Saving to Decumulate: A Lifetime Journey Based on the Perfect Contribution Rate

Andrew Clare, James Seaton, Peter N. Smith, and Stephen Thomas

Andrew Clare
is a professor of asset management in the Bayes Business School at the City University of London in London, UK.
a.clare@city.ac.uk

James Seaton
is a former research fellow in the Bayes Business School at the City University of London in London, UK.
james.seaton@talk21.com

Peter N. Smith
is a professor of economics and finance at the University of York in York, UK, and a research associate in the Centre for Applied Macroeconomic Analysis (CAMA) at the Australian National University in Canberra, Australia.
peter.smith@york.ac.uk

Stephen Thomas
is a professor of finance in the Bayes Business School at the City University of London in London, UK.
stephen.thomas.1@city.ac.uk

KEY FINDINGS

- The perfect contribution rate (PCR) is the annual contribution required over a person's savings' life that will allow the individual to achieve a real wealth target prior to the start of decumulation, assuming perfect foresight of asset returns.
- Mean reversion in asset returns over long time periods is suggested by the positive relation between PCR and subsequent perfect withdrawal rate (PWR).
- The perfect retirement ratio (PRR)—the ratio of the PWR to the previous PCR—is highly time-dependent and provides a means of estimating the future worth of investment contributions at the point that they are being made.

ABSTRACT

This paper examines the relationship between the accumulation (contribution) and decumulation (withdrawal) phases of pension saving across a person's economic life. These are concepts that are usually considered "independent," but for which we find a surprisingly strong empirical relationship, generated most likely by the long-run mean reversion of returns. Using evidence from a long run of US data since 1870, we introduce the idea of a perfect contribution rate (PCR), which is the annual contribution required over a person's savings' life that will allow the individual to achieve a real wealth target prior to the start of decumulation, assuming perfect foresight of asset returns. In particular, there is an interesting positive relation between PCR and subsequent perfect withdrawal rate (PWR), which we believe reflects mean reversion in asset returns over long time periods. In other words, if an agent has to save larger amounts to achieve their target wealth when investment returns are poor, then typically the subsequent (possible) withdrawal rates may also be large. We also introduce the idea of the perfect retirement ratio (PRR)—the ratio of the PWR to the previous PCR—and find this to be highly time-dependent, with significant policy implications.

In this paper, we examine the relationship between the accumulation (contribution) and decumulation (withdrawal) phases of a person's economic life. These are concepts that are usually considered "independent," but for which we find a surprisingly strong empirical relationship, generated most likely by the long-run mean reversion of returns (e.g., see Reichenstein and Dorsett 1995; Aked and Ko 2017; Pfau 2011; Strong and Taylor 2001). The intention here is to go some way to answering the challenge from the financial advisory community that researchers and academics do not offer practical, joined-up thinking in this space, but rather focus on pre- or post-retirement separately.

Much of the literature examining savings choices over the life cycle and professional advice offered to savers focuses on a changing pattern of diversification between stocks and bonds across the life cycle, and the innumerable withdrawal strategies created to sustain withdrawals through decumulation (see Blanchett 2007; Pfau and Kitches 2014, among others). Both strands propose that the young should hold more stocks than the old, and that those approaching retirement should hold an increasing proportion of their savings in bonds. In the market, this pattern is followed by target date funds, as popularized by Vanguard (Daga et al. 2022). Recent research has provided more evidence that challenges these tenets, developing the arguments of Shiller (2005) and others. Using a long, multi-country set of returns, Anarkulova, Cederburg, and O'Doherty (2023) show that an all-equity, balanced portfolio of domestic and overseas stocks dominates target date funds in terms of the level of wealth at retirement, and retirement income. Ennis (2024) similarly challenges traditional diversification. This paper offers a simplified and implementable approach in this vein.

The problem with examining the very long period of accumulation (say, 30–40 years) followed by a retirement period (of, say, 20 years) is the lack of an appropriate, very long run of data and, indeed, independent data periods. To this end, we focus on the long run of US equity data since 1870, studied extensively in Clare et al. (2017) and Shiller (2005), among others. Although this would seem rather limiting in focusing on a single asset, there is increasing evidence that diversification in the direction of multi-assets may well not be as advantageous as previously thought (see, for example, Anarkulova et al. 2023; Ennis 2024). We also focus on the US experience, due to the relative homogeneity of the institutional structures, and the quality and length of the data. Equally, we acknowledge the richness of the international experience and the expanding ability of investors to exploit the opportunities that these markets offer. We leave analysis of the international data to further work.

Using this long run of equity returns as our investment portfolio, we introduce the idea of a perfect contribution rate (PCR). This is the single-value, annual contribution required over a person's savings' life that will allow the individual to achieve a real wealth target prior to the start of decumulation, assuming perfect foresight of asset returns. It is expressed as a proportion of this final real-wealth target sum. The path of these contributions to the retirement pot is primarily determined by the growth of the portfolio of accumulated savings. We focus on portfolios held primarily in equities for the reasons given above. Comparing possible paths for returns over the accumulation period, higher returns will allow the PCR to be lower than otherwise. This is, of course, directly parallel to the much more widely researched perfect withdrawal rate (PWR) (see Suarez, Suarez, and Walz 2015; Clare et al. 2017; Anarkulova et al. 2023; Blanchett, Kowara, and Chen 2012). The PWR is the annual proportion of the accumulated real-wealth sum that can be withdrawn over the retirement period, with or without a bequest. We assume that the individual concerned will contribute in line with the PCR up to the point of retirement, and then immediately switch from contributing to start withdrawing from their retirement pot.

We examine the empirical relationship between the PCR and the subsequent PWR that it finances in practice, and introduce a new concept relating the PCR and PWR, which we call the perfect withdrawal-contribution ratio, or perfect retirement ratio (PRR) for short. Clearly, as noted above, given the working lifetime of accumulation of up to 40 years and subsequent retirement period of up to 30 years, there are a limited number of independent, very long periods of data available to explore. However, some strong findings are surprisingly clear. In particular, there is an interesting positive relation between PCR and subsequent PWR, which we believe reflects mean reversion in asset returns over long time periods. In other words, if an agent has to save larger amounts to achieve their target wealth when investment returns are poor, then typically the subsequent (possible) withdrawal rates may also be large.

Estrada (2020) and Pfau (2011) also ask the question of the direct connection between the savings decision in work and the expenditure experience in retirement, and offer historical calculations involving contribution rates along with subsequent decumulation experiences. Estrada (2020) derives the constant annual real contribution implied by a particular choice of retirement income in a similar vein. Here, we generalize the concepts to create the PRR, which is an informative visual and numerical tool to aid policymakers in understanding important aspects of long-run savings and pensions' behavior over time (e.g., how the length of the working and decumulation periods impact the retiree experience). Whilst it is true that the desired level of income in retirement can deliver a required level of savings in work, there are many uncertainties that undermine a direct read-across from one to the other. These include the paths of investment returns and the length of working life. Structuring our discussion around the size of the investment pot allows us to address these and other uncertainties.

THE PERFECT CONTRIBUTION RATE

The most basic form of accumulation strategy is to save a constant amount of money each year from the start of working life, with the anticipation that at the start of retirement there are sufficient funds available to support a desired standard of living. This is the focus of Estrada (2020). In practice, some researchers prefer a variation on this by allowing income (and therefore savings) to first rise and then fall towards the end of one's working life, in a classic life-cycle, convex pattern. This does not qualitatively change our findings here. In reality, the likelihood of landing on the exact amount of money required at retirement is essentially zero. It is highly probable that one would have either saved too much, possibly as a result of working longer than necessary or forgoing certain wants, or else too little, resulting in a different set of problems. If, however, one could be certain of exactly how much money was required at a future point in time, and—stretching credulity even further—know exactly what future investment returns will be, then one could be very specific in the constant annual amount of contributions required. Suarez, Suarez, and Walz (2015) refer to this condition in a decumulation path as the PWR. Following the same logic, we call the ideal accumulation path the PCR.

During the accumulation period, we assume that the individual saves an amount c every period i of their working life, made up of n periods. They start their savings journey with an amount S_S, which could be zero. Savings accumulate over the working life through further contributions and the returns in any period r_i, up to the point of retirement at the end of period n, when savings are S_E:

$$S_{i+1} = (S_i + c)(1 + r_i)$$
$$S_E = \{(S_S + c)(1 + r_1) + c)(1 + r_2) + c\} \cdots + c)(1 + r_n). \qquad (1)$$

The PCR is, then, the value c/S_E, expressed as a percentage of the final balance, which generates the retirement savings pot S_E, given perfect foresight of the string of returns r_1, r_2, $r_3 \ldots r_n$ over the working life.

$$c = [1 - (K_S/K_E)\prod_{i=1}^{n}(1 + r_i)] / \sum_{i=1}^{n}\prod_{j=i}^{n}(1 + r_j) \qquad (2)$$

$$PCR = c/S_E = [1-(K_S/K_E)\prod_{i=1}^{n}(1+r_i)]/\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_j). \tag{3}$$

If initial starting savings S_S are zero, then:

$$PCR = c/S_E = 1/\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_j). \tag{4}$$

The construction of the PWR, demonstrating its symmetry with the PCR, is shown in Appendix A. A feature of the development of the PCR in Equations (3) or (4) is the measurement of sequence risk. For the decumulation case, Clare et al. (2017) show that the quantity $\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_j)$ captures the impact of ordering sets of returns that are otherwise identical (i.e., they have the same mean and variance). Decomposing the denominator in Equation (4):

$$\sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_j) = (1+r_1)(1+r_2)(1+r_3)\cdots(1+r_n)+(1+r_2)(1+r_3)\cdots(1+r_n)$$
$$+(1+r_3)(1+r_4)\cdots(1+r_n)+\cdots+(1+r_n-1)(1+r_n)+(1+r_n) \tag{5}$$

The interpretation of this is straightforward: for any given set of returns, Equation (5) is smaller if the larger returns occur early in the retirement period and lower rates occur at the end. This is because the later rates appear more often in the expression. Equations (3) and (4) show that this is an example of sequence risk. The contribution rate *c* would, in this case, have to be higher to achieve the same S_E, solely due to the ordering of returns. Along with the analysis of sequence risk in decumulation explored in Clare et al. (2017), this shows the heightened risk of a significant savings shortfall, and consequential impaired income flows in retirement, from poor returns performance in the period immediately before or after the point of retirement.

For the purpose of illustration, we begin by assuming that our investor holds only US equities in the form of the S&P 500 index. Although this may seem rather extreme, research increasingly suggests that so-called alternative assets add little or nothing to portfolios formed for this purpose, and that equities and cash are the crucial asset classes (see Clare et al. 2021a; Anarkulova et al. 2023). Indeed, if we use a popular trend-following investment strategy, we are in effect mixing cash and equities with an overlay of market timing (Clare et al. 2016). Therefore, unless stated otherwise, all values from this point onwards are for US equities, using the annual data from Shiller's website,[1] and are reported in real US dollars for 1870–2020. We add other assets for the sake of exposition below, but question their usefulness in such portfolios.

Before exploring the empirical regularities between accumulation (the PCR) and decumulation (the PWR), it will be useful to work through an example.

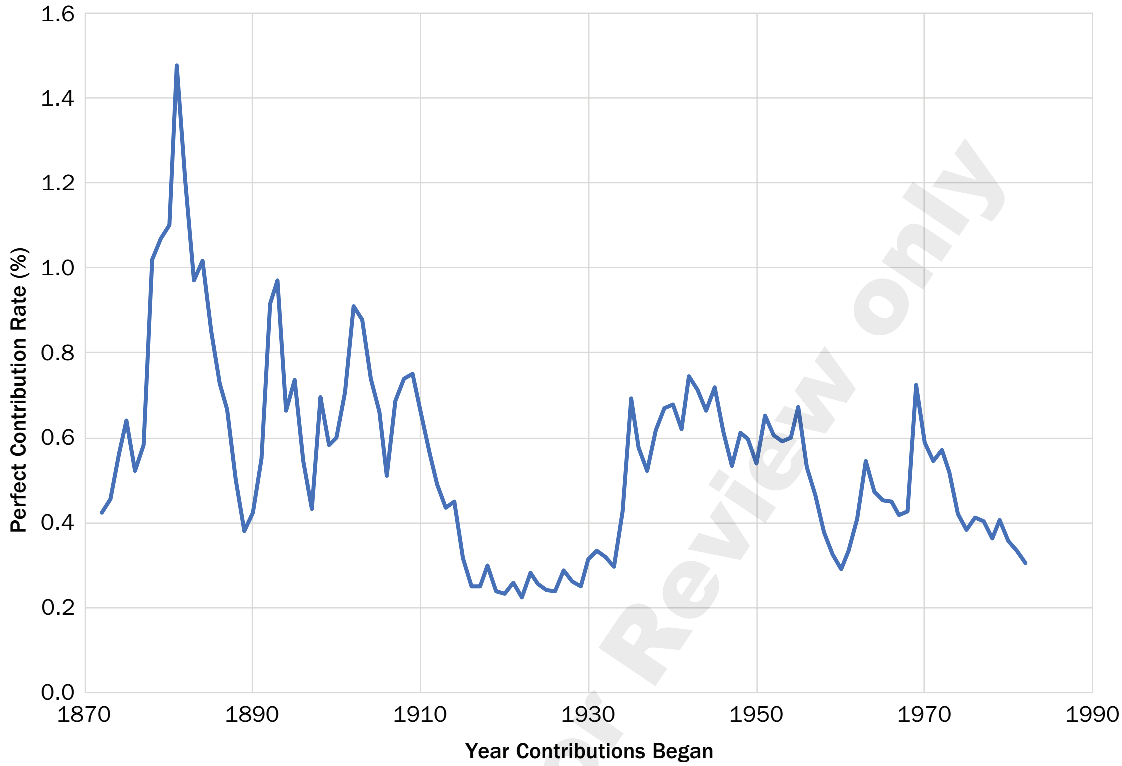
COMBINING ACCUMULATION AND DECUMULATION: A WORKED EXAMPLE

For the sake of illustration, we assume initially that a standard accumulation phase lasts 40 years, with annual contributions, and is then followed by a decumulation period of 20 years, with annual withdrawals. The target accumulation is a sum of $1 million, again chosen for illustration. Exhibit 1 shows how the PCR has varied over time, reflect-

[1] Data from: http://www.econ.yale.edu/~shiller/data.htm.

EXHIBIT 1

40-Year PCR



ing the varying returns on the investment portfolio (here, the S&P 500), with a lower value being preferable for the investor, since they would have been required to contribute less in order to acquire the same size final pot. The mean PCR value is 0.55% per year, so, in order to finish the accumulation period with a sum of $1 million, a consistent annual contribution of $5,500 (0.55% of $1 million) would have been required for 40 years. However, historically, there has been considerable variation around this mean, with a minimum PCR of 0.23% and a maximum of 1.48% for differing 40-year accumulation periods. If someone was particularly unlucky, then the contributions needed during a certain period of history would have been over six times the amount of the most fortunate investor (to achieve the same $1 million pot in real terms).

Looking closely at Exhibit 1, we observe a number of spikes in the PCR line. Notable ones are where contributions begin in the early 1890s and late 1960s. The misfortune of these times, and hence the higher-than-average PCRs, is that the end of the accumulation period coincided with periods of significant market stress: 40 years on from the early 1890s landed people in the Great Depression, and the same length of time from the end of the 1960s found people in the Global Financial Crisis. The common theme here was very large equity losses at the point when the investment pot was at its largest. This is, of course, an example of sequence risk (Clare et al. 2017). If those same losses had occurred at the beginning of an accumulation phase, they would only have affected one or two contributions. As it was, the majority of the 40 payments/contributions received their share of punishment. This is sequence risk in practice.

The flip side of this is that if someone was fortunate enough to near the end of accumulation in a raging bull market, then the PCR was much lower. Note the very low value for a contribution beginning in 1960, which benefited from the exceptional returns of the final years of the dot-com boom in the late 1990s. More recently, PCRs have been favorable compared to historical levels for accumulators just finishing their journey, having started in the early 1980s. The vast money printing in the wake of the pandemic is associated with stock prices rallying just when these investment pots were at their fullest.

The practical implementation of PWRs is explored in Clare et al. (2020), where we consider the case of someone who started their retirement journey on January 1, 2000, aged 65. With the benefit of actual investment returns, we consider their investment and withdrawal rate options, and the lessons we can learn from this experience (see also Clare et al. 2021b).

ACCUMULATION MEETS DECUMULATION

We inevitably start a decumulation analysis by assuming a given pot of wealth and fretting over unknown (expected/required) investment returns and assumed longevity. Additionally, or perhaps alternatively, we suffer angst deciding on the split between guaranteed components (annuity or secure lifetime income?) and the remaining market-related investments. In the latter context, sequence risk is of particular importance as accumulation approaches the decumulation phase. For a given set of investment returns, any particular sequence that is more favorable for accumulation is less favorable for decumulation and vice versa.

As an illustration, Exhibit 2 shows four different annual returns (40%, 5%, 10%, and –25%), and all of the 24 permutations without replacement. Each string has the same compound return, volatility, and maximum drawdown—all that is different is the order in which the individual returns occur. At the very bottom of the table is the zero-volatility equivalent (i.e., imagine the average return occurs each year and volatility is therefore zero). In the right-hand column are the PCRs and PWRs for each string. As a reminder, from the point of view of an investor, a lower PCR implies that they have to contribute less for the same outcome and, with a higher PWR, they receive greater sums: these are the more favorable outcomes.

From Exhibit 2, we observe that the best PWR outcome is when the largest return occurs first (40%), followed by the next largest (10%), and so on with the big loss arriving last. This also gives the highest PCR, where only the very first contribution benefits from the largest return. Near the bottom of the table, we find the reverse of the sequence, with the big loss first and the greatest gain last. Both the PCR and PWR are over 10 percentage points lower, simply due to differences in the order of exactly the same set of returns. We note that the zero-volatility case lies roughly in the middle of these two extremes.

Volatility and drawdown therefore are not penal to investors in this scenario in the sense that they can give rise to both positive and negative "surprises" compared to no variation—rather that, in retirement planning, there is an asymmetry in the desirability of outcomes and volatility increases the probability of unwelcome ones. If a much better result is achieved than one hoped for, then that is pleasant but probably not required, whereas an outcome in the left-tail of the distribution could lead to insufficient funds with which to retire. Choosing an investment strategy with an acceptable volatility to give a high probability of achieving one's goals is almost certainly a prudent approach. Clare et al. (2021a) provide a discussion on forming portfolios with stocks and bonds along these lines.

EXHIBIT 2

Example of Sequence Risk in PCR and PWR

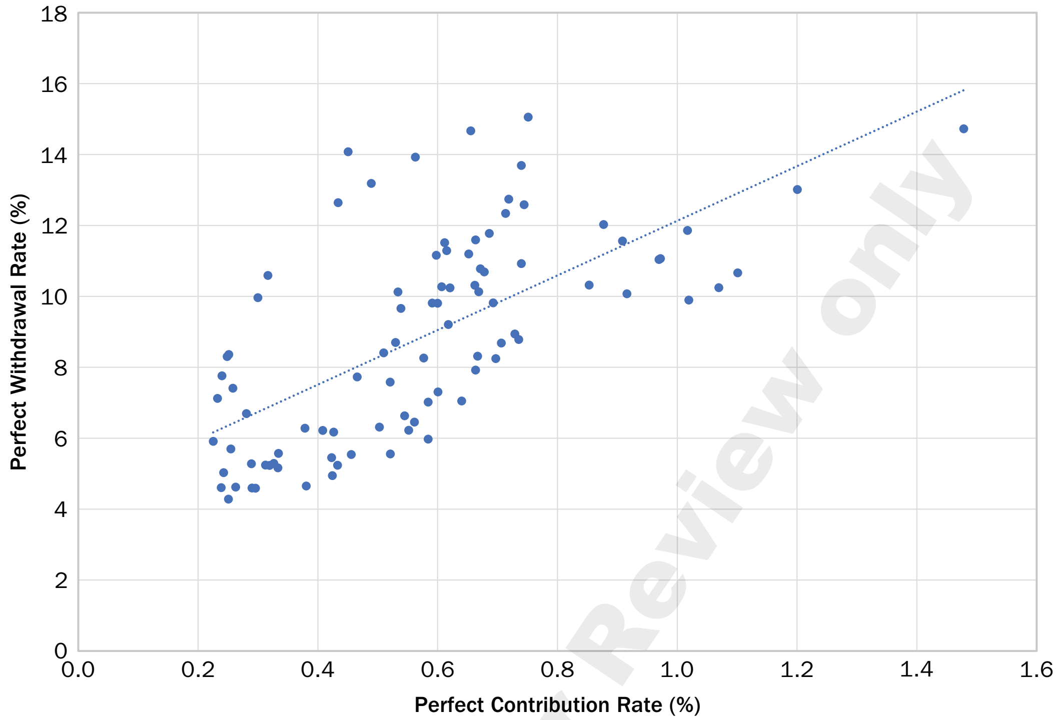
Returns (%)					
Year 1	**Year 2**	**Year 3**	**Year 4**	**PCR (%)**	**PWR (%)**
40	5	10	–25	27.37	33.19
40	5	–25	10	24.98	30.29
40	10	5	–25	27.65	33.53
40	10	–25	5	25.53	30.97
40	–25	5	10	23.07	27.98
40	–25	10	5	23.34	28.31
5	40	10	–25	25.36	30.76
5	40	–25	10	23.30	28.25
5	10	40	–25	23.99	29.10
5	10	–25	40	20.76	25.17
5	–25	40	10	19.97	24.22
5	–25	10	40	18.84	22.85
10	40	5	–25	25.96	31.48
10	40	–25	5	24.08	29.20
10	5	40	–25	24.30	29.47
10	5	–25	40	20.99	25.45
10	–25	40	5	20.68	25.08
10	–25	5	40	19.29	23.39
–25	40	5	10	19.67	23.85
–25	40	10	5	19.86	24.09
–25	5	40	10	18.28	22.17
–25	5	10	40	17.33	21.02
–25	10	40	5	18.69	22.67
–25	10	5	40	17.54	21.28
4.94	4.94	4.94	4.94	22.13	26.84

The Perfect Retirement Ratio

Having finished the accumulation period, the next step is to start drawing the wealth down to provide an income in retirement. Suarez, Suarez, and Walz (2015), and Clare et al. (2017, 2021a), provide a full discussion of the PWR methodology for the US and UK, and we use the same approach here. Exhibit 3 plots each PCR with the associated PWR that follows on as the switch is made from accumulation to decumulation over the period. We see that there is a clear positive correlation, albeit we acknowledge caution due to the large number of overlapping observations. Reflecting this caution with Newey-West heteroskedasticity and autocorrelation robust standard errors, the t-test of the relationship between the two series is 5.54 (coefficient 7.70/standard error 1.39), which is significantly different from zero at any reasonable level of statistical significance. In Exhibit 3, as a robustness check, if one removes the two right-most extreme observations, the fit of the relationship falls while the slope increases to 8.04 from 7.70, with all the data included. The positive relationship is clearly still present and the two outlier observations therefore do not, in themselves, drive the relationship.

One could suggest that there is some "justice" here, in that if someone has been unfortunate in experiencing a high PCR and has therefore made a relatively large contribution (out of their income) to earn their pot, then they have typically been

EXHIBIT 3
40-Year PCR vs. Subsequent 20-Year PWR



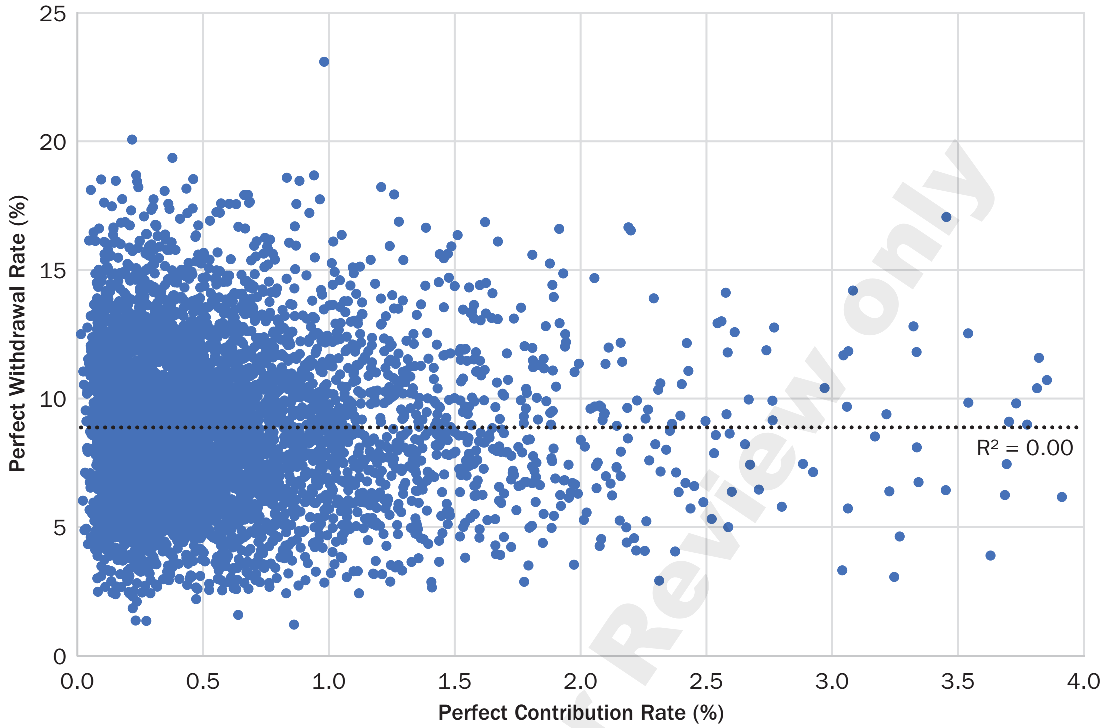
more fortunate in decumulation and had a higher PWR too (i.e., the pot has generated relatively larger annual pay-outs). This is perhaps rather a novel context for mean reversion, but finds support in the time diversification literature (e.g., Reichenstein and Dorsett 1995). We posit that when an accumulation phase has been particularly "successful," equity valuations have probably expanded and therefore expected returns going forward are lower, resulting in a lower PWR and vice versa. Clare et al. (2017) previously demonstrated that a positive relationship exists between earnings yield at the start of decumulation and the subsequent 20-year PWR.

The link we observe between PCR and subsequent PWR has important implications for conducting this type of analysis using Monte Carlo simulations. In Exhibit 4, we plot 5,000 simulated PCRs using annual returns from our data with replacement, and these are then combined with 5,000 PWRs simulated in the same way. Now there is zero correlation between the two variables, as one would expect given that they are generated randomly. Being a Monte Carlo exercise, there are of course also many more extreme observations than occur in real life. For example, see the comparison in Exhibit 4 in Estrada (2021).[2] To an extent, this would be more likely given the increased number of total observations. However, the reversion mechanism is broken in the Monte Carlo simulation as, in reality, after a big negative return, stocks have probably become relatively cheaper on, say, a cyclically

[2] One way of building more realistic behavior into Monte Carlo simulations is to incorporate the observed serial dependence in returns through the use of a block bootstrap. Anarkulova, Cederburg, and O'Doherty (2023), and Anarkulova et al. (2023), use an average 120-month stationary block. They show, perhaps surprisingly, that the main impact is that standard Monte Carlo underestimates drawdowns and ruin probabilities when compared with the block bootstrap. This appears to be partly due to the inclusion of non-US stocks in their calculations. US returns since 1890 similar to those used in this paper show much smaller ruin probabilities in line with lower drawdowns.

EXHIBIT 4

40-Year PCR vs. Subsequent 20-Year PWR: 5,000 Monte Carlo Simulations



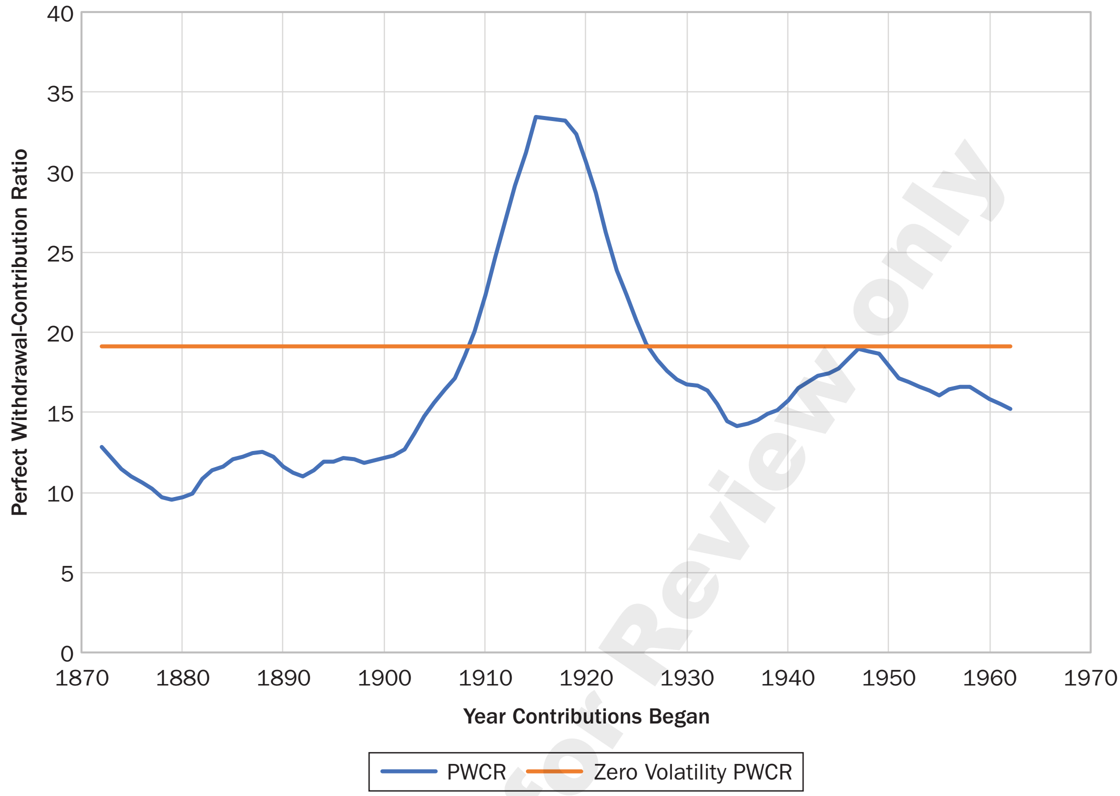
adjusted price-to-earnings measure. The long-term expected return has therefore risen, reducing the probability of another large negative return sequence. In the Monte Carlo analysis, the probability of another large down year remains the same, and the reverse would equally be true for big positive return years. In summary, using simulations of this nature is probably going to overestimate the probability of achieving a low PCR followed by a high PWR, and vice versa. In the concluding comments of a very insightful study, Pfau (2011) suggests that such analysis should involve strings of returns to preserve such statistical dependence. This seems to have been largely ignored in the subsequent literature.

In the same spirit, in Exhibit 5, we directly link both the PCR and PWR by forming a ratio between them. The perfect withdrawal-contribution ratio, or PRR, is simply the PWR divided by the (preceding accumulation) PCR, and reflects the number of times more that the annual withdrawal has exceeded the annual contribution. We find that the mean value is 16.94 with a median of 16.06. However, there has been significant variation around these levels, with observations less than 10 and above 30.

Over the data period in this study, the real annual return of the index (i.e., the investment portfolio) was 7.05%. If there had been absolutely zero volatility, and one had received this constant return year in and year out, the PCR would have been 0.46%. We will call this the zero-volatility PCR. Following the same logic, the zero-volatility PWR would have been 8.85%, giving a zero-volatility PRR of 19.13. This is shown as the horizontal line in Exhibit 5. The point of this is that even if the long-term return of equities had been known in advance, the variation in the returns and the sequence in which they arrived caused substantial deviation for actual PRRs away from the zero-volatility line. This only reinforces the notion that one is best served by building in some conservatism into retirement planning.

EXHIBIT 5

Perfect 20-Year Withdrawal, 40-Year Contribution Ratio



VARYING THE ACCUMULATION AND DECUMULATION PERIOD LENGTHS

Up to this point, we have assumed a 40-year accumulation phase followed by 20 years of decumulation. We denote this as PWR20:PCR40. In Exhibits 6 and 7, we now examine how adjusting the lengths of the two phases, but still assuming a total of 60 years, affects the PRR. As one might expect, the longer the accumulation phase is, and therefore—by definition—the shorter the drawdown period, the higher the PRR becomes. Not only are there more annual contributions, but those contributions are exposed to the compounding of, on average, positive returns for more time.

The flexibility of being able to work for longer is one way of introducing some conservatism to retirement planning. One might ideally look to have a 40-year accumulation phase. However, if investment returns have been low and/or one has not been able to make the desirable level of contributions, then the possibility of working for up to another, say, five years uses one's human capital as a form of insurance.

Example with a Decumulation (Pension) Annual Real Target

In reality, a person is unlikely to make constant contributions over their working life. Some professions will reward experience, and compensation is almost certainly higher in later years compared to when someone is just starting out. This will almost certainly mean that they can save relatively more towards the end of the accumulation phase. For a few jobs—professional sports being a probable candidate—the earning

EXHIBIT 6

Various Perfect Withdrawal-Contribution Ratio Lengths

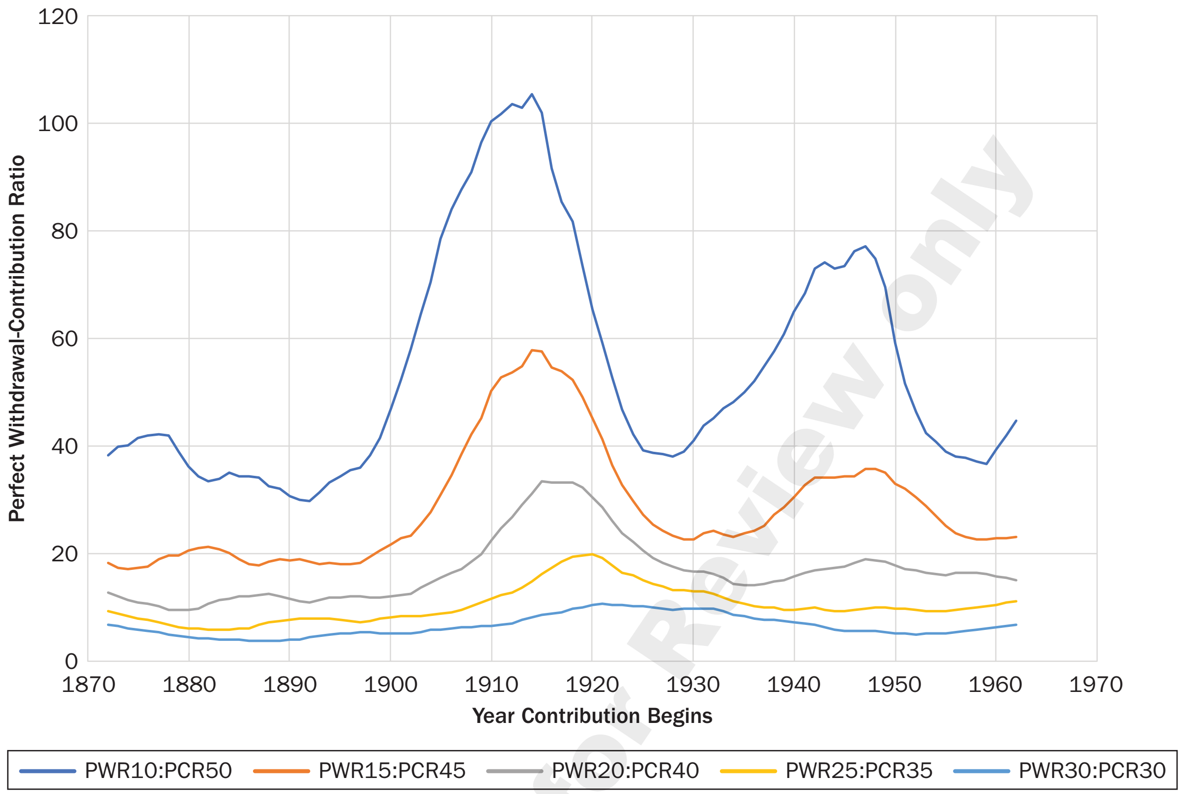


EXHIBIT 7

Summary Statistics for Varying Perfect Withdrawal-Contribution Ratios: 100% S&P 500 Portfolio

	PWR10:PCR50	PWR15:PCR45	PWR20:PCR40	PWR25:PCR35	PWR30:PCR30
Maximum	105.37	57.76	33.45	19.92	10.82
10th Percentile	87.78	49.08	26.96	16.03	9.87
25th Percentile	70.02	34.19	18.31	11.83	7.98
Median	44.72	24.25	16.06	9.65	6.02
75th Percentile	38.20	20.33	12.17	8.04	5.25
90th Percentile	34.17	18.29	11.28	6.94	4.36
Minimum	29.86	17.32	9.58	5.90	3.85

power is greatest near the beginning of one's working career. The upshot of this is that the retirement planning process is ongoing, and annual reviews (or something of a similar frequency) will be required to analyze progress towards the objective and suggest appropriate courses of action for contributions and/or asset allocation. Clare et al. (2023) provide a discussion for adaptive methods in trying to meet accumulation targets.

For the purposes of this example, we are going to assume that the investor has managed to save a small amount of money over their first 10 years (out of a total of 40), such that they have an investment pot of $20,000. The stated goal is to have a fairly high degree of confidence in being able to withdraw at least $25,000 per year during a 20-year decumulation phase. Firstly, we can look at what the $20,000 already saved might generate as a future income. At the start of each year, historically, we can assume that the money was invested for 30 years, with no additions or withdrawals, to give a final pot amount. This sum then generates the PWR for the next 20 years. The final pot amount multiplied by PWR gives the perfect withdrawal amount (PWA), and this is plotted in Exhibit 8. As with earlier graphs, there is quite a bit of volatility in what one could have been able to receive depending on when the investment process began. We are going to be relatively conservative and use the 90th-percentile value, which gives a PWA of $6,653 per year. This is a useful sum towards the stated goal, but it is clearly a long way short of $25,000.

The next step is to try to estimate how much might be required in terms of annual contributions to fund the deficit. We now revert to our PRR metric, but this time there are only 30 years of accumulation remaining, so we need to find a value for PWR20:PCR30. Exhibit 9 displays this withdrawal-contribution ratio over the period of study. Sticking with a 90th-percentile risk tolerance, we find that the value for PWR20:PCR30 is 5.27.

Now, we know from our earlier calculations that the gap to be filled in terms of decumulation withdrawals was the stated goal minus the estimated PWA from the initial investment sum: $25,000 – $6,653 = $18,347. The estimated required annual contribution is therefore this amount divided by PWR20:PCR30, or $18,347/5.27 = $3,481.

This process could be run each year by looking at the current value of the investment pot and estimating what PWA it would generate. The new PRR distribution is calculated, allowing for the year of accumulation that has elapsed, with a resulting contribution rate set in order to provide a tolerable risk of meeting the target. If a large single sum were anticipated to be received at a known point in the future, this too

EXHIBIT 8

20-Year PWA from $20,000 Invested for 30 Years

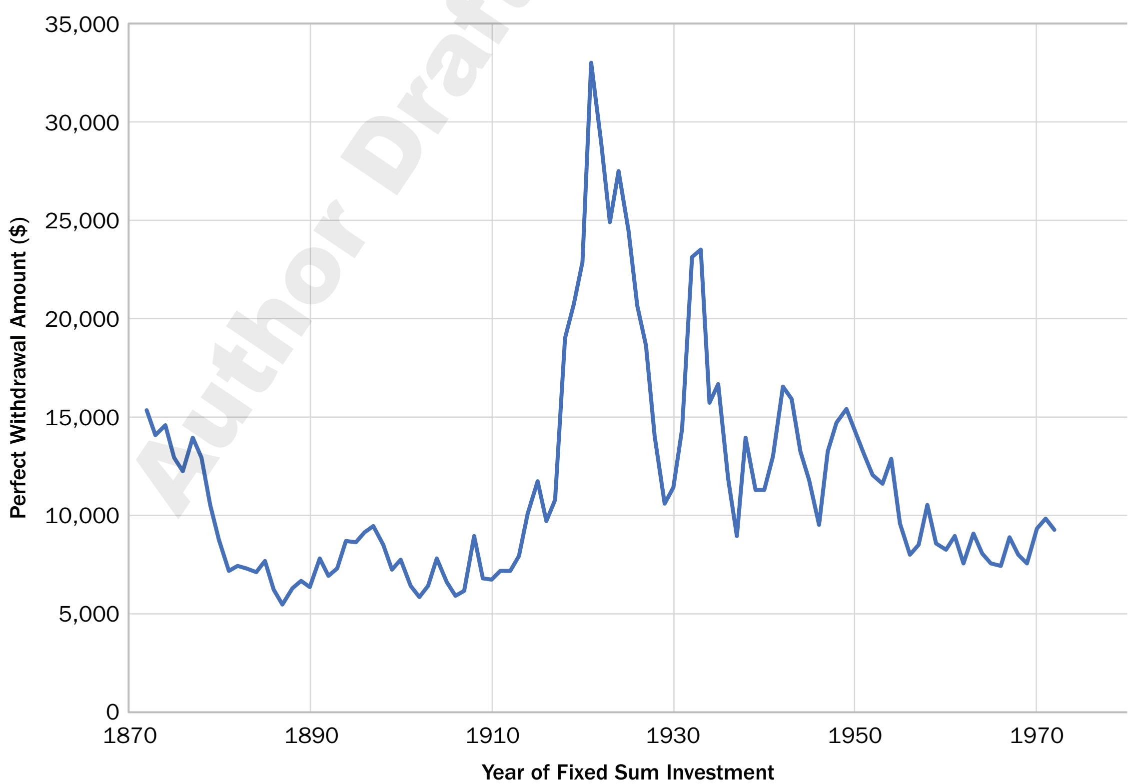
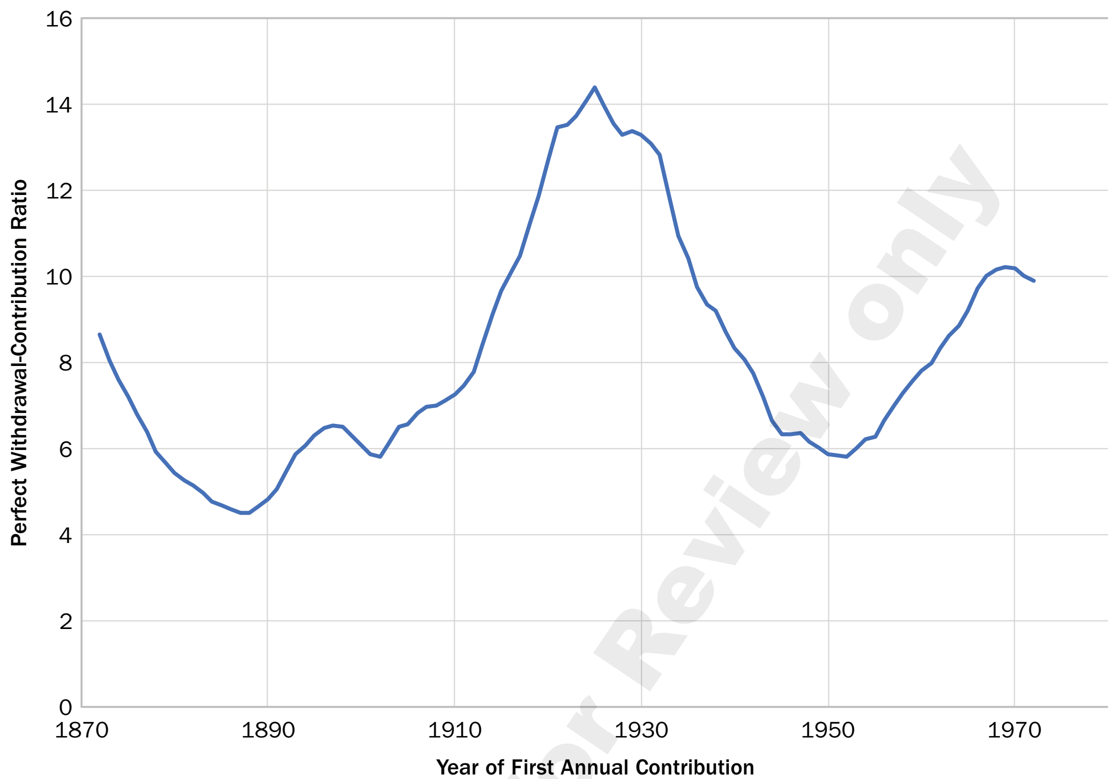


EXHIBIT 9

Perfect 20-Year Withdrawal, 30-Year Contribution Ratio



could have a PWA calculated. For instance, if it were expected to arrive with 15 years of accumulation, then one could generate a PWR20:PCR15 distribution and model its likely withdrawal sum. If one wished to insure the longevity risk, then the purchase of a deferred annuity could be built into the calculation. For example, one could assume that, at the point of switching from accumulation to decumulation, the annuity was bought and the pot available for decumulation would be commensurately smaller.

MAKING THE INVESTMENT PORTFOLIO MORE "REALISTIC": ALTERNATIVE INVESTMENT APPROACHES

We have so far assumed that all investment, through both accumulation and decumulation, has been in equities. Of course, this does not have to be the case. One could assume a different asset allocation during the withdrawal phase. But, here, we are particularly interested in the impact of reducing the volatility of investment returns, and its impact on accumulation and decumulation. We achieve this by "smoothing" returns using trend-following techniques (Clare et al. 2017). In a separate study, we show that, from the point of view of a UK retiree, diversifying to a multi-asset portfolio has no benefit in terms of withdrawal experience (Clare et al. 2021a)—a finding that is consistent with the study by Ennis (2024).

Clare et al. (2021a) provide some examples of multi-asset decumulation portfolios in the UK. Indeed, the accumulation process could also be a multi-asset portfolio with PCRs calculated as described earlier (see Clare et al. 2021a for UK evidence). However, simply adding assets (e.g., "alternatives") seems to offer limited return/risk enhancements (Anarkulova et al. 2023). Rather, we wish to extend the example by investigating "smoother" returns, applying trend-following to the conventional S&P 500 portfolio with the specific aim of reducing sequence risk in returns.

Clare et al. (2017) demonstrate that adding a trend-following filter to US equity investments both improves the withdrawal rates and reduces their volatility (see Exhibit 10). We now extend this to the accumulation phase, adopting the same popular 10-month trend-following rule, whereby a long position is taken in equities if the index is above the 10-month moving average, investing in Treasury bills otherwise. Exhibit 11 provides some summary statistics comparing trend-following with the standard buy-and-hold approach. One can observe the higher return and lower volatility that has been reported previously by Faber (2009), among others.

Exhibit 12 reprises Exhibit 3, but this time with trend-following overlay applied to both the contribution and withdrawal periods. As before, we still observe the positive correlation between PCR and PWR, but it is weaker than with buy and hold. We attribute this to trend-following smoothing out the investment journey. Periods of high valuations are still followed by lower returns and vice versa. However, by not being invested in equities the whole time, the full force of this cyclical effect is not felt. Exhibit 12 does not really show any major outliers. If, however, one takes out the single observation with the PCR above 0.6, then the fit falls with all the data. The slope also declines this time

EXHIBIT 10

The Relationship between 40-Year PCR and Subsequent 20-Year PWR for Stocks

	Original	TF
α	4.43	6.15
	(4.79)	(5.46)
β	7.70	11.37
	(5.54)	(3.08)

Estimation Period: 1872–1982, Newey-West *t*-statistics in Brackets $PWR20_t = \alpha + \beta PCR40_{t-40}$

NOTE: TF: Trend Following.

EXHIBIT 11

Summary Statistics for Trend Following: 1870–2022

	S&P	Trend Following
Annualized Real Return (%)	7.05	8.82
Annualized Real Volatility (%)	17.88	12.81
Maximum Real Drawdown (%)	50.79	28.22

EXHIBIT 12

40-Year PCR vs. Subsequent 20-Year PWR, both with Trend Following

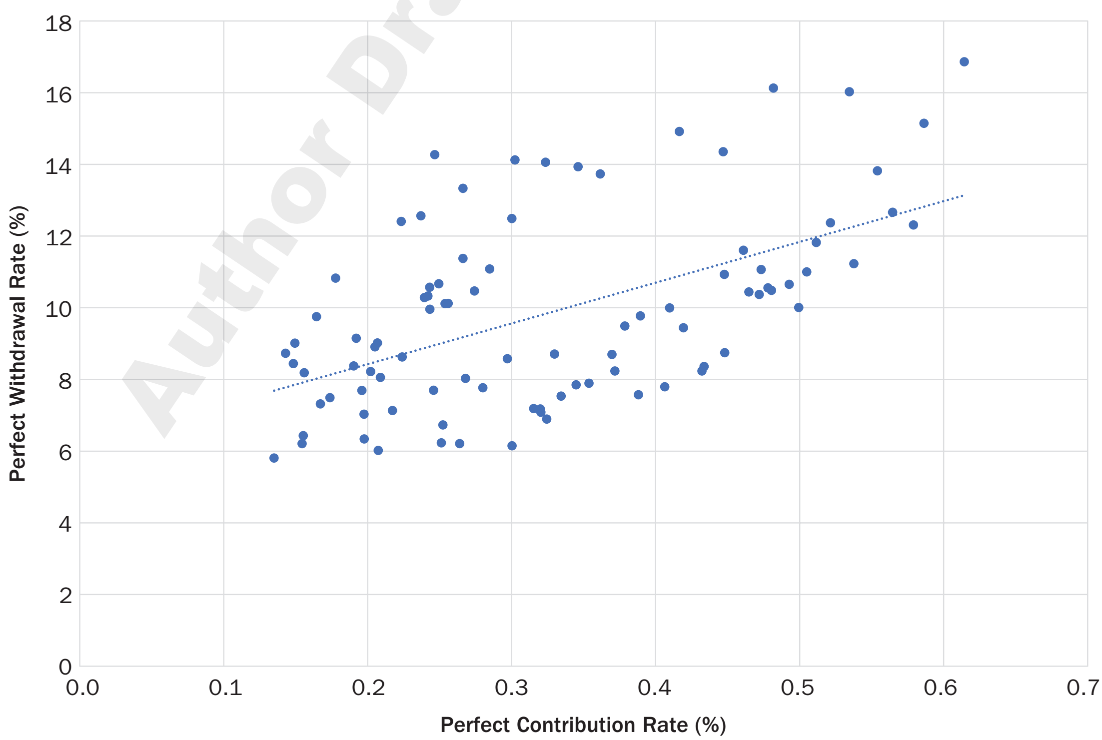
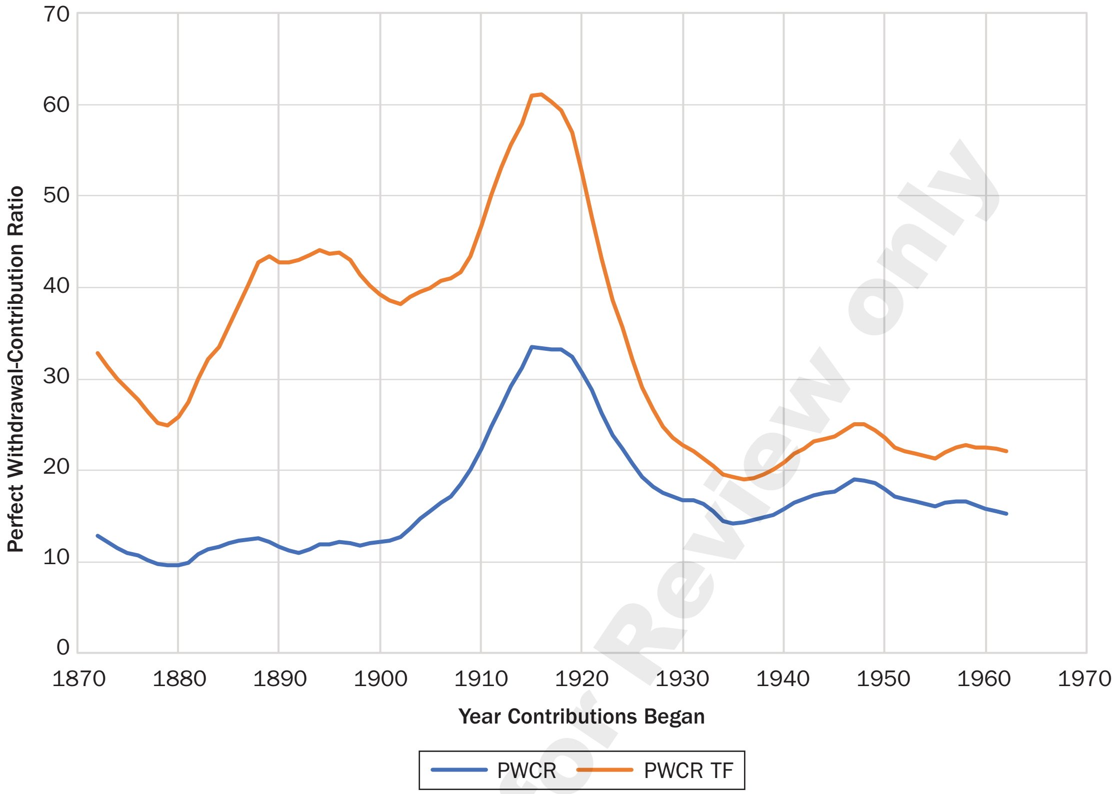


EXHIBIT 13

Perfect 20-Year Withdrawal, 40-Year Contribution Ratio with and without Trend Following



to 10.57 from 11.37 previously, but is still very significant, with a t-statistic of 3.01 compared with 3.08.

Finally, in Exhibit 13, we plot the PWR20:PCR40 for trend-following and compare it to the same ratio for standard equities investment shown in Exhibit 5. The peaks and troughs largely match up between the two lines, but trend-following clearly outperforms across every 60-year period, although the gap has narrowed in more recent decades. We observe that the PRR for equities is below 20 for all but around 15 starting years, whereas the trend-following PRR is above this level for almost the entire data period, except for a few years in the 1930s where it dips just below.

To this extent, using smoothed returns—possibly by adopting trend-following as an investment strategy—appears to be an interesting methodology to consider for both the accumulation and decumulation phases, to enhance the withdrawal rate relative to contribution levels.

CONCLUSION

In this article, we have explored the idea of using the PCR as a means of estimating how much future retirees might need to contribute annually to achieve their investment goals. We have combined this with the PWR to create a new metric—the PRR—and shown how using standard Monte Carlo analysis could lead to overestimating the probability of extreme values.

The PRR joins the investment performance of both the accumulation and decumulation phases together, and provides a means of estimating the future worth of investment contributions at the point that they are being made. We have also developed the framework to allow for fixed-sum receipts and their associated influence on the ultimate withdrawal amounts. Finally, we have suggested a potential way to improve on the basic equity investment approach by including an element of trend-following for the investing portfolio. We find that removing the extreme observations in Exhibits 3 and 12 (the relation between contributions and subsequent withdrawals) fails to alter the story meaningfully in either case. However, and most importantly, both look very different to the Monte Carlo example in Exhibit 4, where the trendline is horizontal and the R-squared is zero: time-diversification certainly matters! Indeed, Exhibit 12 strongly reinforces other work (Clare et al. 2017), showing the advantages of trend-following investing: the trend-adjusted portfolio forcefully shows how the withdrawal-contribution experience is superior with a smoothed portfolio.

Rational investors would want contributions to be as low as possible and withdrawals to be as high as possible, so the PWR:PCR relationship feels rather like a benefit:cost ratio. Would glidepath strategies be as popular if they were expressed in this way? One particularly useful feature of the ratio is that it also allows easy analysis of what happens if the proportions of time allocated to the accumulation and decumulation phases are adjusted (e.g., PWR15:PCR45 vs. PWR25:PCR35). This is, of course, an important public policy issue, although the ratio should get smoothed out somewhat by the mean reversion we have noted. For example, high return accumulation phases are often followed by lower return decumulation phases, and vice versa.

APPENDIX A

THE PERFECT WITHDRAWAL RATE

As shown in Suarez, Suarez, and Walz (2015), and Clare et al. (2017), during the decumulation period, we assume that the individual takes an amount w in income from their retirement savings pot every period i of their retirement, made up of n periods. They start their retirement journey with an amount K_S, which was saved during their working life. The retirement savings pot will reduce over time, but will also be augmented by the rate of return r_i in year i in annual percent in any period up to the end of retirement at the end of period n, when savings are K_E. Formally, K_i decumulates as follows:

$$K_{i+1} = (K_i - w)(1 + r_i) \qquad \text{(A-1)}$$

$$K_E = \{(K_S - w)(1 + r_1) - w)(1 + r_2) - w\} \cdots - w)(1 + r_n) \qquad \text{(A-2)}$$

We solve Equation (A-2) for w to get:

$$w = [K_S \prod_{i=1}^{n} (1 + r_i) - K_E] / \sum_{i=1}^{n} \prod_{j=i}^{n} (1 + r_j) \qquad \text{(A-3)}$$

If $K_E = 0$, no bequests, then:

$$w = [K_S \prod_{i=1}^{n} (1 + r_i)] / \sum_{i=1}^{n} \prod_{j=i}^{n} (1 + r_j) \qquad \text{(A-4)}$$

The PWA w can also be expressed as a percentage of the initial retirement savings pot, as the PWR. In the case with no bequests:

$$PWR = w/K_S = \prod_{i=1}^{n}(1+r_i) / \sum_{i=1}^{n}\prod_{j=i}^{n}(1+r_j) \tag{A-5}$$

Clare at al. (2017) provide a discussion of the properties of the PWR.

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