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Can Risk-free and Zero-Beta portfolios be constructed? UK and US Evidence

Zhen He[a], Fergal O'Connor[b], Jacco Thijssen[c]

a. School of Economics, Shanghai University

b. Cork University Business School, University College Cork, Ireland

c. University of York, UK

Abstract

This paper aims to determine whether a risk-free portfolio can be formed using gold, T-bills, silver, platinum, and palladium. We first construct zero variance portfolios composed of two assets. The results show that it is possible to construct risk-free portfolios based on zero variance. Secondly, we apply Wald tests to Black's (1972) zero-beta CAPM to examine whether these constructed risk-free portfolios qualify as zero-beta portfolios. The empirical results suggest that a risk-free portfolio is not always found to be a zero-beta portfolio, and vice versa. UK and US results show that a risk-free portfolio and a zero-beta portfolio in one market is not necessarily so in another.

Keywords: Risk-free portfolio; Precious metals; Wald test; Zero-beta CAPM.

1. Introduction

He et al. (2022) demonstrated that standard proxies of risk-free assets or assets with a zero-Beta per the Capital Asset Pricing Model (CAPM) do not qualify as such when they are compared to the assumptions of financial theory. Assets such as T-bills, Over Night Interest Rate Swap (OIS), Inter Bank Offer Rates (IBOR), and gold were not consistently found to be zero-beta assets internationally under Blacks's (1972) Zero-Beta CAPM.

This paper continues to pursue a proxy for a risk-free asset by constructing portfolios using a range of assets traditionally perceived as risk-free, such as T-bills, or those exhibiting a stable zero-beta relationship with assets under the CAPM framework. Its goal is to explore the feasibility of constructing risk-free portfolios utilizing gold, T-bills, silver, platinum, and palladium for investors in the UK and US.

The content in this paper will be delivered as follows. Section 2 will review the related literature. Data will be presented in Section 3. Section 4 will describe the methodology. Section 5 will present the empirical results. And the conclusion will be drawn in Section 6.

2. Literature review

Commonly accepted risk-free assets (T-bills and gold) are not zero-beta assets in Black's (1972) zero-beta CAPM (He et al., 2018). In addition, the results in He et al. (2022) suggest that none of: gold, T-bills, OIS and IBOR qualify as a zero-beta asset in the zero-beta CAPM for all markets. To solve this dilemma, another way to construct a portfolio that can be the proxy for the risk-free portfolio for all markets. And a zero-beta portfolio must also be constructed and tested when a risk-free portfolio cannot be constructed.

Batten et al. (2010) provides support precious metals in their portfolio as a diversifier due to their low betas Hillier et al. (2006) investigate precious metals by examining the relationship of precious metals in relation to the S&P 500 and MSCI Australia/ Europe/ Far East index. The low correlation shown in their results provides evidence to conclude that silver, platinum and palladium are not zero-beta assets. Another similar evidence can be found in Lucey and Li (2015). The low correlations between precious metal and other asset prices are mentioned in Vigne et al. (2017) in a review of the literature on financial economics on silver, platinum and palladium. Given these research, white precious metals are worth testing since they might be candidates to construct a risk-free portfolio or zero-beta portfolio.

3. Data

Daily gold price data for the UK and the US in Sterling and Dollars respectively are collected from Federal Reserve Economic Data (FRED). The data of silver prices comes from the LBMA (London Bullion Market Association). The price data for platinum and palladium are collected from the database validated by the LPPM (London Platinum and Palladium Market). The AM fixing price of platinum and palladium are selected to ensure a larger sample size in the data. Other data include the stock prices of every listed company in the UK and US indices (FTSE 350 and S&P 500), a list of these is available for request[1].

[Insert Table 1 here]

4. Methodology

4.1 Single-period risk-free portfolios

Since the CAPM is a static model, we start by constructing single-period portfolios. We, thus, implicitly assume that returns over time are iid. This is, admittedly, a very strong assumption that could be relaxed (see Section 6 below). Let $\widetilde{\boldsymbol{R}} \coloneqq (\tilde{R}_j)_{j=1}^{J}$ be the vector of raw returns on J assets, with (non-singular) variance-covariance matrix $\boldsymbol{\Sigma}$. Assuming admissibility of short-sales, the return on a portfolio $\boldsymbol{\alpha} \coloneqq (\alpha_j)_{j=1}^{J} \in \mathbb{R}^J$ is given by $\tilde{R}_\alpha = \boldsymbol{\alpha}'\widetilde{\boldsymbol{R}}$, with variance $\sigma_\alpha^2 = \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}$. For a risk-free portfolio, $\boldsymbol{\alpha_f}$, it holds that $\sigma_{\boldsymbol{\alpha}_f}^2 = 0$, and, thus, that the (net) risk-free rate is $r_f = \boldsymbol{\alpha}_f'\widetilde{\boldsymbol{R}} - 1$. Since $\boldsymbol{\Sigma}$ is a symmetric matrix, the set of solutions to the zero-variance equations is a cone that contains the null-space of the set of

[1] ADF unit root tests have been undertaken for ever stock price of all constituents in each index for the UK and US. Results available on request.

Σ. In fact, since the matrix **Σ** is positive semi-definite, the set of solutions coincides with the null space of **Σ**. Hence, every solution to the system of equations **Σα**=**0** is a zero-variance portfolio. In what follows we will, for simplicity, restrict attention to portfolios made up of two assets.

4.2 Zero-beta CAPM

Following the zero-beta CAPM introduced by Black (1972), we assume that the risk-free rate is missing or unknown in the model shown in equation (1), which is differs from the classic CAPM. The zero-beta CAPM is written as follows,

$$E\left[R_{i,t}\right] = \gamma(1-\beta_i) + \beta_i E\left[R_{M,t}\right] \tag{1}$$

where $R_{i,t}$ is denotated as the raw return of a listed companies in the UK and US indices, γ is an unknown constant that represents the expected return on a zero-beta asset since there is no risk-free asset. The zero-beta asset is uncorrelated to the underlying market portfolio m.

Let $R_{j,t}$ be the return on other assets where the asset j can be a portfolio composed of gold or T-bills, silver, platinum and palladium as in the empirical tests in Section 5.3 below. To start the analysis in tests in the zero-beta CAPM, the parameter γ in equation (2) is substituted by $E(R_{j,t})$.

$$E\left[R_{i,t}\right] = E[R_{j,t}](1-\beta_i) + \beta_i E\left[R_{m,t}\right] \tag{2}$$

To let the model coincide with the zero-beta CAPM shown in equation (2), the null hypothesis is the following,

$$H_0\colon \alpha_i = E\left[R_{j,t}\right](1-\beta_i) \tag{3}$$

The null hypothesis can also be written as only if beta is not 1,

$$H_0: \frac{\alpha_i}{(1-\beta_i)} = E[R_{j,t}] \tag{4}$$

against the alternative hypothesis,

$$H_A: \frac{\alpha_i}{(1-\beta_i)} \neq E[R_{j,t}] \tag{5}$$

Since $i = 1, \dots, N$ and γ is unknown, there are $(N+1)$ restrictions,

$$H_0{}' : \frac{\alpha_1}{(1-\beta_1)} = \frac{\alpha_2}{(1-\beta_2)} = \cdots = \frac{\alpha_N}{(1-\beta_N)} = E[R_{j,t}] \tag{6}$$

$H_0{}'$ is another form of the null hypothesis. It puts a stricter requirement that this hypothesis in equation (6) must hold at the significant level for portfolios of gold, T-bills, silver, platinum and palladium against every listed company of FTSE 350 in the UK and S&P 500 in the US[2].

5 Empirical Results

5.1 Risk-Free Portfolios

To find the existence of roots in the quadratic equations, it is necessary to examine the results of the discriminant. We must examine whether the discriminant is positive, which can ensure real roots in the quadratic equation. Corresponding solutions can be found using the calculated weights in constructing a risk-free portfolio.

These results are shown in *Table 2*. The discriminant is calculated as the positive results in the portfolios constructed by the pairs of *gold & platinum* and *gold & palladium* in the UK, and by the pairs of *gold & silver*, *gold & platinum*, *gold & palladium* and *T-bills &*

[2] These tests have also been run using the constituents of the FTSE 100 and NASDAQ indices as a robustness check and we find the conclusions of the paper are unchanged. These robustness check results are available on request.

platinum in the US. These pairs are the only pairs that have the real roots for their weights to construct a risk-free portfolio.

[Insert Table 2 here]

As shown in *Table 2*, there are two sets of results for the weights in the risk-free portfolio shown in *Table 3*. It can be seen that gold plays a consistent role in constructing risk-free portfolios, while T-bills are only used in a single case in the construction of risk-free portfolios in the US. Interestingly, the weight of gold and the other asset are almost equal weights, except for the case of *gold & platinum* in the UK. In the UK, *T-bills* cannot be used to construct a risk-free portfolio, while T-bills can be used in the US even though their proportion is far less than platinum. The risk-free portfolio can be constructed by using the weights for the corresponding assets in *Table 3*.

[Insert Table 3 here]

5.2 Results of Zero-Beta Portfolio

Table 4 presents all the portfolio combinations. Using the results from *Table 3*, we apply naïve portfolio construction, using equal weights among the assets in each portfolio expect for the pairs where a risk-free portfolio can be constructed. Portfolios *P1* to *P10* constructed using only two assets, and *P11* to *P26* are using more than two assets. Since there are some asset sets that do not qualify as the risk-free portfolio, we test whether these portfolios can be formed as a zero-beta portfolio from the Black's Zero-beta CAPM.

[Insert Table 4 here]

Table 5 shows the percentage of the insignificant results of the Wald test for each portfolio combination against each individual company in the UK and US. In the UK, there are more portfolio combinations that show the potential to be a zero-beta portfolio. As the cut-off point in our tests is a 5% level of significance, if more than 95% of the portfolios are found to be insignificant this would indicate that a particular portfolio is a zero-beta portfolio. The percentage of insignificant results are all above 95% for portfolios which cannot be constructed as risk-free portfolios (shown in italics in *Table 5*). As shown in *Table 4* portfolios *P3* and *P4* can be zero-beta portfolios in the UK and can also be risk-free portfolios. This is further evidence for the finding that the portfolios constructed using *gold & platinum* (*P5*) and *gold & palladium* (*P4*) can be considered as risk-free portfolios for UK investors.

In the US, the percentage of the insignificant results from the Wald test is shown above 95% in the portfolios *P9*, *P10*, and *P20.* These portfolios could, in a similar way to above, be zero-beta portfolios in the US. However, the portfolios *P2*, *P3*, *P4* and *P6* have percentages lower than 95% in *Table 5* and so the zero-beta portfolios. This is further evidence that the risk-free portfolio and zero-beta portfolio are not the same in practice.

We retested the data using Likelihood Ratio Tests (LRT), as they are asymptotically equivalent to the Wald test, and these provide the same conclusions to those in the Wald test. These test results are available for request.

[Insert Table 5 here]

It is interesting that identical portfolios of assets in the UK and the US, such as portfolios *P2* and *P3* in *Table 4,* are not found to be zero-beta/risk-free portfolio in both markets. This also raises questions about the generalisability of certain assets or portfolios in a broader context, which is why portfolios *P2* and *P3* might be qualified as zero-beta portfolios in the UK but not in the US.

Table 6 show the results of the same Wald Tests using weekly data, run as a robustness check. These show that more portfolios qualify as zero-beta when this less noisy data is used, and these portfolios are highlighted in italics in the table. This indicates that these results are robust to the frequency of data used, and that for financial managers with longer holding periods more zero-beat portfolios may exist.

[Insert Table 6 here]

6 Conclusion

This paper aims to determine whether portfolios constructed using gold, T-bills, silver, platinum, and palladium, can be considered as risk-free and/or the zero-beta portfolios. By calculating the roots of quadratic equations, we show that risk-free portfolios can be constructed using gold *& platinum*, and *gold & palladium* in the UK; and using *gold & silver, gold & platinum, gold & palladium, and T-bills & silver* in the US. Gold appears to play an essential role in the construction of a risk-free portfolio with fairly equal weights when put together with the other asset. A contribution of this paper is that T-bills, the assumed proxy for the risk-free asset in many empirical studies, can be replaced by these real risk-free portfolios constructed as above.

Further, all the portfolio combinations are tested against each listed company in the UK and US indices using the Wald test, based on Black's (1972) zero-beta CAPM. The results show that some constructed portfolios qualified as zero-beta portfolios in both the UK and US. Typically, risk-free portfolios constructed using *gold & platinum* and *gold & palladium* also exhibit zero-beta characteristics in the UK. The constructed risk-free portfolios from Section 5.1 are tested as the zero-beta portfolio in the Wald test in Section 5.2, but a risk-free portfolio is not always found to be a zero-beta portfolio, and vice versa. This conclusion could prevent the misuse and misunderstanding of the risk-free portfolio. Also, risk-free portfolios can be constructed in the real world.

These findings increase the options available to portfolio managers when building portfolios, possibly allowing them to satisfy their need for a risk-free asset while also benefiting from gold's well-researched diversifications benefits (O'Connor et al., 2015). Additionally, while holding T-bills as a risk-free asset requires frequent trading as these bills come to maturity, resulting in costs for funds, the four precious metals once purchased can remaining in a portfolio indefinitely without requiring trading, though they do incur storage costs (Lucey, 2013).

Further research on smaller markets may identify risk-free or zero-beta portfolios, which could be particularly useful in jurisdictions without a default risk-free asset—that is, where the government does not borrow in its domestic currency—allowing investors to allocate capital more efficiently. Another avenue for future research is examining whether the portfolios constructed here can serve the same function during periods of extreme market stress.

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Tables

Table 1 Data source for UK and US

	Start date	End date	Data source
UK			
Gold price	04/01/1968	10/31/2024	Datastream
T-bills	01/04/1985	10/31/2024	Datastream
Silver price	02/01/1990	10/31/2024	LPPM
Platinum price	02/04/1990	10/31/2024	LPPM
Palladium price	02/04/1991	10/31/2024	LPPM
FTSE 350	12/31/1985	10/31/2024	Datastream
US			
Gold price	04/01/1968	10/31/2024	Datastream
T-bills	01/02/1972	10/31/2024	Datastream
Silver price	02/01/1990	10/31/2024	LPPM
Platinum price	02/04/1990	10/31/2024	LPPM
Palladium price	02/04/1991	10/31/2024	LPPM
S&P 500	12/31/1963	10/31/2024	DataStream

Table 2 - Quadratic equation solutions for all portfolio pairs: UK and US-daily data

	UK		**US**	
1st asset and 2nd asset	**Discriminant**	**Real Roots**	**Discriminant**	**Real Roots**
Gold & T-bill	Negative	No	Negative	No
Gold & Silver	Negative	No	Positive	Yes
Gold & Platinum	Positive	Yes	Positive	Yes
Gold & Palladium	Positive	Yes	Positive	Yes
T-bill & Silver	Negative	No	Negative	No
T-bill & Platinum	Negative	No	Positive	Yes
T-bill & Palladium	Negative	No	Negative	No
Silver & Platinum	Negative	No	Negative	No
Silver & Palladium	Negative	No	Negative	No
Platinum & Palladium	Negative	No	Negative	No

Table 3 - Results of weights for the risk-free portfolio: daily data

	Set 1		**Set 2**	
1st asset & 2nd asset	**α**	**β**	**α**	**β**
UK				
Gold & Platinum	0.01	0.98	0.02	0.97
Gold & Palladium	0.48	0.51	0.49	0.51
US				
Gold & Silver	0.48	0.52	0.50	0.49
Gold & Platinum	0.49	0.51	0.49	0.51
Gold & Palladium	0.48	0.51	0.51	0.50
T-bill & Platinum	0.02	0.97	0.03	0.98

Note: α is the weight for the 1st asset, and β is the weight for the 2nd asset.

Table 4 - Portfolios combinations in both the UK and US

	Gold	T-bill	Silver	Platinum	Palladium	Risk-free portfolio
P1	√	√				No
P2	√		√			Only US
P3	√			√		UK & US
P4	√				√	UK &US
P5		√	√			No
P6		√		√		Only US
P7		√			√	No
P8			√	√		No
P9			√		√	No
P10				√	√	No
P11	√	√	√			
P12	√	√		√		
P13	√	√			√	
P14	√		√	√		
P15	√		√		√	
P16	√			√	√	
P17		√	√	√		
P18		√	√		√	
P19		√		√	√	
P20			√	√	√	
P21	√	√	√	√		
P22	√	√	√		√	
P23	√	√		√	√	
P24	√		√	√	√	
P25		√	√	√	√	
P26	√	√	√	√	√	

Note: √ presents the assets that are used to construct the portfolio. So, the diagonal line is drawn for the portfolio from P11 to P26 as we cannot use the method outlined above to examine whether these portfolios can be a risk-free portfolio.

Table 5 - Percentage of the insignificant results from daily Wald Tests: UK and US

	UK	US		UK	US
P1	0.005	*0.056*	***P14***	0.986	0.090
P2	*0.957*	0.040	***P15***	0.983	0.080
P3	*0.968*	*0.036*	***P16***	0.981	0.084
P4	*0.977*	*0.046*	***P17***	0.011	0.078
P5	0.006	0.070	***P18***	0.020	0.070
P6	0.008	*0.060*	***P19***	0.029	0.080
P7	0.006	0.064	***P20***	0.014	0.976
P8	0.965	0.832	***P21***	0.029	0.106
P9	0.977	0.972	***P22***	0.034	0.114
P10	0.971	0.954	***P23***	0.042	0.120
P11	0.011	0.098	***P24***	0.982	0.082
P12	0.014	0.104	***P25***	0.032	0.086
P13	0.008	0.110	***P26***	0.028	0.166

Table 6 - Percentage of the insignificant results from weekly Wald Tests: UK and US

	UK	US		UK	US
P1	0.004	0.002	***P14***	0.978	0.024
P2	*0.991*	0.003	***P15***	0.988	*0.976*
P3	0.982	0.007	***P16***	0.996	0.001
P4	0.978	0.009	***P17***	0.002	*0.997*
P5	0.004	0.997	***P18***	0.003	*0.978*
P6	0.007	0.998	***P19***	0.004	0.981
P7	0.004	0.973	***P20***	*0.997*	0.945
P8	0.991	0.917	***P21***	0.007	0.003
P9	0.997	0.961	***P22***	0.006	0.004
P10	0.996	0.934	***P23***	0.009	0.006
P11	0.004	0.004	***P24***	0.993	0.018
P12	0.003	0.003	***P25***	0.005	*0.974*
P13	0.006	0.001	***P26***	*0.967*	0.004