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4 **NOVEL GREY-VECTOR OPTIMIZATION OF DESICCATION-INDUCED SHRINKAGE AND**  
5 **STRENGTH OF INDUSTRIAL WASTE-BASED SOIL COMPOSITE BINDER AS SUSTAINABLE**  
6 **CONSTRUCTION MATERIAL**  
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21 **ABSTRACT**

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24 Infrastructural development geared towards sustainable cities and communities requires the adoption of sustainable  
25 construction materials. As a contribution towards this, optimization of the unconfined compressive strength (UCS)  
26 and desiccation-induced shrinkage (DS) behavior of expansive soil stabilized with alkali-activated waste materials  
27 was executed. A novel approach was applied for the multiple characteristics optimization of the developed industrial  
28 waste-based soil-composite binder for use in construction. Laboratory experiments were done employing Taguchi  
29 design and considering several process parameters. The outcome of the single response optimization based on Taguchi  
30 signal-to-noise ratio (SN) analysis resulted in conflicting optimal levels for the UCS and DS. Hence, a novel multi-  
31 response optimization was applied to concurrently optimize the UCS and DS based on grey-vector analysis (GVA).  
32 The outcome of the GVA identified the greatest factor effects on the vector resultant to be waste quantity, type and  
33 preparation method. The new method was compared with the renowned Taguchi-grey relational analysis and was  
34 found to be more computationally efficient, in addition to matched performance based on Euclidean distance between  
35 the optimal solutions. Thus, the new method can serve construction engineers in development of alkali-activated  
36 waste-treated soil as sustainable construction material for embankments and waste containment.  
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44 **Keywords:** desiccation-induced shrinkage, grey-vector analysis, industrial wastes, optimization, unconfined  
45 compressive strength.  
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48 **Introduction**

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51 Expansive soils, which are widely distributed globally potentially constitutes an integral component of many civil  
52 engineering infrastructures such as roads, embankments, fills, buried pipelines, railways and waste containment  
53 facilities. Indeed, the utilization of such material in construction applications is clearly inevitable. Great concerns  
54 however, arise due to the adverse volume change orchestrated by the proliferation of moisture infiltration in contact  
55 with the soil, which causes severe distress to overlying infrastructure and other geo-structures. Instructively, this nature  
56 of expansive soils obviously encourages various mechanisms of infrastructure deterioration, one form of which is the  
57 renowned desiccation-induced shrinkage (DS) cracking.  
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4 Superficial and subterranean cracking of expansive clays resulting from desiccation effect on environmental exposure  
5 has been reported as the cause of various geo-hazards including failure of pavements, waste containment  
6 infrastructures, reservoirs, natural and engineered slopes [1, 2, 3, 4]. In the case of road pavements, macro-cracks  
7 constitute pathways for moisture infiltration and the subsequent loss of bearing strength of the subgrade layer that acts  
8 as the pavement foundation, ultimately resulting to pavement failure [4, 5, 6]. In waste containment infrastructures,  
9 desiccation – induced volumetric shrinkage leads to increment in the permeability of the liner material, which can  
10 permit leachate, mine tailings or other toxic wastes to find its way to the groundwater, in addition to the shear failure  
11 of engineered landfills [1, 8, 9]. Similarly, surficial slope failures occur due to cracks, which not only increases  
12 infiltration to increase pore water pressures, but also causes suction loss that ultimately results in shear strength  
13 attenuation with time [4, 10, 11]. These considerations therefore necessitate the adoption of sustainable technologies  
14 to mitigate the DS cracking of expansive soils, while improving strength for durability as a veritable construction  
15 material.  
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23 In this regard, various technologies have been experimented in recent years to improve expansive soils in a sustainable  
24 manner. This is often attempted by adopting the use of industrial waste materials, which not only serve as a soil  
25 improvement strategy, but also serves the complementary purpose of waste management. Several industrial wastes  
26 have been used in soil improvement, including rice husk ash, fly ash, quarry waste (QW), tailings, waste tires and  
27 others [12]. In the study by Ahmed and Naggari [13], basanite-tire waste mixtures were deployed to improve the  
28 strength of a silty soil. The results show that the addition of recycled basanite waste had a more positive effect on the  
29 compressive strength improvement, while the rubber tire waste increased the tensile resistance of the soil. In a related  
30 study [14], various percentages of waste marble powder additive (5, 10, 20, 30 and 50%) and curing time (0, 7, 30 and  
31 60 days) were considered for soil strength improvement. The UCS of the uncured soil samples attained peak values at  
32 5% marble waste content, whereas the cured specimens only showed strength gain at 7 days, which suggest a threshold  
33 for the occurrence of chemical reactions within the soil-additive matrix. The stabilized soil generally resisted freeze-  
34 thaw cycles up to the 5<sup>th</sup> cycle before failure. It was also reported that mass loss increased with increase in the freeze-  
35 thaw cycles; however, for each cycle, the stabilized soil had lower mass loss in comparison with the untreated soil,  
36 indicating its durability for practical application. Other studies have similarly reported strength improvement with the  
37 addition of industrial waste materials [15, 16, 17, 18].  
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47 Instructively, these studies suggest that soils can be improved using industrial wastes, albeit the extent of improvement  
48 achievable is less when compared with the conventional materials such as cement and lime, which are considered to  
49 be eco-unfriendly. The search for a more robust technology birthed the utilization of geopolymer technology in soil  
50 improvement, which has comparative strength improvement with cement and lime, while maintaining eco-friendliness  
51 [19, 20]. Geopolymer soil stabilization has emerged in recent years in the area of soft ground improvement and has  
52 been widely reported. Murmu et al. [19] improved the properties of black clay using fly ash waste as geopolymer  
53 binder, considering different binder contents and curing ages (0, 7, 14, 28 and 90 days). The outcome of the study  
54 revealed that with increase in geopolymer content and curing time, the UCS and bearing strength of the soil  
55 significantly improved. The UCS increased from an initial value of about 200 kPa to an ultimate high of about 3250  
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4 kPa, while the bearing strength improved from 3% to about 42% under unsoaked condition. Furthermore in a similar  
5 study [21], the shrinkage limit of black clay was minimized from a value of about 32% to less than 1%. Similar studies  
6 have also shown that geopolymer soil stabilization using industrial waste materials significantly improves soils for  
7 various construction applications [22, 23, 24].  
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11 Previously, it has been reported that several process parameters influence the development of geopolymer binders for  
12 soft ground improvement, which necessitates the costly and time-consuming experimental trials [20, 25]. Thus, this  
13 motivates the search for robust optimization approaches for their development. Taguchi method has shown to be the  
14 preferred and reliable method so far and is typically opted for in the robust development of geopolymers for  
15 construction applications [26, 27, 28]. However, the Taguchi method can only optimize responses singly and hence;  
16 a multi-response technique has to be integrated into it for the concurrent optimization, which inspired a novel aspect  
17 of the present study.  
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21 In the present study, geopolymer binders based on industrial wastes including QW and palm oil fuel ash (PFA) were  
22 used to minimize the DS of an expansive soil while maximizing the UCS, adopting the Taguchi method. The novel  
23 highlight of this study showcases an optimization method, which has been developed from the simple concept of  
24 vector analysis. Furthermore, a comparative analysis is conducted with a renowned multi-response method based on  
25 grey relational analysis (GRA), and the advantages of the new method have been emphasized.  
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## 30 31 **Optimization Background**

### 32 33 **Taguchi method**

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35 Design of experiment that is based on the Taguchi method is a renowned robust approach for product development.  
36 The method employs the use of orthogonal arrays, which ensures that the experimental space is navigated in a balance  
37 manner to achieve an optimized product [29]. Moreover, the signal-to-noise ratio (SN) optimization function is used  
38 to ensure minimum deviation from a specified target to ascertain the overall quality of the product development  
39 process [29]. These unique attributes, make the Taguchi method a very simple and reliable method for industrial  
40 product development. Lately, ample applications of the method have been reported in different ground improvement  
41 schemes for single response optimization [29, 30, 31]. Thus, it is a highly favored method for the construction industry  
42 in single response scenarios.  
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49 On the other hand, when multiple responses are involved, other multi-response methods have to be integrated with the  
50 Taguchi method. Some of these methods include genetic algorithm weighting, GRA, data envelopment analysis, utility  
51 concept, to mention but a few. More often than not, in addition to the issue of lack of distinctiveness in the optimal  
52 solution; these methods require great computational effort and are often based on advanced analytical or numerical  
53 approaches, which make them inapt for direct practical use [7, 32]. For instance, in the analytical method based on  
54 GRA, several computations are required for the multi-response attribute known as grey relational grade (GRG). These  
55 computations include: comparability sequence (grey relational generation), deviation sequence, grey relational  
56 coefficients and then the GRG. Moreover, a weighting factor, which is more commonly denoted as the identification  
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4 coefficient,  $\zeta$  is required for weighting the prioritization of each multiple attribute [33]. As such, various methods are  
5 adopted for its selection namely intuitionist approach, entropy weighting method, analytic hierarchy process, et cetera.  
6 This further compounds the GRA and could introduce bias depending on the level of subjectivity inherent in the  
7 adopted response prioritization method. This challenge is similarly shared by other multiple response methods such  
8 as the utility concept [28]. Apparently, there is need for improved techniques that will surmount the aforementioned  
9 drawbacks associated with Taguchi multi-response optimization. In view of this, a new approach is proposed in the  
10 present study.  
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### 15 **Proposed grey-vector analysis**

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18 In this study, a method is presented, which is based on the processes of grey-based normalization and vector analysis.  
19 In an orthogonal array experiment, each individual response data representing the quality attributes of a product is  
20 obtained as a  $n \times 1$  vector. Instructively, this data is rightly a vector since it has magnitude and a direction, which is  
21 defined by the optimization objective. As such, it is amenable for vector analysis using algebra of vectors. Since the  
22 concurrent attainment of the desired optimum is required for the product development, the concurrent effect of all the  
23 vectors representing the quality attributes is determined. In vector physics, this overall effect of vectors is known as  
24 the resultant vector,  $R$  and can be expediently obtained from vector algebra. The method proposed in the present study  
25 is mathematically represented thus:  
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31 Consider a product, in which the overall quality  $Y$  is based on  $X_k$  attributes defined as the function in Equation 1.

$$32 \quad Y = f(X_1, X_2, \dots, X_k) \quad (1)$$

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38 Each  $X_i$  attribute is considered as a  $n \times 1$  vector representing the product's experimental data space, while the overall  
39 product quality is also represented as a  $n \times 1$  vector. Hence, Equation 1 is aptly represented in vector notation as  
40 shown in Equation 2.  
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$$44 \quad \vec{Y} = f(\vec{X}_1, \vec{X}_2, \dots, \vec{X}_k) \quad (2)$$

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49 The quantities representing each *ith* row entry of the  $X_j$  column are the scalar elements for each vector, which defines  
50 the magnitude, while the optimization objective defines the vector direction. These scalar elements are the normalized  
51 SN, known as the comparability sequence. Thus, the analytical function that defines the overall effect of each  
52 component vector is the resultant vector. This resultant represents the overall multi-characteristics quality attribute,  $Y$   
53 and can be expressed as the sum of each vector component as shown in Equation 3, based on a priori definitions  
54 explained subsequently.  
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$$\vec{Y} \equiv \vec{R} = \sum_{j=1}^k \vec{X}_j \quad (3)$$

In order to assuage bias towards vectors of higher magnitude, grey normalization is applied on the data prior to the vector analysis. The min-max grey normalization functions are shown in Equations 4 and 5 for the respective minimization and maximization objectives.

$$\eta_i^*(j) = \frac{\max \eta_i^0(j) - \eta_i^0(j)}{\max \eta_i^0(j) - \min \eta_i^0(j)} \quad (4)$$

$$\eta_i^*(j) = \frac{\eta_i^0(j) - \min \eta_i^0(j)}{\max \eta_i^0(j) - \min \eta_i^0(j)} \quad (5)$$

Where  $\eta_i^0(j)$  is the base sequence, which represents the SN;  $\min \eta_i^0(j)$  and  $\max \eta_i^0(j)$  are taken as the respective minimum and maximum of the sequence (that is min and max of SN);  $\eta_i^*(j)$  is then the comparability sequence, which represents the normalized SN.

The SN for the respective minimization and maximization respectively denoted as smaller-the-better (STB) and higher-the-better (HTB) functions are given as shown in Equations 6 and 7.

$$\eta = -10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^n (X_i + X_0)^2 \right) \quad (6)$$

$$\eta = -10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^n \left( \frac{1}{X_i + X_0} \right)^2 \right) \quad (7)$$

Where the number of response data per experimental run is  $n$ ,  $X_i$  is the response value and  $X_0$  is the target value. In the case of the HTB and STB SN, the unstated target is zero.

It is noteworthy at this juncture that Equation 3 holds as long as the SN functions are used. This is true because an implicit requirement of Equation 3 is that all the  $\vec{X}_j$  have the same direction, which is attainable for SN functions that

are always maximized. Hence, when SN is used, the vector directions are the same. In contrast, when the individual responses require either of minimization or maximization, the appropriate vector resultant function has to be used in Equation 3 since the vector directions differ in an opposing manner. Similarly, Equation 5 must be used with the SN; else, either of Equations 4 or 5 should be employed for the respective minimization or maximization objective. However, the SN functions are strongly recommended for use to prevent computation errors and ensure a robust optimization design. The final optimization Equation is represented in Equations 8-9.

Maximize:

$$\vec{R} = \sum_{j=1}^k \vec{X}_j \tag{8}$$

Subject to

$$0 \leq \eta_i^*(j) \leq 1$$

$$\forall \vec{X}_j^k \tag{9}$$

The overall product quality is thus obtained from the optimization shown in Equations 8-9 taking the resultant function to be the HTB SN function.

## Methodology

### Materials

The data used in the present study is based on the Taguchi-based experiment conducted by Nwonu [35] for the development and application of expansive soil geopolymer binders. In the study by Nwonu [35], a locally sourced expansive soil with geotechnical properties as shown in Table 1 was treated with alkali-activated binders made from two industrial waste materials QW and PFA. These waste materials were also sourced locally from their point of disposal and were then characterized to obtain their chemical oxide composition summarized in Table 2.

### Laboratory experiments

Laboratory experiments were conducted to determine the UCS and DS of the alkali-activated treated samples as reported by Nwonu [35]. The UCS test was conducted in a compression testing machine based on the specifications of the British standard institute. The tested samples were initially compacted in the laboratory based on the British standard light compactive energy at the required moisture content and then cured for 7 days under humidity controlled condition for strength development. The required moisture content was obtained from the moisture density

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4 relationship for the geopolymer mixtures, which was generally within the optimum moisture content for the natural  
5 soil  $19.7 \pm 2\%$  as reported in previous study [9]. The compaction was done in the standard Proctor mold, after which  
6 cylindrical 38 mm diameter by 76 mm height specimens were extruded for the UCS test.  
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10 The DS test was conducted as per previous studies [36, 37]. The samples were prepared at the required moisture  
11 content using the previously described compaction method. Then, three cylindrical specimens of 38 mm diameter by  
12 76 mm height were extruded from the compaction mold. The cylindrical specimens were allowed to air-dry on the  
13 laboratory bench in order to closely simulate the in situ environmental condition. At least, three measurements of the  
14 height and diameter were taken using a vernier calliper until there was no further reduction in mass, representing no  
15 further moisture loss. The average measurements were used to compute the final volume and determine the volumetric  
16 shrinkage, which is the DS in this study.  
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## 20 21 **Experimental design and optimization**

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23 The aforescribed laboratory experiments were performed based on a designed experiment by Nwonu [35],  
24 employing the L8 Taguchi orthogonal array. The process parameters considered were the waste type (factor A), waste  
25 content (factor B), activator type (factor C), ratio of activator to waste content on mass basis (factor D) and method of  
26 mixing (factor E). the two levels used in factor C are the sodium hydroxide (NaOH) and potassium hydroxide (KOH),  
27 while two mixing methods M1 and M2 were used. In M1, the soil and industrial wastes were homogenously mixed  
28 before the activator was added, whereas in M2, the activator and industrial waste were mixed prior to adding to the  
29 soil. The alkali-activator comprised of an equal amount of alkali hydroxide and water glass. The factors and their two-  
30 level designations are shown in Table 3, while the orthogonal array containing both the coded and uncoded variables  
31 is shown in Table 4 along with the obtained SN. The SN was obtained for the UCS using HTB function for  
32 maximization, whereas that of the DS was computed using the STB for minimization.  
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39 The flow charts for the GVA and the GRA used for the multi-response optimization are clearly presented in Figs 1  
40 and 2 respectively. It is clear that the GVA is more computationally efficient because it requires less computation  
41 processes than the GRA for the multi-response optimization.  
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## 45 **Results & Discussion**

### 46 47 **Individual response optimization**

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49 Assessment of the effect of process factors on the DS is clearly shown in the SN plot of Fig 3. It can be seen that  
50 factors E, A and B have the most significant effect on the variation of the DS based on the magnitude of the difference  
51 in SN between the factor levels. Apparently, the method of soil stabilization that involves mixing the industrial waste  
52 with the soil prior to alkaline activation is more effective for the reduction of the DS. Furthermore, the choice of PFA  
53 as a preferable industrial waste for DS minimization is also inferred. Moreover, the use of a higher percentage (20%)  
54 of the industrial waste gives a better improvement in the DS behavior of the soil. These factors are very pertinent in  
55 achieving a stabilized soil with minimum DS. Based on the Taguchi SN analysis, the optimal factor levels is obtained  
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4 from Fig 3 as A1 – B2 – C2 – D1 – E1. Using this optimal, the predicted DS based on the Taguchi additive model is  
5 1.1% (SN = -8.849). This represents a tremendous reduction in the DS of the soil from an initial value of 20.3%, which  
6 demonstrates the stabilization capability of the alkali-activated additive used in the present study.  
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10 In a related development, a similar representation of the factor effects on the UCS of the stabilized soil is displayed in  
11 Fig 4. The factors B and C clearly had the most significant influence on the soil strength behavior. This implies that  
12 the quantity of the industrial waste and the alkali type have notable effect on the strength development of the stabilized  
13 soil. The potassium-based activator appears to give an improved soil of higher strength than the sodium-based  
14 activator. The possible underlying mechanisms which are responsible could be similarly attributed to the effect of  
15 moisture content on the ionic changes which occur at the diffuse double layer of the expansive clay [9]. This causes  
16 significant modifications in the strength of the bonds at the interlayer, which is dependent upon the nature of the ions  
17 in contact with the diffuse double layer of the soil within the clay-pore fluid media. The optimal factor level as shown  
18 by the maximum SN in Fig 4 corresponds to A2 – B2 – C1 – D2 – E1. The predicted UCS value obtained at this  
19 optimum using the Taguchi additive model is 489. 25 kN/m<sup>2</sup> (SN = 54.206). This represents a huge increment of about  
20 276% in the UCS of the stabilized soil.  
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27 Despite these significant improvements in the DS and UCS of the soil achieved via alkali-activation, it is also  
28 noteworthy that the optimal levels for factors A, C and D are conflicting. This presents a dilemma to the mix design  
29 engineer charged with the responsibility of prescribing the optimum mixing method in field application. To resolve  
30 this identified conflict, further analysis was executed using the novel method developed in the present study and is  
31 explicitly presented in the subsequent section.  
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### 35 **Grey-vector analysis multi-response optimization**

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37 The outcome of the novel GVA executed for the multi-response optimization of the DS and UCS using the steps  
38 shown in Fig. 1 produced the result summarized in Table 5. Determination of the factor effects on the Taguchi SN  
39 computed using the HTB function due to the desirability of higher resultant produced the results shown in Fig. 5 and  
40 Table 6. The Figure clearly shows that the factors which had the most significant effect on the vector resultant are A,  
41 B, and E. The extent of their effect on the vector resultant can be summarized in descending order as B > E > A > D >  
42 C as can be clearly observed from Table 6.  
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48 Furthermore, analysis of variance (ANOVA) was conducted by computation of the sum of squares (SS), mean squares  
49 (V), variance ratio (F-ratio) and percentage contribution (P), taking into account the degrees of freedom (DOF) of  
50 each process parameter [29]. The result, which is summarized in Table 7 presents a quantitative assessment of the  
51 factor effects and is found to be consentient with the Taguchi analysis. It can be seen that the factor B contributed the  
52 most to the vector resultant variation with 41%, followed by factor E (38.5%) and then A (10.4%). Factors C and D  
53 have very negligible contribution to the variation of the vector resultant.  
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58 Apparently, a higher quantity of the industrial waste is needed to minimize the DS of the expansive soil, while  
59 maximizing the UCS. This reveals the high degree of waste reuse inherent in the geopolymer technology utilized in  
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the present study for the expansive soil improvement. Furthermore, it is pertinent to apply the correct preparation method for field application. Vividly, the result of the present study show that homogeneously mixing the soil and industrial waste material prior to alkali-activation produces a stabilized soil of higher strength and minimized DS. Furthermore, the choice of PFA as a better industrial waste material for the soil improvement process has also been shown and is largely attributed to its large alumino-silicate content anticipated to be more amorphous than that of the QW. Therefore, future studies can explore the pretreatment of the QW (for instance heat treatment) to derive a more amorphous alumino-silicate framework before use.

Furthermore, to obtain the predicted optimum values of the DS and UCS, the optimal factor levels in Fig. 5, which is given as A1 – B2 – C1 – D2 – E1 is used for the response prediction. The optimum predicted DS is 1.85%, while that of the UCS is 458 kN/m<sup>2</sup>. To verify the accuracy of the obtained optimum value, 90% confidence interval of the confirmation experiment was computed with Equation 10 [28].

$$CI = \pm \sqrt{F_{\alpha}(1, f_e) V_e \left[ \frac{R + N_{eff}}{RN_{eff}} \right]} \quad (10)$$

Where  $F_{\alpha}(1, f_e)$  is the F-ratio at confidence level of  $1 - \alpha$  against DOF 1 and error DOF,  $f_e$ ; R is the sample size for the confirmation experiment and  $N_{eff} = N/1 + DOF_u$ ;  $V_e$  is the error variance; N is the total number of trials, and  $DOF_u$  is the total degrees of freedom associated with the estimate of the vector resultant value

The predicted optimum vector resultant, is given as  $2 \pm 0.5895$ . The obtained SN from the confirmatory experiments for the respective DS and UCS were -4.7106 and 53.9794. Similarly, their vector resultant was gotten using Fig. 1, including the predicted and confirmatory experimental values in the comparability sequence of the orthogonal array. The computed vector resultant is 1.977, which is lucidly within the confidence interval of  $2 \pm 0.5895$ ; hence, the prediction accuracy is verified.

### Comparative analysis of the novel GVA and GRA

Multi-response optimization using GRA as applied in soil stabilization [38] is pertinent for comparison with the novel GVA. The required computations done for the GRA are summarized in Table 8, showing the comparability sequence, deviation sequence, grey relational coefficient, GRG and grey ordinal value (GOV) by applying the sequence in Fig.2. The factor effects are shown in Fig 6, which suggests the existence of two optimum solutions. The first solution is given as A1 – B2 – C2 – D2 – E1, while the second solution is A2 – B2 – C2 – D2 – E1. The predicted optimum for UCS and DS obtained with the first solution are 364.25 kN/m<sup>2</sup> and 1.275% while that of the second solution gave UCS of 366 kN/m<sup>3</sup> and DS of 3.7%. Rationally, the first solution is a better multi-response compromise to the single response optimization solution and thus was used further in the comparative analysis.

Confidence interval for the confirmatory experiment was similarly obtained using Equation 10. The predicted optimum solution was also verified at the 90% confidence interval of the confirmatory tests. In order to execute a direct comparison of the aptness of the compromise solution gotten using both GVA and GRA, a new approach was adopted in the present study by consideration of the Euclidean distance between the single response optimization solution (ideal solution) and each of the obtained multi-response optimization solution (compromise solution). The Euclidean distance is a robust approach, which is typically used as a similarity index measure in hierarchical cluster analysis [20], as well as in machine learning such as K-nearest neighbors. The general formula for the computation as applicable in the comparison of the optimization solutions is provided in Equation 11.

$$\Delta = \left[ \sum_{j=1}^k (\eta_s^j - \eta_m^j)^2 \right]^{\frac{1}{2}} \quad (11)$$

Where the Euclidean distance is  $\Delta$ ,  $\eta_s^j$  is the grey-based normalized single response predicted value for the  $j$ th response and  $\eta_m^j$  is the grey-based normalized multi-response predicted value for the  $j$ th response.

It is noteworthy that the normalized responses are used in Equation 11 to resolve any inherent bias in responses of large magnitude. The grey-based normalization is executed using a comparability sequence comprised of the single response predicted values and all the multi-response solutions involved in the comparison. The outcome of this analysis is summarized in Table 9. The result clearly reveals the introduction of bias in the raw Euclidean distance computed using the non-normalized response. On the other hand, the normalized Euclidean distance clearly shows that the two multi-response methods exhibit a matched performance in their predicted optimum values. In a related development, the optimal solution obtained from the GVA is the same with that obtained using the utility concept [39] for the main effects. The upshot of this study also highlights the issue of non-uniqueness in the optimal solution for multi-response methods and is an aspect that requires further attention to create room for improvement. However, the GVA method developed in the present study is more computationally efficient than both GRA and the utility concept, because it requires fewer computations for multi-response optimization.

### Microstructural analysis of geopolymers structure

Evidence of geopolymerization from the alkali-activation of alumino-silicate rich industrial waste materials can be revealed via microstructural analysis. The results of Fourier transform infrared spectroscopy (FTIR) and energy dispersive spectrophotometer elemental (EDSE) analysis are presented for the natural soil and the optimally stabilized soil in the respective Table 10 and Table 11, Fig 7.

The significant peaks for the natural soil from FTIR analysis show the presence of weakly bonded water molecules, typically expected within the clay-pore fluid media of expansive soils. Furthermore, peaks for inactive sites of alumino-silicate and quartz are present due to the crystalline nature of the alumino-silicate content of the natural soil,

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4 in addition to the presence of significant amount of quartz mineral in the soil. The stabilized soil exhibited nuances,  
5 which give an indication of the occurrence of geopolymerization in the stabilized soil. The peaks for the water  
6 molecules slightly dropped to lower wave numbers, became broad, and sharp as presented in previous study [28].  
7 These suggest that the water molecules have become strongly bonded. This is likely attributed to the stronger –OH  
8 bonding anticipated from the alkali activator [28, 39]. The inactive sites occurring in the stabilized soil could likely  
9 have resulted from that in the natural soil and some residual industrial waste particles in the stabilized soil matrix.  
10 Further indication of the geopolymerization reaction is provided by the carbonate functional group due to reaction of  
11 atmospheric CO<sub>2</sub> with the alkali activator (KOH) during mixing to form K<sub>2</sub>CO<sub>3</sub> [28, 40]. The occurrence of the  
12 alumino-silicate gel band associated with the T–O silicate and siloxo framework confirms the formation of geopolymer  
13 binders in the stabilized soil [41, 42, 43].  
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20 Furthermore, the EDSE in Figure 7a show that the major elements prominent in the natural soil are Si, Al and Fe,  
21 based on the peaks. The atomic concentration of the elements is also shown in Table 11. This is as expected due to  
22 the rich content of the oxides of Si, Al and Fe in the oxide composition of the natural soil (Table 2). The stabilized  
23 soil EDSE peak show slight variation in the prominence of relevant elements in the soil. The peak for Al slightly  
24 dropped, while the peaks for Na and K became more prominent. These nuances are attributed to the formation of more  
25 siloxo alumino-silicate framework structures; in addition to the contributions of the sodium-based water glass  
26 (Na<sub>2</sub>SiO<sub>3</sub>) and potassium hydroxide (KOH) activator used for the alkali-activation.  
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## 32 **Conclusion**

33  
34 Sustainable and improved construction materials are attainable by harnessing the reuse of industrial waste materials.  
35 This has been shown in the present study via the adoption of alkali-activation of PFA and QW for the improvement  
36 of the DS and UCS of an expansive soil. A robust and systematic experimental design and novel multi-response  
37 optimization approach was further adopted to ensure a replicable process when considered for field application. The  
38 developed composite material achieved very low volumetric DS (less than 4%) and high UCS value (far above 200  
39 kPa) and can be employed as a sustainable construction material in embankments and waste containment facilities.  
40 The following summarizes the key findings of this study.  
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- 45 1. The key process factors which significantly influence the DS and UCS of the stabilized expansive soil  
46 using alkali-activated waste materials are the industrial waste quantity, method of preparation and waste  
47 type.  
48
- 49 2. A computationally efficient novel multi-response optimization for the DS and UCS of the stabilized soil  
50 was developed based on grey normalization and vector analysis (GVA).  
51
- 52 3. The optimally stabilized soil based on the GVA predicted significant improvement in the UCS (from 130 to  
53 458kPa) and DS (from 20.3 to 1.85%). Moreover, the prediction was verified to be accurate at 90%  
54 confidence interval of the confirmation experiments.  
55
- 56 4. The multi-response optimization and prediction using the novel GVA matched the performance of GRA  
57 based on the computed Euclidean distance between the single response prediction (ideal solution) and  
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4 multi-response prediction (compromise solution). In addition, the GVA gave the same optimization  
5 solution as the utility concept.  
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5. The GVA is regarded to be particularly advantageous, more desirable and expedient for construction and mix design engineers because it requires less computational effort than other multi-response methods.
  6. Evidence of the geopolymerization process achieved via alkali-activation was confirmed based on FTIR and EDSE analysis, which revealed the formation of alumino-silicate gels in the stabilized soil.

### Declaration of Interest Statement

The authors declare that there is no known conflict of interest associated with this study.

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Fig 7. EDSE peaks for (a) natural soil (b) optimally stabilized soil

Figure

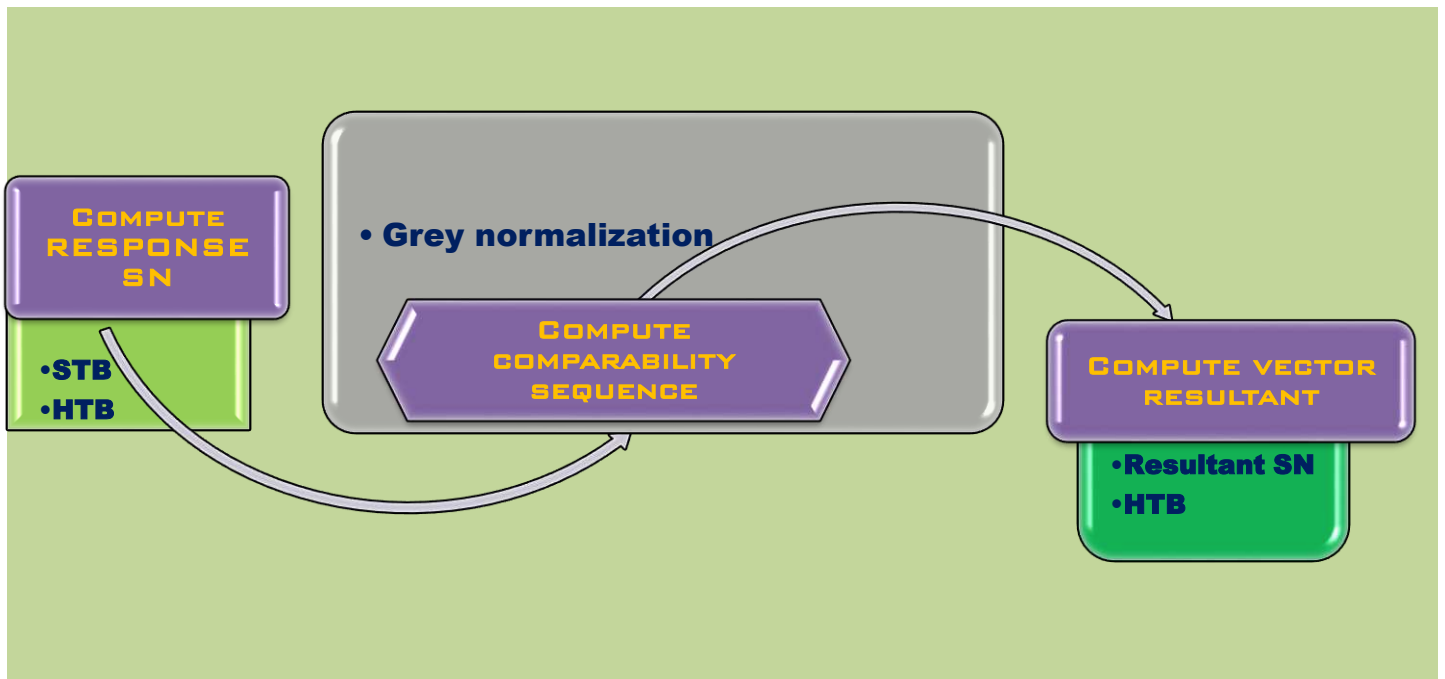


Fig 1. Flowchart for the novel multi-response optimization based on Taguchi-GVA

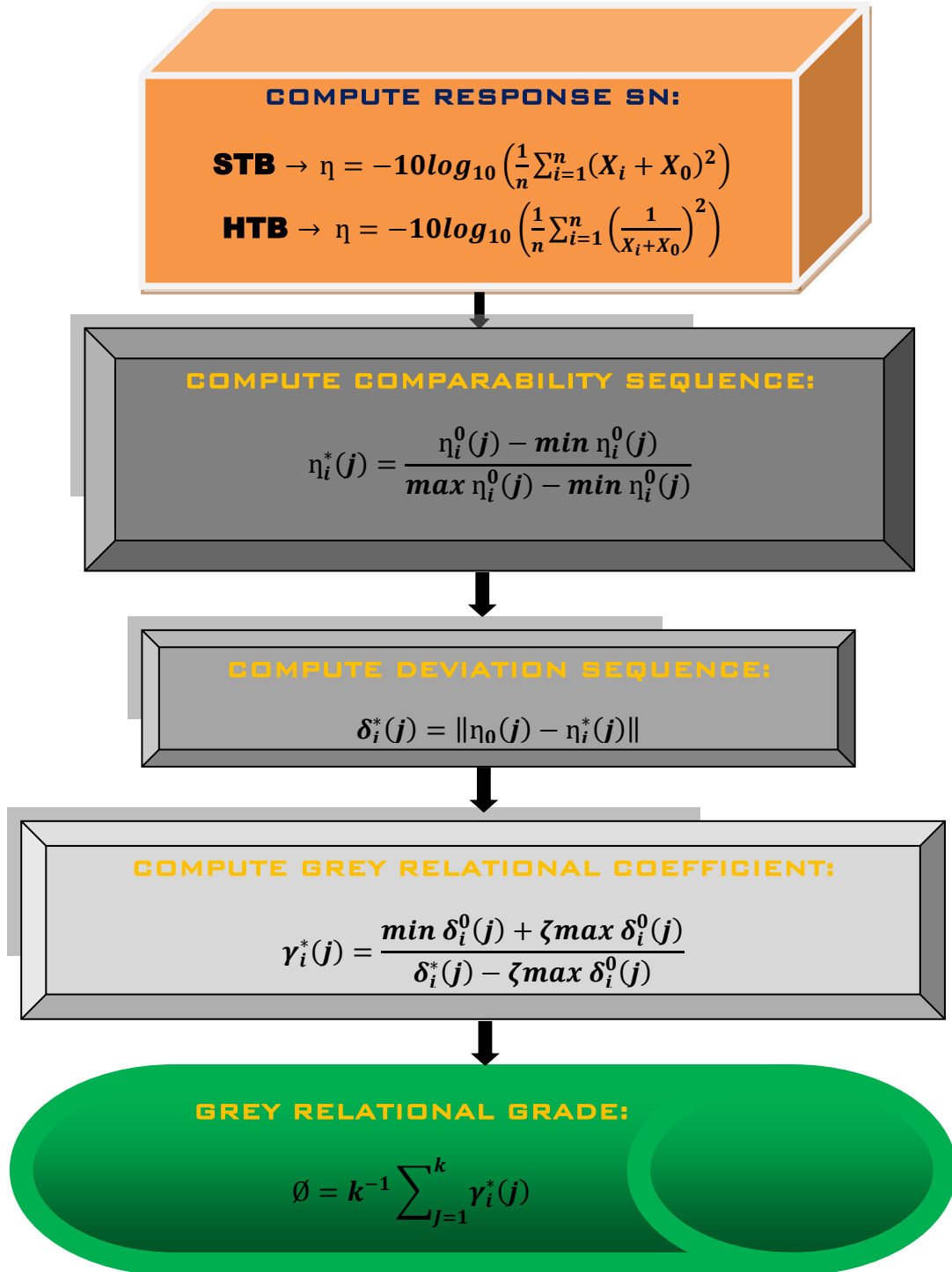


Fig 2. Flowchart for the multi-response optimization based on Taguchi-GRA

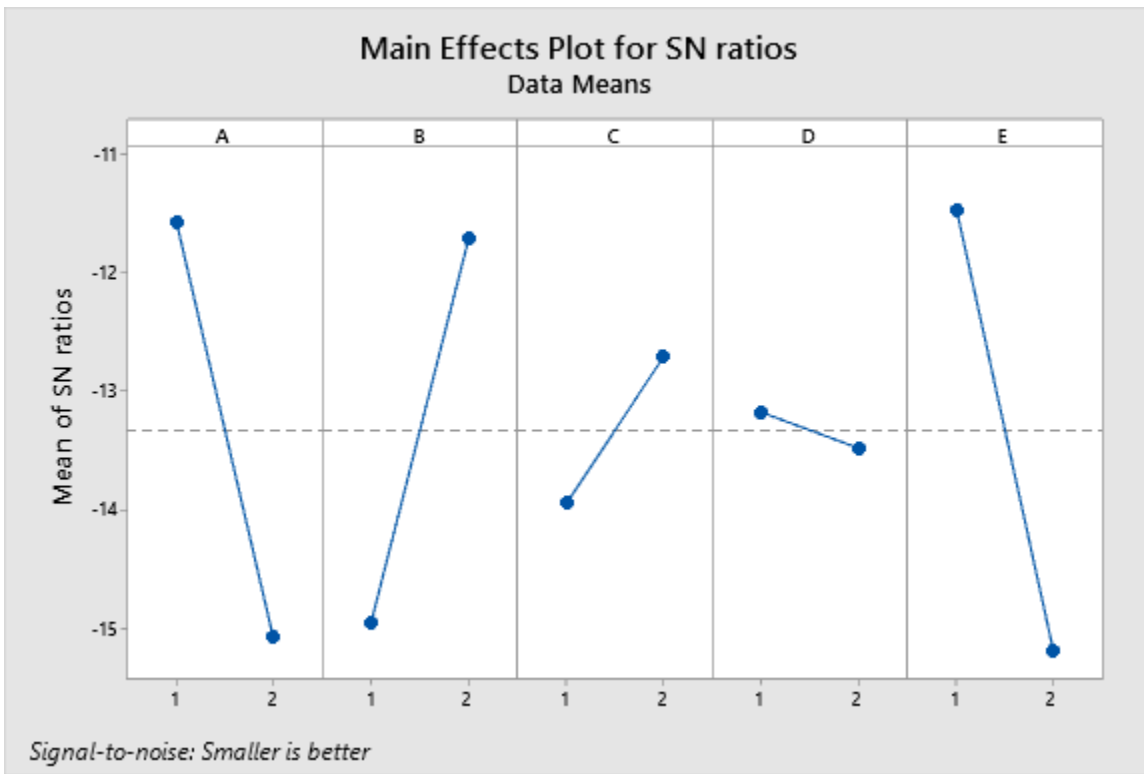


Fig 3. Plot of factor effect on the SN of DS

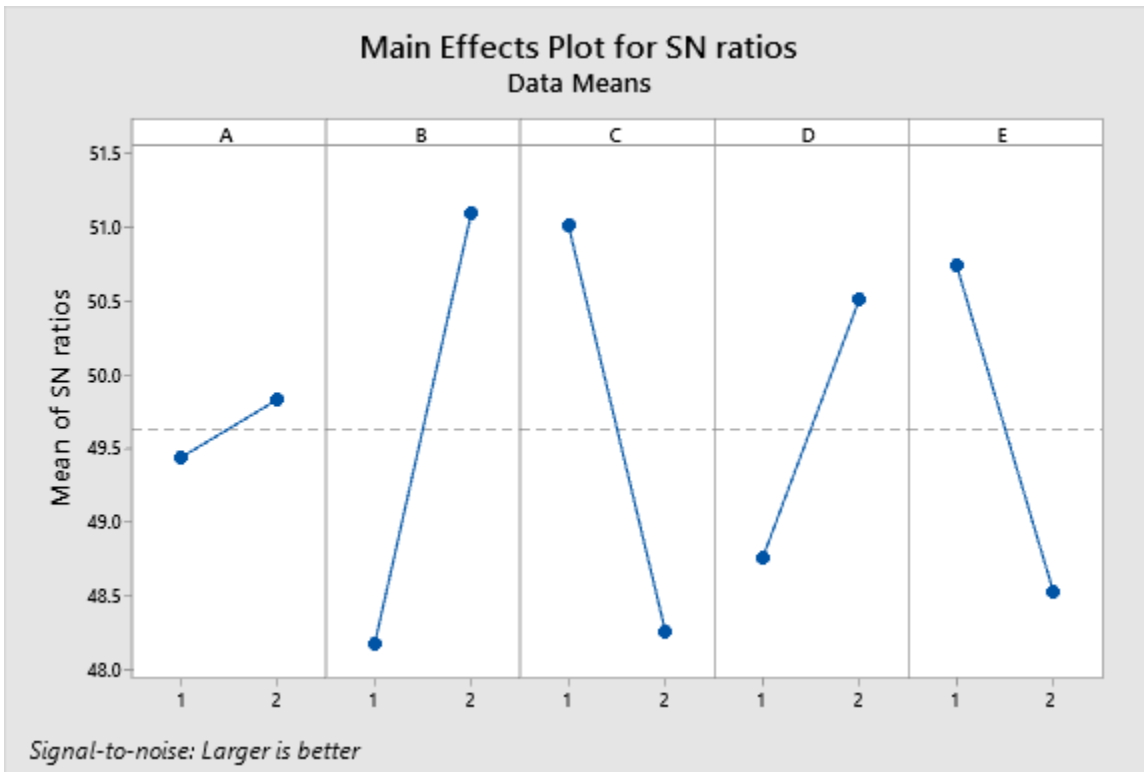


Fig 4. Plot of factor effect on the SN of UCS

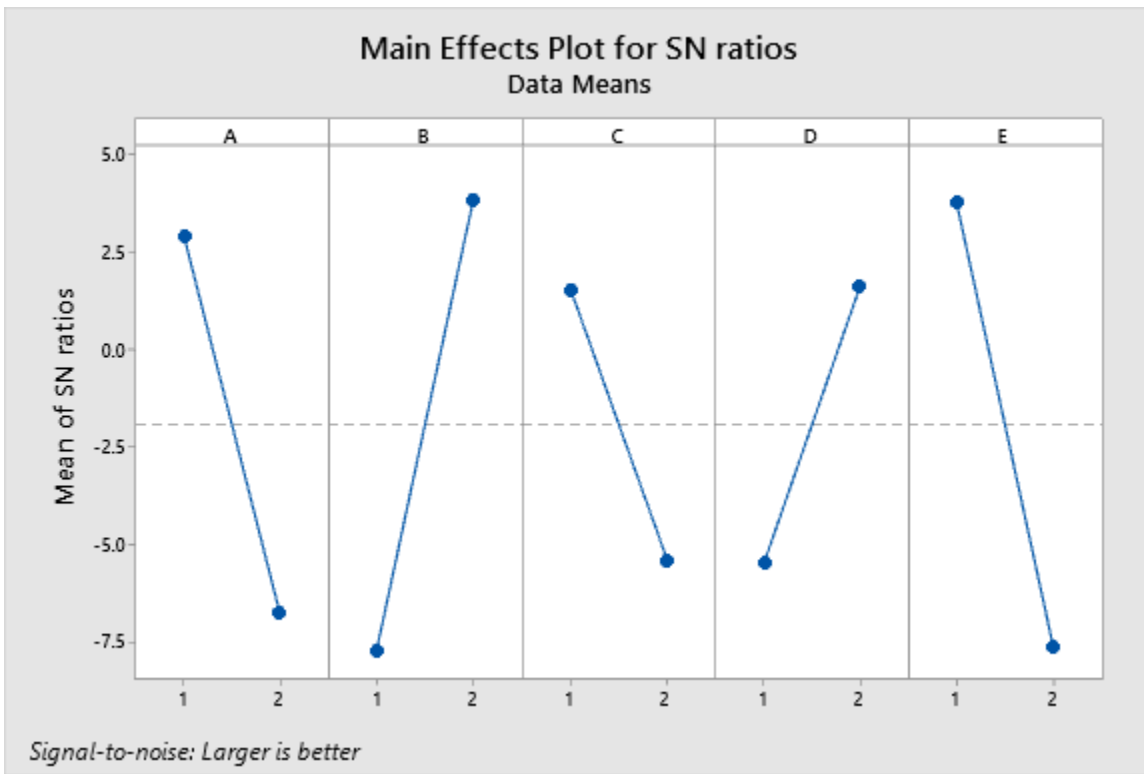
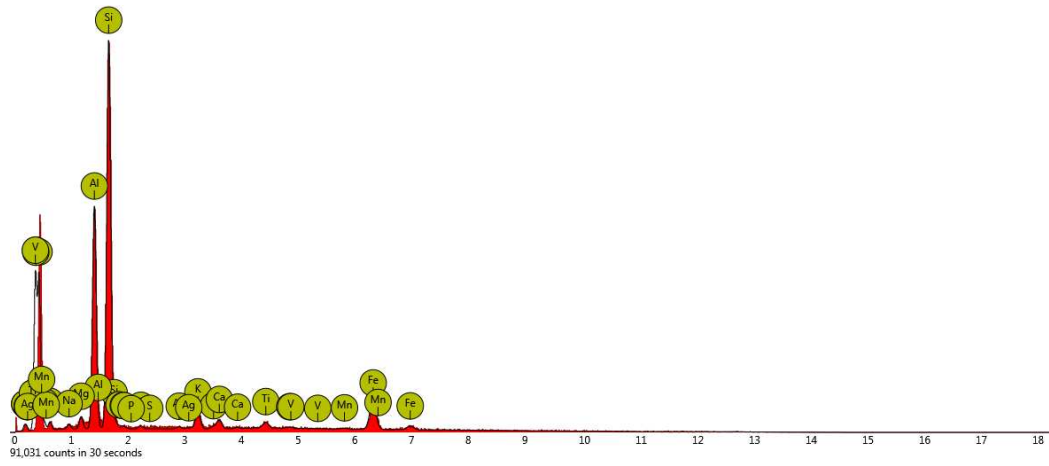


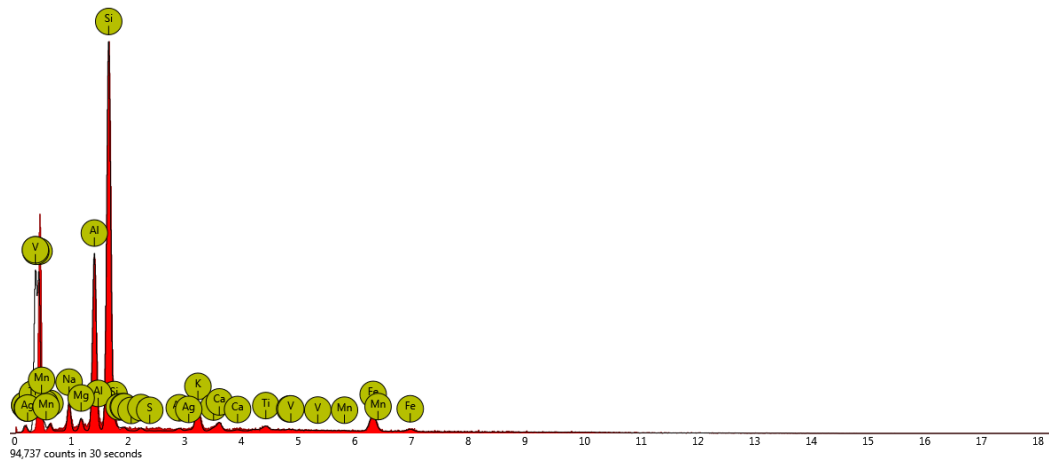
Fig 5. Plot of factor effect on the SN of vector resultant



Fig 6. Plot of factor effect on the SN of grey ordinal value



(a)



(b)

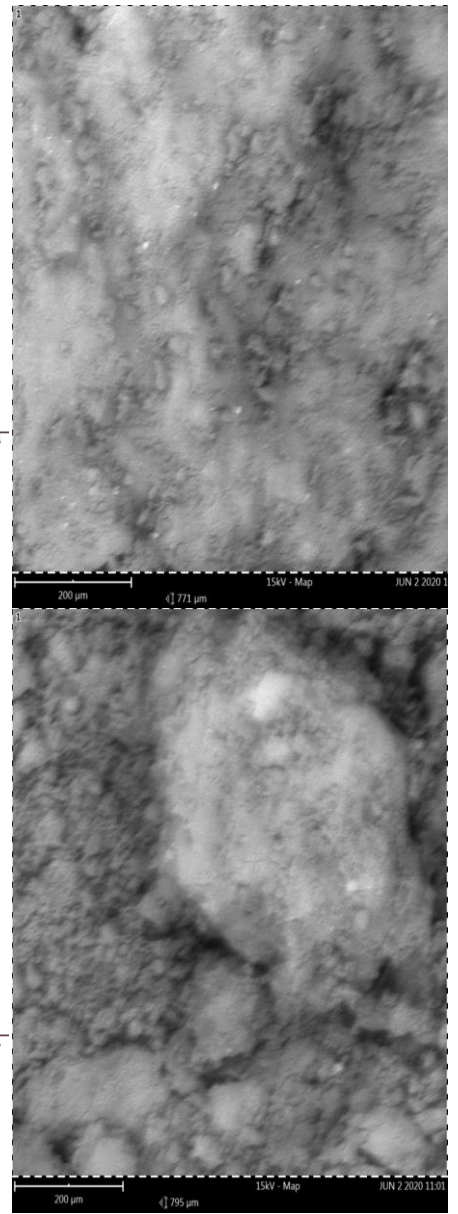


Fig 7. EDSE peaks for (a) natural soil (b) optimally stabilized soil

**Table 1.** Geotechnical/physical properties of the TBC soil

S/No	Property	Description
1	Specific gravity	2.62
2	Natural moisture content	9.9%
3	Fines	80.1%
4	Sand	19.9%
5	Liquid limit	66.0%
6	Plastic limit	34.9%
7	Plasticity index	31.1%
8	<u>Optimum moisture content (OMC)</u>	19.7%
9	<u>Maximum dry density (MDD)</u>	1.59g/cm <sup>3</sup>
10	AASHTO classification	A-7-5
11	USCS classification	CH
12	Permeability	2.025x10 <sup>-9</sup> ms <sup>-1</sup>
13	UCS	130 kN/m <sup>2</sup>
14	DS	20.3%

**Table 2.** Major oxide composition of the soil and industrial waste materials

Compound (%)	Soil	QW	PFA
Al <sub>2</sub> O <sub>3</sub>	17.7	20.4	16.3
SiO <sub>2</sub>	59.3	50.3	59.8
Fe <sub>2</sub> O <sub>3</sub>	5.4	0.7	9.6
CaO	0.8	1.5	0.8
K <sub>2</sub> O	2.1	2.3	1.2
TiO <sub>2</sub>	0.7	0.2	0.5
SO <sub>3</sub>	0.3	<0.1	<0.1
Loss on ignition	<10	<10	<10



**Table 3.** Designation of process parameters and their levels

Factor Designation	Level	
	1	2
A	PFA	QW
B	10%	20%
C	NaOH	KOH
D	0.5	0.7
E	M1	M2

**Table 4.** Design orthogonal array with coded and uncoded variables and SN

Run	A	B	C	D	E	A	B	C	D	E	UCS SN	DS SN
1	PFA	10%	NaOH	0.5	M1	1	1	1	1	1	48.9432	-12.041
2	PFA	10%	KOH	0.7	M1	1	1	2	2	1	49.218	-10.63
3	PFA	20%	NaOH	0.7	M2	1	2	1	2	2	51.5957	-13.255
4	PFA	20%	KOH	0.5	M2	1	2	2	1	2	47.9935	-10.37
5	QW	10%	NaOH	0.7	M2	2	1	1	2	2	49.9662	-18.69
6	QW	10%	KOH	0.5	M2	2	1	2	1	2	44.5577	-18.486
7	QW	20%	NaOH	0.5	M1	2	2	1	1	1	53.5339	-11.821
8	QW	20%	KOH	0.7	M1	2	2	2	2	1	51.2696	-11.364

**Table 5.** Output for the multi-response GVA

Run	Comparability sequence		Vector resultant
	$\eta_i^+(UCS)$	$\eta_i^+(DS)$	R
1	0.799	0.489	1.288
2	0.969	0.519	1.488
3	0.653	0.784	1.437
4	1.000	0.383	1.383
5	0.000	0.603	0.603
6	0.025	0.000	0.025
7	0.826	1.000	1.826
8	0.881	0.748	1.628

**Table 6.** Response table for the vector resultant HTB SN

Level	A	B	C	D	E
1	2.904	-7.736	1.544	-5.488	3.778
2	-6.783	3.857	-5.423	1.609	-7.657
Delta	9.686	11.593	6.967	7.098	11.435
Rank	3	1	5	4	2

**Table 7.** Analysis of variance for vector resultant

Process factor	DOF	SS	V	F-ratio	P (%)
A	1	0.200	0.200	4.703	10.4
B	1	0.805	0.805	18.915	41.7
C	1	0.501	0.501	1.192	2.6
D	1	0.046	0.046	1.076	2.4
E	1	0.741	0.741	17.424	38.5
Error	2	0.085	0.043		4.4
Total	7	1.927			100.0

**Table 8.** Output for the multi-response GRA

Run	Deviation sequence		Grey relational coefficient		GRG	Ordinal values (rank)
	$\delta_i^*(UCS)$	$\delta_i^*(DS)$	$\gamma_i^*(UCS)$	$\gamma_i^*(DS)$	$\phi$	
1	0.201	0.511	0.713	0.494	0.604	3
2	0.031	0.481	0.941	0.510	0.726	6
3	0.347	0.216	0.590	0.698	0.644	4
4	0.000	0.617	1.000	0.448	0.724	5
5	1.000	0.397	0.333	0.557	0.445	2
6	0.975	1.000	0.339	0.333	0.336	1
7	0.174	0.000	0.741	1.000	0.871	8
8	0.119	0.252	0.807	0.665	0.736	7

**Table 9.** Euclidean distance for performance comparison

Designation	UCS	DS	$\eta^{UCS}$	$\eta^{DS}$	$\Delta_{raw}$	$\Delta_{norm}$
Single response	489.25	1.1	1	1	-	-
Multi-response GVA	458	1.85	0.75	0	31.2589987	1.03
Multi-response GRA	364.25	1.275	0	0.766667	125.0001225	1.03

**Table 10.** Summary of relevant FTIR peaks and related functional groups

	Wave number (cm <sup>-1</sup> )	Functional group
<b>Natural soil</b>	3698, 3623, 3392, 1636	- OH and H - O - H weakly bonded water molecules
	1979, 775	Inactive alumino-silicate sites
	995, 682	Quartz band
<b>Stabilized soil</b>	3694, 3619, 3366, 1636	H - O - H strongly bonded water molecules
	2099	Inactive alumino-silicate site
	1394	O - C - O carbonate group
	999, 746	T - O (T = Si/Al) alumino-silicate gel band, siloxo Si-O-Si alumino-silicate framework structures

**Table 11.** Summary of atomic concentration for relevant EDSE peaks

Element Symbol	Natural soil atomic concentration	Stabilized soil atomic concentration
Si	50.1	51
Al	27.6	23
Fe	10.8	7.2
K	2.9	4.4
Ti	1.4	0.9
Ca	1.6	1.6
Mg	2.1	2.3
Na	1.4	7.7

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