Numerical Study of Bevelled Disk Settling Hydrodynamics Using Structure-Resolved Simulations

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Abstract. Numerical studies of bevelled circular disk settling dynamics with various shapes and sizes have been investigated in a quiescent fluid. Direct numerical simulations, together with an immersed boundary method, have been used to develop understanding of the settling process for these disks. High resolution and accuracy were facilitated using the spectral element method, as applied using the Nek5000 solver. The size of the particles (based on diameter, d) was considered in the 1-2 mm range, which falls in the intermediate flow regime with a maximum particle Reynolds number, $Re_p = 835$. A sensitivity analysis was performed for the particle and fluid mesh resolution. Settling velocities, disk orientations, and the local particle Reynolds numbers were recorded over time. The disks demonstrate motions in all three dimensions after release from rest, with a reciprocation during translation, and rotations about their three orthogonal axes. The upper limit of the Galileo number, Ga, was 370, which represents the relative importance of either gravitational or viscous forces, based on size, shape, and density ratio. Disks with sharp and bevelled edges were simulated and their settling rates compared, with enhanced settling rates found for the latter.

INTRODUCTION

The motions of settling solid particles are important due to their occurrence in many situations. Polymers have gained attention in recent years due to the detection of microplastics (MPs) on and near the surface of the oceans [1]. Investigations into particle settling rates is highly interdisciplinary, with classifications generally based on material type such as glass, metals, and polymers [2]. Several transport mechanisms and modelling methodologies for particle settling have been identified and demonstrated, with particle motion in both the vertical and horizontal planes observed [3]. Surface transport has been studied using a two-dimensional wave profile based on the advection-diffusion equation [4]. Such studies are useful for estimating plastic budgets, but to date they do not provide a detailed description of the settling dynamics. Settling velocity is a key parameter in studying the vertical deposition of MPs [5]. However, complexities in the settling process due to the influence of many parameters pose challenges. The settling of disks is three-dimensional regardless of the fluid type, with particle diameter also an important factor in the impact on their resulting motion. Smaller disks (within the Stokes regime) fall steadily without the presence of any secondary motions and may be analysed in two dimensions [6]. As their size increases, however, particles exhibit oscillations in all three directions, thus posing a three-dimensional problem [7]. Circular disks have rarely been studied in detail. A discretised immersed boundary method was utilised in [8] to study the settling of non-spherical particles, with predictions comparing well with results obtained using finite element methods. Also, a recent study on settling disks [9] used the fractional area-volume obstacle representation method, which eliminates flow fluctuations and irregularities by smoothly blocking out the 'fractional portions' of grid cell faces and volumes, with this quantity related to the numerical grid refinement. Grid-independence studies in such cases are, however, difficult. Despite these studies, however, the literature is sparse, and further knowledge would be beneficial to settling applications in nature and industry. This work aims to improve understanding of non-spherical particle settling by using first principles modelling, facilitated by direct numerical simulation (DNS) and the immersed boundaries approach. The aim is to both demonstrate the approach, and to uncover the influence of system parameters on the resulting motions. The variation of important non-dimensional parameters such as the Reynolds (*Re*) and Galileo (*Ga*) numbers is discussed, and the effect of using sharp edged rather than bevelled disks on the resulting dynamics is also determined.

METHODOLOGY

This study uses the direct numerical simulation solver, Nek5000 [10], an open-source computational fluid dynamics code based on the spectral element method, which provides high accuracy for complex geometries and flow conditions. The immersed boundary method leverages the flexibility of choosing a separate mesh for the particle and fluid, allowing for optimisation of accuracy. For the continuous phase, a three-dimensional vertically aligned cuboid is used for the entire analysis. Particles are released near the top surface allowing for a small gap $(2d)$ to eliminate surface effects. All boundaries of the column are periodic. The column's physical size is variable for each case, depending on the particle size, and the fluid is assumed to be Newtonian. The number of spectral elements is chosen to be between 10,800 and 26,500. Three-time steps (dt) are investigated (10^{-5} s, 10^{-4} s, and 10^{-3} s). Furthermore, two particle shapes are chosen for analysis, consisting of a disk with either bevelled or sharp edges. The aspect ratio of each disk is 4:1 (AR, ratio of diameter to depth), and kept fixed for all cases. The particle mesh for a sharp-edged disk possesses 1536 triangular faces, whilst the bevelled edged disk has 1116 faces. To capture the orientations of a particle, orthogonal axes in real space are taken as a reference, with α and γ measured in the horizontal plane, from the x and z axes respectively, while β is the angle from the y axis. The size of the disks is obtained in terms of the volume equivalent to a sphere with diameter d_{es}, $g = 9.81ms^{-2}$ is the gravitational acceleration, the fluid kinematic viscosity $\nu = 10^{-6}m^2s^{-1}$, and $\rho_p = 2650\ kgm^{-3}$ and $\rho_f = 1000\ kgm^{-3}$ (equivalent to water) are the densities of the particle and fluid phases, respectively. The mass moment of inertia (I) for a particle is calculated based on the density and size of a particle. I for the vertical axis $(\beta = 0^0)$ lies between $1.256 \times 10^{-11}kgm^2$ and $4.021 \times 10^{-10}kgm^2$.

The settling of spheres was first considered as a validation, with comparisons made to previous studies [11] [12]. In this case, the blocking ratio (BR) is defined as the ratio of the particle diameter to the edge length of the square base of the column. For spheres, the Corey shape factor = 1.0, AR = 1, and $BR = 0.3125$ are considered, with the lower the BR, the greater the distance to the wall. Further, the number of spectral elements per particle diameter $(SEPD)$ varied between 1 to 8. This range is sufficient to test the sensitivity of the resulting motion to the grid resolution. $SEPD = 6.25$ is chosen for plotting settling histories with time, which are presented in Fig. 1 (left). Here, the vertical and horizontal axes are normalised with the terminal velocity (U) and time required to achieve 95% of the maximum velocity (t_{95}), respectively, and excellent agreement is obtained for the settling velocity (w) data. The settling mechanism of disks is unlike that of the spheres, as demonstrated in Fig. 1 (right).

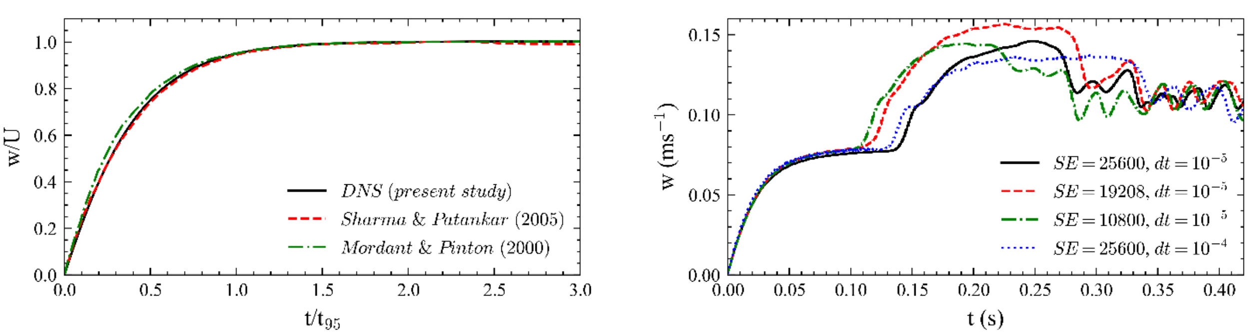

FIGURE 1. Comparison of settling rates with time for spheres with $\rho_p/\rho_f = 3.0$ (left) and sensitivity of DNS analysis against SE and dt for $d = 1mm$ disks with sharp edges and $\rho_p/\rho_f = 2.65$ (right).

In the work below, a blocking ratio of $BR = 0.125$ was used in all cases, with the methodology adopted identical to that used for spheres except for the geometry and flow properties. The bevelled edges in the particle mesh were made with an outer diameter equivalent to the depth of the disk, such that the cross section exhibits smooth semi-circle like curvature at the edges. The motion of circular disks is shown in Fig. 1 to be sensitive to grid resolution and time step. As a result, several attempts were made to test the relationship between the vertical components of velocity and time for a range of spectral elements per water column (SE) and dt. It is observed that larger SE resulted in a lower maximum settling velocity in the initial stage. The higher value of SE (26500) resulted in the maximum velocity being achieved at later times, at the point the particle changed its orientation. A step size $dt = 10^{-4}\ s$ appears to be less informative comparatively and fails to capture the particle oscillations over time.

RESULTS AND DISCUSSION

The motion of bevelled disks was simulated and compared with [9], although the geometric features of the particle mesh are missing in [9], hence an exact comparison is difficult. Further comparisons were made with disks with sharp edges, with the settling velocities presented in Fig. 2 (left). The plots are analysed by splitting each into several regions separated by crosses on each curve. Region I begins from the origin to the first cross, region II is between the first and second crosses, etc. Initially, [9] observed a smooth variation of settling velocity, attaining a terminal value for a period before it decreased due to the development of a reciprocating particle wake. The present study found additional details on the motion dynamics of disks. For instance, the disk accelerates immediately on release from rest (region I), then starts oscillating when the position is still in a horizontal configuration, $\alpha, \beta, \gamma = 0^0$(region II), and shows oscillations without changes in orientation. Finally, the rotation of the particle is seen in region III, which exhibits significant development in the velocity (> 2 times the magnitude in region I) due to particle rotation. For the initial rotation, the particle experiences a higher drag relative to the previous reduced drag configuration. This is also confirmed in the comparison study [9]. The trends in settling velocity obtained in this study behave like step curves (regions II and III). These variations are completely missing in [9], likely due to the numerical methodology adopted. Furthermore, from Fig. 1 (right) it was demonstrated that the behaviour in region II is sensitive to the number of spectral elements, hence discrepancies could also be due to insufficient numerical resolution in their computations.

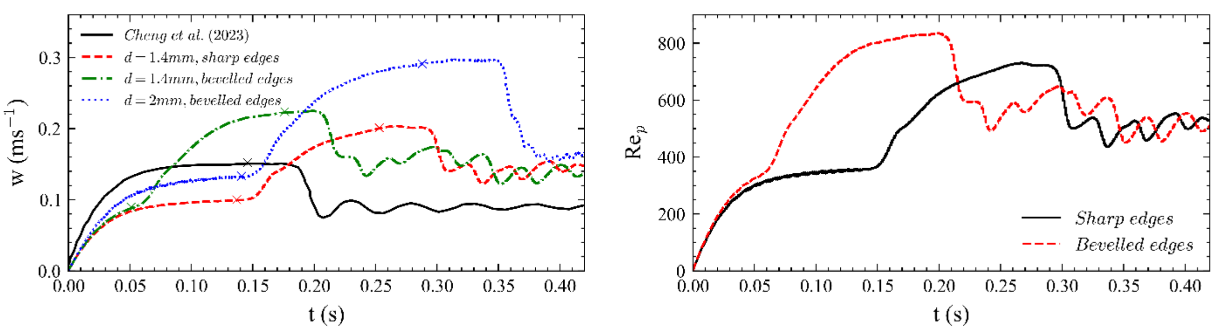

FIGURE 2. Settling rate of disks with sharp and bevelled edges for various sizes and $\rho_p/\rho_f = 2.65$ (left) and local particle Reynolds number for $d_{es} = 1\ mm$, maximum $Re_P = 806$ (right).

It is also important to consider the effect of the bevelled edge on the resulting dynamics (Fig. 2). A smooth edge leads to the maximum settling velocity increasing by more than double in Region II. The sharp corner creates additional drag and results in a lower settling rate. However, due to being the same size, the order of magnitude of velocity is the same in region III, and there is little effect on settling rate when the particle achieves a stable state. The trends in Fig. 2 demonstrate both mean and fluctuating components in the vertical velocity. The maximum in $Re_p \approx 835$, and small fluctuations about the mean velocity are expected at this value. This is due to resultant net viscous forces acting on the curved surfaces, which force the particle to rotate locally, with sufficient mean drag force available to fully turn the particle. A maximum value of $Re_p \approx 732$ was obtained for the sharp-edged disk, hence both particles are in the transitional regime. Ga ranges between 127 and 370, meaning that chaotic motion would be expected depending on the Re range [13], although the low Re (subcritical regime, Re<1000) means that significant irregularity is missing from the plots. For higher Re, the fluctuating components would grow further and oscillations in the mean components would produce small-scale motions leading to more turbulent flow around the particles.

Orientations of both types of disk are compared in Fig. 3, with orthogonal components of angle parameters, α, β and γ, for $d_{es} = 1mm$. Initially, all values are zero when the particles are at rest. The onset of rotation appears at $t \approx 0.15s$, where α reaches 90^0 for sharp edged disks, although this is not observed for bevelled disks (Fig. 3(a) and (d)). This shows that the magnitude of the drag force induced by the sharp corners is sufficient to overcome the torque about the particle's geometric axis in the vertical direction. There are smooth transitions of forces for the bevelled disk at low times. The sharp corners impose slightly different physics, causing a lower frequency oscillation compared to the bevelled edges over time, with a delay in the start of the rotation of sharp-edged disk, and a shift in the degree of rotation. The distribution of all forces is aligned in the direction of motion for bevel-edged particles and, as a result, these settle faster. However, the dynamics of the rotation of the disks in both cases are qualitatively similar.

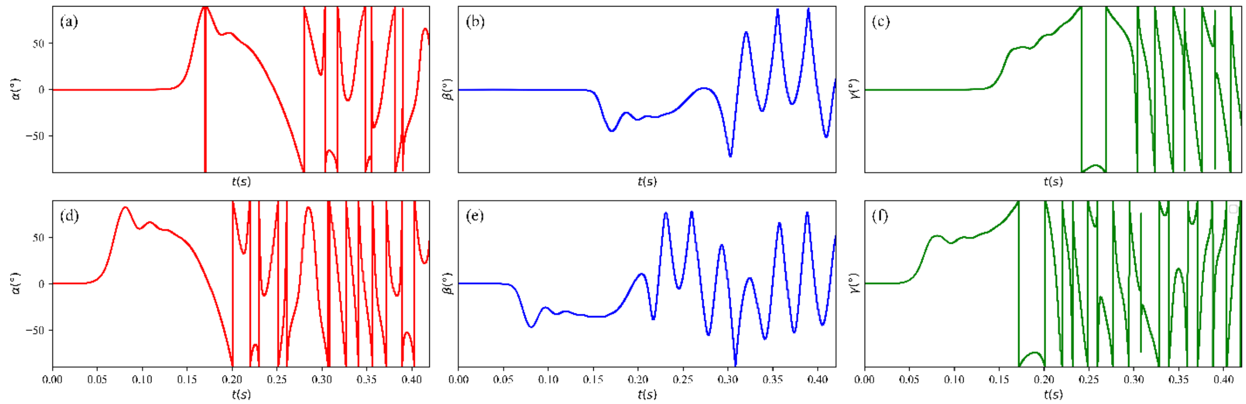

FIGURE 3. Tracking of orientations in three orthogonal axes for $d_{es} = 1mm$, (a)-(c) – sharp edges, (d)-(f) – bevelled edges.

CONCLUSIONS AND FUTURE WORK

Disks initially dropped with a horizontal alignment exhibited lower settling velocities due to drag, while vertically aligned disks settle faster for the same particle size. The settling velocity of bevelled disks is higher than for sharp edged particles, increasing by more than double for the same particle size, due to the additional drag imposed by the sharp corners. An enhancement in the settling rate is also caused by continuous changes in the orientation of the particles, such that hydrodynamics effects have a strong influence on settling rate. The predicted changes in orientation and the eventual settling velocities clearly depend upon the methodology adopted for the numerical analysis. This study suggests that estimation of the fate of polymers in a quiescent fluid needs to be reconsidered due to the influence of the shape of the particles in response to hydrodynamic effects. Future work will simulate other non-spherical particles, with application above the subcritical regime to explore the influence of turbulence around the particles.

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