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Tribo-Dynamics Digital Twins (TDDT): Prediction of Friction and Frequency Response Function (FRF) in a Dry Tribological Contact under Reciprocating Motion

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ABSTRACT
Assembled systems typically contain mechanical joints that are in physical contact and heavily influenced by friction and vibration. Friction is affected by contact stress, temperature, material, and roughness of contacting parts, From geometrical features at the macro- to nanoscale. Understanding and predicting the friction of contact helps to create designs that reduce wear, crack propagation, damage, and energy consumption. Recently, digital twins have been used in different mechanical engineering mechanisms and systems to predict crack, damage and frequency response functions. Digital twins, with their system-level thinking, have promoted the idea of cross-industry development and ideology. The aim of the current study is to develop the digital twin-enabling technology for a simple dry contact under reciprocating motion. This enabling technology (digital twins) is the development of a grey-box model using conventional tribometer experimental data under cyclic loading and advanced multi-scale (contact mechanics to macro-scale dynamics) finite element analysis to provide an accurate estimation in a realistic time scale for digital twins. To demonstrate this, a ball and a flat plate made of steel (304) were used to create a physical twin. The test was run using a Universal Mechanical Tester (Broker UMT-3 tribometer) under speed and load sweep conditions to determine the coefficient of friction at different operating conditions. The experimental data for friction were collected and used for machine learning along with an FEA model using Abaqus which makes the digital twin. The machine learning part of the digital twin was used to predict the coefficient of kinetic friction under different operating conditions and can interoperate with other models to greatly expand the digital twin functionality. The predicted coefficient of friction was fed to FEA model to predict the mechanical behaviour of the system such as Frequency Response Function (FRF).

Keywords: Digital Twin, Friction, Tribology, Multi-scale

INTRODUCTION

Tribology is the science of friction, wear, and lubrication. The behaviour of the contacting parts and surfaces in mechanical engineering mechanisms is complex and influenced by numerous factors such as the mechanical properties of the contacting surfaces, surface finish, substrate material, and operating conditions (e.g., temperature, speed, and load). Tribology has become more in demand as the new composite materials used in complicated mechanisms require more study. However, there are engineering challenges in dealing with mechanical engineering mechanisms such as reducing the development cost (experimental testing, total time, etc.) and sustainability. This can be done by predicting the behavior of the systems prior to reaching certain or critical conditions. For example, predicting the damage of the wind turbine bearing or the friction between the rail and wheel. In addition to making accurate predictions, a large amount of data is typically generated especially when the whole lifespan is considered. One transformative tool for dealing with this large amount of data and reducing prediction time is machine learning. Machine learning has been studied and used in order to predict the coefficient of friction and wear [1–4]. The results are promising and show that machine learning can be used to reduce energy consumption and forecast damage.

One approach for taking a holistic and life-long understanding is to use digital twins. Digital twins represent a step up from traditional modelling approaches where there is a direct relationship between the physical system and the model. This allows for accurate predictions, historical traceability, multi-physics expertise, and other aspects when considering the system as a whole instead of a single-discipline perspective. For those readers that might be interested in the history, development and applications of digital twins, there are multiple detailed descriptions of these (and many other) topic areas in the growing number of review papers on the topic of digital twins including, but not limited to [5–21].

The digital twins proposed by the authors for the current study (Tribo-Dynamics Digital Twins (TDDT)), covers the interaction between tribology, dynamics, machine learning and digital twins. In assembled systems, it is impossible to neglect the effect of tribology and dynamics when dealing with the contacting parts. The aim of this study is twofold: first, to investigate an accurate machine learning algorithm to predict the coefficient of kinetic friction when the contact pressure is in a plastic region; second, to propose a digital twins model for dry tribological contact with kinetic friction. The Tribo-Dynamics digital twins presented here to predict the friction are based on a machine learning algorithm. The predicted friction is fed frequently into the Finite Element Analysis (FEA) model to predict other analyses such as Frequency Response Function (FRF), heat at the contact, contact pressure, and wear to name a few.

ANALYTICAL APPROACH

Friction

Friction is a tangential resistance force between the contacting surfaces under relative (or impending) motion [22]. Amontons introduced two basic empirical laws for friction [23]:

1. The tangential friction force and the normal force are proportional
2. The friction force is independent of the nominal contact area.

The coefficient of friction μ is defined as a dimensionless ratio of the tangential (friction) force to the normal load. Two factors in a dry contact contribute to the friction coefficient during the sliding (or impeding sliding) known as adhesion and deformation [22, 24]. The opposing asperities of the surfaces under normal load in dry contact and in the absence of superficial contamination (e.g., oxides), experience plastic deformation and cold weld [22–24]. For example, the adhesion occurs at the workpiece-cutting tool contact in machining [23].

The real contact area in dry contact takes place at the peak of the asperities of contact surfaces and is much smaller than the nominal contact area. This makes the real contact pressure larger than the yield strength and creates plastic deformation at the contact [23, 24]. After several consecutive loading/unloading cycles, the real contact area increases hence the contact pressure falls below the yield strength and contact becomes elastic [23, 25–27]. An increase in the real contact area (as Amontons states) and elastic deformation of contact (which relates to the interfacial stiffness [26, 27]) influences the coefficient of friction. The coefficient of friction is distinguished as the static or kinetic (also known as dynamic) coefficient of friction. Static friction is the resistance force just before the sliding occurs while kinetic friction is the one during the sliding motion.

Machine Learning

Regression analysis is a category of machine learning algorithms with the requirement of continuous datasets. The most successful regression (or classification) type is supervised learning, as the trained algorithm can be tested and its accuracy is measured [28]. In this kind of machine learning, inputs (features) are imported to an algorithm to obtain a desired output. To do this, a portion of the dataset is chosen as the training dataset (70%-80%) to derive a machine learning algorithm. The accuracy of this algorithm is required to be tested. Therefore, the rest of the dataset (the last 30%-20%) is imported into the machine learning algorithm defined by the training dataset. The predicted value is compared with the test dataset to show the accuracy of the algorithm. There are numerous machine learning algorithms for regression. However, some of them are suitable for linear analysis, while others are for both linear and nonlinear problems.

In the current study, two machine learning algorithms for regression due to either their simplicity or accuracy and being practical for both linear and nonlinear problems were used to predicate the coefficient of kinetic friction. These algorithms are Linear Regression (Ordinary Least Squares (OLS)) and Artificial Neural Network (ANN). These algorithms are capable of being employed for a single input (one feature) or multiple inputs (multiple features). In the current study, the machine learning models are designed to use two inputs (sliding speed and applied load) and a single output (kinetic coefficient of friction).

Linear Regression (Ordinary Least Squares (OLS))

One of the simplest and most straightforward machine learning algorithms is linear regression [28]. The computational time of this model is low compared to ANN and alternatives.

A linear regression with multiple inputs (so-called multivariate linear regression) and a single output is given by [29]:

$$y = w_0 + w_1 x_1 + w_2 x_2 + \ldots w_n x_n = w_0 + \sum_i w_i x_i \tag{1}$$

where w_0 is an offset and and w_i is the response sensitivity (also called weight or coefficients [28]) corresponds to the input i.

This model has a main advantage which also can be a disadvantage, namely the complexity control [28]. OLS does not have any parameters that can modify the complexity of the system, since the inputs and orders must be selected *a priori*. But because of this, the results of the training can be easily explained and understood, an issue that other algorithms have.

Artificial Neural Network (ANN)

ANN is a nonlinear statistical method and one of the most productive machine learning algorithms as it is capable of predicting both linear and nonlinear problems [30]. A multilayer perceptron (MLP) (also known as feed-forward neural networks) are the simplest model of ANN [28]. It can be defined as a multi-layer of linear algorithms to predict the output and consists of either single or multiple inputs, single or multiple hidden layers and single or multiple outputs [28]. Fig. 1 shows a schematic diagram for MLP with two inputs (sliding speed and applied load), a hidden layer and only one output (coefficient of friction).

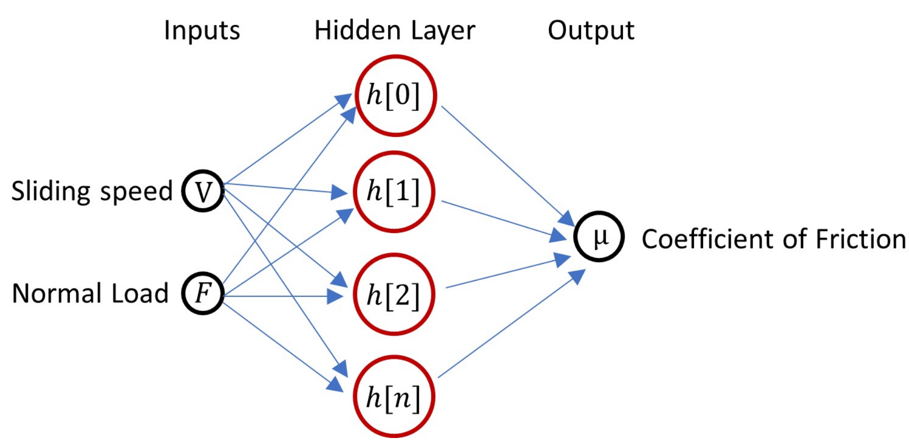

Figure 1: Schematic diagram of a multilayer perception for the coefficient of kinetic friction with two inputs, a single hidden layer and one output.

The weighted sum of the inputs and outputs are determined by [28]:

$$h[0] = \tanh(w[0,0] * V + w[1,0] * F + b[0])$$

$$h[1] = \tanh(w[0,1]*V + w[1,1]*F + b[1])$$
$$h[n] = \tanh(w[0,n]*V + w[1,n]*F + b[n])$$
$$\mu = s[0]*h[0] + s[1]*h[1] + ... + s[n]*h[n] + b \tag{2}$$

where w are weights between the input and hidden layer, and s are weights between the hidden layer and output. Despite being really accurate for complex problems, the main disadvantages of ANN are the computation time and the amount of data needed to tune the nonphysical parameters [28].

Accuracy and Error

The accuracy of a machine learning algorithm and the predictions are determined by different metrics such as mean squared error (MSE); mean absolute error (MAE) and R-squared score (R^2 score).

Mean Squared Error (MSE):

One of the metrics to measure the error of a regression machine learning algorithm is MSE (also known as L2 error norm) and given by [28]:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_{i,pred} - y_{i,actual})^2 \tag{3}$$

where y_{pred} is the predicted value and y_{actual} is the actual value of the output. In this study, y_{actual} is the output of the test set. The MSE has a value in the interval $[0,+\infty)$, where zero shows no error [31].

Mean Absolute Error (MAE):

MAE is a metric for regression that can be used when there are outliers in the test dataset and is given by [28]:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}|y_{i,pred} - y_{i,actual}| \tag{4}$$

Similar to MSE, the MAE value is positive and the closer value to zero shows a better machine learning algorithm.

R^2 score:

R^2 score or the coefficient of determination illustrates how fit the predicted and test datasets are, and is given by [28, 31]:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_{i,actual} - y_{i,pred})^2}{\sum_{i=1}^{N}(y_{i,actual} - y_{mean})^2} \tag{5}$$

where y_{mean} is the average of the output of the test dataset. R^2 is in the interval $(-\infty,1]$. Negative values of R^2 show anti-correlated prediction, while $R^2 = 1$ shows perfect prediction [28]. Chicco et al. [31] shows that although the accuracy of the machine learning algorithm cannot be judged using only a single metric, R^2 gives a more reliable value that can decide the performance of the machine learning algorithm.

Data Processing for Machine Learning

One of the most important parts of machine learning analysis is data processing. This part includes cleaning undesired data to reduce the computational time and accuracy of the prediction and splitting the dataset into training and test sets. In the current study, Python was used for analysis as it is equipped with libraries and packages available for machine learning such as sklearn, matplotlib, numpy, pandas, Jupyter Notebook and tensorflow.

The real-time data was imported into Python for data processing. The coefficient of friction, normal and tangential loads were sorted, analysed and plotted.

Digital Twins

Fig. 2 shows a schematic diagram for the Tribo-dynamic digital twins. This approach uses both a physical twin and a digital twin. The physical twin is the ball-flat square plate with the dry contact under a reciprocating motion that was used for the experimental set-up. The digital twin consists of:

(1) Data store for sensors and actuators to measure the coefficient of friction (and possibly real-time data in the future)

(2) CAD and FEA models of the physical twin

(3) Coding software for data processing (e.g., Python)

(4) Machine learning algorithm(s)

(5) Accuracy, uncertainty and error expressions

(6) Visualisation

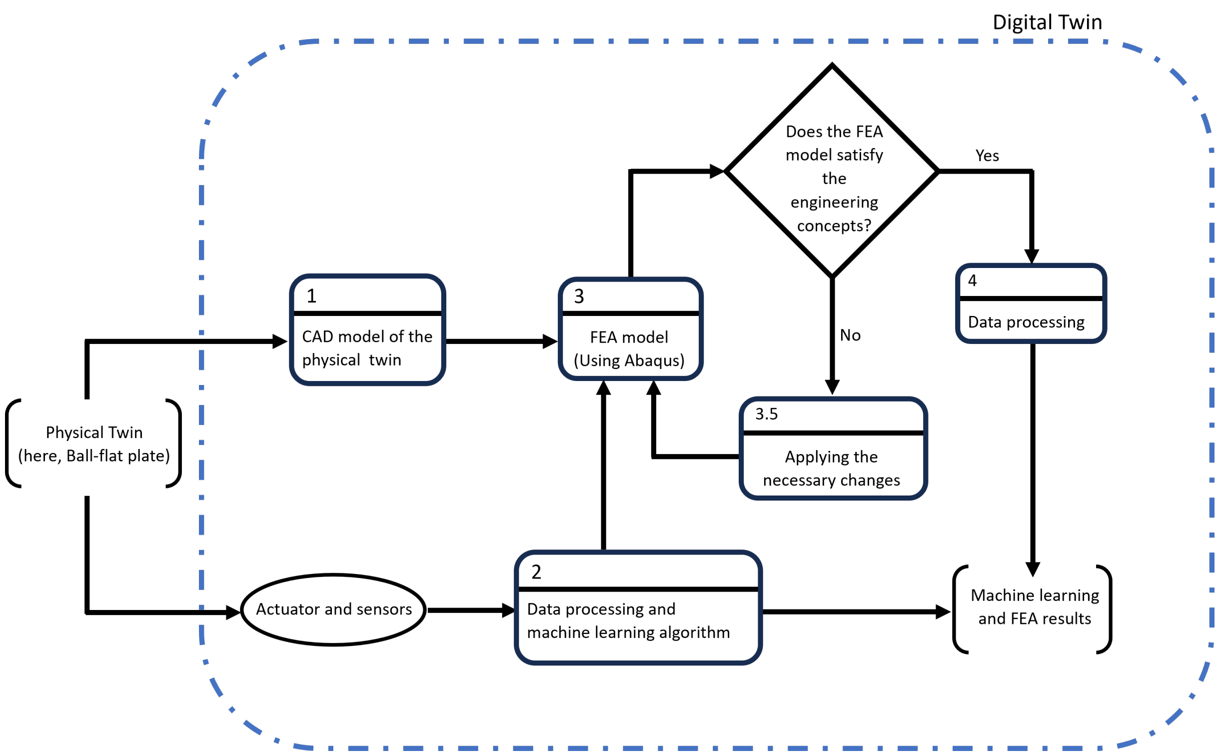

Figure 2: Schematic diagram of Tribo-dynamics digital twins.

Prior to running the experiment, CAD and FEA models of the experimental set-up are constructed (steps 1 and 3 shown in Fig. 2). The data measured with the sensors and actuators are used for data processing. The processed data are inputs for the machine learning process (step 2). A few machine learning algorithms (at least one nonlinear algorithm as the coefficient of friction varies nonlinearly against the operating conditions) are chosen (typically based on expert opinion) to predict the contact's tribological parameter (e.g., coefficient of friction, oil film thickness and wear). The test data set is defined to the

FEA model and the results are compared with the experimental data to validate the FEA model. As shown in step 3.5 of Fig. 2 if the error of the FEA is not acceptable based on the engineering concepts and expectations, the model is updated until an acceptable value of the error is achieved. The coefficient of friction is then predicted for the future condition (for example, the next two minutes of the operating condition with the machine learning algorithm). The predicted coefficients of friction are fed into the FEA model to predict the mechanical behaviour of the system such as oscillation and vibration, contact pressure, oil film thickness, and heat at the contact. The predicted values are used to make decisions, for example changing the operating conditions such as load or speed. This prediction can be used to reduce the possibility of damage and energy consumption or inform the operator to stop the engineering system. The Tribo-Dynamics digital twins can be used as an online and web-based platform. The accuracy of the digital twin can be measured using the error metrics of machine learning and the comparison between the FEA result and experimental data.

Frequency Response Function (FRF)

A frequency response function (FRF) is a transfer function in the frequency domain and defined as the ratio of the response of a system to the excitation [32]. FRF is a complex number consisting of both real and imaginary parts. Fig.3 illustrates a schematic diagram of a linear system. The input of the system is an excitation (an impulse force here) and the output is the response of the

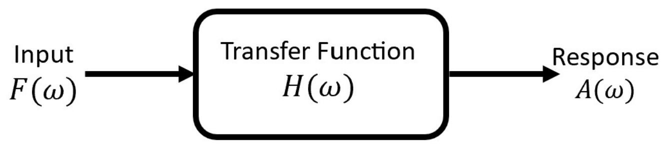

Figure 3: Schematic diagram of FRF of a linear system.

system to the excitation. The $H(\omega)$ is the transfer function (FRF), $F(\omega)$ is the excitation (impulse force here) and the response can be acceleration $A(\omega)$. The FRF of acceleration (also known as accelerance or inertance) is used to determine the stiffness, damping ratio, natural frequencies, and mode shapes of the structure and is given by [32]:

$$H(\omega) = \frac{A(\omega)}{F(\omega)} \tag{6}$$

In the current study, a Fast Fourier Transform (FFT) of the time domain of the impulse (force) and acceleration was used to determine the FRF.

EXPERIMENTAL SET-UP

A stainless steel (304) ball with a diameter of 6.35 *mm* was placed on a stainless steel square flat plate with a thickness of 10 *mm* and a length of 75 *mm*. The Elastic modulus and Poisson's ratio of the ball and plate were 195 *GPa* and 0.29, respectively. The ball was attached to a holder of a Bruker UMT-3 tribometer. The UMT was equipped with a reciprocation motor to generate linear motion. The ball was loaded at a constant normal load while its linear speed varies from 1 mm/s to 30 mm/s in a step of 1 mm/s. The experiment was undertaken at every speed run for a five-cycle with a stroke of 20 *mm* (200 *mm* sliding distance at every speed step). This allows us to average the fiction coefficient over a longer sliding distance and results in more reliable results. The normal load applied to the ball was 0.5 *N*, 0.7 *N*, 1.5 *N* and 2.5 *N*. These loads create the maximum pressure measured with the Hertz contact theory when the contacting surfaces are at stationary (477.3 *MPa*, 534.0 *MPa*, 688.5 *MPa*, and 816.2*MPa*). The contact pressure is greater than the yield strength of the stainless steel (215 *MPa*) and the plastic deformation occurs. This test was repeated for the nominal contact pressure. The coefficient of friction was recorded and averaged to use machine learning algorithms to predict the coefficient of friction.

FINITE ELEMENT ANALYSIS (FEA) OF FRICTIONAL CONTACT

Kinetic friction is vital for the FEA model when the surface comes into contact. As a result, ignoring the friction or using an incorrect value, gives an unreliable FEA model far away from the real contact. The aim of this FEA model as part of a digital twin is to build a model that can use the predicted coefficient of kinetic friction (with machine learning) to predict the other mechanical behaviour and parameters such as frequency response function (FRF), heat at the contact surface, contact pressure and any other desired parameters. This helps to predict the behaviour of the contact (not only limited to a simple ball on flat plate contact) in order to reduce the wear, damage, and cost and even to be more sustainable in terms of energy.

Fig. 4 shows the FEA model of a frictional contact under normal load F and sliding speed V in a reciprocating motion. The initial FEA model was made using the mechanical properties and dimensions of the ball and plate. The velocity, the normal force and the sliding distance were introduced to the FEA model according to the experimental test. The analysis was set as explicit dynamics. For the simplicity of the model, the surface roughness of the contacting parts was ignored. This simplification is realistic, as the coefficient of friction was defined at the contact. The measured coefficient of friction of the test set and predicted by the machine learning algorithm was defined at the contact of the FEA model. As the computational time was significantly large, only 4 sets of coefficient of friction were used. The output of the FEA model was the FRF. The results of the test and prediction were compared. Future plans is to train an additional machine learning algorithm using the FEA model to use the input of coefficient of friction and output the FRF.

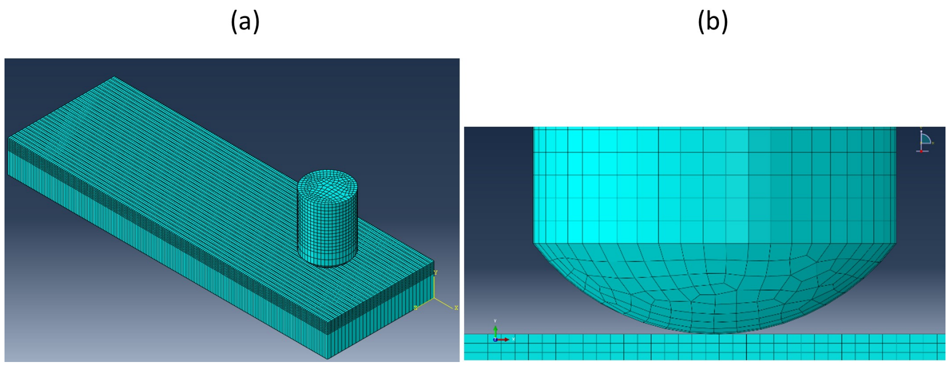

Figure 4: FEA model for a frictional contact.

RESULT AND DISCUSSION

Although the coefficient of friction in dry contact is complex and many factors such as temperature have an influence on it, sliding speed and normal load are among the most important features that affect kinetic friction. It should be noted that some of these parameters may have insignificant effects on the coefficient of friction depending on the contacting materials and operating condition. For example, Burwell et al. [33] show the coefficient of kinetic friction of a lead block on steel at the sliding speed up to 10^{-4} m/s is constant [23]. Bowden et al. [34] also show that the coefficient of kinetic friction of dry copper-copper contact at the sliding distance 600-650 m/s is constant [23]. Fig. 5 shows the coefficient of kinetic friction against the sliding speed at the normal load 0.5N and 0.7N.

The machine learning algorithm for linear regression and ANN consists of two inputs (features) and a single output. The inputs are sliding speed and normal load and the output is the coefficient of kinetic friction. Fig. 6 illustrates the predicted and actual coefficient of kinetic friction for the linear regression and ANN. Although the linear algorithms required low computational time, they poorly predicted the coefficient of kinetic friction (see Fig 6a). Unlike linear algorithms, nonlinear machine learning shows a higher correlation between the actual and predicted coefficient with R^2 scores of 0.88 for the ANN. However, R^2 should not only be considered to decide the more efficient algorithm. Table. 1 compares the error and accuracy metrics for the algorithms. It is seen from Table 1 that ANN gives lower MSE and MAE errors and higher R^2 score compared to the other machine learning model. Therefore, despite the longer computational time of ANN, it is a more accurate model to predict the coefficient of kinetic friction as the physics behind the mechanisms is nonlinear.

Once the coefficient of friction is predicted using a machine learning algorithm, they are used to predict the mechanical behaviour of the system. Four data points of actual and corresponding predicted coefficients of kinetic friction were defined at the contact of the FEA models. The FRF of the ball-flat plate contact at different coefficient of friction were determined as shown in Fig. 7. The actual and predicted FRF shown in Figs. 7b and 7c show really similar while their R^2 score is 0.88. There is a slight dissimilarity in the FRF of the actual and predicted coefficient of coefficient for Figs.7a and 7d for the higher mode

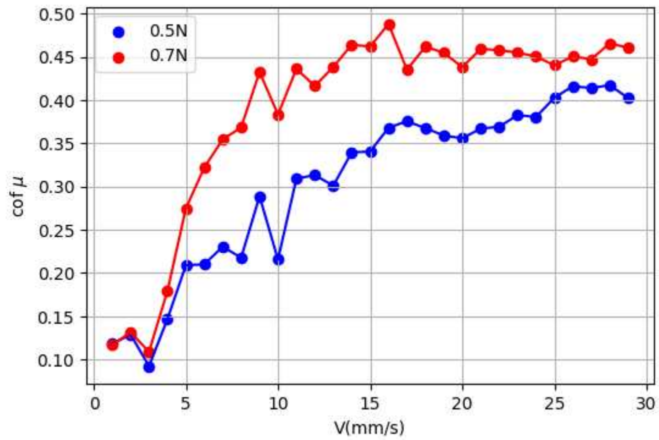

Figure 5: Coefficient of kinetic friction against sliding speed at normal load $0.5N$ and $0.7N$.

Algorithm	Mean Squared Error (MSE)	Mean Absolute Error (MAE)	R Squared score R^2
ANN	0.00120	0.026	0.88
Linear Regression	0.00453	0.052	0.64

Table 1: The accuracy and error metrics of the machine learning.

(larger than 700 Hz and 800 Hz, respectively) when the difference in their coefficients is almost 4.7% and 10 % respectively.

CONCLUSION

In this study, digital twins as a multidisciplinary tool between tribology and dynamics named Tribo-dynamics Digital Twins (TDDT) was proposed to use the predicted coefficient of friction to predict the behaviour of the system such as FRF. TDDT consists of a physical twin and a digital twin. For the digital twin, machine learning was used to predict the coefficient of friction, python was used for data processing and FEA for the prediction of other mechanical behaviours. Two different machine learning algorithms for linear and nonlinear problems due to either their simplicity or accuracy were used to predict the coefficient of kinetic friction as the sliding speed and normal load vary. These models were linear regression and Artificial Neural Network. It was shown that for this study, ANN gives more accuracy despite the high computational time. The predicted values are then imported into FEA model to predict the FRF. The comparison between the actual and predicted values shows promising results that TDDT can be used to reduce the damage and energy consumption of the system.

Some of the future works for the current study are:

1. Comparing more machine learning algorithms to investigate the best trade-off between accuracy and computation time
2. Considering more factors and parameters that influence friction and wear (i.e., temperature, surface roughness, surface finishes, coatings)
3. Applying the concepts of TDDT to lubricated contacts in all lubrication regimes (i.e., boundary, mixed, hydrodynamic, and elastohydrodynamic regimes)
4. Developing a web-based platform (i.e., HTML) for online monitoring and user interface to control the operation of the operating conditions (e.g., sliding speed, normal load, lubricant film thickness)

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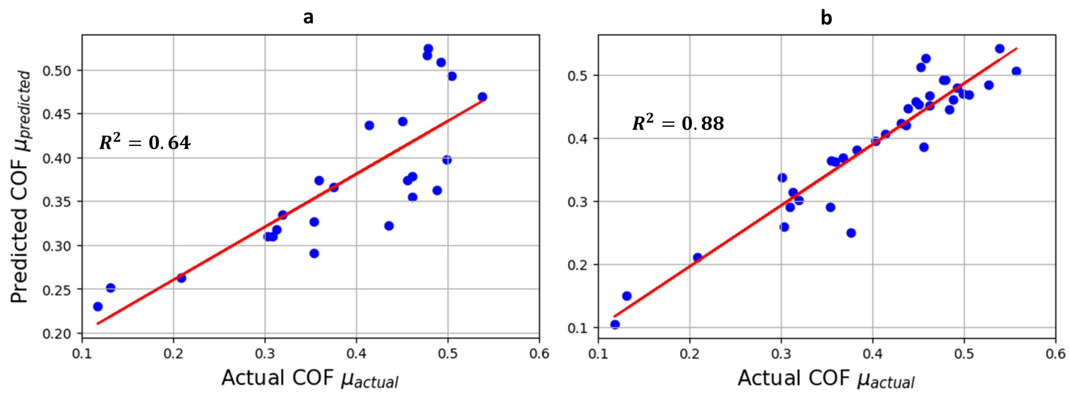

Figure 6: Correlation between the predicted and actual coefficient of kinetic friction: (a) linear regression; (b) ANN.

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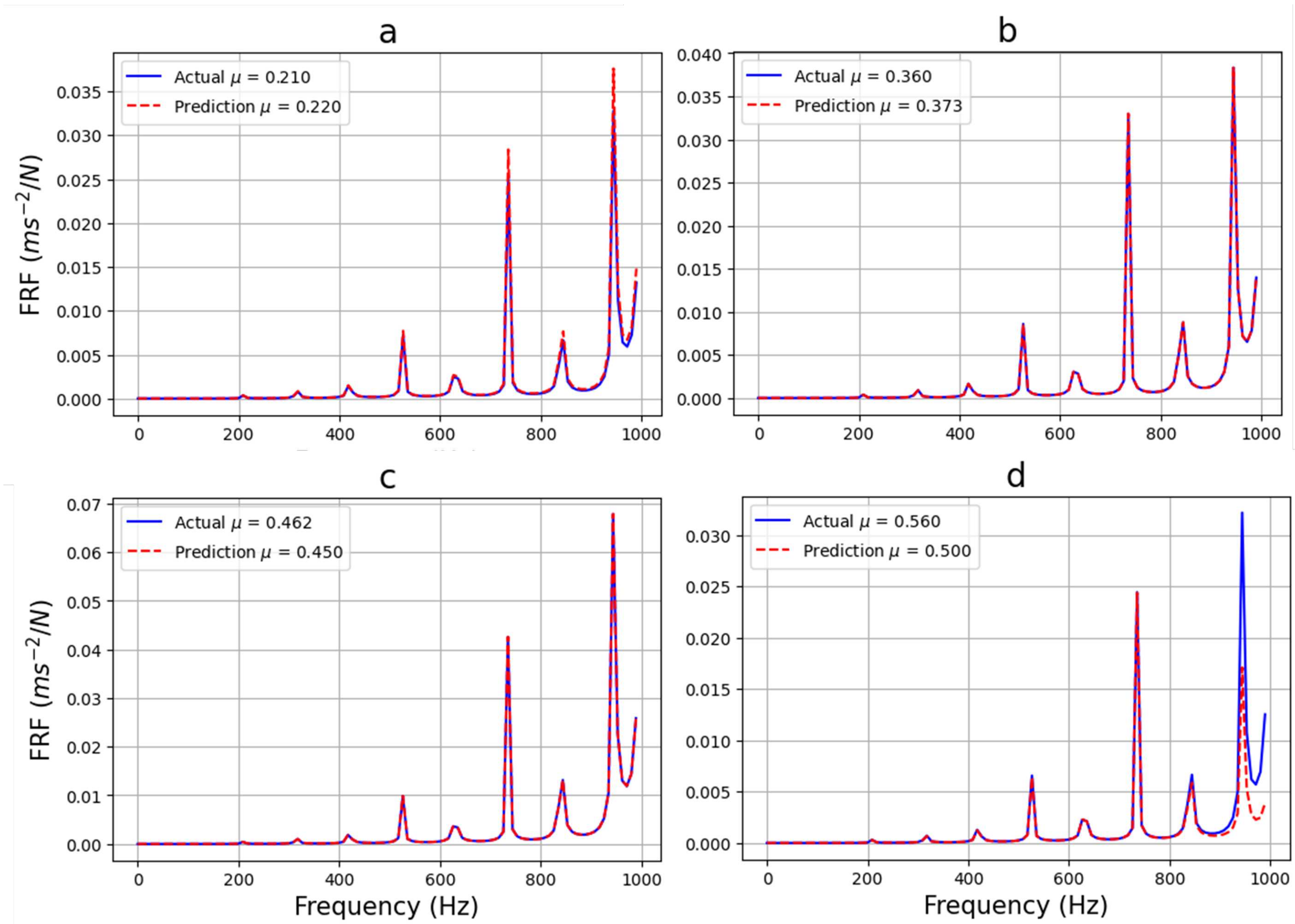

Figure 7: Comparison between actual and predicted FRF with FEA using actual and predicted coefficient of friction.

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