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Towards nonlinear model predictive control of flexible structures using Gaussian Processes

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Abstract. In recent years, there is a growing interest of using and implementing data driven control in structural dynamics. This study considers applying Nonlinear Model Predictive Control (NMPC) to flexible structures by utilising recent developments in models which have been learnt from example data, i.e. machine learning approaches. The Gaussian process (GP) is a Bayesian machine learning algorithm identified for use as a black-box model in NMPC; it provides both the prediction of the system output and the associated confidence. In a control context, a GP can be utilised as a discrepancy model for linear or nonlinear flexible dynamic structures within MPC or even as the nonlinear model of the system itself. The Nonlinear Output Error model (GP-NOE) is a popular GP structure for dynamic systems that is utilised in predictive control strategies and requires predictions to be propagated to the control horizon. This novel framework is evaluated on a cantilever beam with light damping, and the results demonstrate robust control performance in both tracking and regulator tasks. The positive results inspire additional investigation into the proposed technique, particularly in the setting of a fully nonlinear system with unknown dynamics, such as an actuator within the flexible structure.

1. Introduction

Vibration control in structural dynamics is still an active area of research, yet data-driven approaches to modelling and control in active vibration applications have not been thoroughly explored [1]. One of the reasons for the lack of implementation of these recent advances is the difficulty in developing a reliable dynamic model of linear or nonlinear dynamic systems, particularly when applied in control systems [2]. Furthermore, flexible structures are susceptible to environmental effects, resulting in excitations, as well as issues related to damage, degradation, and uncertainty [3]. Therefore, there is an urgent need to investigate whether data-driven control can improve the control performance of an active vibration system, particularly using nonlinear model predictive control (NMPC) and Gaussian process (GP) techniques.

GP is a probabilistic nonparametric modelling approach and its popularity in dynamic system identification stems from its ability in predicting the output value of the system associated with the measure of its confidence. More importantly, the GP can be used in a time series model to highlight parts of the input space where there is insufficient data and model systems which exhibit nonlinearity [4]. The Gaussian Process Nonlinear AutoRegressive model with eXogenous input (GP-NARX) has been extensively employed in structural dynamic applications such as wind turbines [5], and bridges [6]. In the structural health monitoring context, GP-NARX can

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be formed into One Step Ahead (OSA) prediction, and Model Predicted Output (MPO) [7]. While MPO is a GP structure in which previous predictions are fed back into the model, OSA prediction involves a model making a single-step prediction into the future based only on previous measured outputs. Having said that, the GP-NARX model is only one step ahead prediction in control systems, whereas the multiple predictions ahead while when previous predictions are fed back into the model is called the Gaussian Process Nonlinear Output Error model (GP-NOE) [11]. Consequently, this work follows the terminology in control systems engineering.

Furthermore, the popularity of system identification using GPs has sparked interest in NMPC, particularly given the efficacy of data-driven modelling in control engineering. One of the main advantages of NMPC is its ability to deal with practical constraints, such as input energy or state limits. As a result, GPs have been integrated into NMPC to benefit from its prediction uncertainty, with control input adjusted to take the region of uncertainty into account. This control strategy was originally used in a first-order process system as a theoretical approach in [8], and later in a chemical system in [9]. Although the use of a GP model in a control system is not new in control applications such as car racing [2] and unmanned quadrotor [10], including a fully offline GP model into an NMPC has been rare [11]. The contribution of this work is implementing GP as a black box model trained and fixed offline in which it gives the prediction of the output while optimising inside NMPC. A simulated cantilever beam serves as a case study to validate the proposed methods.

The layout of this paper is as follow, Section 2 provides a brief introduction to GP and fundamental procedures for system identification of dynamic systems. Section 3 presents the NMPC theoretically, as well as the controller and reference trajectory designs. Section 4 describes the application of the GP-NMPC framework to a flexible structure, namely a cantilever beam. Section 5 contains the concluding remarks and future work from this project.

2. Gaussian Process in system identification

To help comprehend the proposed framework, a brief overview of GP and dynamic system identification is provided by summarising the existing literature [11] and [12].

2.1. Gaussian Process Regression

The Gaussian process is a Bayesian approach and it is defined as a collection of random variables which have a joint multivariate Gaussian distribution [12]. Within the context of regression problems, a GP formulates a prior over the latent function $f(x)$, which is depicted in Equation (1). Here, x represents a vector of training inputs and X represents a matrix comprised of multivariate input data, and y represents the associated vector of output data obtained for training. The noise component is modeled as a Gaussian-distributed random variable with zero mean and noise variance σ_n^2, $\epsilon \sim \mathcal{N}(0, \sigma_n^2)$.

$$y = f(x) + \epsilon \tag{1}$$

where,

$$f(x) \sim GP(\mu(x), k(x, x_*)) \tag{2}$$

The function f represents a hidden variable that is not directly observable. A GP is comprehensively described by its mean function, denoted by $\mu(\cdot)$, and its covariance function, denoted by $k(\cdot, \cdot)$. These functions encapsulate the prior assumptions about the nature of the underlying latent function. The mean function can be articulated as any linear combination of basis functions dependent on x, with the flexibility to extend to different input spaces.

Meanwhile, the covariance function is responsible for quantifying the extent of covariance between any two points within the input space.

As we obtain a training dataset $\{X, y\}_{n=1}^{N}$, we can update the GP prior based on this dataset to establish a posterior distribution y_* for a new unseen input x_*. Following the methodology outlined in [12], standard Gaussian equations provide us with an explicit formulation for the posterior distribution of y_*,

$$p(y_*|x_*, X, y, \theta) = \mathcal{N}(\mu[y_*], \Sigma[y_*]), \tag{3}$$

where the expected mean value μ and variance Σ are defined as:

$$\mu[y_*] = K(x_*, X)(K(X, X) + \sigma_n^2 I)^{-1} y, \tag{4}$$
$$\Sigma[y_*] = K(x_*, x_*) - K(x_*, X)(K(X, X) + \sigma_n^2 I)^{-1} K(X, x_*). \tag{5}$$

Defining a covariance function typically requires selecting several hyperparameters θ, which can include coefficients for a mean function. These hyperparameters modify the kernel's behaviour and usually have an interpretable meaning. For instance, a common parameter in many kernels is the length scale, which practically determines the required proximity of inputs in the same dimension to affect each other. The Gaussian process framework allows for a systematic approach to estimate these hyperparameters by maximizing the model's marginal likelihood, also known as the model evidence.

2.2. GP model setup

The challenge with GP system identification is that selecting a GP model set is quite hard especially when the relationship between the input and target training data is nonlinear. For simplicity, this section provides a general overview of establishing an accurate model. The first step in the identification process begins with the purpose of the model in which it leads to level of details. Since the aim of this work is control, that requires a higher number of sample data. However, obtaining a high number of sample data leads to extensive computing.

Identifying the GP structure is the second step. As mentioned, GP-NOE model has been identified because GP-NOE provides prediction for a number of steps ahead . GP-NOE used a regressor that included output estimates $\hat{y}(k-i)$ and input values $u(k-i)$. The mathematical description is as follows:

$$\hat{y}(k) = f(\hat{y}(k-1), \hat{y}(k-2), \dots, \hat{y}(k-L), u(k-1), u(k-2), \dots, u(k-L)) \tag{6}$$

where L represents the number of lags. There are two types of simulations when GP-NOE utilised as GP structures: naive simulation and approximation. The main difference is that naive simulation does not propagate uncertainty and Equations (4) and (5) can be used. Although the approximation simulation has been applied in [12], this work uses naive simulation for computational efficiency. The key success of the GP-NOE is also based on choosing a suitable model order. In other words, the number of lags required to capture the dynamics of the system. This step requires a number of objective measures for comparison such as the standardised Mean Square Error (SMSE) [12].

Finally, the remaining step to complete the set up of the GP model is selecting the covariance function $k(x, x_*)$. In practice, squared exponential (SE) is the most used covariance function in modelling physical systems. However, SE with automatic relevance determination (ARD) is the most commonly used covariance function in predictive control with GP, especially when the model is fixed and trained offline. The SE-ARD is shown as follows:

$$k_{SE-ARD}(x, x_*) = v_1 \exp\left(-\frac{1}{2}\sum_{d=1}^{D} w_d (x_{d_i} - x_{d_j})^2\right) + v_0 \tag{7}$$

where $[w_1, \ldots, w_D, v, v_0]^T$ represents a hyperparameter vector θ. While D represents the length of the regressor vector x, parameter v determines the magnitude of the covariance, and w_i represents the relative importance of each component x_d of the regressor vector. v_0 denotes the white noise variance. The importance of this function is that the length scale is optimised for each lag of the delayed output and input values.

3. Nonlinear model predictive control

The conventional MPC is often based on using the system model to predict the system output for a number of predictions ahead at the present output measurement. The system's prediction of a preset cost function, while meeting input and state constraints, is then utilised to determine the optimal values for future control inputs. In this work, a GP-NOE model serves as the system model within MPC. The key factor of this GP model is being trained and fixed offline, meaning the performance of NMPC is relying on prior knowledge of the system dynamics and the model's hyperparameters. The general description of NMPC method is described in great details in [11] and the block diagram is depicted in Figure 1.

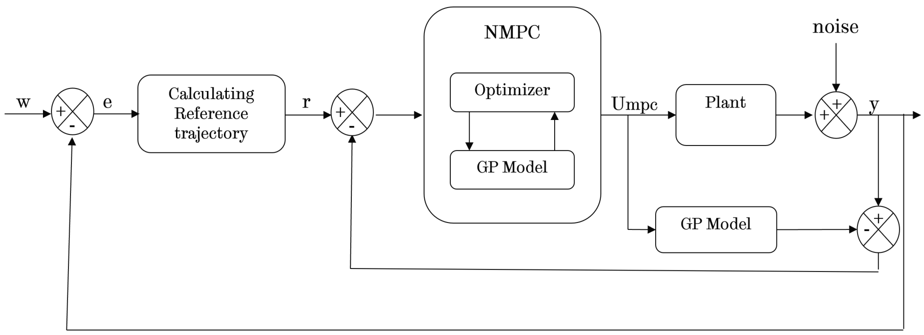

Figure 1. General block diagram of GP-NMPC. The main part of this method is based on the algorithm within the NMPC block.

3.1. Controller design

The selection of appropriate cost function is a critical step in designing an NMPC controller. Generally, NMPC solves a constrained or unconstrained control problem and can be written in the unconstrained form as follows:

$$\min_u J(u, x(k), r(k)) \tag{8}$$

where u is the control input, $x(k)$ is the state at time k, and $r(k)$ is the reference value at time k. The NMPC, however, utilised a GP as the system model in which the NMPC is based on a nonlinear, discrete-time system described mathematically in Equation (6). As a result, The optimisation problem is

$$\min_u J(u, \hat{y}(k), r(k), u(k-1)) \tag{9}$$

where

$$J(u, \hat{y}(k), r(k), u(k-1)) = [r(k+P) - E(\hat{y}(k+P))]^2. \tag{10}$$

Equation (10) is based on predictive functional control technique. This method requires that the value of the reference trajectory at the coincidence point P matches to the estimated output value of the system. Although the method's significance comes from the fact that the closed loop response is expected at the coincidence point, selecting the appropriate coincidence point remains challenging. The control sequence is obtained by finding the optimal solution of Equation (9):

$$u_o = [u_o(k), u_o(k+1), \ldots, u_o(k+P-1)]. \tag{11}$$

Once this optimisation problem is solved, the control law can be formed based on the receding horizon principle. This states that the system is only excited by the first control value in the optimised sequence, before the optimisation problem is re-solved. This repeated action of applying the first optimised control in the sequence defines a feedback control law:

$$\kappa_N(x) = u_o(k). \tag{12}$$

3.2. Design of a reference trajectory

The remaining control design is now generating the NMPC controller's reference trajectory. The goal of developing the reference trajectory is not only to determine the trajectory that the plant should take to return to the set-point trajectory, but it is also crucial in determining the controlled plant's closed loop behaviour. Determining the reference trajectory is firstly set by identifying the current error between the set point trajectory $w(k)$ and the current output measurement value $y(k)$ as shown below:

$$e(k) = w(k) - y(k), \tag{13}$$

Since it is assumed that the reference trajectory is considered to approach the set point exponentially from the current measurement output value, the next step is to calculate the error at the number of steps (i) in the following way:

$$e(k+i) = e^{-iT_s/T_{\text{ref}}} e(k), \tag{14}$$

where T_s is the sampling time and T_{ref} is the time constant of the exponential defining the speed of the response. Then, the reference trajectory ended up being completely defined, as shown below:

$$r(k+i) = w(k+i) - e(k+i) = w(k+i) - e^{-iT_s/T_{\text{ref}}} e(k). \tag{15}$$

Finally, the general algorithm used with NMPC is shown in Figure 2.

4. Identification and controlling of a cantilever beam

To demonstrate GP-NMPC in structural systems, a linear continuous cantilever beam was employed as an illustrative example. Even though the MPC controller is nonlinear, the ultimate purpose of this work at this stage is to develop a robust control system that provides an adequate understanding of GP-NMPC and leads to a fully nonlinear system. The dynamics of this flexible structure are detailed in great depth in [13], and the system is formed up of a uniform beam with a point force actuator and sensor at the tip. Table 1 displays all of the important parameters. For simplicity, this section is divided into three subsections. It starts with generating dynamic data to form a relationship between the input and output of the system. Then, the process of selecting the GP model is described in the model identification. Finally, the control design and its performance is described in the control performance section.

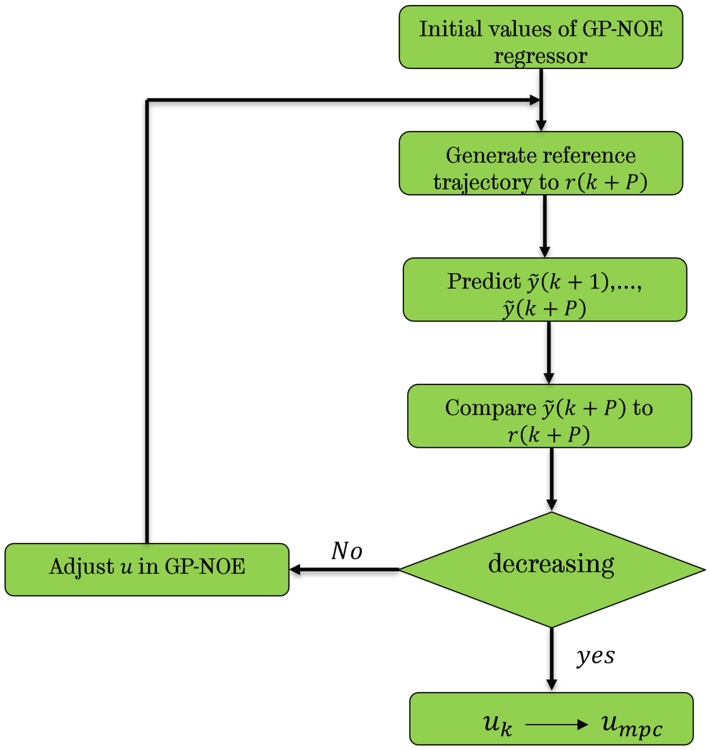

Figure 2. Steps of the GP-NMPC. The decision box represents the cost function of the NMPC optimisation.

Table 1. Parameters of the dynamics and control system.

Element (Symbols)	Value (Units)
Height (h)	25 mm
Width (b)	1 mm
Length (L)	350 mm
Modulus of Elasticity	210 GPa
Density of steel (ρ)	7850 (kg/m^3)
Damping ratio (ζ)	1%

4.1. Obtaining data
Two separate sets of dynamic data were obtained as a result the GP model intended to be used in control system. The first set was collected by exciting the cantilever beam using a random input generator that lasted 7 seconds. The input magnitude ranged from -7 to 7 N, with a total of 350 training points. The validation data was obtained using the same method but over extended periods of time. The input signals were 9 seconds long, with 450 points for validation. To obtain a full experimental design, white noise with a variance of 0.5 and a mean of zero was applied to both the identification and validation sets of data. Figure 3 displays the cantilever beam responses and input signals for both data sets. It is worth noting that the number of training points affected the dimension of the covariance matrix. The drawback of obtaining a high number of sample points is leading to longer computation time. Consequently, a thumb rule is to choose a number of training points that capture the system dynamics sufficiently.

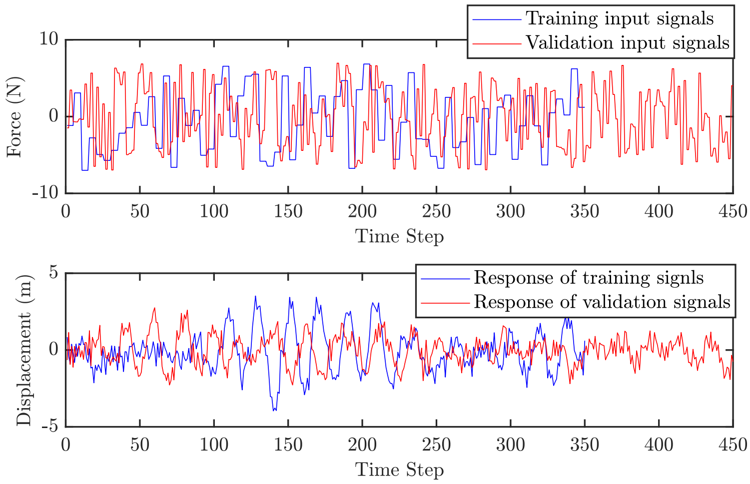

Figure 3. Upper figure shows the random input signals for both sets of data. Lower figure depicts the responses of a cantilever beam for both data sets, where the white noise effects were visualised.

4.2. Model identification

After acquiring dynamic data, the process of selecting an appropriate GP model began by determining the system's order. Intuitively, the cantilever response will be dominated by its first mode of vibration, which would imply the use of a GP-NARX model with at least two lags. Table 2 compares several higher order models, based on some validation features. It is not clear, however, whether the GP is able to capture richer dynamics than just the first mode since it learns a nonlinear function of the lags rather than a linear one. In contrast, for an ARX model, there is a direct equivalence between the number of lags and the number of dynamic modes. The squared exponential ARD covariance function was then used, providing a specified scale for each lag. Conjugate gradient was chosen as the optimisation approach for this project due to its convergence properties. The hyperparameters were then trained using the maximum likelihood method. The selected set of hyperparamters were $\theta = [6.5, 6.07, 10.57, 13.9, 6.23]$.

Figure 4 shows the GP model for the training data and it is clear that it fits the system response quite well. The final step in this modal identification is to validate the GP model setting with a different dynamic data sets. Even though the validation signal was longer than the training signal, Figure 5 shows that the selecting GP model setting was also suitable for validation data. Having said that, the GP model is now suitable for NMPC as an offline and fixed GP model.

Table 2. Validation of the identification data based on the selecting set of GP model.

Order	SMSE	MSLL
1	0.092	–1.09
2	0.052	-2.62
3	0.053	-2.58

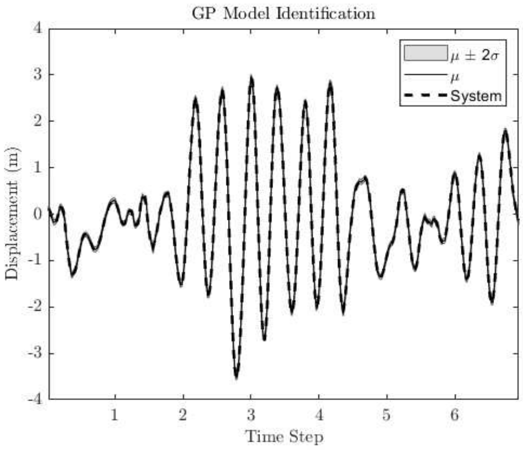

Figure 4. The GP model fits the response of the flexible structure by using training data.

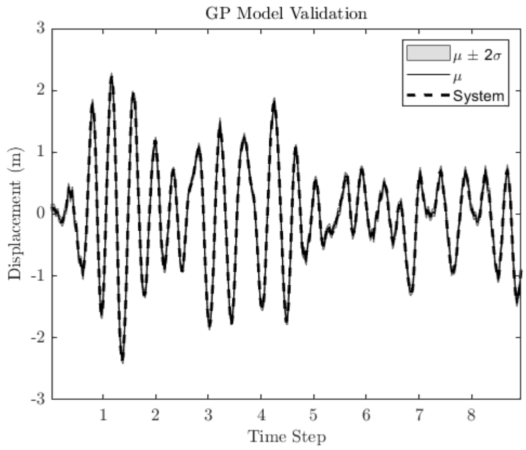

Figure 5. The GP model fits the response of the flexible structure by using validation data.

4.3. Control performance

Before demonstrating the control performance of GP-NMPC applied to the cantilever beam, several control settings must be defined. First, selecting an adequate sample time T_s is an important step in control design. Equation (14) sets T_s to be 0.01 second for the reference trajectory design. The number of predictions is 9. Despite the general rule that more prediction leads to better control performance, particularly when the system is linear, it makes the computation time longer. Finally, the control horizon is one.

The closed loop responses of unconstrained control in both tracking and regulator tasks are given in Figures 6 and 8. It is clear that the GP model was able to predict the dynamics of the reference trajectory in which it leads the NMPC optimiser to provide the best control action.

5. Conclusions

This paper presents a novel GP-NMPC controller that has been applied to the dynamics of flexible structures. The introduced framework uses GP-NOE as an offline, fixed model within the NMPC controller. The GP predicts the output values and their uncertainty, allowing the control optimiser to avoid regions with high uncertainty. Numerical results demonstrate the effectiveness of this novel framework in the cantilever beam.

The value and contribution of this work can be summarised in two points. First, it is an initial step towards clarifying some GP structure terminologies used in the GP community in the fields of structural dynamics and control systems. Second, this framework obtains vibration control by utilising the most recent advancements in data-driven modelling and control. The next step with GP-NMPC control framework is to explore the efficacy of this method when the dynamics of the system is nonlinear.

6. Acknowledgement

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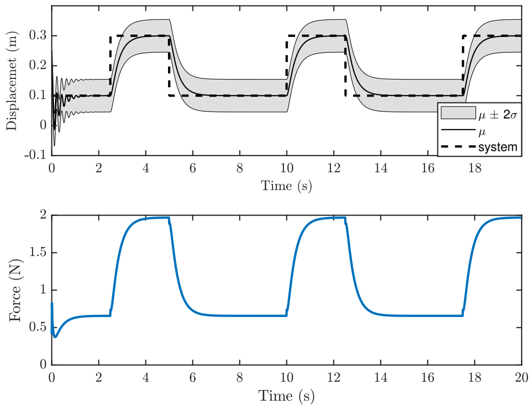

Figure 6. Upper figure shows the response of GP-NMPC with highlight the uncertainty region of the prediction. Lower figure depicts required control inputs in order to track the setting points.

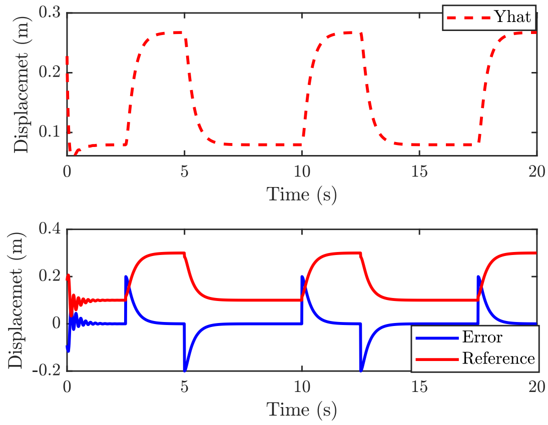

Figure 7. Upper figure shows the GP prediction within NMPC and it is clear the model was able to predict the dynamic changing. Lower figure depicts the generated reference within NMPC (red line) and its errors (blue line).

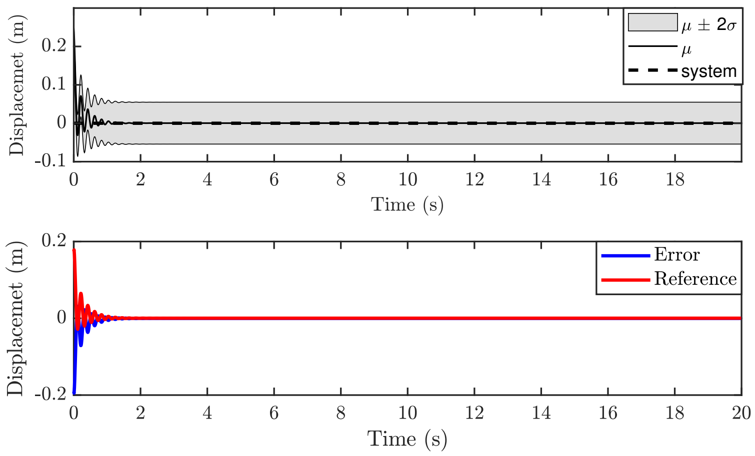

Figure 8. This figure shows the response of GP-NMPC of regulator tasks.

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