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Editorial

Forward to the Special Issue on "The Applied Mathematics of Machine Learning"

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This special issue revises some of the state-of-the-art in the applied mathematics of machine learning and its variants from theory to practice. Of particular interest is the application of rigorous mathematical techniques for the development of convergent, stable and robust learning algorithms. It is also hoped that establishing connections with other related fields of interest such as the regularization theory of inverse and ill-posed problems, optimisation and design will help clarifying the benefits and limitations of machine learning techniques.

The analyses on machine learning undertaken by the illustrious contributors to this special issue made advances in machine learning, while pointing out their advantages and limitations. In particular, establishing error bounds for hyper-parameter optimization, constructing a unified framework for the universality of neural network activation functions, the use of bi-level optimization for model learning in variational imaging and practical computation of source condition elements of variational regularization for inverse and ill-posed problems have been realized. Further achievements concerned solving inverse problems for dental cone-beam computed tomography and river water quality. At the other end, vulnerabilities of neural networks to adversarial attacks have been identified and discussed. Finally, an insightful comparison has been performed between physics-informed neural networks and the finite element method for solving initial/boundary value problems for partial differential equations.

This special issue is only a stepping stone in trying to establish a firm rigorous basis for the rapidly-appearing topics, theories and applications of machine learning in applied mathematics.