis approach in conjunction with amplification factors to compensate for the increased stresses due to dynamic action. McKay et al. [33] reviewed guideline suggestions for amplification factors, noting several inconsistencies and the over-conservative nature of using the standard values. Kang et al. [59] also criticised the general use of load factors for dynamic problems due to their inability to capture the multi-directional nature of the structural responses. Based on these shortcomings, researchers have proposed ways to approximate more accurate dynamic amplification values for truss structures in certain situations [57, 60]. For problems such as sudden column removal in frame buildings, pseudo-static analysis methodologies have been developed, such as the energy balance method proposed by Izzuddin et al. [61], with its application being demonstrated in [62].

3. Fail-safe optimization

3.1. *Brief Introduction to Optimization & Section Aims*

Investigations into the limit of material efficiency to transmit a given set of loads to a set of supports were first investigated by Michell [63], developing on the work of Maxwell [64]. Since the establishment of the theory, numerical methods for finding approximate solutions to these problems have become an extensive engineering research area. The research topics can be roughly split into continuum optimization and truss optimization based on the structural model employed. For either of these, three optimization problems can be addressed: size, shape, and topology [65]. Size optimization refers to the identification of element dimensions, shape optimization refers to alterations of the structure's geometry, and topology refers to the connectivity of elements. Such characteristics are thus used to minimise or maximise a design objective (usually structural volume or compliance) whilst ensuring the structure meets design requirements. Structural optimization problems may also be categorized as deterministic or probabilistic depending on the degree of certainty of the system parameters, such as the material properties and applied load.

Fail-safe optimization introduces the requirement to resist defined damage events, thus ensuring that design requirements are met after such an event and that the structure maximises the given objective, e.g. material efficiency. For deterministic problems, the degree and nature of damage events are usually predefined, typically through either damage regions/zones or explicit damage to defined elements. Their occurrence is guaranteed, and so satisfactory behaviour must be obtained for every case. Guaranteed damage models may also be applied to probabilistic optimization problems, with the uncertainties arising elsewhere. Alternatively, probabilistic frameworks may consider models where damage cases have associated probabilities < 1. It is reiterated here that behaviour after damage has occurred is a fundamental part of fail-safe design, and so consideration of aleatory uncertainties alone does not make a problem one of fail-safe design. Furthermore, due to the high number of possible damage cases of any given structure, as highlighted in Sect. 2.3, creating optimum structures which meet fail-safe design requirements poses computational challenges. However, with the recent increase in computing power and the importance of collapse-resistant characteristics, fail-safe optimization has become an area of growing interest.

This section looks to review the state-of-the-art in fail-safe optimization research. Its application to continuum structures is presented first, followed by trusses and other structure typologies. A critique of the literature is then presented, identifying some disconnects between the current fail-safe optimization tools and the recommended fail-safe design methodologies, along with general areas which have yet to be extensively explored.

3.2. Continuum Structures

3.2.1. General Method

The optimization of solid continuum structures revolves around assigning material at any location within a design domain to meet the design requirements while simultaneously minimising or maximising an objective function [66]. The design domain is typically discretized into a finite number of elements which can be removed or maintained to establish a solid structure. This general process is shown in Fig 5. To accomplish this complex task, the current leading approach is the Solid Isotropic Material with Penalisation method, where the density of the finite elements is allowed to vary continuously between 0 (void) and 1 (material present) but are penalised when non-binary values are taken [67]. Another popular approach is the Evolutionary Structural Optimization method [68], where the material within the design domain is iteratively removed from under-utilised regions. For both methods, a structure's response for some given design variables is determined through finite element analysis.

3.2.2. Initial Studies

The first introduction of fail-safe design in continuum optimization was by Jansen et al. [70], who introduced a failure-patch approach, where a structural damage case is modelled by a void zone in which no material can exist. Square-shaped patches were applied at every finite element grid individually, leading to a high number of damage cases for a given problem and thus leading to significant computational cost.

The method was developed by Zhou & Fleury [15], extending the formulation to solve 3-dimensional problems, where cube damage volumes replaced square damage patches. The number of damage cases was also reduced by applying the damage zones to a grid size equal to the size of the patches, thus ensuring no damage cases overlap. For both studies, including damage cases in the optimization showed that the resulting optimised structures possess increased levels of complexity over their nominal cases (see Fig. 6). Since these studies, continuum fail-safe optimization has received significant attention from researchers, extending the basic approach in various ways.

3.2.3. Improving Computational Efficiency

Many studies have looked to improve the computational efficiency of the method by reducing the necessary damage cases which need to be considered, employing methods similar to those discussed for critical member identification in Sect. 2.3. Ambrozkiewicz & Kriegesmann [71] introduced an active-set method, reducing damage cases by identifying the active constraints within the optimization problem. Wang et al. [72] employed the use of von Mises stresses to dictate the position of damage cases for each iteration of the optimization process. Others have reduced the number of damage cases by using search strategies to locate critical damage regions. Hederberg & Thore [73] proposed the use of 'moving morphable components' to allow the damage patches to move to the locations of critical failure. Alternatively, Zhang et al. [74] propose a framework for placing local damage patches using a stochastic sampling strategy to locate regions of critical damage for each iteration quickly. Although Herrero-Pérez & Picó-Vicente [75] do not directly reduce the number of damage cases, computation effort is reduced by employing a hierarchical parallelization scheme and thus allows the optimization to be applied to high-resolution structures. By establishing a methodology for assessing fail-safe qualities of continuum structures, Kranz et al. [27] looked to investigate the influence of failure patch sizes and shapes for solutions using a max stress minimisation objective function. Dense and computationally expensive damage patch sets were considered unnecessary for creating good fail-safe solutions. Furthermore, non-fail-safe solutions using local volume constraints [76] to establish additional load paths were tested but were considered less favourable due to their dependence on more heuristic parameter tuning.

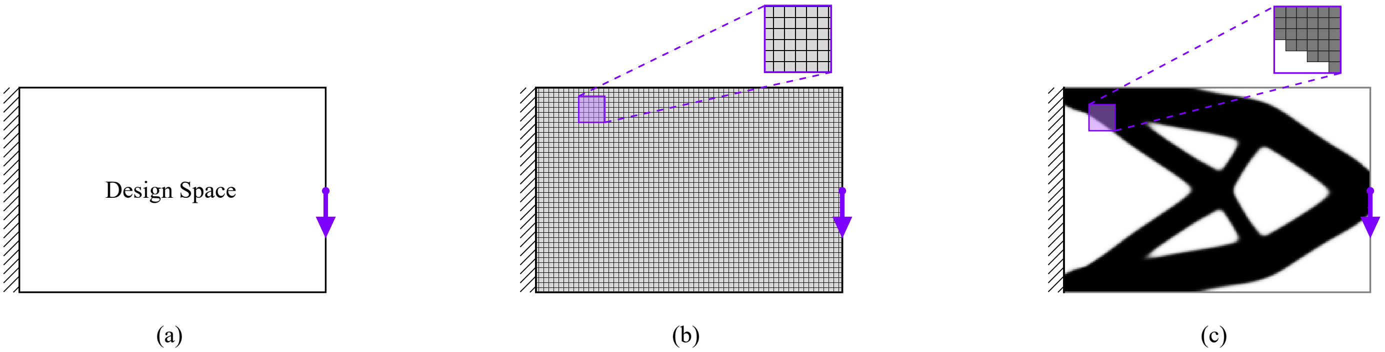


Figure 5: General process for topology optimization of continuum structures. (a) Defining boundary conditions and the design domain. (b) The design domain is discretized into a mesh of finite elements. (c) The optimization problem is solved by removing and adding material in the design domain to produce an optimum structure. Optimized structure produced using Interactive 2D TopOpt App [69].

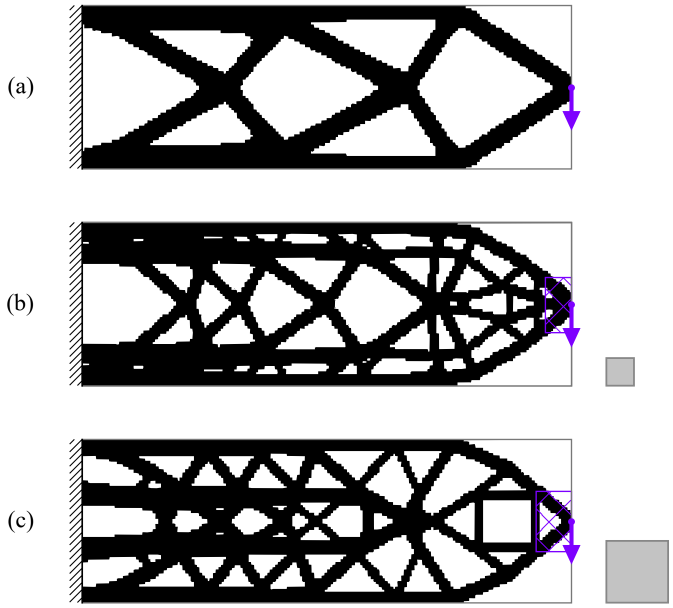


Figure 6: Continuum fail-safe optimization solutions using a failure patch approach on a cantilever problem within increasingly large damage cases, adapted from Zhou & Fleury [15]. The hatched zone is excluded from damage. (a) Nominal non-fail-safe solution. (b) Fail-safe solution using small damage patches. (c) Fail-safe solution using large damage patches.

3.2.4. Additional Design Considerations

Other researchers have looked to introduce additional factors into the fail-safe optimization problem. Viewing the problem from a probabilistic 'robust' optimization approach, Long et al. [77] looked to consider uncertainties in the system's loading concerning both the force's direction and magnitude. Addressing the problem of uncertainty in damage cases, Martínez-Frutos & Ortigosa [78] attribute probabilities to pre-defined damage locations, along with uncertainties to their size. In addition, they introduce the minimisation of the standard deviation of structural compliance, thus helping to ensure that each damage case inflicts a similar level of structural capacity loss. Cui et al. [79] considered a reliability-based topology optimization problem, integrating the uncertainty of design parameters within a problem and thus looked to integrate reliability-based topology optimization into a fail-safe formulation. Similarly, da Silva & Emmendoerfer Jr [80] introduced the considerations of imperfections due to manufacturing error, where eroded and dilated topologies represent extreme manufacturing error [81].

da Silva et al. [82] looked to address the problem of designing against failure induced by overloading whilst ensuring predictable points of failure, a requirement for designing large landing gear systems. By utilising a stress-constrained formulation and defining a predefined 'damage zone', the resulting optimized solutions were shown to experience excessive stresses in the defined region during overloading situations, thus providing a predictable point of failure, and were capable of redistribution of the loads after the material in the over-stressed region had been removed.

Peng and Sui [83] considered incorporating deflection-based constraints into a volume minimization problem with traditional failure patches. The effects on optimum topologies considering different failure patch shapes and sizes were also investigated.

Zhao et al. [84] looked to introduce fatigue considerations into the optimization problem, subjecting their structures to cyclic loading conditions and employing equal life curves for material failure evaluation. The resulting optimised structures were found to be sensitive to the applied cyclic load conditions.

3.2.5. Multiscale Structures

While most continuum fail-safe optimization looks to alter the macro-structure, others have investigated the optimization of cellular micro-structures and their contribution to structural robustness (also known as two-scale topology optimization). The methodology involves the replacement of the solid continuum finite elements with substructures that have been optimized to resist the local stress states, as shown in Fig. 7. See Wu et al. [85] for a comprehensive review of this optimization type applied to non-fail-safe problems.

A study by Qiu et al. [86] optimised continuum structures subject to standard loading and no damage cases using a set of predefined micro-structures. Applying the methodology to an example case, Qiu et al. [86] tested the resulting structure's robustness against a typical solid continuum optimised structure, modelled damage by a triangular void domain. Due to the larger member sizes of the micro-structure optimised solution, lower strain energy increases were found, suggesting improved structural robustness. Similarly, Do Quang et al. [88] investigated using Voronoi tessellation for infill optimization, comparing the structural robustness properties of the resulting structure to that of other structures optimised using alternative tessellation strategies. Damage was inflicted on the structures through small regular failure patches at a defined location within the sub-structure. Recently, work by Yang et al. [89] looked to incorporate cellular micro-structures into continuum fail-safe optimization fully, considering lattice structures of fixed topology. It was found that structures optimised using micro-structures could be more structurally robust than those optimised using solid material. However, the performance is strongly related to the configuration of the lattice micro-structure. Similarly, Huang et al. [90] considered a two-parameter-based lattice microstructure, thus allowing greater cell topology change without the need to adjust predefined unit cell configurations. Whilst lattice approaches provide computational benefits, the structural performance is still constrained by the predefined cell topology. Addressing this, Ding et al. [91] recently proposed a multiscale concurrent topology fail-safe optimization formulation, allowing both the macro and cell topologies to be optimized in parallel. As a consequence, the influences of the initial microstructure were negligible.

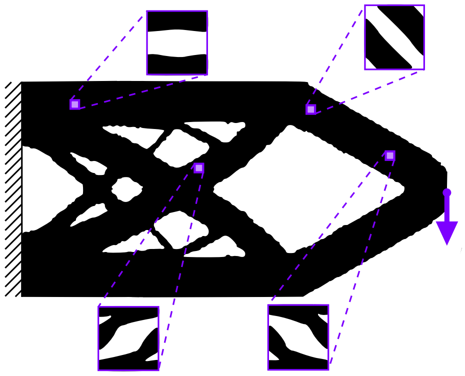


Figure 7: Example of a non-fail-safe continuum optimization solution using micro-structures for a cantilever problem. Adapted from Ferrer et al. [87]

Concerning optimising structures subject to impact loading, Shen et al. [92] looked to undertake a supervised machine-learning approach for the inverse design and optimization of auxetic honeycomb structures with gradient structural properties, looking to maximise specific energy absorption. Such auxetic honeycomb structures are known for their unique negative Poisson's ratio effect, which consequently can result in superior impact resistance [93].

3.3. Pin-jointed Truss Structures

3.3.1. General Method

Although many methodologies for finding optimum structural topologies of truss structures exist (e.g. [94]), the most popular approach is the ground structure method [95]. This method's most straightforward approach creates a dense structure of potential members from which a size optimization problem can be solved. The member's cross-sectional areas can take a range of continuous values from zero to infinity. It is common for most members to take a zero cross-section value, effectively removing them from the structure and establishing a new topology. This process is illustrated in Fig. 8 for a simple cantilever problem. Truss optimization problems involving the transfer of a single static load case (without consideration of damage or manufacturing tolerances) commonly result in statically determinate structures. Due to these characteristics, the structure possesses low structural robustness to resist abnormal loading and damaging events.

In the context of fail-safe optimization, two general types of constraints exist to resist disproportionate and progressive collapse. The typical method, which follows from non-fail-safe optimization problems, uses element-based constraints such that no further elements will fail after a damaging event (e.g. stress constraints). Alternatively, system-level constraints may be considered, where the condition of the entire structure is integrated, such as progressive collapse failure. The latter method is less popular and is primarily considered in probabilistic studies discussed in Sec. 3.3.4.

3.3.2. Initial Studies

The first attempt to address the lack of structural robustness of standard optimised trusses through the incorporation of fail-safe design considerations was by Sun et al. [3], where the complete removal of members defined the damage cases, and optimization was achieved through sizing the elements. Due to the necessity to evaluate the structure for all prescribed damage and load cases, the optimization problem sizes increase rapidly, and consequently [3] considered only relatively small structures with a limited number of damage cases (up to 72 bars, 5 damage cases).

Arora et al. [96] considered a slightly larger problem (108 bars, 6 damage cases), grouping members to take the same cross-sectional areas and thus reducing the size of the optimization problem. This was later developed by Nguyen & Arora [97], who incorporated the division of a given truss structure into several smaller substructures to reduce computational work associated with alterations to the system's stiffness matrix due to the member removal damage cases.

Furthermore, [97] introduced a 'worst violated constraint' method to reduce the number of constraints the program needs to consider, further improving computational efficiency. A more general damage definition was introduced by Achtziger et al. [98], where a damage case is defined by an elastic modulus degradation field acting over the design domain. This general damage model was then applied to the fail-safe optimization of truss structures by Achtziger & Bendsøe [99].

Shechter [100] incorporated an elastoplastic material model into the fail-safe optimization formulation; however, it only considered a very simple 3-bar problem. Considering the problem from a more classical engineering perspective, Feng & Moses [2] used the different load paths activated due to the failure to size the truss members, employing a 'fully stressed design' approach. Feng & Moses [2] also highlighted the importance of considering brittle elastic and plastic material models and their impact on determining optimum structural volumes.

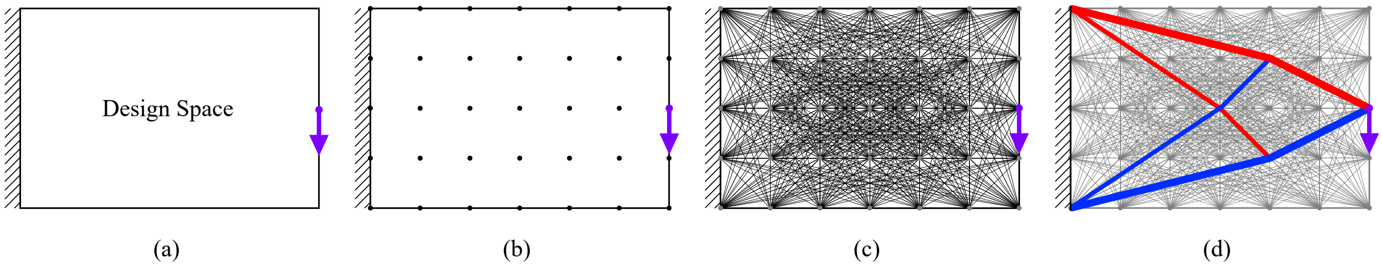


Figure 8: Ground structure method for topology/ size optimization of pin-jointed truss structures. (a) Define boundary conditions and design domain. (b) The design domain is discretized with a grid of nodes. (c) The ground structure is constructed by connecting every node to every other node, establishing every possible member. (d) The optimization problem is solved, sizing and removing bars from the ground structure to give an optimum solution.

3.3.3. Recent Developments - Deterministic Optimization

This section explores the recent developments towards optimum fail-safe truss design utilizing deterministic optimization formulations, where known constants define system parameters. Structural damage is defined as certain events (unconditional probability of 1) and are known before solving the optimization problem.

Kanno [101] presented a widely applicable fail-safe optimization formulation where the objective function is minimized to the worst-case damage scenario. Although the formulation is generalizable to multiple structural typologies, the study predominantly concerns truss structures for illustration. The formulation considers all possible damage scenarios for various damaged elements (e.g. Fig. 2). The problems are generally classified as mixed interior linear programming problems; however, the objective function may not be differentiable due to introducing the worst-case constraint. Thus, a derivative-free algorithm was developed and employed based on the sequential quadratic programming method and simplex gradients.

Stolpe proposed a working-set-based approach [105] to reduce the number of required damage conditions, formulating the problem into a classic minimum compliance problem with volume constraints, thus guaranteeing global optimality. Complete member removal and partial member damage definitions were considered and applied to diagonal ground structures up to 272 members in size. Stolpe [105] considered each damage case of 1 and 2 bars, thus considering up to $\binom{272}{2} = 36856$ total damage cases, requiring a computational time of around 13151s. It was found that high levels of partial damage (i.e. where an element is almost but not quite completely damaged) gave almost identical results to complete removal of the element. However, later work by Dou & Stolpe [102] showed that the introduction of stress constraints into an equivalent volume or compliance-based fail-safe truss optimization problem with variable partial damage removes the continuity of the objective function as the damage definition tends from partial (99% cross-section reduction) to complete (complete member removal), as shown in Fig. 9.

This results from the elements experiencing elevated stress when subject to near complete damage, thus demanding enlarged member sizes. This characteristic is similar to the ‘singular optimum’ issue seen in stress-constrained truss topology optimization, where the transfer from near-zero to zero cross-

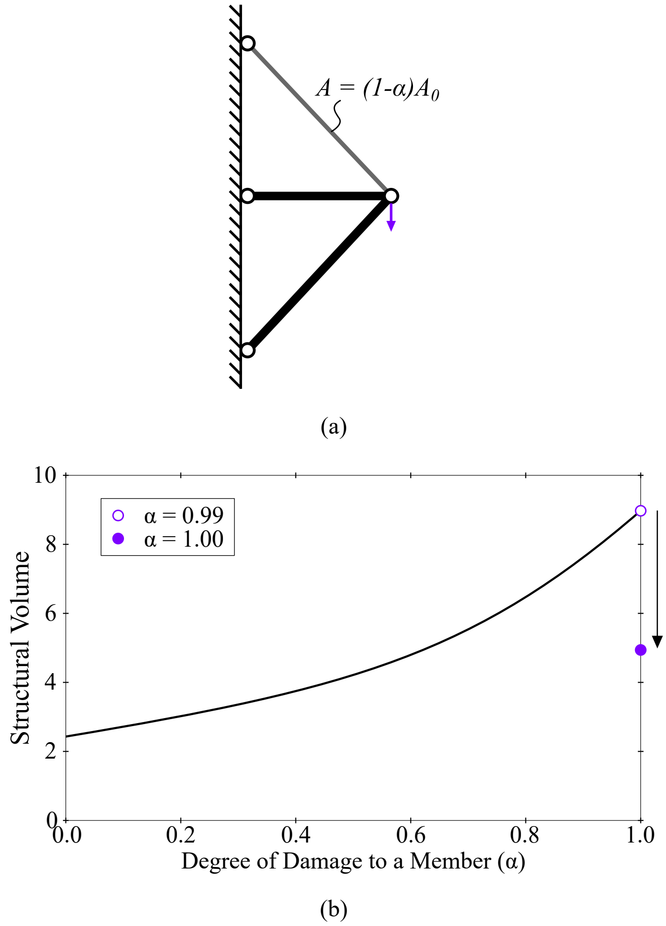


Figure 9: Singularity characteristic of the objective function (structural volume) for a 3-bar stress-constrained fail-safe truss optimization problem. (a) A 3-bar problem, where each bar is subject to a degree of damage, defined by α. (b) Plot of the objective function over the degree of damage. (adapted from Dou & Stolpe [102]).

sectional area of a member results in a sudden jump in improvement of the objective function [106, 107]. A similar phenomenon was seen in the fail-safe frame optimization work by Dou & Stolpe [108] (see Sect. 3.4).

Kirby et al. [103] investigated some of the characteristics of fail-safe optimal structures subject to single and multiple-member damage scenarios. A simple 3-bar cantilever prob-

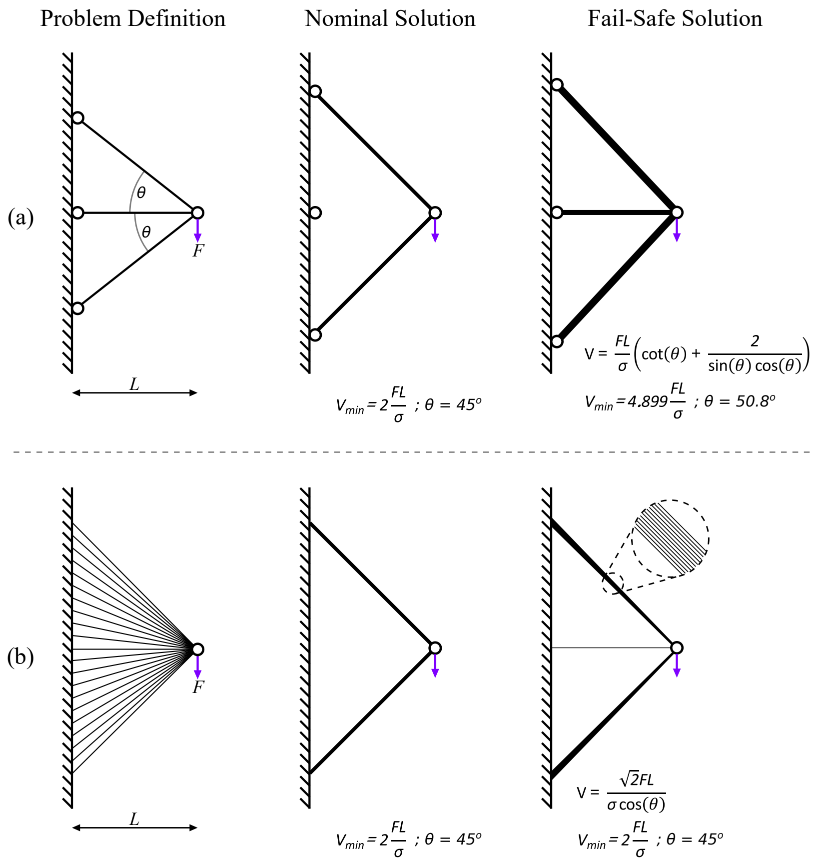


Figure 10: Findings from simple cantilever fail-safe truss optimization studies by Kirby et al. [103], and Fairclough et al. [104]. (a) Three bar topology with single member damage cases. (b) Infinite bar topology with finite member damage cases.

lem was considered to demonstrate some of the geometric and topological differences between fail-safe and nominal designs. Furthermore, by considering the same boundary conditions but with an infinite number of bars, Kirby et al. [103] showed that provided the number of elements being removed for each damage case remains finite, the solution will tend towards the nominal solution. Similar conclusions were found in a study by Fairclough et al. [104]. The findings from [103] are illustrated in Fig. 10.

Using the findings from the infinite bar case, Kirby et al. [103] proposed that theoretical optimum fail-safe structures will naturally tend towards the nominal cases, where instead of solid members, the bars will be composed of bundles of near parallel elements. However, this theoretical optimal is a consequence of the structural damage definition (as shown in Fig. 2) of a single member. This may be an adequate reflection of damage caused by certain material issues, such as crack propagation, but it does not necessarily reflect realistic damage events where damage may act over a zone as opposed to individual bars, such as that implied by accidental loading (impact/blast etc.) or environmental factors such as fire or corrosion. The limitations of this damage definition are addressed by Fairclough et al. [104], who defined damage cases using a damage patch method, similar to the method commonly used in continuum fail-safe optimization, where all elements within a given zone are said to have zero capacity, as shown in Fig. 11).

Furthermore, the previously mentioned fail-safe optimization studies have mainly employed a standard linear elastic material model in which the elements cannot exceed a limiting strain. However, as stressed by Frangopol & Curley [24], the considering of plasticity behaviour of materials can be of great importance for fail-safe design. Fairclough et al. [104] addresses this by considering a rigid-plastic material model, thus better approximating a structure's ultimate limit state. Computational costs were reduced through a member-adding formulation [109] and a damage-case-adding formulation, thus allowing large-scale problems to be tackled of up to 16,290 potential members (e.g. see Fig. 11).

When considering design methodologies which involve deterministic modelling and optimization, such as the studies mentioned in this section, design guidelines, including Eurocodes [110], compensate for certain system uncertainties with the inclusion of partial safety factors. Such factors in the Eurocodes have been derived from probabilistic theory and calibrated to historic engineering practice [111]. Furthermore, deterministic studies require that the occurrence of the damage events to be certain, yet in reality, such events are commonly considered through a probability of occurring. System uncertainties and probabilistic damage may be addressed more directly through probabilistic optimization approaches.

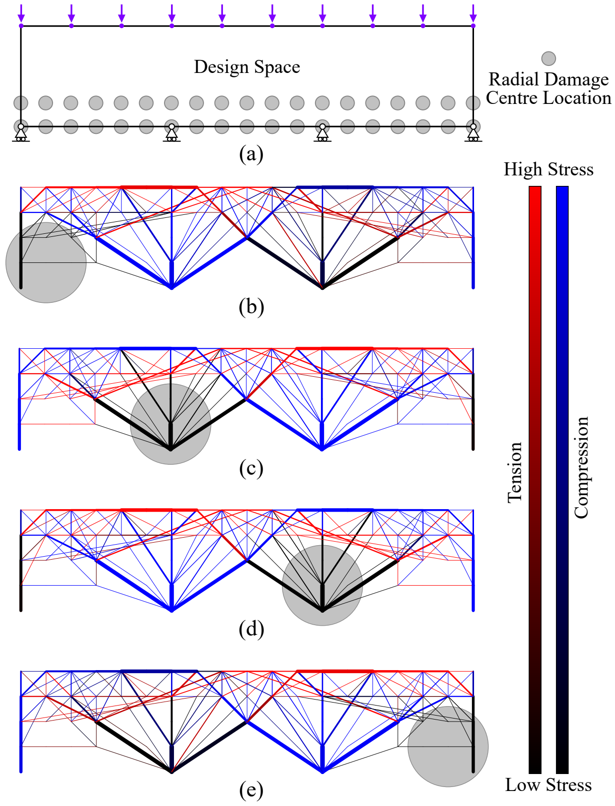


Figure 11: Multi-spanning optimum fail-safe truss structure (Adapted from Fairclough et al. [104]). (a) Problem definition (b)-(e) Optimised structure with the different damage cases along with the load paths of the structure. Other load paths/ damage cases are excluded for brevity.

3.3.4. *Recent Developments - Probabilistic Optimization*

This section looks at the recent development made in applying probabilistic optimization to fail-safe truss design. Unlike deterministic optimization, the problems are constrained through failure/constraint violation probabilities, which must be less than a target value. Typically, such problems consider system parameter uncertainties such as material properties and design loads, which are often assigned probability distributions. However, in the context of fail-safe optimization, system parameters can also include probabilities of structural damage.

Mohr et al. [112] looked to incorporate aleatory uncertainty of design parameters (such as the applied loads) into a 'robust' optimization formulation. Structural redundancy is achieved by combining multiple complete structures for a given load case to create a single complete structure. As such, the resulting fail-safe structures are resistant to the loss of complete substructures and are likely to be over-conservative compared to the single-member damage case formulations. Similarly, Cid Bengoa et al. [113] also considered statistical uncertainty of problem parameters, creating a reliability-based design optimization formulation, but modelled damage through predefined removal of members to form a more classic material minimisation fail-safe optimization problem. Consequently, they incorporated a multi-model optimization technique whilst considering load uncertainties.

The consideration of structural damage through subjective system uncertainties through events such as abnormal loading or human error in design and construction, also known as epistemic uncertainties, and aleatory uncertainties on reliability-based design optimization and risk optimization for collapse-resistant design, was investigated by Beck [114]. Epistemic uncertainties were introduced through 'latent failure probabilities' [115], formed from risked topology analysis and could be applied to each element independently. The problem is formed considering system-level progressive collapse failure constraints. By considering a simple two-parallel-element structural system, it was illustrated that considering the subjective uncertainties has significant consequences on the optimum structural configuration, showing that to achieve a suitable failure probability, the structure must possess redundancy. The application of latent failure probabilities to system reliability and risk-based optimization problems was extended by da Silva et al. [116] to consider larger truss-based problems, confirming the findings of [114]. It was observed that an increase in the epistemic system uncertainties led to a change of structural topologies, going from simple statically determinate solutions when such uncertain were small to indeterminate and redundant solutions when large, as illustrated in Fig. 12. The risk-based optimization formulations are of particular interest due to their cost-consequence objective function, where, in addition to material costs, expected costs of failure are included (calculated as a cost coefficient multiplied by the probability of the failure mode). Refs. [114, 116] differentiated between progressive collapse and instantaneous failure modes, with the latter incurring higher costs. It was found in [116] that when certain latent probabilities were applied, producing a non-redundant indeterminate structure was most economical, thus allowing slow progressive collapse in the event of element failure.

It should be noted that some of the mentioned collapse probability formulations used in [116] are highly non-linear, non-convex, and non-continuous. Consequently, the functions are not differentiable; thus, mathematical gradient-based optimization methods could not be applied, and meta-heuristic optimizers were used to handle the challenging properties. Considering a slightly different problem, with sequential failure being induced by fatigue damage, Biton et al. [117] proposed a system-reliability-based design optimization framework solvable through a sequential gradient methodology. The gradient calculations are semi-analytical, employing a modified sequential compounding method [118] and the Chun-Song-Paulino method [119] to determine the sensitivity information. The method was compared against a typical meta-heuristic optimizer, showing improved computational efficiency. However, limitations in its application to large problems were noted, with the error of the method increasing with the number of elements.

The introduction of probabilistic failure paths induces high computation cost and generally results in only small-scale problems (in terms of the number of design variables compared to the studies using deterministic optimization methodologies) being addressed.

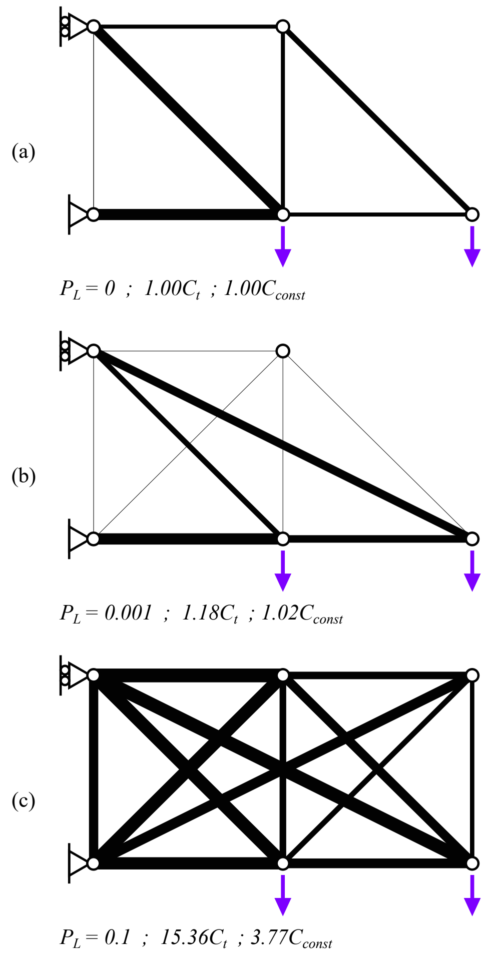


Figure 12: Optimum structures illustrated using data from da Silva et al.'s [116] risk-based topology optimization (table 9). P_L is the 'latent failure probability' used to measure the structural system's subjective (epistemic) uncertainty. C_t and C_{const} are the total cost and construction cost of the non-epistemic results shown in (a), respectively, taking values of 108.29 and 105.85. (a) $P_L = 0$. (b) $P_L = 0.001$. (c) $P_L = 0.1$.

3.4. Other Structure Typologies

Whilst most fail-safe optimization research has focused on continuum and pinned truss typologies, few studies have been conducted on alternatives.

Lüdeker & Kriegesmann [120] looked to incorporate beam elements into elastic volume minimisation, where stress constraints were measured through Euler-Bernoulli beam analysis and damage cases were defined by the removal of single beam elements. An intuitive engineering approach was first used, where, for a given topology, a size optimization was solved for each damage case, creating multiple structures that were then combined. It was found that such a method could not guarantee the fail-safe criteria since the indeterminate nature of the topology would result in an alternative load redistribution to what the structure was optimised for, thus causing some elements to fail after a damage event. A holistic approach was then taken, introducing p-norm and constraint reduction methods to reduce computational demand.

Similarly, Dou & Stolpe [108] presented a study on the fail-safe sizing optimization of tubular frame structures within the context of offshore wind turbines, incorporating member thickness degradation and partial removal of a member as damage models. The beam elements were divided into multiple sub-elements to allow damage to only parts of the whole bar. Stress and modal eigenvalue constraints were considered. Due to the partial damage definition and the consideration of stress constraints, the singularity phenomenon, as discussed in [102], occurred. Recent work by Lan et al. [121] has also looked to address the problem of offshore wind turbine structures, using a continuum topology optimization approach to establish the layout of a structural jacket that could then be translated into a beam-element type FEA model to allow for more rigour assessment.

Furthermore, Esfandiari & Urgessa [122] investigated the fail-safe optimization of reinforced concrete frame structures employing meta-heuristic optimization techniques by using a modified version of the particle swarm optimization algorithm [123] and machine learning as a decision maker. Similarly, da Rosa Ribeiro et al. [124] looked to perform a risk-based optimization of reinforced concrete beams when a frame structure possesses probabilities of experiencing column removal, optimizing the average dimension properties of the beams. Moreover, risk-based two-variable optimization problems looking to optimize column and beam strengthening factors of frame structures subject to sudden column removal and abnormal blast loading were presented by Beck et al. [125] and Beck & Stewart [126].

Considering the fail-safe optimization of cable-stayed bridges, Soto et al. [127] explored the impact of accidental cable breakage on the material optimization of the cable system. The formulation enables the alteration of the cable's cross sections, their anchorage position on the deck and their pretensioning force. Qasi-static analysis, as highlighted in [128], accounted for the dynamic behaviour of cable rupture, where the broken cable's axial force is statically applied at the deck and the pylon.

3.5. Critique

The above review has demonstrated that a significant amount of high-quality fail-safe optimization research has been conducted. However, when viewing the working through the lens of current fail-safe design methodologies and findings (as presented in Sect. 2), several disconnects can be identified. This subsection critiques the reviewed fail-safe optimization literature through this context and identifies other gaps within the current knowledge.

3.5.1. Critique Against Fail-safe Design Methodologies

A characteristic common to all current fail-safe optimization tools is their utilisation of static analysis. As noted by Refs. [33, 21], contemporary guidelines promote dynamic structural analysis, and whilst these guidelines are curated towards the application of frame-type structures, their principles hold for continuum structures. The response after fast-acting damage is naturally a dynamic event [28], resulting in heightened stresses compared to static analysis [45], as highlighted in Sect. 2.4. Although some researchers have noted the importance of dynamics within their work and recognised their static analysis assumptions as a limitation, in the event of sudden damage, current fail-safe optimised structures are potentially unsafe or, if amplification factors are used, overly conservative [33]. Thus, a significant gap exists in extensively exploring the effects of dynamics on fail-safe optimum design and its integration into efficient problem formulations. It remains an open research question whether simpler dynamic approaches e.g. modal analysis will be sufficient to incorporate these effects or whether full dynamic analysis is requred. Furthermore the influence on the resulting optimum solutions remains to be investigated, along with the impact on the geometric and gradient characteristics of the optimisation problem. Furthermore, from a risk-based optimization perspective, authors (e.g. [114]) have looked to distinguish between progressive and instantaneous failure and their probability consequence costs, noting the former collapse mechanism to be more desirable due to its temporal nature and allowing for human reaction. However, the rate of the collapse should naturally influence its desirability, and so quantifying consequence costs for such a collapse mechanism may benefit from the integration of dynamics.

Amongst similar lines, researchers such as Frangopol & Curley [24] have stressed the importance of considering plasticity in the design of collapse-resistant structures. Some recent researchers (e.g. [104]) have considered plasticity through rigid and bi-linear plastic material models; however, the vast majority have placed focus on a purely elastic analysis. The consideration of analysis non-linearity may not only help to unlock additional capacity concerning a material's ductile properties but also through its geometric effects. Investigations into the structural robustness of typical truss and standard frame designs highlight the importance of certain non-linear mechanisms, such as the catenary effect. The non-linear response of the bending elements enables an alteration to their load redistribution method, transitioning from bending to axial loading. Although it is unclear whether the inclusion of such a method into fail-safe optimization would yield improved optimality, it is interesting to consider how the nature of certain elements might change as a consequence of excessive deformation and how non-linear effects may be leveraged to improve optimal designs.

Although not unique to fail-safe design, buckling effects remain important. Results from experiments [48] and numerical modelling of planar trusses [51, 129] have illustrated the criticality of local buckling during progressive collapse. Such a failure mechanism has yet to be integrated into a topology-based fail-safe optimization. Although local element buckling considerations are common within typical structural design, their integration into mathematical optimization problems causes difficulties [130]. Most local buckling constraints can be described as 'conditional constraints' or 'topology-dependent constraints' since they are only valid when their associated element geometry variable (typically cross-sectional area) is non-zero, thus introducing the singularity phenomenon [131]. However, its implementation into non-fail-safe optimization has been successful for both continuum-based topology optimization (e.g. [132]) and truss-based optimization (e.g. [131]).

Moreover, although global buckling failure is not a common mechanism experienced in standard structures that undergo disproportionate collapse, the lack of its consideration in an optimization problem creates potential vulnerability. Its importance in non-fail-safe optimization has been illustrated for problems with certain boundary conditions [133, 134], and thus is therefore likely to be critical for similar fail-safe related problems.

The presented literature also showed a bias towards simple load redistribution as the primary mechanism against disproportionate collapse. The non-probabilistic optimization constraints dictate that no other elements within the undamaged part of the structure (or the damaged elements if they are subject to only partial damage) may fail after the initial damaging event. This is likely due to many fail-safe optimization formulations being adaptations from the common non-fail-safe problems. Consequently, current work in fail-safe optimization, especially from a deterministic perspective, has not adequately considered the true concept of disproportionate collapse and how failure to a proportion of the structure does not necessarily lead to complete structural failure. In particular, the singularities observed by [135] (Fig. 9) imply that this may lead to over-conservative results. Considerations of leveraging standard disproportionate collapse prevention conceptions such as compartmentalization and structural fuses have yet to receive implementation or consideration. Hence, it would seem that there is scope to consider optimization formulations that permit partial collapse of the structure to prevent failure propagation, even within deterministic frameworks.

Finally, the importance of bending behaviour within fail-safe truss optimization still requires greater investigation. Numerical modelling and experimental studies of current truss designs (see Sect. 2.4) suggest that bending is often one of the main mechanisms of trusses to prevent progressive collapse (e.g. see Zhao et al. [48]), and the use of continuity (i.e. avoiding pinned joints) is a very common practical strategy for improving structural robustness. The works by Lüdeker & Kriegesmann [120] and Dou & Stolpe [108] have begun investigations into the use of frame structures, but further work looking to compare its material efficiency over axial structures and its role in creating more rationalised solutions is still required.

3.5.2. General Areas Requiring Further Investigation

Recent studies have proven that modern computing technology and techniques have enabled large multi-variable and multi-constraint-based problems with dense ground structures/design space resolution to be addressed. Consequently, the resulting optimal structures have become increasingly complex, which naturally presents difficulties for construction. Simplifying such truss structures through applying joint-costs [136] and other rationalisation methods [137] into the formulation may be of interest for producing more practical designs. Shape feature control [138] or moving morphable component approaches [139] are example methods for rationalization of continuum structures.

Furthermore, most truss and frame typologies studies have purely considered a size optimization problem. Very little consideration has been given to optimizing the positions of the nodal joints (geometry optimization). This is likely due to the nonlinear and nonconvex nature of geometry optimization problems, which exacerbates their already computationally demanding nature. However, applying such a technique may enhance simplified structures without drastically impacting constructability.

4. Future Development in Fail-Safe Optimization: Incorporating Advanced Analysis Methods

4.1. Section Motivation and Aims

The review and critique above have highlighted several discrepancies relating to recommended fail-safe design methodologies. Many related to the analysis methods utilised in the optimization formulation/methodology. Although buckling and bending analysis themes were identified to require further development, these problem types have already been extensively investigated in the literature for non-fail-safe optimization problems. However, the themes of dynamic and non-linear analysis are arguably more complex and thus are given further attention within this section.

The consensus from researchers and design guides for collapse-resistant design advocates using advanced dynamic and non-linear methods capable of capturing the temporal natures of load redistribution, along with material and geometric non-linearity. However, the majority of fail-safe optimization tools have employed linear static methods. Thus, a natural progression is for the optimization tools to incorporate these more advanced analysis methodologies. The following section discusses the feasibility of such, reviewing optimization methodologies and some of the challenges which may be faced. Table 1 presents the generalised findings from this review. The subsequent subsections highlight these methods and their characteristics in greater detail. Where available, examples of the application of these approaches in structural problems bearing some relevance to the fail-safe problems targetted by this paper, such as in the optimization of structures subject to time-varying loads, are given.

4.2. Flexibility and Efficiency of Optimization Algorithms

When looking to solve structural optimization problems, researchers and engineers must utilise a set of algorithms and methodologies. However, the properties of the optimization problem hold a bearing on what algorithms may be used and thus influence the computational efficiency of solving said problem. For example, using the interior point method, whilst highly computationally efficient and capable of handling many problem constraints and variables, requires formulating optimisation problems to possess convex geometries. Problems that cannot be formulated to conform to certain geometric qualities or simply lack any clear mathematical formulation must then utilise algorithms or other optimization methodologies that do not rely on the problem's geometrical properties. From a more general perspective, one can consider Wolpert & Macready's 'No Free Lunch' theorem [140], which suggests that for any algorithm, both deterministic and stochastic, performance improvement over one problem class is offset by performance in another, insinuating that for an algorithm to gain applicability to a wide range of problems, it must sacrifice computational efficiency. Thus, engineers are challenged to determine cleaver formulations that may leverage the efficiency and accuracy of first-order solvers whilst ensuring the desired optimization aims are still met. Alternatively, they may consider more general problems that do not possess the necessary characteristics for such solvers and, thus, must be solved through less computationally efficient and accurate means (e.g. zero order methods). As more complex analysis and modelling methods are considered, the difficulty in formulating the problems to fix certain geometric qualities will naturally increase and, as a result, may require consideration of more flexible optimization approaches. However, it is worth noting that developments in first-order algorithms and methodologies may allow for a greater range of problem properties in the future, as historically shown with the interior point method, which was originally exclusive for linear problems.

4.3. Mathematical Programming Optimization

Mathematical programming generally looks to solve an optimization problem where the objective function and constraints are all clearly defined through mathematical relations, such as the general example problem given as such:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \geq b_i, \qquad i = 1, ..., m \end{aligned} \tag{2}$$

where $\mathbf{x}$ is a vector containing the optimization variables, the function $f_0(\mathbf{x})$ is the objective function, $f_i(\mathbf{x})$ are constraint functions and b_i is the bound for the constraint i, and m is the total number of constraints. Structural optimization problems can be formed into these mathematical problems, where the objective function will typically be a structure's total volume or compliance, and the constraints will account for the design requirements, such as global equilibrium, material failure and deflection limits, and load and state cases. As discussed in Sec. 4.2, the formulations must possess certain geometric and gradient qualities to apply first-order algorithms but will solve the problems efficiently and accurately. Consequently, mathematical programming is the preferred method by many researchers for solving structural optimization problems.

A review of structural optimization problems with transient loads solved through mathematical programming methods is given by Kang et al. [141]. Dynamic response optimization for continuum structures are commonly formulated so sequential gradient-based algorithms may be employed. However, Kang et al. [59] note the high difficulty and computational inefficiency of such gradient-based methods due to the requirement of large-scale numerical integration computations, thus making them unsuitable for large-scale problems. Furthermore, such methods have yet to be applied to multiple load cases/ multiple structural states due to their difficulty and computational

Optimization Method	Advanced Analysis Integration	Speed	Accuracy	General Ease of Implementation
Mathematical Programming	Difficult	Fast / Moderate	High	Difficult
Meta-heuristic Algorithms	Easy	Slow	Low	Easy
Machine Learning Methods	Easy / Moderate	Moderate	Low / Moderate	Difficult
Nelder Mead with Meta-heuristic	Easy	Slow	Moderate	Moderate

Table 1: Authors interpretation of the relative performance of different optimization methods for incorporating more advanced structural analysis procedures.

demand, thus limiting the method's application to dynamic response fail-safe optimization.

As a consequence of such computational expense, equivalent static load methods have been a popular alternative, where sets of static loads which replicate the dynamic/ nonlinear response at critical periods are applied to a standard static optimization problem [142, 59, 143, 144]. Another approximate approach is the response surface method, where a polynomial curve surrogate model of the structure's non-linear dynamic response is formed from statistical data. The curve can then be used to gain the sensitivity information required for first-order algorithms. The method is commonly used in the area of crashworthiness [145].

Mathematical programming has also been applied to minimum frequency constraint problems, where the structure's modal frequencies are limited to defined values. Such a constraint causes the problem to become non-linear, although still convex, with Refs. [146, 147, 148] using semi-definite programming to solve the problem.

Although challenging, the above methods suggest a possible feasibility for integrating dynamic and non-linear analysis into a mathematical fail-safe optimization formulation. However, due to the difficulties and potential limitations, the subsequent subsections explore optimisation algorithms that do not require such precisely formulated mathematical constraints.

4.4. Meta-heuristic Optimization Algorithms

Meta-heuristic algorithms employ basic heuristic methods to search a domain for an optimal solution [149]. The optimization problems are typically formulated into an unconstrained problem using penalty functions [150], where the entire problem can be expressed through a single equation often known as the fitness function. To determine if a structural solution violates the problem's constraints, the solution is typically simulated through numerical modelling, retrieving information which can then be directly compared against the problem constraint values. Azad & Hasançebi [151] suggest that the processes of these algorithms have three main strengths: (i) Independent of gradient information; (ii) inherently capable of dealing with continuous and discrete variables; (iii) applicable to highly complex problems.

Many different meta-heuristic algorithms have been derived and applied to the problem of structural optimization, with the literature mapping by Renkavieski & Parpinelli [152] finding 71 different algorithms from 179 articles. Some popular examples include genetic algorithms [153, 154], particle swarm algorithms [123, 155, 35], the big bang-big crunch algorithm [156, 157, 158], and differential evolution [159, 152]. In addition, researchers have sought to create hybrid algorithms, combining the methodology of two or more approaches to exploit the beneficial characteristics of each [160, 161, 162]. Researchers such as Charalampakis [163] have looked to compare these algorithms in their performance of size optimization for truss structures. However, there is no consensus on a superior algorithm for structural optimization. A review by Stolpe [135] notes that most of their applications rarely utilise the algorithms' strengths, often solving non-noisy and differentiable problems.

In general, the algorithms are reliable in locating the local area of an optimum; however, they struggle to refine the exact position. Hence, although such algorithms are great at dealing with poorly defined problems, making them suitable for incorporating more advanced analysis methods, their utilisation will exacerbate the computational demand of fail-safe optimization problems.

4.5. Machine Learning

The application of machine learning for structural optimization is relatively new; however, current results suggest it is a potential alternative to evolutionary-based algorithms. A review of machine learning techniques can be found in Ramu et al. [164]. Machine learning approaches can be divided roughly into three categories: supervised, unsupervised, and reinforcement.

Concerning the application of supervised machine learning that looks to determine connections and correlations between input data to predict an output, Sun & Ma [165] have applied a generative design approach to the topology optimization of continuum structures, allowing multiple possible solutions to a given problem to be generated quickly. Alternatively, Mai et al. [166] combined a machine-learning approach with a differential evolution algorithm for the optimization of trusses with geometrically non-linear behaviour, using the surrogate model to predict the non-linear static analysis results which inform the meta-heuristic algorithm. However, Mai et al. [167] later critiques the use of surrogate-based machine learning to solve structural optimization problems, noting issues with estimating suitable training data sizes and that the accuracy and reliability of the model are highly dependent on the quality and quantity of data.

Concerning the application of unsupervised machine learning, which looks to determine connections and patterns within the data input without using any provided output data, Mai et al. [167] looked to use a deep neural network framework for the size optimization of trusses. The findings indicated a significant reduction in the number of FEA evaluations from differential evolution. This was later developed by Mai et al. [168] to incorporate Bayesian optimization for self-tuning the model's hyperparameters. The method was applied to trusses with geometrically non-linear behaviour, finding substantial gains in convergence rate and overall computational time.

Finally, concerning the application of reinforcement learning, which looks to train an agent (model) to accomplish a task within a dynamic environment using a reward-based system, Hayashi & Ohsaki [169] applied the methodology to binary topology truss optimization. The study found that the method could substantially reduce computational load, however, in some cases missed the global optimums.

Based on the findings from these researchers, machine learning shows promise as an alternative methodology to meta-heuristic optimization algorithms, indicating a substantial reduction in computational demand. Their functionality with poorly defined optimization problems makes them a potential candidate for incorporating dynamics. However, such methods are substantially more complex to implement and don't avoid the computational constraints set by the 'No Free Lunch' theorem [140].

4.6. Approximate Gradient-based Methods

Although introducing advanced analysis methods makes obtaining gradient information more challenging, certain methods can approximate local gradient information and thus inform the movement of potential solutions. One such method employed in structural optimization is the Nelder-Mead method [170, 171], which samples the local environment of the search space around a current solution. Due to their focus on local search, the Nelder-Mead method is frequently used in conjunction with meta-heuristic algorithms to improve the overall exploration. The introduction of the Nelder-Mead method also tackles the solution refinement issue of meta-heuristic algorithms. Rahami et al. [172] and Assimi & Jamali [173] present hybridisation of the Nelder-Mead method and a genetic algorithm for truss optimization. Improvements in accuracy and convergence were found.

5. Summary & Outlook

This paper reviewed historic and state-of-the-art literature on fail-safe optimization – the process of optimising a structure while allowing for some form of defined damage. A brief overview of general fail-safe design was also given to provide context to the work of fail-safe optimization, focusing on critical member identification and dynamic response of truss structure due to their importance within fail-safe optimization.

The research area has been developed extensively for continuum and truss-based structural typologies, alongside a few studies concerning frame, cable-stayed bridge, and reinforced concrete design. Recent studies have shown that large, multi-variable and multi-constraint-based problems can be tackled without excessive computational cost, illustrating the development of the research field over the years. Beyond improving computational efficiency, researchers have innovated the field through the implementation of additional design considerations, including, but not limited to, uncertainties of system parameters, risk-based costing, fatigue from cyclic loading, and effects on modal vibration frequencies. Intending to gain a fuller understanding of the underlying characteristics of fail-safe optimization, other recent studies have investigated different methods of modelling damage and their influence on the optimized solution. The summation of all the mentioned work results in a great development in designing efficient collapse-resistant structures.

The reviewed work was also subject to a critique against the recommended guidelines of fail-safe design, looking to identify where areas may be developed to improve coherence. In addition, general requirements for further investigation were identified. The main outcomes of this critique are summarised below:

- Current tools rely upon a static-based analysis method, contrasting the recommendation of a dynamic analysis by design guidelines.
- Considerations of plasticity and geometric non-linearities have been shown to improve the capacity of the damaged structures, meaning the current tool's dependency on linear analysis potentially limits their gains in material efficiency.
- Considerations of local and/or global buckling failure have yet to be implemented.
- The nature of typical fail-safe optimization failure constraints makes simple load redistribution the sole method for disproportionate collapse avoidance, meaning scope exists to consider partial collapse-based constraints by considering alternative methods such as compartmentalisation or structural fuses.
- The role of bending as a mechanism against collapse in truss-based structures still requires additional investigation.
- Qualities such as constructibility have yet to be considered, creating scope for implementing structural rational-

ization methods into optimization formulations. For example, consideration of geometry optimization for truss typologies may be beneficial.

Since a significant number of the current shortcomings of fail-safe optimization tools relate to their analysis methods, the authors have suggested that a natural progression to merge fail-safe design and structural optimization practices is to incorporate more advanced analysis methods considering dynamic and non-linear effects, as shown in Fig. 1. As a means of a preliminary feasibility study for achieving this, a short critique of the available optimization methods was undertaken, reviewing the algorithms and methodologies through the lens of incorporating advanced analysis methods. Implementation of first-order optimization algorithms will likely be challenging; however, due to their computational efficiency, solution accuracy, and capabilities of handling many variables and constraints, efforts should be made to use these. Contrasting this, algorithms that deal with poorly defined problems enable more straightforward implementation. However, such algorithms are inherently inefficient, thus potentially creating a problem of high computational load. Yet, developments in zero-order methods, particularly in the area of machine learning, show promise in tackling the problem of computational demand.

This review and its identification of current knowledge gaps and shortcomings should aid in developing fail-safe optimization, allowing future structures to be material efficient and providing us safety against the uncertainties and hazards of the real world.

CRediT Author Contribution Statement

Edward A. Whiteside: Conceptualization, Data curation, Investigation, Visualization, Writing – original draft, Writing – review and editing.

Helen E. Fairclough: Conceptualization, Formal analysis, Methodology, Supervision, Writing – review and editing

Samuel E. Rigby: Conceptualization, Formal analysis, Methodology, Supervision, Writing – review and editing

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The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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