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Salehi Dobakhshari, A. and Azizi, S. orcid.org/0000-0002-9274-1177 (2025) On Estimation of Zero-Sequence Impedances of Parallel Transmission Lines from Fault Data. IEEE Transactions on Power Delivery. ISSN 0885-8977

https://doi.org/10.1109/TPWRD.2025.3527738

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On Estimation of Zero-Sequence Impedances of Parallel Transmission Lines from Fault Data

Ahmad Salehi Dobakhshari, Sadegh Azizi, *Senior Member, IEEE*

***Abstract*—The problem of estimating zero-sequence parameters of a parallel transmission line from fault data is considered. This paper analytically demonstrates that the zero-sequence impedances of a parallel transmission line are not, in general, attainable using the synchronized measurements taken at the line terminals following a ground fault on the line. To this end, Kirchhoff's voltage law (KVL) and current law (KCL) are employed to establish the system of equations that relate the measurements to the zero-sequence impedances of the line. The paper also highlights two rather theoretical exceptions to this generalization: first, the scenario of bolted faults, and second, situations where the fault resistance value is known beforehand (although this assumption is not valid in practice). A lemma is introduced and proved demonstrating that under specific conditions the zero-sequence reactances of the line can be accurately estimated while the zero-sequence resistances of the line remain unattainable. Simulation results, under a variety of conditions such as time-varying fault resistance and untransposed parallel lines, support the theoretical findings that zero-sequence resistances cannot be obtained from fault data while for short transposed lines or untransposed lines without earth wire zero-sequence reactances can be estimated quite accurately. Realistic measurement errors undermine the reliability of estimates, further questioning the attainability of zero-sequence parameters of parallel lines from fault data.**

***Index Terms*—Faults, parallel transmission line, sequence networks, zero-sequence mutual impedance.**

NOMENCLATURE

A. Known/Measured Quantities

- V_l^+ Positive-sequence voltage at line terminal l.
- V_r^+ Positive-sequence voltage at line terminal r.
- V_l^- Negative-sequence voltage at line terminal l.
- V_r^- Negative-sequence voltage at line terminal r.
- V_l^0 Zero-sequence voltage at line terminal l.
- V_r^0 Zero-sequence voltage at line terminal r.
- I_{lr1}^+ Positive-sequence fault current through the 1^{st} circuit of the line flowing from terminal l to r.
- I_{lr2}^+ Positive-sequence fault current through the 2^{nd} circuit of the line flowing from terminal l to r.
- I_{rl1}^+ Positive-sequence fault current through the 1^{st} circuit of the line flowing from terminal r to l.
- I_{rl2}^+ Positive-sequence fault current through the 2^{nd} circuit of the line flowing from terminal r to l.
- I_{lr1}^- Negative-sequence fault current through the 1^{st} circuit of the line flowing from terminal l to r.
- I_{lr2}^- Negative-sequence fault current through the 2^{nd} circuit of the line flowing from terminal l to r.
- I_{rl1}^- Negative-sequence fault current through the 1^{st} circuit of the line flowing from terminal r to l.
- I_{rl2}^- Negative-sequence fault current through the 2^{nd} circuit of the line flowing from terminal r to l.
- I_{lr1}^0 Zero-sequence fault current through the 1^{st} circuit of the line flowing from terminal l to r.
- I_{lr2}^0 Zero-sequence fault current through the 2^{nd} circuit of the line flowing from terminal l to r.
- I_{rl1}^0 Zero-sequence fault current through the 1^{st} circuit of the line flowing from terminal r to l.
- I_{rl2}^0 Zero-sequence fault current through the 2^{nd} circuit of the line flowing from terminal r to l.
- Z_l^+ Positive-sequence impedance of the line.
- Z_l^- Negative-sequence impedance of the line.

B. Unknown Quantities

- V_{sl} Thevenin voltage source at terminal l.
- V_{sr} Thevenin voltage source at terminal r.
- Z_{sl}^+ Positive-sequence Thevenin impedance at terminal l.
- Z_{sr}^+ Positive-sequence Thevenin impedance at terminal r.
- Z_{sl}^- Negative-sequence Thevenin impedance at terminal l.
- Z_{sr}^- Negative-sequence Thevenin impedance at terminal r.
- Z_{sl}^0 Zero-sequence Thevenin impedance at terminal l.
- Z_{sr}^0 Zero-sequence Thevenin impedance at terminal r.
- m Fault distance from terminal l in pu.
- R_f Fault resistance.
- I_f Fault current through R_f.
- V_f^0 Zero-sequence voltage at the fault point on the 1^{st} circuit of the line (faulted circuit).
- Z_l^0 Zero-sequence self impedance of the line representing $R_l^0 + jX_l^0$.
- Z_m^0 Zero-sequence mutual impedance of the line representing $R_m^0 + jX_m^0$.

I. INTRODUCTION

TRANSMISSION line parameters have been traditionally calculated by Carson's equations, that use conductor dimensions, tower geometry and an assumed value for ground resistivity [1]. Zero-sequence parameters are in particular sensitive to the ground resistivity, which in turn depends on soil type, temperature, and moisture content in soils [2]. Accuracy of the zero-sequence parameters of the line are crucial to the protection of the line and fault location along it.

With the advent of measurement technologies in power systems, estimating transmission line parameters by using

A. Salehi Dobakhshari is with the Faculty of Engineering, University of Guilan, Rasht 4199613776, Iran (e-mail: salehi_ahmad@guilan.ac.ir).

Sadegh Azizi is with the School of Electronic and Electrical Engineering, University of Leeds, Leeds LS2 9JT, U.K. (e-mail: s.azizi@leeds.ac.uk).

measurements from line terminals has become attractive and been scrutinized in several studies. One approach uses multi-scan measurements in different normal operating conditions from line terminals equipped with Phasor Measurement Units (PMUs). The formulation used is either based on least-squares estimation [3]–[15] , Kalman filter [16]–[21] or evolutionary algorithms [22]–[24]. Another approach uses fault data recorded at line terminals by digital relays equipped with PMU functionality [25]–[29]. The first approach yields positive-sequence parameters of the line while the second approach focuses on ground faults which involve zero-sequence parameters of the line.

The majority of existing attempts in estimating transmission line parameters are concerned with single-circuit transmission lines. However, double-circuit or parallel transmission lines are increasingly utilized due to the shared tower and right-of-way requirement [30]. Measurement-based estimation of the zero-sequence self and mutual impedances of a parallel transmission line has been shown to be the most difficult and only feasible in certain conditions. The usual routine includes a single-phase voltage source applied to the line when the line is out of service [31]. In [32], this approach is followed by accurately modeling the transmission line. Amongst the limited research works devoted to the estimation of parameters from fault data, in [33], the zero-sequence self and mutual impedances of a parallel transmission line are obtained in a closed form under two conditions: 1) One of the circuits is out of service and 2) The fault occurs beyond the investigated parallel line. Similarly, in [34], a nonlinear-estimation approach is presented for when the fault is not on the line itself. This approach also requires fault data from more than a single fault event. Reference [35] shows that even when the fault is beyond the examined line, the zero-sequence mutual impedance of the line cannot be obtained separately from its zero-sequence self impedance, when both circuits are in service.

Recently, in [36], it is attempted to find the zero-sequence self and mutual impedances of a parallel transmission line using synchronized fault data recorded by digital relays at the line terminals. Despite previous literature, the presented method calculates the zero-sequence self and mutual impedances when a fault occurs on one of the circuits of the line. The advantage of the presented method is that the fault data extracted from the protective relays are utilized. As will be shown in this paper, however, there are some queries around equations of [36], invalidating the possibility of obtaining self and mutual zero-sequence impedances from fault data.

In this paper, different conditions under which the zero-sequence parameters of a parallel transmission line can be or cannot be obtained are scrutinized. The presented analysis determines when and which zero-sequence parameters can be obtained from fault data. The main contribution of the paper is proving that, under realistic scenarios, complete and accurate determination of zero-sequence parameters of parallel lines remains infeasible. Nevertheless, the paper also explores theoretical exceptions and discusses specific conditions under which limited estimations might be possible. More specifically, through comprehensive analysis of circuit equations, this paper

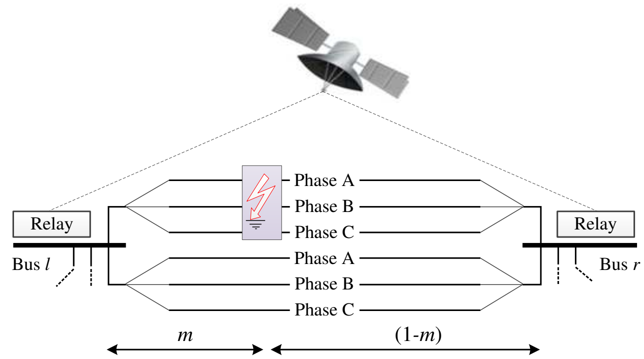

Fig. 1. A single line-to-ground fault on a parallel transmission line.

- Demonstrates that determining the zero-sequence impedance of parallel transmission lines solely from synchronized terminal measurements is impossible.
- Exposes the inherent limitations (rank deficiency) of the equations used in previous research work. This directly contradicts claims that such parameters can be estimated from fault data.
- Introduces and proves new lemmas, which demonstrate the possibility of approximating the zero-sequence reactance of short parallel lines under specific conditions. To this end, the line must be either transposed (conductors regularly switched positions) or untransposed without an earth wire. However, realistic measurement errors exacerbate the accuracy of these estimates.
- Emphasizes that estimating the zero-sequence resistance of parallel lines from fault data is not viable.

The rest of this paper is organized as follows. In Section II, single-line-to-ground faults on a parallel transmission line are analyzed and corresponding formulas for estimating zero-sequence impedances of the line are developed. The general case as well as a special case where zero-sequence parameters can be estimated are analytically derived. Section III extends the presented analysis to double-line-to-ground faults. In Section IV, the validity of a previously presented algorithm is scrutinized. Section V proves that zero-sequence reactances of the line can be accurately estimated. Simulation results are presented in Section VI, followed by a summary of conclusions drawn in Section VII.

II. Single-Line-To-Ground Faults on Parallel Transmission Lines

This section is aimed at estimating the zero-sequence self and mutual impedances of a parallel line using the data taken following a single-line-to-ground (SLG) fault on one of the parallel circuits. Fig. 1 shows a SLG fault located at fraction m from terminal l on the first circuit of a parallel transmission line. Two protective relays located at terminals l and r record synchrophasors of fault voltages and currents. This is the standard practice in protective relaying where recorded data may be used later for event analysis. It is assumed that the two relays are equipped with the time synchronization technology.

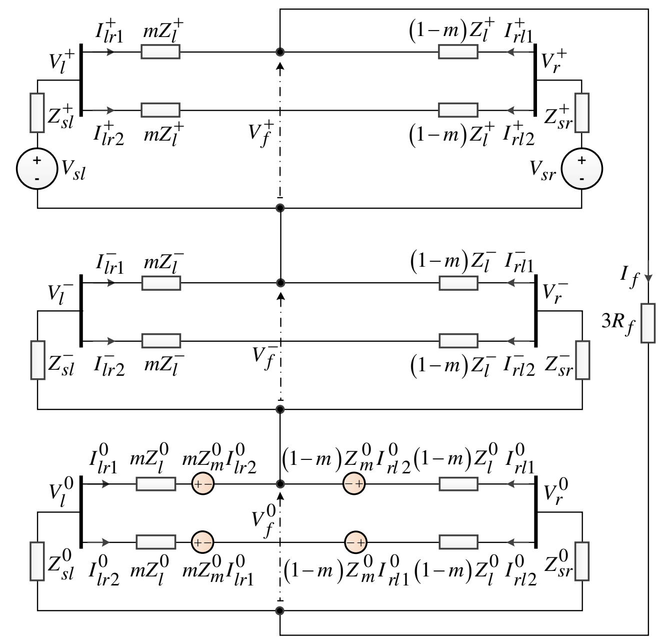

Fig. 2. Sequence network connection for a SLG fault.

A. Circuit Derivations

To simplify asymmetrical fault analysis, the sequence networks can be interconnected in a way as to satisfy the fault type constraints and provide an integrated single-phase circuit to solve. Fig. 2 presents the fault loop for the SLG fault in Fig. 1. The loop includes positive-, negative- and zero-sequence networks connected in such a way to model post-fault voltages and currents in symmetrical components. In addition to voltage sources V_{sl} and V_{sr}, which model Thevenin voltage sources, four dependent voltage sources are considered in the zero-sequence network in order to represent the impact of zero-sequence mutual impedance of the line. According to Fig. 2, the following KVL equation can be written for the inner loop of the zero-sequence network:

$$\begin{aligned}&-mZ_l^0 I_{lr1}^0 - mZ_m^0 I_{lr2}^0 + (1-m)Z_m^0 I_{rl2}^0 + (1-m)Z_l^0 I_{rl1}^0 \\ &-(1-m)Z_l^0 I_{rl2}^0 - (1-m)Z_m^0 I_{rl1}^0 + mZ_m^0 I_{lr1}^0 + mZ_l^0 I_{lr2}^0 = 0\end{aligned} \tag{1}$$

where Z_l^0 and Z_m^0 are zero-sequence self and mutual impedances of the line. Equation (1) can be reordered as:

$$\begin{aligned}&I_{lr1}^0(-mZ_l^0 + mZ_m^0) + I_{lr2}^0(-mZ_m^0 + mZ_l^0) \\ &+I_{rl1}^0[(1-m)Z_l^0 - (1-m)Z_m^0] \\ &+I_{rl2}^0[(1-m)Z_m^0 - (1-m)Z_l^0] = 0\end{aligned} \tag{2}$$

which can be further simplified as follows given that $I_{lr2}^0 + I_{rl2}^0 = 0$ based on Fig. 2.

$$(Z_m^0 - Z_l^0)[mI_{lr1}^0 + I_{rl2}^0 - (1-m)I_{rl1}^0)] = 0 \tag{3}$$

Table I presents typical parameters for two parallel lines, implying that $Z_m^0 \neq Z_l^0$. Therefore, based on (3), fault location can be obtained as

$$m = \frac{I_{rl1}^0 - I_{rl2}^0}{I_{lr1}^0 + I_{rl1}^0} \tag{4}$$

TABLE I
TYPICAL VALUES FOR Z_l^0 AND Z_m^0 [37].

Sequence Impedance	$132KV$ Parallel Line	$245KV$ Parallel Line
$Z_l^0(\Omega)$	$1.18\angle 71°$	$0.95\angle 76°$
$Z_m^0(\Omega)$	$0.63\angle 71°$	$0.52\angle 75°$

As shown, a measurement-based parameter-free formula is obtained for calculating fault location m, which will be used later in the derivations. It can be concluded that the KVL equation for the inner loop of the zero-sequence network does not give any information regarding Z_l^0 and Z_m^0.

With reference to Fig. 2, another KVL equation can be written for the outer loop of the zero-sequence network as follows:

$$\begin{aligned}&V_r^0 - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 - (1-m)Z_l^0 I_{rl1}^0 \\ &-(1-m)Z_m^0 I_{rl2}^0 = 0\end{aligned} \tag{5}$$

which can be simplified as follows given that $I_{lr2}^0 + I_{rl2}^0 = 0$.

$$V_r^0 - V_l^0 + mZ_l^0(I_{lr1}^0 + I_{rl1}^0) - Z_l^0 I_{rl1}^0 - Z_m^0 I_{rl2}^0 = 0 \tag{6}$$

By substituting m from (4) into (6) we have

$$V_r^0 - V_l^0 - (Z_l^0 + Z_m^0)I_{rl2}^0 = 0 \tag{7}$$

Therefore, the zero-sequence network equations yield:

$$Z_l^0 + Z_m^0 = \frac{V_r^0 - V_l^0}{I_{rl2}^0} \tag{8}$$

Another KVL equation involving the zero-sequence voltage at fault point can be written according to Fig. 2 as follows:

$$mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 = V_l^0 - V_f^0 \tag{9}$$

where V_f^0 is the zero-sequence voltage at fault point. In a matrix form, the two independent equations (8) and (9) that involve Z_l^0 and Z_m^0 can be written as follows:

$$\begin{bmatrix} 1 & 1 \\ mI_{lr1}^0 & mI_{lr2}^0 \end{bmatrix} \begin{bmatrix} Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} \frac{V_r^0 - V_l^0}{I_{rl2}^0} \\ V_l^0 - V_f^0 \end{bmatrix} \tag{10}$$

Even if the fault location m is known from field inspection or output of the fault locator of the relay or (4) or other equations in positive or negative sequence networks, Z_l^0 and Z_m^0 cannot be obtained unless V_f^0 is known; otherwise, (10) will be a system of two linear equations in three unknowns which cannot have a unique solution. Since having the knowledge of V_f^0 as input is not possible in practice, zero-sequence self and mutual impedances of a parallel transmission line are generally unattainable form fault data.

B. Special Case of Bolted Faults

There is a special case in which fault data can give zero-sequence self and mutual impedances of a parallel transmission line. Consider a bolted ground fault for which $R_f = 0$ in

Fig. 2. In this case, V_f^0 in (10) can be obtained using positive network impedance data. According to Fig. 2, the following KVL can be written:

$$V_f^0 + V_f^- + V_f^+ = 3R_f I_f \tag{11}$$

where V_f^- and V_f^+ can be obtained from negative and positive sequence networks, respectively, as follows:

$$V_f^+ = V_l^+ - mZ_l^+ I_{lr1}^+ \tag{12}$$

$$V_f^- = V_l^- - mZ_l^- I_{lr1}^- \tag{13}$$

Provided that $Z_l^- = Z_l^+$ are already known, V_f^- and V_f^+ can be obtained from fault data. Now in the case of a bolted ground fault, i.e. $R_f = 0$, V_f^0 obtained from (11) can be substituted in (10) as follows:

$$\begin{bmatrix} 1 & 1 \\ mI_{lr1}^0 & mI_{lr2}^0 \end{bmatrix} \begin{bmatrix} Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} \dfrac{V_r^0 - V_l^0}{I_{rl2}^0} \\ V_l^0 + V_f^+ + V_f^- \end{bmatrix} \tag{14}$$

In this especial case, Z_l^0 and Z_m^0 can be directly obtained from (14) as follows:

$$Z_l^{0,Bolted} = \frac{(m-1)V_l^0 - mV_r^0 - (V_f^+ + V_f^-)}{m(I_{lr2}^0 - I_{lr1}^0)} \tag{15}$$

$$Z_m^{0,Bolted} = \frac{(1 - \frac{mI_{lr1}^0}{I_{lr2}^0})V_l^0 + \frac{mI_{lr1}^0}{I_{lr2}^0}V_r^0 + V_f^+ + V_f^-}{m(I_{lr2}^0 - I_{lr1}^0)} \tag{16}$$

where V_f^+ and V_f^- are given by (12) and (13), respectively. It must be emphasized that if (15) and (16) are used in a general case of non-zero fault resistance, the estimation results will be in error.

III. Double-Line-To-Ground Faults on Parallel Transmission Lines

The fault loop for a double-line-to-ground (DLG) fault on a parallel transmission line is presented in Fig. 3. It can be seen that (1) and (5) written for respectively the inner and outer loops of the zero sequence network still holds for this fault. Therefore, (8) is still valid for this fault. On the other hand, (9) is also obtained for the zero-sequence network and therefore also holds for this fault. The system of equations (10) is therefore also valid. However, according to Fig. 3, (11) should be replaced with the following equations:

$$V_f^+ - V_f^0 = 3R_f I_f \tag{17}$$

$$V_f^+ = V_f^- \tag{18}$$

By analyzing (10) in the same way as what we did at the end of Section II-A, one can conclude that Z_l^0 and Z_m^0 cannot be solved by (10) given that V_f^0 is dependent on fault resistance as can be seen in (17).

The only exception is the case of $R_f = 0$, in which case, $V_f^0 = V_f^+ = V_f^-$. Assuming that the positive-sequence impedance is reliably known and provided that (12) also holds for this fault according to Fig. 3, (10) will be uniquely solvable in this case and Z_l^0 and Z_m^0 can be obtained from fault data.

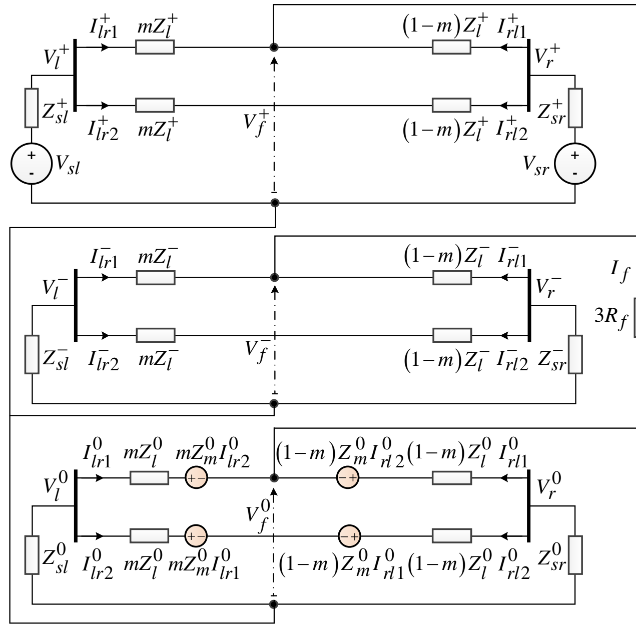

Fig. 3. Symmetrical component network for a double-line-to-ground fault on a parallel transmission line

The assumption of $R_f = 0$, however, is impractical and therefore (10) cannot give Z_l^0 and Z_m^0. In summary, the only difference compared to the case of SLG faults is that (11) has to be replaced with (17) and (18) when DLG faults are concerned.

IV. Assessment Of Previous Algorithms

In Section II, the derivations for zero-sequence parameters of a parallel transmission line were presented. Reference [36] concerns zero-sequence parameter estimation for the the same type of transmission lines. Therefore, in what follows, the derivations of [36] are scrutinized and compared with those presented earlier.

A. Single-Line-to-Ground Faults

In [36] (Section III, part C), three equations (13), (14) and (15) are written based on KVL applied to Fig. 4, which is a duplicate of Fig. 6 of [36]. However, the original circuit, and thus the reproduced one in Fig. 4, are neither complete nor reliable. The circuit must change to the zero-sequence circuit of Fig. 2 (with four voltage sources) to account for all mutual couplings of the two circuits.

Let us examine the process of finding the zero-sequence parameters of a parallel line using fault data in [36]. Based on Fig. 4, (13) in [36] is duplicated here as:

$$\begin{aligned} &(1-m)Z_l^+ I_{rl1}^+ + 3R_f I_f - V_r^0 + (1-m)Z_l^0 I_{rl1}^0 \\ &-V_r^- + (1-m)Z_l^- I_{rl1}^- - V_r^+ = 0 \end{aligned} \tag{19}$$

which should have been written according to the zero-sequence network shown in Fig. 2 as follows:

$$\begin{aligned} &(1-m)Z_l^+ I_{rl1}^+ + 3R_f I_f - V_r^0 + (1-m)Z_l^0 I_{rl1}^0 \\ &+(1-m)Z_m^0 I_{rl2}^0 - V_r^- + (1-m)Z_l^- I_{rl1}^- - V_r^+ = 0 \end{aligned} \tag{20}$$

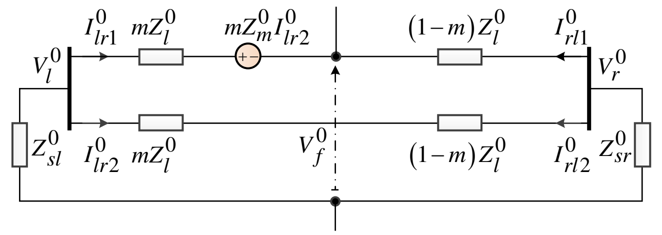

Fig. 4. Zero-sequence network for a ground fault on a parallel line, as presented in [29, Fig. 6].

The difference between correct KVL in (20) and (13) in [36] is the induced voltage $(1-m)Z_m^0 I_{rl2}^0$. Likewise, based on Fig. 4, (14) in [36] is duplicated here as:

$$V_r^0 - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 - (1-m)Z_l^0 I_{rl1}^0 = 0 \quad (21)$$

which should have been written according to the zero-sequence network shown in Fig. 2 as follows:

$$\begin{aligned} &V_r^0 - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 - (1-m)Z_l^0 I_{rl1}^0 \\ &-(1-m)Z_m^0 I_{rl2}^0 = 0 \end{aligned} \quad (22)$$

It can be seen that the induced voltage $-(1-m)Z_m^0 I_{rl2}^0$ is missing from (21). In [36] (Section III, part C) it is claimed that *"By solving equations (13), (14) and (15) for Z_l^0, Z_m^0 and R_f, line zero-sequence, mutual zero-sequence and fault resistance are obtained."* This claim is questionable. Equations (13) and (14) in [36] have already been duplicated here as (19) and (21), respectively. Let us duplicate (15) in [36] as follows:

$$\begin{aligned} &mZ_l^+ I_{lr1}^+ + 3R_f I_f - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 \\ &-V_l^- + mZ_r^- I_{lr1}^- - V_l^+ = 0 \end{aligned} \quad (23)$$

Now let us write (19), (21) and (23), i.e. (13), (14) and (15) in [36], in a matrix form to solve for Z_l^0, Z_m^0 and R_f.

$$\begin{bmatrix} 3I_f & (1-m)I_{rl1}^0 & 0 \\ 0 & mI_{lr1}^0 - (1-m)I_{rl1}^0 & mI_{lr2}^0 \\ 3I_f & mI_{lr1}^0 & mI_{lr2}^0 \end{bmatrix} \begin{bmatrix} R_f \\ Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (24)$$

where y_1, y_2 and y_3 depend on measurements. It can be seen from (24) that sum of the first two rows of the coefficient matrix equals its third row. This means the equations are linearly dependent and the rank of the coefficient matrix in (24) cannot be greater than 2. It follows that Z_l^0, Z_m^0 and R_f cannot be uniquely obtained as claimed in [36].

B. Double-Line-to-Ground Faults

Using a similar analysis to the previous subsection, one can show that (16) and (17) in [36] are also incorrect due to missing induced voltages from the adjacent circuit. They are duplicated as follows:

$$(1-m)Z_l^- I_{rl1}^- + R_f I_f - (1-m)Z_l^0 I_{rl1}^0 + V_r^0 - V_r^- = 0 \quad (25)$$

$$V_r^0 - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 - (1-m)Z_l^0 I_{rl1}^0 = 0 \quad (26)$$

which should also be modified according to Fig. 3, respectively, as follows

$$\begin{aligned} &(1-m)Z_l^- I_{rl1}^- + 3R_f I_f - (1-m)Z_l^0 I_{rl1}^0 - \\ &(1-m)Z_{ml}^0 I_{rl2}^0 + V_r^0 - V_r^- = 0 \end{aligned} \quad (27)$$

$$\begin{aligned} &V_r^0 - V_l^0 + mZ_l^0 I_{lr1}^0 + mZ_m^0 I_{lr2}^0 a \\ &-(1-m)Z_l^0 I_{rl1}^0 - (1-m)Z_m^0 I_{rl2}^0 = 0 \end{aligned} \quad (28)$$

which show the induced voltage $-(1-m)Z_m^0 I_{rl2}^0$ is missing from both (25) and (26). Equation (18) in [36] is duplicated here as

$$mZ_l^+ I_{lr1}^+ + R_f I_f - mZ_m^0 I_{lr2}^0 - mZ_l^0 I_{lr1}^0 + V_l^0 - V_l^+ = 0 \quad (29)$$

Now let us write (25), (26) and (29), i.e. (16), (17) and (18) in [36], in a matrix form to solve for Z_l^0, Z_m^0 and R_f.

$$\begin{bmatrix} I_f & -(1-m)I_{rl1}^0 & 0 \\ 0 & mI_{lr1}^0 - (1-m)I_{rl1}^0 & mI_{lr2}^0 \\ I_f & -mI_{lr1}^0 & -mI_{lr2}^0 \end{bmatrix} \begin{bmatrix} R_f \\ Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (30)$$

where where z_1, z_2 and z_3 depend on measurements. It can be seen from (30) that sum of the last two rows of the coefficient matrix equals the first row in this matrix. Therefore, similar to the analysis presented above, Z_l^0, Z_m^0 and R_f cannot be solved by (16), (17) and (18) in [36], where fault data recorded during a DLG fault is used.

Even if the complete zero-sequence circuit of Fig. 3 is used, (27), (28) and (29) will be written as

$$\begin{bmatrix} 3I_f & -(1-m)I_{rl1}^0 & -(1-m)I_{rl2}^0 \\ 0 & mI_{lr1}^0 - (1-m)I_{rl1}^0 & mI_{lr2}^0 - (1-m)I_{rl2}^0 \\ 3I_f & -mI_{lr1}^0 & -mI_{lr2}^0 \end{bmatrix} \begin{bmatrix} R_f \\ Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (31)$$

where, similar to (30), the sum of the last two rows of the coefficient matrix equals its first row and hence Z_l^0, Z_m^0 and R_f cannot be uniquely obtained either. Equation (31) clearly shows that even if all induced voltages from the two circuits are considered, zero-sequence impedances cannot be obtained using the three KVL equations used in [36].

V. Estimation of X_l^0 and X_m^0 for the General Case of Non-Bolted Faults

Previously, it was shown in Section II that Z_l^0 and Z_m^0 cannot be directly obtained unless fault resistance is zero. In what follows, it will be shown that infinitely many sets of zero sequence parameters can lead to the same measurement set when different fault resistance values are considered. First consider that $Z_l^{0,Bolted}$ and $Z_m^{0,Bolted}$ are the same as those given in (15) and (16), respectively. These are the estimates of zero-sequence impedances with the assumption that R_f=0.

Now let us consider the same set of measurements for a general case with non-zero fault resistance R_f. With reference to (10), the zero-sequence impedances of the parallel line under this new condition are given by

$$\begin{bmatrix} Z_l^0 \\ Z_m^0 \end{bmatrix} = \frac{1}{m(I_{lr2}^0 - I_{lr1}^0)} \begin{bmatrix} mI_{lr2}^0 & -1 \\ -mI_{lr1}^0 & 1 \end{bmatrix} \begin{bmatrix} \frac{V_r^0 - V_l^0}{I_{rl2}^0} \\ V_l^0 - V_f^0 \end{bmatrix} \quad (32)$$

Substituting (11) into (32) and using (14) leads to:

$$\begin{bmatrix} Z_l^0 \\ Z_m^0 \end{bmatrix} = \begin{bmatrix} Z_l^{0,Bolted} \\ Z_m^{0,Bolted} \end{bmatrix} + \frac{1}{m(I_{lr2}^0 - I_{lr1}^0)} \begin{bmatrix} 3R_f I_f \\ -3R_f I_f \end{bmatrix} \quad (33)$$

where $I_f = I^0_{rl1} + I^0_{lr1}$ depends on measurements. Since there are infinitely many feasible values for R_f, different sets of Z^0_l and Z^0_m can be estimated with the same set of measurements. This clearly rejects the claim that zero-sequence parameters of the line can be obtained from fault data.

In what follows, it will be shown that the imaginary parts of Z^0_l and Z^0_m , i.e. X^0_l and X^0_m, can be directly estimated using (10) for the general case of non-zero fault resistance.

We proceed using the following lemma, which holds for any ground fault on a parallel transmission line, based on Fig. 2.

Lemma 1: $Imag\{\frac{-3R_f I_f}{m(I^0_{lr2} - I^0_{lr1})}\} = 0.$

Proof: See Appendix A.

The difference between $Z^{0,Bolted}_l$ and Z^0_l, as well as Z^0_m and $Z^{0,Bolted}_m$, is $\frac{-3R_f I_f}{m(I^0_{lr2} - I^0_{lr1})}$ according to (33). On the other hand, the imaginary part of this term is zero according to Lemma 1. Therefore, X^0_l and X^0_m can be estimated without the knowledge of the value of R_f by assuming that R_f is zero. It is worth noting that the real part of $\frac{-3R_f I_f}{m(I^0_{lr2} - I^0_{lr1})}$ is not zero. Therefore, in the general case of non-zero R_f, self and mutual resistance of the zero-sequence network, i.e. R^0_l and R^0_m, cannot be obtained by (10).

VI. Evaluation Studies

A parallel transmission line, whose actual data are tabulated in Table II, is used for evaluation studies. The aim is to estimate zero-sequence impedances using the data collected following a fault on either of the parallel circuits of this line. Table II also presents the parameters of the equivalent networks at the line terminals. The same line parameters as those studied in [36] are adopted for the sake of comparison. Although the studied line is a short line, the distributed parameter model of the line is simulated in the DIgSILENT Power Factory environment in order for the results to be as accurate as possible. RMS simulations are adopted for case studies, except where transients have to be considered, in which case EMT simulations are carried out. To save space, only results for SLG faults on phase A are presented.

A. Bolted Ground Faults

Table III presents the output of the proposed method as well as the algorithm in [36]. As expected, the algorithm of [36] fails to give meaningful results due to the rank-deficiency of the equations employed as discussed in Section IV. In contrast, the proposed method estimates the zero-sequence parameters of the line by (15) and (16). The reason for the estimation error is the neglect of the shunt capacitive currents of the line during the fault event. Estimation of self and mutual zero-sequence capacitance of the line is not covered by the paper since these have limited application in protection engineering. More importantly, with the same set of measurements, addition of more unknowns will not positively impact the rank deficiency of the developed system of equations in [36]. This is still in line with the key message of the paper that zero-sequence parameters of a parallel line cannot be estimated from data of a fault on either circuit of the line. Nevertheless, if the zero-sequence capacitance data of the line is reliably available, it can readily be taken into account in the derivation to increase the accuracy of estimated zero-sequence impedances, provided that fault resistance is zero.

TABLE II
PARALLEL TRANSMISSION LINE DATA [36]

Length	50 (km)
Positive-sequence Impedance (Z^+_l)	0.0186+j0.3766 (Ω/km)
Positive-sequence Susceptance (B^+)	4.3879 (μS/km)
Zero-sequence Self Impedance (Z^0_l)	0.3618+j1.2278 (Ω/km)
Zero-sequence Mutual Impedance (Z^0_m)	0.1206+j0.4092 (Ω/km)
Zero-sequence Susceptance (B^0)	2.8977 (μS/km)
Positive-sequence Impedance of Network at Terminal $l(Z^+_{sl})$	0.76+j35.93 (Ω)
Positive-sequence Impedance of Network at Terminal $r(Z^+_{sr})$	1.58+j42.49 (Ω)
Zero-sequence Impedance of Network at Terminal $l(Z^0_{sl})$	3.28+j39.98 (Ω)
Zero-sequence Impedance of Network at Terminal $r(Z^0_{sr})$	6.91+j62.56 (Ω)
Thevenin Voltage at Terminal $l(V_{sl})$	$1\angle10°$
Thevenin Voltage at Terminal $r(V_{sr})$	$1\angle0°$

TABLE III
ESTIMATION ERRORS FOR ZERO-SEQUENCE PARAMETER ESTIMATION FROM BOLTED SLG FAULT DATA

Par.	true value (Ω)	Actual m (pu)	Proposed Method	[36]
R^0_l	18.09	0.1	17.99	-2.2×10^{14}
		0.5	17.77	-5×10^{14}
		0.9	16.36	-1.2×10^{14}
X^0_l	61.39	0.1	61.14	1.3×10^{15}
		0.5	60.72	-9.4×10^{12}
		0.9	57.89	-1×10^{15}
R^0_m	6.03	0.1	5.26	2.1×10^{15}
		0.5	8.99	5.5×10^{15}
		0.9	8.41	4×10^{15}
X^0_m	20.46	0.1	18.90	-1.7×10^{16}
		0.5	26.87	9×10^{13}
		0.9	25.28	3.5×10^{16}

Fig. 5 presents the estimated zero-sequence parameters for short and line longs. The true value of each parameter is drawn as a dashed line with the same color as that designated to the estimated parameter. As expected, with the increase in the line length, the estimation error increases significantly, while for very short lines the estimation results are quite accurate. This observation confirms that the shunt capacitive currents ignored in the developed formulation play an important role for the accuracy of the estimates if long lines are concerned. As mentioned, Z^0_l and Z^0_m cannot be uniquely obtained for the general case of non-bolted ground faults, regardless of whether or not the zero-sequence self and mutual capacitance of the line are accurately known.

B. Non-Bolted Faults with Unknown Fault Resistance Values

In order to demonstrate the non-uniqueness of the estimates when the fault resistance value is unknown, a fault at the middle of the line is simulated as an example.

Fig. 6 presents the locus of estimated Z^0_l and Z^0_m parameters with the same measurements when the fault resistance varies

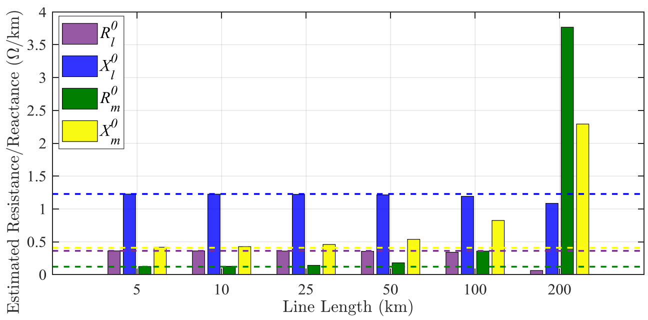

Fig. 5. Estimation of zero-sequence parameters for different line lengths using bolted fault data.

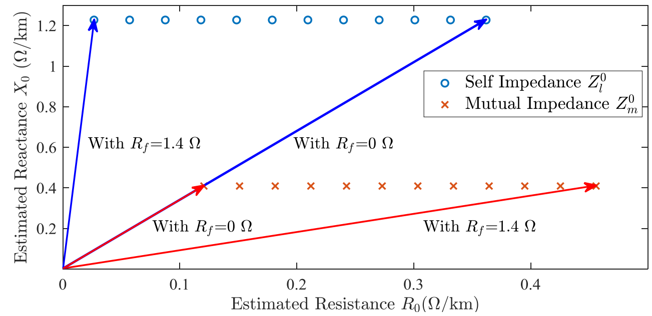

Fig. 6. Estimated self and mutual zero-sequence impedances of the line with varying fault resistance value but the same fault data (m=0.5 pu).

TABLE IV
SENSITIVITY OF ZERO-SEQUENCE PARAMETER ESTIMATION TO FAULT RESISTANCE

$R_f(\Omega)$	0	10	20	30	40
$R_l^0(\Omega)$	17.766	137.77	257.78	377.79	497.79
$X_l^0(\Omega)$	60.725	60.738	60.752	60.765	60.779
$R_m^0(\Omega)$	8.9961	-111.01	-231.02	-351.03	-471.03
$X_m^0(\Omega)$	26.874	26.861	26.847	26.834	26.82

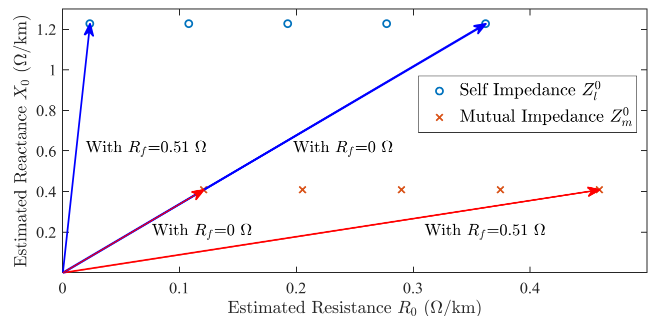

Fig. 7. Estimated self and mutual zero-sequence impedances of the line with varying fault resistance value but the same fault data (m=0.1 pu.)

from zero at equal intervals. In practice, the fault resistance value is unknown, and therefore, zero-sequence parameters of the line cannot be estimated from fault data recorded by relays at line terminals. However, as a result of Lemma 1 and (33), the zero-sequence self and mutual reactances can be estimated accurately when R_f is assumed to be zero .

The same procedure is followed when a fault occurs at 10% of the line length. Similar results are obtained as shown in Fig. 7. The difference is that the maximum fault resistance that leads to a positive value for R_l^0 is less than the previous case. This can be justified by Lemma 2.

Lemma 2: $R_l^{0,Bolted} - R_l^0 = \dfrac{3R_f}{m(1-m)}$.

Proof: See Appendix B.

It is evident from Lemma 2 that, for the same set of measurements, R_f can change in a wider range for m=0.5 than for m=0.1 before R_l^0 becomes negative.

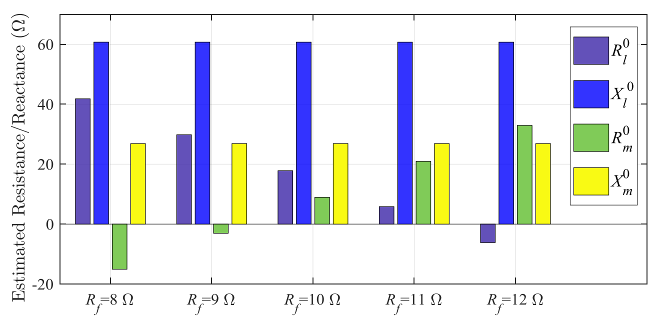

Fig. 8. Estimated self and mutual zero-sequence impedances of the line with different R_f values assumed (True $R_f = 10$ Ω).

C. Non-Bolted Faults with Known Fault Resistance Values

From (10) and (11) one can see that if R_f is known, Z_l^0 and Z_m^0 can be uniquely estimated. Fig. 8 presents the estimation results for a fault at the midpoint of the line when actual fault resistance is 10 Ω. It can be seen that for $R_f = 10\Omega$ the estimation results very much resemble those in Table III. However, when fault resistance is not known precisely, the estimation results for R_l^0 and R_m^0 may be far from the true value and even become negative. This sensitivity of R_l^0 and R_m^0 to small changes of the assumed fault resistance was already observed in Figs. 6 and 7. Similarly, the estimated values for X_l^0 and X_m^0 remain the same for different R_f values assumed according to Section V. Table IV shows that X_l^0 and X_m^0 are estimated quite accurately even for large fault resistance values while the estimation error is pronounced for R_l^0 and R_m^0 as the fault resistance value increases. These two observations are in agreement with Lemma 1 and Lemma 2, respectively.

D. Impact of Nonlinear Fault Resistance

Thus far, fault resistance has been assumed to be constant. Although the fault resistance often remains constant over the course of fault [38], [39], there are few cases in which fault resistance changes, for example during arcing faults. A nonlinear fault resistance is used in order to examine the estimation results. Electromagnetic transient (EMT) simulations involving the time-varying fault resistance are carried out. Fig. 9 shows the voltage of the faulted phase at terminal l. It can be observed that the magnitude of post-fault voltage slightly reduces over time due to the change in fault resistance. Fig. 10 presents the estimation results for X_l^0 and X_m^0 previously shown not to be influenced by the fault resistance. It can be seen that after the initial transients in fault measurements, the estimated parameters tend to their respective actual values.

Due to the rank deficiency of equations in [36] as well as non-zero fault resistance, neither the formulation of [36]

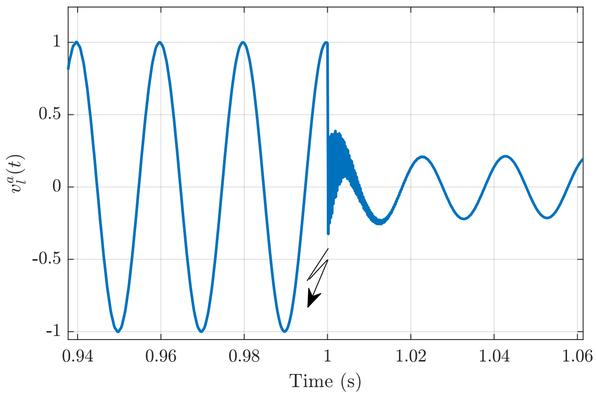

Fig. 9. Terminal voltage of the faulted phase following a fault with time-varying fault resistance.

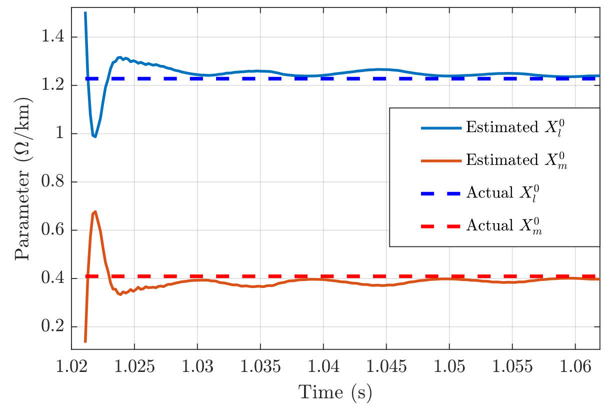

Fig. 10. Estimated zero-sequence reactances of the line following a fault with time-varying fault resistance.

nor that of this paper is able to find zero-sequence resistance values. Therefore, as expected, the zero-sequence parameters of the line are not entirely attainable from fault data in this case.

E. Zero-Sequence Parameter Estimation for Untransposed Parallel Lines

Fig. 11 presents a typical double-circuit tower used in the extra high voltage transmission network. Transmission lines that are shorter than a threshold, e.g. 40 km [31], are often operated without transposition. Two cases of presence and absence of the earth wire are discussed in the following.

1) Untransposed Lines without Earth Wire: The line in this case includes 6 phase wires while the ground as the path for zero-sequence fault current is assumed to have a resistivity of 100 $\Omega.m$. The actual values of zero-sequence self and mutual impedances are obtained from the output result of the software by setting the ground voltage equal to zero and obtaining the symmetrical impedance matrix of the line [1], [40]. For an SLG fault with $R_f = 10\ \Omega$, it can be seen from Table V that only the proposed method is able to estimate the zero-sequence reactances of the line quite accurately.

2) Untransposed Lines with Earth Wire: This case is similar to the previous case, except for an earth wire at the top of the tower in Fig. 11. This significantly affects the zero-sequence parameters of the line since there is a new path for the zero-sequence current to flow through the earth wire. This can be seen by comparing the actual parameters of the line in Table V with those in Table VI, which also includes parameter estimation results. In this case the estimated X_l^0 and X_m^0 are much less accurate compared to those listed in Table V.

A detailed analysis in Appendix C demonstrates that the impact of earth wire is crucial to the circuit equations since the mutual impedances between zero, positive and negative sequence components are not negligible in contrast to the case of transposed lines. This also emphasizes that in the case of untransposed parallel transmission lines, no zero-sequence parameter of the line can be estimated from fault data, accurately.

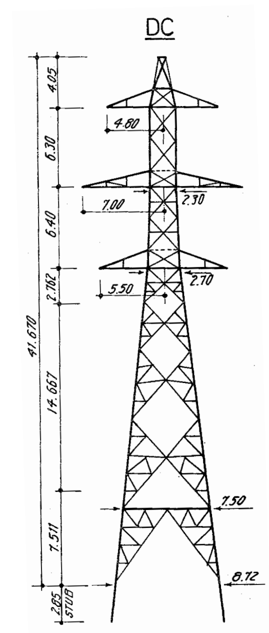

Fig. 11. A typical tower for untransposed parallel transmission lines (All dimensions are in meters.) [41]

TABLE V
PARAMETER ESTIMATION FOR THE PARALLEL UNTRANSPOSED LINE IN FIG. 11 WITHOUT EARTH WIRE

Par.	Actual Value (Ω/km)	Estimated by Proposed Method	Estimated by [36]
R_l^0	0.189	6.19	2.1×10^{12}
X_l^0	1.312	1.306	9.2×10^{11}
R_m^0	0.139	-5.86	–2.7×10^{13}
X_m^0	0.808	0.858	–1.2×10^{13}

TABLE VI
PARAMETER ESTIMATION FOR A PARALLEL UNTRANSPOSED LINE IN FIG. 11 WITH AN EARTH WIRE

Par.	Actual Value (Ω/km)	Estimated by Proposed Method	Estimated by [36]
R_l^0	0.126	6.125	5.1×10^{12}
X_l^0	0.976	0.872	4.7×10^{11}
R_m^0	0.076	-5.907	–6.8×10^{13}
X_m^0	0.472	0.604	–6.2×10^{13}

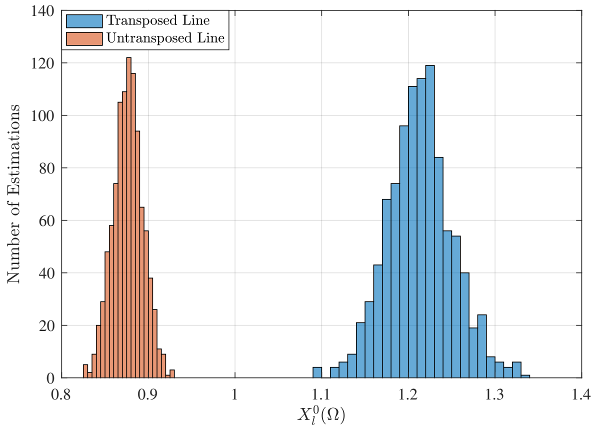

Fig. 12. Estimation of zero-sequence self reactance of parallel lines under measurement error.

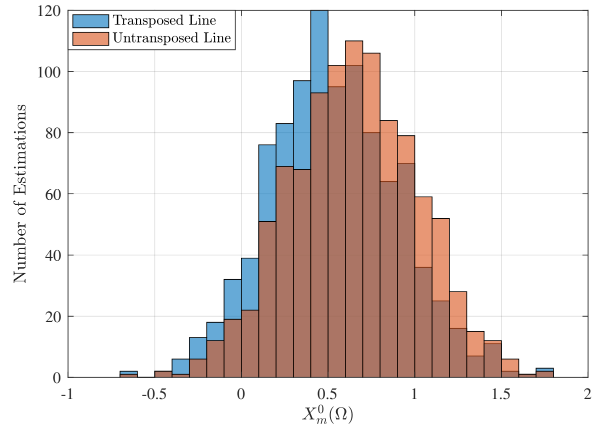

Fig. 13. Estimation of zero-sequence mutual reactance of parallel lines under measurement error.

F. Impact of Measurement Error

Thus far, perfect measurements have been considered in order to investigate the possibility of parameter estimation. Here, we present a more realistic situation where measurement errors introduced mainly by instrument transformers [42] are present. It is expected that the estimation results are aggravated when measurement errors are considered.

To examine the impact of instrument transformers, current transformers (CT) and voltage transformers (VT) are considered to have Gaussian errors so that the maximum CT or VT error does not exceed that of the respective accuracy class with a confidence interval of 99.7% [43]. The accuracy classes for CTs and VTs are assumed to be 5P and 3P, respectively [44], [45]. Thus, the measurement error for each of the magnitude and phase angle is modeled as a Gaussian random variables with zero mean and a standard deviation equal to one third of the associated maximum error [12]. To capture the stochastic nature of the estimation error, 1000 Monte Carlo simulation cases [46] are run for estimating zero-sequence reactances of the transposed line of Table II and untransposed line of Fig. 11, separately. The estimation results for the self and mutual zero-sequence reactances are reflected in Figs. 12 and 13, respectively. It can be seen that under realistic conditions estimation results are unreliable even for X_l^0 and X_m^0 parameters, which earlier were shown to be estimated quite accurately under perfect measurements. Other phenomena such as CT saturation are not considered, although they may aggravate the estimation results.

VII. Conclusion

This paper investigates the possibility of the estimation of zero-sequence parameters of a parallel transmission line from fault data. It shows that the zero-sequence self and mutual impedances of a parallel transmission line cannot, in general, be obtained from recorded data of a short-circuit fault on either circuit of the line. Specifically, for transposed parallel lines,

- Z_l^0 and Z_m^0 cannot be estimated from synchronized measurements at line terminals. The only exception is the case where the fault resistance value is known beforehand, which is impossible in practice.
- X_l^0 and X_m^0 are estimated accurately for short lines regardless of the fault resistance value.

For short untransposed parallel lines,

- X_l^0 and X_m^0 are approximated accurately if the line has no earth wire.
- In presence of the earth wire, X_l^0 and X_m^0 are estimated with much less accuracy compared to the case where the line has no earth wire.

For both transposed and untransposed parallel lines, it is proved that despite claims made in the literature, it is impossible to estimate zero-sequence parameters of the line due to the rank deficiency of the equations employed. Besides, the reliability of zero-sequence reactance estimation deteriorates if realistic measurement errors are taken into account. The theoretical findings of the paper are supported by extensive simulations under various fault conditions.

Appendix A
Proof of Lemma 1

From (4) and given that fault distance m is real-valued, one can write:

$$\begin{aligned}
&Imag\{\frac{I_{rl1}^0 - I_{rl2}^0}{I_{lr1}^0 + I_{rl1}^0}\} = 0 \\
&= Imag\{\frac{I_{rl1}^0 + I_{lr1}^0 - I_{lr1}^0 - I_{rl2}^0}{I_{lr1}^0 + I_{rl1}^0}\} \\
&= Imag\{\frac{I_f - I_{lr1}^0 + I_{lr2}^0}{I_f}\} \\
&= Imag\{1 + \frac{I_{lr2}^0 - I_{lr1}^0}{I_f}\} \\
&= Imag\{\frac{I_{lr2}^0 - I_{lr1}^0}{I_f}\} \\
&= Imag\{\frac{I_f}{I_{lr2}^0 - I_{lr1}^0}\} \\
&= Imag\{\frac{-3R_f I_f}{m(I_{lr2}^0 - I_{lr1}^0}))\}
\end{aligned} \tag{A.1}$$

where the third line is written based on the zero-sequence network shown in Fig. 2 and the last line is based on the fact that both R_f and m are real-valued.

Appendix B
Proof of Lemma 2

By taking the real part of the first equation in (33) it follows that

$$Re\{Z_l^0\} = Re\{Z_l^{0,Bolted}\} + Re\{\frac{3R_f I_f}{m(I_{lr2}^0 - I_{lr1}^0)}\} \quad \text{(B.1)}$$

where the second term in the right hand side has no imaginary part according to Lemma 1. Therefore, (B.1) can be rewritten as

$$R_l^0 = R_l^{0,Bolted} + \frac{3R_f I_f}{m(I_{lr2}^0 - I_{lr1}^0)} \quad \text{(B.2)}$$

On the other hand, from (4) it follows that

$$\frac{1}{1-m} = \frac{I_{lr1}^0 + I_{rl1}^0}{I_{lr1}^0 + I_{rl2}^0} \quad \text{(B.3)}$$

Substituting $I_f = I_{lr1}^0 + I_{rl1}^0$ and $I_{rl2}^0 = -I_{lr2}^0$ based on Fig. 2 into (B.3) yields

$$\frac{1}{1-m} = \frac{I_f}{I_{lr1}^0 - I_{lr2}^0} \quad \text{(B.4)}$$

Substituting (B.4) into (B.2) yields Lemma 2.

Appendix C
Influence of Earth Wire on Untransposed Parallel Line Parameters

Tables V and VI present different accuracy levels for zero-sequence reactance estimation, suggesting the crucial impact of earth wire on untransposed line parameters. From the output results of the software simulator, the following reactance matrices are calculated for the untransposed line in Fig. 11 for cases of without and with earth wire, respectively.

$$X_{NEW} = \begin{array}{c} \text{Circuit I} \\ \text{Circuit II} \end{array} \left[\begin{array}{cccccc} \multicolumn{3}{c}{\overbrace{\qquad Circuit \quad I(0,1,2) \qquad}} & \multicolumn{3}{c}{\overbrace{\qquad Circuit \quad II(0,1,2) \qquad}} \\ 1.31 & -0.006 & -0.007 & 0.808 & -0.002 & -0.002 \\ & 0.41 & 0.014 & 0.004 & -0.014 & -0.007 \\ & & 0.41 & 0.005 & -0.007 & -0.014 \\ & & & 1.31 & -0.007 & -0.007 \\ & & & & 0.41 & 0.013 \\ & & & & & 0.41 \end{array} \right] \quad \text{(C.1)}$$

$$X_{EW} = \begin{bmatrix} 0.98 & -0.028 & -0.029 & 0.47 & 0.015 & 0.016 \\ & 0.41 & 0.013 & -0.017 & -0.012 & -0.007 \\ & & 0.41 & -0.017 & -0.006 & -0.012 \\ & & & 0.98 & 0.01 & 0.011 \\ & & & & 0.41 & 0.013 \\ & & & & & 0.41 \end{bmatrix} \quad \text{(C.2)}$$

Comparing (C.1) and (C.2), one can see that although the positive-sequence self reactances remain intact after introducing the earth wire, zero-sequence self and mutual reactances and to a lesser extent mutual inductances in all sequences between the two circuits change. The reason is that in the presence of the earth wire, part of the zero-sequence current flows through the earth wire, which happens to be closer to the phase wires compared to the ground. Of particular interest is the first row of (C.2) where the mutual inductances between the zero sequence of Circuit I and the positive and negative sequences of Circuit I as well as Circuit II are more than corresponding entries of (C.1). This implies that mutual inductances between the zero-sequence circuit and the positive- and negative- sequence circuits neglected in transposed lines undermine the equations utilized to estimate the zero-sequence impedances. This justifies less accuracy of the estimates of X_l^0 and X_m^0 in Table VI compared to those in Table V. On the other hand, from (C.1) it is evident that the assumption of decoupling of sequences is valid with a good approximation when there is no earth wire. This explains the accurate results for X_l^0 and X_m^0 in Table V.

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Ahmad Salehi Dobakhshari received his B.Sc., M.Sc., and Ph.D. degrees from Sharif University of Technology, Tehran, Iran, in 2006, 2008, and 2014, respectively, all in electrical engineering. He was with Monenco Iran Consulting Engineers Company, Tehran, from 2009 to 2013, and with Mapna Group, Tehran, from 2014 to 2015. He joined the Faculty of Engineering, University of Guilan, Rasht, Iran, in 2016, as an assistant professor. Now, he is an associate professor of electrical engineering at the same university. He is also an Associate Editor for Elsevier's *e-Prime*. His current research interests include wide-area measurements, state estimation and digital protection applied to power systems.

Sadegh Azizi (Senior Member, IEEE) received the B.Sc. degree in electrical power engineering from the K. N. Toosi University of Technology, Tehran, Iran, in 2007, the M.Sc. degree in electrical power engineering from the Sharif University of Technology, Tehran, in 2010, and the Ph.D. degree in electrical power engineering from the University of Tehran, Tehran, in 2016. He is currently a Lecturer of smart energy systems with the School of Electronic and Electrical Engineering, University of Leeds, Leeds, U.K. From June 2016 to January 2019, he was with The University of Manchester, Manchester, U.K., as a Postdoctoral Researcher leading their work on the protection Work Package of the EU H2020 MIGRATE project, in collaboration with more than 20 European Transmission System Operators and research institutes. He was with the Energy and System Study Center, Monenco Iran Consulting Engineers Company, Tehran, from 2009 to 2011, and the Iran Grid Management Company, Tehran, from 2013 to 2016. His research interests include wide-area monitoring, protection and control systems, digital protective relays, and applications of power electronics in power systems. He is the Managing Editor of Elsevier's *e-Prime* and an Associate Editor for *The International Journal of Electrical Power and Energy Systems*. He is also a Task Leader of Cigré WG B5.57, which is investigating new challenges of frequency protection in modern power systems.