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Is there an association between frequency of Home Mathematical Activities (HMA) and Children's Mathematical Outcomes? Data Harmonisation and Secondary Analyses of UK-based Datasets.

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Abstract

Early mathematical skills lay an important foundation for later academic success. Substantial variation in mathematical skills can be observed in young children and these differences have been related to family socioeconomic circumstances (SEC). The type and frequency with which parents engage in home mathematical activities (HMA's) with their children has been suggested as a key mechanism explaining inequalities in early mathematical skills, they may also be a potential target to narrow attainment gaps. However, evidence for the relation between HMA's and mathematical skills, and whether there is an SEC gradient in HMA engagement, remains mixed. In the present, pre-registered study, we conducted harmonisation and latent profile analyses on nine UK-based datasets (containing n= 969 dyads; mean child age= 46.83 (SD= 5.41) months; child age range 35-69 months). These analyses identified three profiles based on the frequency of engaging in HMA's (i.e., low, intermediate, high). Children in the high HMA category had significantly higher mathematical skills than those in the intermediate and low categories. While SEC correlated with mathematical skills, no SEC differences were found in engagement with HMA's. This suggests that families that engage in a higher frequency of HMA's have children that tend to have higher mathematical skills, but SEC does not predict engagement with HMA's. We discuss the implications of these findings for narrowing early attainment gaps and how to best measure and capture the home mathematical learning environment.

Public significance statement

Findings suggest that there is an association between the frequency that families (with children aged 3-5 years) engage with home mathematics activities and their children's maths skills. Secondly, contrary to previous research, families experiencing lower socioeconomic circumstances did not engage less often in home mathematics activities with their children. Our study suggests that engagement with home mathematics activities may be a useful target for intervention and that lower parental education levels may not be a barrier.

Introduction

Mathematical skills in the early school years have been shown to be a powerful predictor of later academic achievement, over and above other important school-related factors, such as reading skills and attention (Duncan et al., 2007). Furthermore, mathematical skills are related to later health, income, and quality of life (National Numeracy, 2015; Ritchie & Bates, 2013; Wagstaff et al., 2001). There is substantial variation in children's mathematical skills prior to starting school (Manolitsis et al., 2013; Sirin, 2005), with children who start schooling with low mathematical skills tending to fall behind academically (Aubrey et al., 2006; Cahoon et al., 2021). These differences in mathematical skills are associated with a child's socioeconomic circumstances (SEC) from as young as age three (Sarama & Clements, 2009; James-Brabham et al., 2023; Blakey et al. 2020). Given that disparities in mathematical development begin forming prior to the start of formal education, and have lasting implications on performance, it is important to understand how to best support foundational mathematical skills.

Early mathematics learning is shaped by contextual factors that can influence children's development from birth. These factors include, families' material and economic resources, their practices, and cultural capital (e.g., home learning environment; LeFevre et al., 2010), as well as individual-child level factors (e.g., genetics, vocabulary, cognitive skills; Ribner et al., 2018; Ruthsatz et al., 2014). One factor that has received much recent attention is the home mathematics environment (HME). The HME encompasses the type and range of activities that parents and children undertake that involves mathematics learning, parental beliefs and confidence in mathematics, and maths talk (Anders et al., 2012; Ayala et al., 2024; Blevins-Knabe & Musun-Miller, 1996). The HME is thought to support the development of children's early mathematical skills and lays crucial foundations for future mathematical skill development (Anders et al., 2012; Byrnes & Wasik, 2009; Elliott & Bachman, 2018; LeFevre et al., 2010). Researchers have recently shown that this aspect of the home environment may be modifiable and therefore may be a factor that could feasibly be an intervention target to improve early mathematical skills (Galindo & Sheldon, 2012) and potentially reduce SEC disparities. The HME is comprised of multiple factors such as parent beliefs, thoughts, and confidence about mathematics and general learning (Zippert & Rittle-Johnson, 2020), as well as the degree to which maths-related language is used in the home (e.g., Maths talk; Levine et al., 2010). Whilst engagement with maths activities only provides a partial view of the HME, the majority of studies have investigated the use of activities in the home and have consequently operationalised this construct as the *frequency* of activities that parents and children engage in that involve mathematical skills or content (see Elliot & Bachman, 2018; Daucourt et al., 2021). These activities are subsequently referred to as "Home Mathematics Activities" (HMA's).

Recent systematic reviews have found that there is an overall small, positive relation between engagement with HMA's and children's mathematical skills *(r*= .13)*,* both when taking a broader approach to the measurement of the HME (e.g., observations, parental attitudes/ beliefs scales; Daucourt et al., 2021), and when examining only studies that include frequency-based engagement scales (James-Brabham et al., 2024). It should be noted that there is wide variation between individual studies, with some studies reporting positive relations between frequency of engagement with HMA's and children's outcomes (Lefevre et al., 2009; Skwarchuk et al., 2014), some reporting no relation (James-Brabham et al., 2023; Missall et al., 2015; Zhou et al., 2006), and some even reporting a negative relation between these variables (Blevins-Knabe et al., 2000; Ciping et al., 2015). However, a recent multi-country (n=54) study containing secondary data analyses of the Trends in Mathematics and Science Study (TIMSS) dataset established a weak, positive correlation between engagement with HMA's and mathematical attainment across the included countries (*r*=.15, Ellis et al., 2023). This study was important, as it suggests that an underlying level of consistency exists.

Although there is overarching consistency in the relation between the frequency of engagement with HMA's and mathematics skills, there is an emerging body of evidence that suggests that there may be subtle differences in how different families engage with HMA's. Past research has highlighted how children from lower SEC families tend to, on average, start school with lower mathematical skills compared to their peers from higher SEC families. These disparities in mathematical skills not only remain but widen throughout school (Caro et al., 2009; Sarama & Clements, 2009). HMA's have been proposed as a mechanism through which SEC disparities in early mathematics skills may develop (Elliott & Bachman, 2018). Differences in mathematical skills related to SEC are likely influenced by a range of proximal and distal factors, as well as structural inequalities that may restrict support and opportunities available to families experiencing lower SEC (Golinkoff et al., 2019). Various SEC indicators may differentially influence mathematical outcomes, for instance income levels may influence parent stress and material resources available; whereas education level may influence parent beliefs about the importance of mathematics, as well as confidence in mathematics itself (see Davis-Kean et al., 2021). One influential model of SEC-related differences that attempts to consolidate these indicators in explaining childhood cognitive outcomes is the Family Investment Model (e.g., Conger & Donnellan, 2007; Davis-Kean, 2005; Duncan et al., 2014). This model proposes that parents from higher SEC's are able to invest more in their children due to a greater access to resources and capital. In the context of the HME, higher SEC parents may have more time and resources to engage in HMA's with their children (Blevins-Knabe & Musun-Miller, 1996; Muñez et al., 2021). Indeed, some evidence suggests that SEC is associated with the frequency of HMA's that parents do with their children (Galindo & Sonnenschein, 2015), their complexity (Saxe et al., 1987), as well as the range and consistency of activities (DeFlorio & Beliakoff, 2015; Stipek et al., 1992). However, the literature is mixed, with some studies finding no relation between HMA frequency and SEC (e.g., DeFlorio & Beliakoff, 2015; James-Brabham et al., 2023; Pan et al., 2018). Overall, it remains uncertain whether HMA frequency does vary according to family SEC. Before cost and time intensive interventions are implemented focusing on HMA frequency, it is essential we better understand whether HMA frequency is related to mathematical skills and family SEC.

As already mentioned, the *frequency* with which HMA's are undertaken by parents has been the focus of correlational research, where parents are asked to complete a questionnaire detailing the frequency with which they engage in a pre-determined list of mathematical activities (Daucourt et al., 2021).

Total frequency scores are then calculated, or factors derived, and then researchers test whether these total scores or factors are related to children's mathematical skills. It is plausible that these frequency measures of HMA's, where scores for the frequency of engaging in specific activities are summed or condensed using data driven approaches, may fall short of adequately representing the breadth of parental engagement in mathematics at home. For example, parents may undertake mathematical activities with their children which are not captured by the pre-determined list (Andrews et al., 2022). Equally, frequency is only one way to capture mathematical activities, and variables such as the range of activities or type of activities may be equally important to consider, but have largely been neglected to date within this literature (Hornberg et al., 2021). Moreover, consolidating the frequency of various activities into a single score can overlook the nuanced relation between specific mathematical skills and corresponding activities. For instance, not all mathematical activities may equally contribute to the development of specific mathematical competencies: for example, activities focused on shapes and spatial reasoning may not enhance children's counting abilities. In a critique of the use of quantitative methods to condense mathematics activities into arbitrary variables (such as direct vs. indirect activities), Andrews et al. (2022) suggested that this approach may have exacerbated the lack of consensus for a likely positive relation between HMA's and children's mathematics skills. Hence, rather than grouping HMA's arbitrarily, it might prove advantageous to categorise them based on the specific skills they aim to foster (Andrews et al., 2022).

Another factor that may affect the reporting of HMA's is related to how activities are viewed by parents compared to researchers. Given that the items on the scales rarely provide context to ground the activity, it could be that parents doing an activity do not consider it focused on mathematical learning. For example, parents may be playing with shapes, but the focus of the task is to learn colours rather than learning the names of shapes per se. This mismatch, between how researchers and parents consider (and classify) an activity, is likely to introduce uncertainty into the measurement and interpretation of HMA frequency. This uncertainty may be reflected in the findings on the relation between certain parent-child interaction constructs and SEC (DeFlorio & Beliakoff, 2015; Muñez et al., 2021). Whilst this issue would not be straightforward to counter, and may require a new approach to studying HMA's altogether, increasing the power of the statistical model by bolstering the sample size, as we have done in this study, may help to some degree (Mascha & Vetter, 2018).

In addition to this, the focus of scales of HMA-related caregiver support tends to be on early numeracy and may exclude other important concepts such as spatial and pattern understanding (Zippert & Rittle-Johnson, 2020). Indeed, a principal components analysis conducted by Lefevre et al. (2009) found that statistically derived categories of items, such as shape, size, and colour, had a stronger association with number skills than categories including printing numbers and identifying the names of written numbers. Interestingly, the former activities were found to be employed in the home less frequently than the latter two activities. Therefore, the *type* of HMA's parents choose to engage in appears to be important where later mathematical skills are concerned (DeFlorio & Beliakoff, 2015). This therefore underlines the importance of ensuring that any grouping of HMA items makes *conceptual* as well as *statistical* sense.

One issue that should be addressed, is that research that has focused on the association between frequency of HMA's and family SEC has tended to consist of underpowered studies, suggesting a need for high powered studies to confirm these initial findings. One cost and time effective solution would be to conduct secondary analyses on harmonised datasets obtained from existing studies on HMA's and mathematics skills.

Data harmonisation refers to the combining of data from multiple sources in a manner that makes them appropriate for comparison (Adhikari et al., 2021). To achieve this, statistical procedures are typically employed to calibrate similar, rather than identical, measures across studies, so that quantitative analytical techniques can be performed on larger datasets comprised of smaller ones collected in different contexts (Vonk et al., 2022). This procedure provides researchers with the opportunity to conduct analyses on larger, more demographically heterogeneous datasets, which would strengthen confidence in the interpretation of findings (Cohen, 2013). Although some important exceptions exist (e.g., Vasilyeva et al., 2018), previous research on HMA's has tended to rely on factor analysis models to identify clusters of activities, with subsequent analyses typically conducted on these clusters (Andrews et al., 2022). In contrast, latent profile analysis (LPA) is a person-centred statistical technique that aims to identify underlying, latent subpopulations of participants, by grouping individuals based on the probability that they possess common attributes (Grunschel et al., 2013; Spurk et al., 2020). The fundamental difference between the two approaches is that factor analyses operate by clustering *variables* into factors, whereas LPA models group *individuals* into distinct categories (Gomez & Vance, 2014). In the context of HMA's, LPA has the advantage of being able to identify homogenous subgroups of dyads who engage in certain types of mathematical activities undertaken in the home (Hickendorff et al., 2018). This approach can lend unique insights into the way in which indicator behaviours (e.g., SEC) relate to latent categories of individuals. For instance, latent models that have identified subgroups of patients with eating disorders have been found to possess better rates of predictive validity of mortality than DSM-IV based classifications (Crow et al., 2012).

To date, a small number of studies have applied this approach to understanding mathematical development. For example, Cahoon et al. (2021) used a latent longitudinal model to examine learning pathways over time, and Jordan et al. (2007) assessed number sense development (a composite variable measuring numerical skills such as counting and calculation) over six time points from kindergarten to the middle of the first year of schooling. In both studies, subtypes of individuals were discerned from similar patterns of behaviour, which enabled identification of precursor skills that lead to more complex mathematical skills, which variable-focused models (such as factor analysis) are not typically designed to do.

The current study

The principal aim of the current project was to investigate the types, range, and frequency of HMA's that relate to children's mathematical skills and family SEC. To this end, we collated nine datasets obtained from previous, similar studies conducted in the UK on HMA's and mathematical skills of 3-to-5-year-olds. Daucourt et al. (2021) identified 64 studies focused on the relation between HMA's and mathematical skills, only one of which was conducted in the UK. Therefore, we had an opportunity to synthesise several unique, recent studies using UK samples within this study.

To proceed with analyses, the HMA measures were placed into five higher order categories (e.g., operations), based on the measures' conceptual likenesses (the process is described in further detail below). Previous research has tended to place a focus on numeracy (Blevins-Knabe et al., 2000), although there are several other important mathematical domains (Ehrman et al., 2023) for this age group. Therefore, the present study took a more comprehensive approach to measurement of engagement with HMA's by including concepts representative of wider mathematical skills such as understanding of shape and pattern, for example, in addition to the more frequently explored concept of numeracy. Confirmatory Factor Analysis (CFA) was then conducted to verify that the conceptual categories were a good fit for the data, before employing these in an LPA. The LPA was then used to explore underlying subgroups of HMA engagement, as we were interested in exploring whether subgroups of HMA engagement were characterised by the content of HMA's (e.g., shape vs. number), the breadth of their experience, or frequency of HMA's undertaken. Finally, we explored the extent to which these latent subgroups were associated with SEC and children's mathematical skills using ordinal regression analysis.

Due to the exploratory nature of this secondary data analysis, no specific, directional hypotheses were proposed regarding children's membership of HMA profiles. However, we wished to explore whether there were SEC differences in HMA engagement and expected both SEC and mathematical skills to be significantly associated with LPA profile membership.

Method

This study, including the study design and analysis plan, was pre-registered on Open Science Framework (OSF; for more details on the primary data collection procedures see Hunt, B. W., Cahoon, A., Blakey, E., James-Brabham, E., Matthews, D., & Simms, V. (2025, April 3). Data Harmonisation and Secondary Analyses of how Home Mathematical Activities (HMA) Associate with Children's Mathematical Outcomes: UK based Datasets. https://doi.org/10.17605/OSF.IO/7TJCR).

Data acquisition

An advertisement was posted across different lab social media channels, and targeted mathematics-development focused conference channels. The request stipulated the inclusion criteria for the study, that all data sets should (a) have been collected in the UK, (b) focus on children between the age of 3-5 years old, (c) include a parent-report measure HMA engagement frequency, (d) include a direct measure of children's mathematics skills and (e) preferably include a measure of SEC[1]. The datasets were collected over a span of one month, starting from 24th of August 2023, when the advertisement was posted, until 22nd of September 2023 when the final dataset was received. Researchers who believed their dataset aligned with our research aim contacted our team, and we followed up with them. Additionally, the research team directly contacted researchers who we knew may have a dataset that met our aims. In total we collated nine datasets that met our inclusion criteria.

Participants

Out of a potential total of 1358 participants across nine datasets, only those who provided both mathematical outcomes (i.e., child data) and HMA activities (i.e., parent data) were included in the study. Therefore, a total of 969 dyads (i.e., parent-child participants) were included in the current analysis. Of these 969 dyads, 921 provided SEC data in the form of parents' highest educational qualification.

[1] Due to the nature of the study (i.e., secondary data analysis) access to materials should be sought from the originating labs.

Parent participants were found to have an overall mean age of 35.33 (SD= 5.51) years. The mean age of the child participants was 46.83 months (SD= 5.41; range 35-69). Table 1 shows the gender-age distribution for child participants and Table 2 shows descriptive statistics for SEC.

Table 1. Summary statistics of the gender-age distribution across the nine datasets for the child participants (in months), according to dataset

Dataset	***n***	**Mean (SD)**	**Minimum**	**Maximum**
Cahoon (Ulster)	M= 58	48.67 (3.57)	43	54
	F= 70	48.40 (3.13)	43	54
James-Brabham 1 (Sheffield)	M= 37	43.43 (3.91)	36	50
	F= 32	45.22 (4.31)	37	52
James-Brabham 2 (Sheffield)	M=47	45.11 (3.99)	38	52
	F= 57	45.35 (4.04)	37	52
Van Herwegen (UCL)	M= 46	43.93 (5.19)	36	57
	F= 33	45.21 (5.27)	36	56
Bennett (Loughborough)	M= 34	46.81 (6.43)	35	59
	F= 34	48.12 (7.06)	37	59
Trickett (Loughborough)	M= 87	43.09 (5.92)	36	59
	F= 78	44.78 (6.05)	36	64
Duncan (Ulster)	M= 20	54.90 (6.65)	44	69
	F= 20	53.05 (6.07)	44	67
Simmons (LJM)	M= 128	48.46 (3.42)	41	54
	F=146	48.18 (3.81)	41	55
Chang (Queens)	M= 20	47.55 (7.72)	37	57
	F= 19	49.84 (6.52)	37	57

Total	M= 477	46.46 (5.62)	35	69
	F= 489	47.20 (5.18)	36	67
Note: M = Male, F = Female				

As can be seen in Table 1, an approximately equal split between male and female child participants was observed in this sample that ranged in age from 35 to 69 months.

Variables

The key variables from each data set were extracted (see https://doi.org/10.17605/OSF.IO/7TJCR for more details on the key variables extracted from the nine datasets). These variables were 1) HMA activities with the question, items and response answers extracted; 2) the child outcome measure with the standardised and/or unstandardised items extracted and 3) all available SEC variables. Across all datasets the most frequently used measure of SEC was maternal education. The extracted variables were then harmonised (see supplementary materials for details on the harmonisation process) across datasets (see below).

Socio-economic circumstances (SEC)

Parents highest educational qualification was used to index SEC. There were 6 categories (no qualifications, GCSE, A levels, undergraduate degree, master's degree, PhD; see Supplementary Materials Table [Data Dictionary] for more information). In one of the datasets, master's and PhD levels had been combined into a single response. However, this was only a small number of participants (*n*=19), thus, the decision was made to code these participants as having a master's degree. The SEC variable was treated as a continuous variable in statistical analyses.

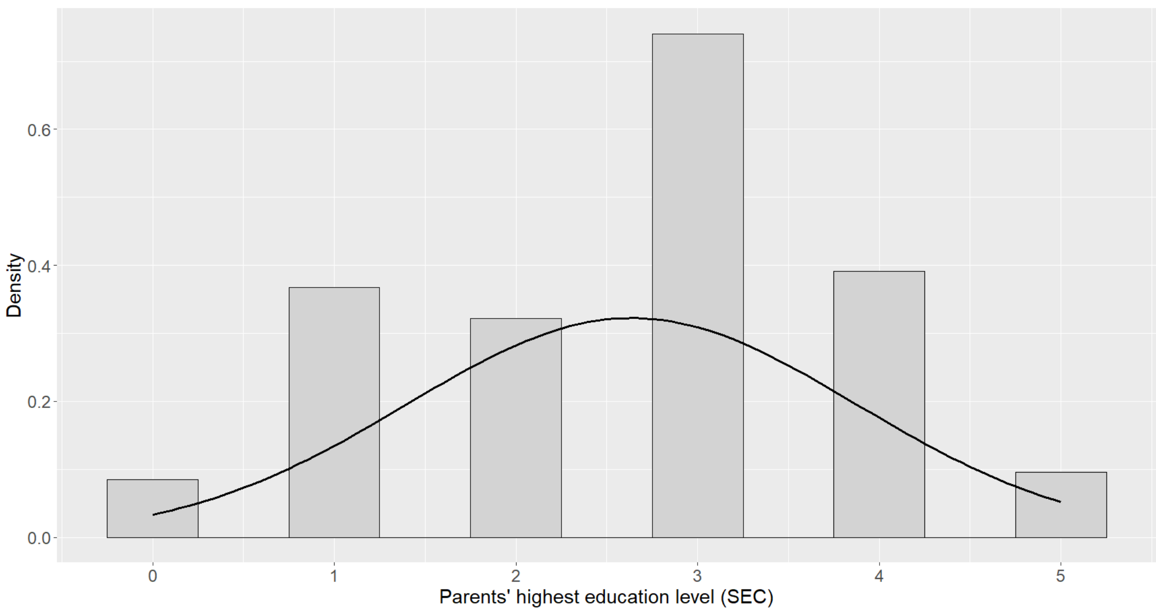
Table 2. Descriptive statistics for SEC according to dataset and overall.

Dataset	**Mean (SD)**	**Minimum**	**Maximum**

Cahoon (Ulster)	2.44 (1.37)	0	5
James-Brabham 1 (Sheffield)	2.04 (1.52)	0	5
James-Brabham 2 (Sheffield)	2.04 (1.28)	0	4
Van Herwegen (UCL)	2.89 (1.17)	0	5
Bennett (Loughborough)	2.93 (1.34)	0	5
Trickett (Loughborough)	2.83 (1.04)	0	5
Duncan (Ulster)	3.51 (1.03)	1	5
Simmons (LJM)	2.65 (1.07)	0	5
Chang (Queens)	2.72 (1.34)	1	5
Total	2.72 (1.24)	0	5

As shown in Table 2, SEC data spanned the range of the socioeconomic spectrum across all nine datasets. Figure 1 shows the distribution of Parents' highest education level (our proxy for SEC) according to the six response options (see Data dictionary in Supplementary materials for possible response options). SEC data was found to be normally distributed.

Figure 1. Distribution of Parents' highest education level (SEC).



Home Mathematics Activities (HMA): Data harmonisation of HMA items

A total of 208 items from the HMA measures across the nine datasets were eligible for harmonisation. Initially, two authors (AC and BH) harmonised a practice item (i.e., “counting objects”). Subsequently, for validity purposes, (AC) independently harmonised the items across the nine datasets. The Duncan dataset was used as a baseline for harmonising items across datasets since the HMA scale in this study contained the most items. Any unmatched items across the datasets were then matched to ensure that “every item would find a home”. Items that did not align with any other item in another dataset remained unmatched and were thus excluded (n= 29 out of a total of n=208; 13.9%; reasons for exclusion are summarised in Supplementary Materials Table 3. The HMA questions differed across the different datasets in terms of how they asked about the frequency of engagement. Some questionnaires used scale metrics “In the past month...” while others stated, “In an average week...” (see Supplementary materials Table 2 for all 9 questions). Although this discrepancy was identified, the primary issue during data harmonisation was reconciling the units of time within the response Likert scale. Regarding units of HMA measurement, Likert responses were adjusted so that data from measures capturing the frequency of HMA engagement on a weekly basis (e.g., 3 times per week) were multiplied by four to give a corresponding value that equated to an approximate monthly frequency (e.g., 12 times per month). See supplementary materials Table 2 for a table of harmonised frequencies. After having completed the harmonisation process, the original 179 items were mapped onto 36 variables for further analyses.

Mathematics skills

Each dataset contained either a standardised measure of mathematics skills (i.e., British Ability Scale; Early Number Concepts, TEMA-3 or Wechsler Preschool and Primary Scale of Intelligence (WPPSI) Mathematics measure) or a combination of domain-specific mathematics skills (i.e., an unstandardised measure). For datasets with a combination of experimental domain-specific mathematics skills, accuracy scores were used to calculate a mathematics skills outcome score (see the harmonised variables Excel for more detail on what variables were used). In cases where a dataset contained both standardised and unstandardised mathematics measures, the standardised measure was selected for inclusion in the final dataset.

The anonymised dataset and other files (key harmonised variables) are available on OSF and can be found at https://doi.org/10.17605/OSF.IO/7TJCR.

Data preparation

All analyses were conducted using R (version 4.3.2). Only dyads who had data for both parent-reported HMA frequency engagement and children's mathematics skill outcome measure were included in the final combined dataset. To be included, participants had to meet a missing data threshold, which was calculated as follows: participants who responded to >90% of the HMA items and completed >90% of the mathematical outcomes were included in the analyses.

Justification of variables

By harmonising across the 9 datasets, 36 HMA variables were generated. After close scrutiny of the literature - specifically Gilmore (2023), Purpura et al. (2013), and Milburn et al. (2019), as well as early years curriculum guidance (*CEA Curricular Guidance for Pre-school Education*, 2018) the HMA engagement variables were initially classified into 6 conceptual categories (early number, shape and space, relationships, pattern, sequencing and time, and size and quantity). VS and EJB independently classified each of the 36 variables into one of the categories, then the two researchers met to discuss any discrepancies and to reach consensus. Through this process it was identified that the majority of variables fell into the conceptual category of "Early number", there was a clear divide in these variables, with some focusing on written number and others focusing on counting and cardinality. Thus, this conceptual category was split to generate two separate categories: "Early number: Written number and Early number: Counting and cardinality. In addition, there were relatively few items that contributed to the "Shape and space" and "Patterning" conceptual categories. Therefore, these categories were merged to form one category: "Pattern, shape and space".

Following this, only those variables that matched across the majority of datasets (i.e. 5 or more) were used in the following analyses. This led to 16 variables being selected for inclusion for the further analyses. The 14 variables that were excluded had between 70.1% to 87.8% missing data. One other variable that had above 70% missing data was identified and was also removed, specifically, "Arithmetic games (using technology and non-technology)". "Time terminology" was the only variable in the "S*equencing and time"* conceptual category, therefore this variable was removed to run the factor analyses. Table 3 shows the final variables that were included within the conceptual categories.

Table 3. Variables included in each conceptual category.

Conceptual Categories	Variable names	No. of included variables
Early number: Written number	Identifying names of written numbers; Write numbers; Board games; Mathematics activities books	4
Early number: Counting and cardinality	Verbal counting; Counting objects; Using fingers to count; Rhyming related to numbers	4
Pattern, shape, and space	Sorting objects; Building blocks	2
Operations	Scenarios number games; Practice simple sums	2
Size and quantity	Measurements; Discussing quantities with everyday objects	2

Measurement invariance

We aimed to conduct an LPA on the harmonised dataset, and to then determine the extent to which the identified subpopulations were related to mathematical skills outcomes and SEC. This process involved comparing mean differences of latent variables from across datasets. One challenge associated with using data collated from multiple sources is the potential lack of measurement invariance across groups (i.e., datasets) (Chen & West, 2008; Millsap & Olivera-Aguilar, 2012). Therefore, we checked whether the measures that were employed across the studies measured the same underlying construct (Eremenco et al., 2005).

Initially, one-factor and five-factor CFA models were performed in R (using the lavaan package (Rosseel, 2012) with full information maximum likelihood (FIML) applied. The FIML function is widely used in CFA models when data are missing (Enders, 2008). Of the two models, the five-factor model (comprising the latent variables: Early number: Written number; Early number: Counting and cardinality; Pattern, shape, and space; Operations; and Size & quantity) was found to be a better fit for the data. The selection of the most appropriate model was based upon goodness of fit statistics (e.g., Chi-square (X^2), comparative fit index (CFI), Tucker Lewis index (TLI), root mean square error of approximation (RMSEA), standardised root mean square residual (SRMR); see Table 4), and their corresponding cutoff values outlined by Hu and Bentler (1999).

To test for measurement invariance, we followed recommendations outlined by Widaman and Reise (1997) by applying increasing levels of constraint (i.e., to factor loadings, intercepts, then residuals) to the five-factor CFA model, whilst also employing a grouping variable. This method determines whether factor structure remains consistent across the nine datasets. The increasingly stringent levels of measurement invariance are referred to as configural, metric, scalar, and strict (i.e., residual) invariance. To compare mean differences of latent variables on the nine collated datasets, partial scalar invariance was required (Vandenberg & Lance, 2000). This suggests that mean differences in the latent variables capture all mean differences in the shared variance of at least two of the observed (indicator) variables (Putnick & Bornstein, 2016).

Both full configural and metric invariance was achieved, as both models met the cutoff criteria on the comparative fit index (CFI; >0.9 suggests satisfactory fit; Awang, 2012; Hair et al., 2010), Tucker-Lewis index (TLI; >0.9 suggests satisfactory fit; Awang, 2012; Forza & Filippini, 1998) and root mean square error of approximation (RMSEA; <.05; Awang, 2012; Hair et al., 2010). Further, the two models did not significantly differ on a test of Chi-square difference (Van De Schoot et al., 2012; see supplementary materials Table 1). To obtain partial scalar invariance, the intercepts of four variables (HMAID, HMAWN, HMABG, and HMAVC) were allowed to be freely estimated across the datasets. These variables were selected for this purpose as they were found to be the most influential in the model. Again, the partially invariant scalar model met the cutoff criteria on the CFI, TLI, and RMSEA, and was not found to differ significantly from the metric model (see supplementary materials Table 1). Finally, we tested for strict (i.e., residual) invariance, however, this was not achieved. Despite not achieving strict invariance, we nevertheless concluded that there was clear evidence for partial scalar invariance, which was sufficient for our purpose of testing mean differences across the latent variables on the nine collated datasets, between these and both mathematical skills outcomes and SEC.

Model one proposed a one-factor model and model two proposed a five-factor model.

Table 4. *Model fit statistics for the alternative models of HMA variables*

Model no.	Model explained	χ^2(p)	*df*	CFI	TLI	RMSEA (90% CI) p	SRMR	AIC	BIC	Sample-Size Adjusted BIC
1	One-factor model	264.385 (<.001)	71	.906	.880	.053 (.047 - .060) .194	.066	25966.435	26199.646	26047.200
2	Five-factor model	123.703 (<.001)	61	.970	.955	.033 (.024 - .041) 1.00	.046	25845.754	26127.551	25943.345

Note: N = 136; Estimator = MLR; n = 136; *χ2* = Chi-square Goodness of Fit statistic; *df* = degrees of freedom; p = Statistical significance; CFI = Comparative Fit Index; TLI = Tucker Lewis Index; RMSEA (90% CI) = Root-Mean-Square Error of Approximation with 90% confidence intervals; BIC = Bayesian Information Criterion; AIC = Akaike information criterion.

Results

Relations between HMA, mathematics outcomes and SEC

After the factor analysis, but before completing the LPA, we explored the correlational relations between the HMA variables, children's mathematics skills and SEC.

Table 5. Correlational relations between HMA conceptual categories, child mathematical outcome score, and SEC.

	1	2	3	4	5	6
1. Child maths skills score	-					
2. SEC	.19***	-				
3. ENWN	.20***	.001	-			
4. ENCC	.01	-.01	.41***	-		
5. PSS	-.08	.01	.37***	.47***	-	
6. O	.23***	.02	.51***	.28***	.29***	-
7. SQ	.04	.12***	.40***	.37***	.37***	.42***

*Note: *** p<.001.* ENWN, Early number: Written number; ENCC, Early number: Counting and cardinality; PSS, Pattern, shape, and space; O, operations; SQ, Size and quantity.

The five conceptual HMA categories, mathematical outcome score, and SEC were entered into a correlational analysis. As can be seen in Table 5, significant associations were observed between mathematical outcome scores and SEC, "Early number: Written number", and" Operations". SEC was also associated with "Size and Quantity" variables. Several significant associations were observed between the HMA conceptual categories.

Latent profile analysis (LPA)

Mean scores for each conceptual category were created by calculating the mean score for all variables within a category.

Table 6. LPA with conceptual categories (14 variables)

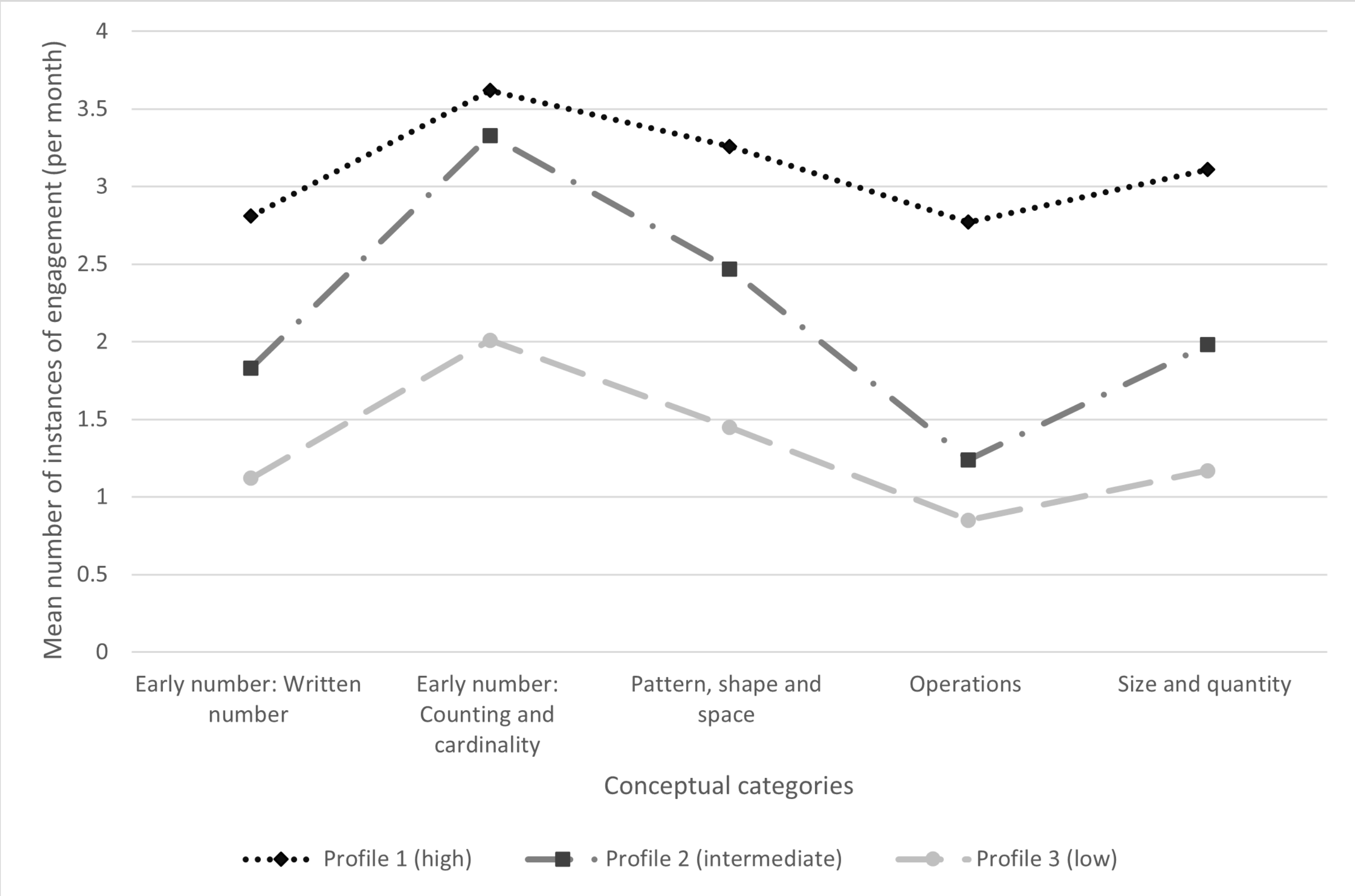
Note: Bold column indicates the chosen solution class. Values on top row refer to number of profiles for each solution.

Fit criterion	1	2	3	4	5	6	7
Akaike information criterion (AIC)	14104.73	13197.25	**12967.66**	12883.71	12860.30	12820.62	12808.48
Consistent AIC	14172.74	13297.97	**13083.80**	13028.93	13015.89	13017.28	13028.54
Bayesian information criterion (BIC)	14153.49	13275.27	**13074.94**	13020.24	13026.09	13015.67	13032.79
Sample size - adjusted BIC	14130.98	13231.16	**12991.93**	12912.00	12873.90	12850.24	12836.45
Entropy		0.74	**0.72**	0.66	0.65	0.70	0.68
Vuong-Lo-Mendell		0.001	**0.001**	0.001	0.001	0.001	0.001
Parametric bootstrapped		0.01	**0.01**	0.01	0.01	0.01	0.01

An LPA was performed and the model fit indices for 1 to 7 profile solutions were extracted in order to evaluate the model that best fit the data (see Table 6 for profile solutions). The three information criteria (IC) indices that were used for model selection were Akaike's information criterion (AIC), the Bayesian information criterion (BIC) and the sample size - adjusted BIC (aBIC), where lower values indicate the best fitting model. The model with the smallest IC values was selected. Following recommendations made by Nylund-Gibson et al. (2014) the elbow of the BIC value (the last large decrease in the BIC value) was used as a guide (Fryer, 2017) and a three-profile model was supported. This was also supported by the entropy value settling at a relatively high amount (s= 0.72) for this model, suggesting satisfactory separation of the profiles (Fryer, 2017; Nylund-Gibson et al., 2014). This finding suggests that dyads could be meaningfully grouped into three separate subgroups based on HMA engagement level.

The model identified 405 dyads in Profile 1, 418 in Profile 2, and 146 in Profile 3. The profiles were designated as being high, medium, and low HMA engagement, due to their respective values for HMA frequency: Profile 1 was named high HMA engagement as dyads in this profile had the highest scores in each of the five conceptual categories. The dyads in Profiles 2 and 3 had the second highest and lowest scores in each of the five categories respectively. See Figure 2 for HMA engagement profile membership according to the five conceptual categories.

Figure 2. Estimated means for the conceptual categories according to high, intermediate, and low HMA engagement profiles.



As shown in Figure 2, dyads identified as having high levels of HMA engagement had relatively high scores across all five conceptual categories, with the reverse being true for dyads with low levels of engagement. Interestingly, dyads with intermediate levels of engagement had relatively high scores for Counting and cardinality (comparable with high HMA engagement), but relatively low scores for Operations (comparable with low HMA engagement).

Ordinal regression

An ordinal logistic regression model was performed in R (R Core Team, 2022) using the 'Ordinal' package (Christensen, 2023) to assess the bivariate relations between both children's mathematics skills and SEC, on the likelihood of being in the high, intermediate, or low HMA engagement profile. The model was significant $X^2(6)= 113.59$, $p<.001$, and met the assumption of parallel lines (proportional odds). Further, the model correctly identified 49% of cases and explained 13% of the variance in HMA engagement profile membership (Nagelkerke pseudo R^2= .133). Dyads with higher children's mathematics skills were found to be 29% more likely to be in a higher engagement profile (B= .26; SE= .07; 95% CI= .13 to .39; OR= 1.29; 95% CI= 1.14 to 1.45; p<.001). SEC was not found to be related to engagement profile membership (B= .002; SE= .05; 95% CI= -.10 to .10; OR= 1.00; 95% CI= .90 to 1.10; p=.961).

With regards to differences between engagement profiles, when holding SEC constant, dyads with higher children's mathematics skills were approximately 83% more likely to be in the high engagement profile compared to the intermediate and low profiles combined (OR= 0.169).

Discussion

In this study, we aimed to better understand how HMA's relate to children's mathematics skills, and whether family SEC relates to the learning activities families do at home. To examine these questions, we adopted a person-centred statistical approach in a large data set. We conducted secondary data analyses on nine datasets (*n*= 969) obtained from studies conducted in the UK that investigated the association between children's mathematics skills and HMA engagement. We then employed LPA, which is a latent variable approach that takes a person-centred approach to clustering participants into homogenous subgroups based on their continuous scores on an outcome (Gomez & Vance, 2014). The purpose of the LPA was to identify whether there are subgroups of individuals who engaged in different types of mathematics learning at home. These subgroups could have been distinguished by the content the activities focused on, such as shape or name, or the breadth of their experience. Instead, the analysis established that the best fitting model was one where parent-child dyads were grouped into high, intermediate, and low engagement profiles based on their degree of HMA engagement, as this was the most salient characteristic among these participants.

We found that children's mathematics skills were significantly, positively associated with LPA profile membership, such that, as mathematics skills increased, dyads were significantly more likely to be in a higher profile of HMA engagement. These results align with previous research indicating a positive relation between frequency of engagement with HMA's and children's mathematical skills. For instance, LeFevre et al. (2009) found parent reported frequency of engagement in mathematical games to be a significant predictor of mathematics knowledge and fluency, as measured by numeration and operations skills. Similarly, Skwarchuk et al. (2014) found frequency of engagement with formal (e.g., practising simple sums) and informal (e.g. exposure to games with numerical content) home numeracy activities to be associated with children's symbolic and non-symbolic skills respectively.

Importantly, while the previous studies focused on home *numeracy* activities, the present study deliberately extended the conceptualisation of HMA's to include broader mathematical concepts, such as shape and size. This extension is important, as these significant aspects of mathematics cognition have been relatively underexplored compared to numerical skills (Zippert & Rittle-Johnson, 2020). Further, our conceptualising the HMA variables into higher order categories based on their conceptual likenesses was a novel approach, and allowed us to avoid the pitfall of relying on factor analyses to group activities into higher order categories arbitrarily (Andrews et al., 2022). It will be interesting to explore the use of conceptual categorisation further. At present, there is little causal evidence linking HMA frequency to children's mathematical skills. Furthermore, there is undoubtedly variation among families whereby some families engage in more HMA's because their child is good at mathematics and likes maths, and some families engaging in HMA's because their child might be struggling. Two important next steps would be to identify whether there is a causal relation, and to explore inter-family differences in their approach to undertaking HMA's.

One important finding from the present study was that SEC was significantly correlated with children's mathematical skills, supporting previous work highlighting early socioeconomic attainment gaps in mathematics (Blakey et al., 2020, James-Brabham et al. 2023). Interestingly, HMA engagement was not related to a families' SEC. Previous empirical evidence regarding this association is mixed. For example, studies have reported a positive relation between HMA frequency and SEC

(Galindo & Sonnenschein, 2015; Muñez et al., 2021), with others reporting no association (Pan et al., 2018), the latter with which our findings are consistent. Our current findings are important, because they suggest – using a well powered sample – that frequency of engagement with HMA's may not be responsible for driving attainment gaps in early mathematical skills. Instead, there may be other influential factors at play.

It is clear that there is a lot more work that needs to be done to understand why early attainment gaps arise and our work offers a clear avenue for future research. Specifically, researchers may want to explore how the quality of mathematical interactions between parents and children play a role, perhaps in terms of mathematical language, the resources available, or scaffolding that parents are able to engage in. However, for now, it is conceivable that families in our sample, living in low SEC are doing just as much as families living in high SEC, which is particularly noteworthy as these families are likely to be under more constraint. The findings also offer caution to interventions that solely focus on boosting the quantity of HMA's between parents and children. Our findings suggest that such interventions may not necessarily help narrow attainment gaps.

The results of this study also add to a growing body of literature showing early attainment gaps in mathematical skills related to family SEC (e.g. Muñez et al. 2021, Zadeh et al. 2010, and James-Brabham et al. (2023)). For instance, Blakey et al. (2020) found attainment gaps in mathematical skills in children from as young as age 3. Together, this body of research suggests we need to urgently understand why these early attainment gaps arise. Our results suggest that focusing on frequency to narrow attainment gaps would not be effective.

It remains possible that there are other features of the wider home learning environment that vary according to family SEC and explain differences in early mathematical skills. Discussion has tended to focus on frequency of HMA's, proposing that parents from higher SEC's may be better prepared to teach educational skills (Blevins-Knabe & Musun-Miller, 1996). While it might be intuitive to expect higher levels of parental SEC to be related to higher levels of engagement with HMA's, the home learning environment is multifaceted (Skwarchuk et al., 2014). Thus, assessments solely focused on measuring the frequency with which HMA's take place may fail to capture other SEC-related aspects of the home learning environment that play an important role in children's mathematics learning (Zippert & Rittle-Johnson, 2020), such as the *quality* of HMA interactions between caregivers and children.

When considering the quality of parent-child interaction, research on parental scaffolding may provide important insights. Mothers with higher levels of education have been shown to engage in more 'scaffolding' behaviour with their children. Scaffolding involves providing support during tasks that are too challenging for the child to accomplish alone, but can be completed with support (Lowe et al., 2013). In the context of HMA's, this higher quality provision of support for fewer mathematics activities might therefore be more beneficial – and more strongly related to mathematics skills – than higher levels of overall engagement in HMA's with little parental scaffolding. Moreover, the degree to which the home learning environment provides cognitive stimulation – a concept that refers to opportunities for play and learning and the provision of developmentally appropriate learning materials (Bradley & Caldwell, 1984; Lurie et al., 2021) – has been found to vary according to SEC, with children who experience more stimulating home environments being found to have higher mathematics scores at 4-11 years-old (Crosnoe et al., 2010). Provision of cognitively stimulating materials is in turn associated with both parent education and income (Christensen et al., 2014; Hackman et al., 2015).

Taken together, these findings suggest that quality of HMA's, rather than quantity (which frequency-based measures tap into [Andrews et al., 2022]) is likely to be critical for mathematics skills in young children, and that the relation between mathematics skills and quality of HMA's is perhaps mediated by parental SEC. Whilst the possibility of higher quality mathematics activities being more beneficial than frequently employed but poorer quality activities has previously been acknowledged (Daucourt et al., 2021; Zippert & Rittle-Johnson, 2020), this specific area of mathematical cognition research is notably lacking. Future research may wish to explore the association between mathematics skills and the interplay between frequency and quality of HMA's.

There were a number of strengths associated with this study. Firstly, we tested a novel hypothesis in an innovative way using LPA. While LPA is increasingly being used in developmental science to take a person-centred approach, very few studies have applied it to understanding home learning. While our resulting finding replicates and extends recent findings, namely, that there is a small relation between HMA's and children's mathematical skills, by using this approach we were able to test alternative models. Specifically, we were able to assess if HMA's should be best considered as being grouped by activity type profiles. It is plausible that the groups identified through the LPA could have been related to activity type profiles e.g., parents doing lots of counting and less patterning compared to parents doing a range of activities across domains. Instead, we found that grouping the profiles around levels of engagement was the best characterisation of the data. In addition to clarifying this, we were able to investigate the role of SEC. Prior work that brings together studies (e.g., in meta-analyses) have been limited in their approach to understanding the relation between home learning and SEC, because the measures of SEC used in the included studies have varied so much due to measurement differences or differences in geographical areas making comparisons less meaningful (e.g., see James-Brabham et al., 2024). In this study, we were able to test the relation in a large sample across studies using the same measure. We found that SEC was positively related to children's mathematical skills but not frequency of HMA's. This is an important finding as it suggests that aiming to narrow attainment gaps by increasing the frequency of HMA's may not be effective.

A further strength of the present project was using data harmonisation to bring to together and analyse nine UK based studies. The measures that were obtained also included wider mathematical concepts, rather than numeracy alone. Finally, the obtained data contained dyads representing the entire SEC spectrum. This is a strength compared to many previous studies which, likely for practicality reasons, have tended to recruit samples from one or two areas of the spectrum (e.g., low versus high or exclusively middle SEC; see DeFlorio & Beliakoff, 2015; Manolitsis et al., 2013).

It is important to note that the obtained data was limited to HMA engagement frequency. As previously mentioned, frequency of take up of HMA's may overlook other important aspects of the home mathematics environment, such as quality of HMA's over quantity undertaken. Further, the obtained data did not contain variables relating to domain-general constructs such as executive function and language abilities, which have been shown to mediate the relation between SEC and mathematical skills (Blakey et al., 2020; James-Brabham et al., 2023). Therefore, we were unable to control for these factors in the analyses. Due to the nature of the design of the present study, it was not possible to infer the directionality of the relation between HMA frequency and children's mathematical skills. We conducted our analyses on a sample with dramatically increased power as a result of the harmonisation, enabling us to run sophisticated statistical analyses to answer unique research questions. However, investigating the association between frequency of engagement with HMA's and children's mathematical skills at a single time point poses an issue, as any observed effect may be the result of other factors, such as the environment or genetic effects. We of course recognise that our data remains correlational, and therefore, does not provide causal insights into the relation between HMA's and children's mathematics skills. Longitudinal research, or even better, randomised control trials would allow us to better understand whether HMA frequency and early mathematical skills are causally related (Cahoon et al., 2023).

We acknowledge that a potential limitation of harmonisation approaches is that it is necessary to exclude some items as they do not conceptually relate to other items in different original data sets. There may be concerns that this means that the resulting data set that may not reflect the intended construct in terms of validity. However, in the case of our analyses, 29 items (13.9%) were excluded as they could not be harmonised across datasets. Thus, this reflects minimal exclusion of items, and of course, any of these items that were excluded were infrequently measured in original datasets.

A key finding from the present results is that family SEC does not relate to frequency of HMA's. Whilst caution should be exercised when extrapolating from a null result, based on the present findings, policies that encourage lower SEC parents to engage in additional mathematics activities in the home in a bid to reduce the mathematics attainment gap may thus be ineffective. In answer to the question posed in the title of this paper, it appears that there is an association between HMA engagement and children's mathematics skills, in keeping with the meta-analysis of HMA's and mathematics skills conducted by Daucourt et al. (2021). Importantly, this relation appears not to depend on SEC, despite some authors arguing that lower SEC parents do not engage in as much home learning as higher SEC parents (e.g., Muñez et al., 2021), which is an important implication of the present work.

In summary, the present study used LPA to establish that children's mathematical skills were found to positively predict HMA profile engagement, such that dyads with higher children's mathematics scores were more likely to be in a higher profile of engagement. SEC was not related to frequency of HMA engagement, despite being correlated with children's mathematics skills. Therefore, this study, utilising large-scale harmonised data, supports previous research suggesting parents that are lower in SEC's are not necessarily engaging in fewer mathematical activities with their children. Instead, interventions may be more effective if they address factors such as access to resources or the quality of parent-child interactions during mathematical activities in the home.

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