Collision-avoidance target-tracking actuator command generation for UAVs*

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Abstract— **Monitoring moving ground vehicles using vision sensors mounted on Unmanned Aerial Vehicles (UAVs) in a densely populated area with high-rise buildings is a min-max problem. The cost function minimized or maximized by a pursuer or an evader is the integral of the distance between two vehicles over a finite time interval. One of the common approaches to the optimal tracking problem for UAVs is to minimize the cost function in the worst case caused by the evader. Extending the algorithm for one UAV to an algorithm for multiple UAVs requires collision avoidance capability. The cost function degradation to achieve collision avoidance must be minimal. We present an optimal collision avoidance target tracking algorithm combining the tracking command and the collision avoidance command. Finding the optimal combination becomes a line search optimization problem that can be solved with little computation. The resulting algorithm provides acceleration commands that can be directly used in the low-level controller of UAVs. The performance of the algorithm is demonstrated by multiple computer simulations.**

I. INTRODUCTION

Developing moving target tracking algorithms for autonomous aerial vehicles is one of the fundamental research questions of interest in the civil, security, and defence areas. Groups of unmanned aerial vehicles (UAVs) that cooperatively track moving ground vehicles have a plethora of applications and present many difficult control challenges. Guidance algorithms for UAVs have been intensively studied over the past few decades.

In [1], they compare four tracking algorithms of multiple UAVs to track a moving target. They focus on maintaining the separation angle between two UAVs to achieve cooperative tracking of a target vehicle in windy conditions. A guidance algorithm for one UAV to track a ground target with the heading angular rate constraint in windy conditions is presented in [2]. A modification version of [2] focuses on target detection algorithms and the tracking guidance [3].

On the other hand, [4] and [5] use a quadcoptor instead of a fixed-wing aircraft. The tracking algorithm in [4] mainly addresses the target detection problem where deep-learning neural network algorithms process a stream of vision sensor images. In [4], they present a tracking algorithm for a moving target in cluttered indoor environments, using an online Quadratic Programming formulation. The algorithm minimizes tracking error and assure safety efficiently and highlight the challenges in real-world experiments.

A circling tracking method that UAV tracks the ground vehicle in a circular trajectory is well-known. In [6], they achieve cooperative target tracking of a moving target in a balanced circular formation. The algorithm is centralized and does not consider inter-vehicle collision avoidance. To avoid centralized communication, [7] presents a vector field approach for decentralized multi-UAVs. The heading rate controller is used to obtain the desired circular tracking trajectory. The commanded heading design is based on a low-order UAV model. Furthermore, [8] considers the minimum turning radius of the fixed-wing UAV. When the distance between the UAV and the target exceeds a certain range, the circular path is updated. The algorithm does not consider controlling the speed of UAVs.

A ground target tracking in densely populated areas by high-rise buildings poses a challenging guidance problem. [9] proposes the solution for minimizing the line of sight between the UAVs and the target by a random sampling method. The algorithm provides the optimal circular path and the separation angles between the UAVs. The algorithm is extended for multiple target tracking cases in [10].

Most of the target tracking algorithms provide flight paths to follow and assume the existence of a low-level controller in the UAVs. Hence, it is the low-level controllers to make the UAVs fly into the desired flight path. In addition, there are issues with how frequently the flight paths are updated and how the stability is affected.

In this paper, we tackle the target tracking problem in urban areas and the algorithm is to provide the low-level control action directly so that the flight path update issue is removed. The paper is organized as follows: firstly, we summarize the urban area target tracking problem and the algorithm for one UAV in [11]; secondly, the algorithm is extended for multiple UAVs providing the collision avoidance capability between the UAVs; thirdly, the efficiency of the target tracking and the collision avoidance is demonstrated by multiple computer simulations; finally, conclusions & future works are presented.

II. PROBLEM FORMULATION

A UAV at its maximum altitude minimizes the chance that high-rise buildings or obstacles in the city block the monitoring camera's field of view towards the target. Given

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weather conditions, monitoring sensor capacities and UAV flight envelope, we assume that the UAV always flies at its maximum altitude. The target tracking problem in 3-dimensional space becomes a 2-dimensional one.

This section is the summary of the tracking problem formulation and the worst-case tracking algorithm shown in [11].

A. Target tracking problem

The target tracking problem between one UAV and one ground vehicle becomes a min-max problem whose cost function is given by

$$\underset{\mathbf{w}(t)\in\mathbb{W}}{\text{Maximize}}\ \underset{\mathbf{u}(t)\in\mathbb{U}}{\text{Minimize}}\ J = \int_{t=t_0}^{t=t_f} \left[\mathbf{y}(t)-\mathbf{z}(t)\right]^T \left[\mathbf{y}(t)-\mathbf{z}(t)\right] dt \tag{1}$$

where t is the time in seconds, $\mathbf{w}=[w_x,\, w_y]^T$ is the velocity control input of the ground vehicle in each direction of the global coordinates system (x-y), which allows the ground vehicle to make discontinuous changes in its velocity, the superscript, $[\ ]^T$, is the transpose, $\mathbb{W}$ is the feasible set of the ground vehicle velocity, $\mathbf{u}(t)=[u_x,\, u_y]^T$ is the acceleration control input of the UAV in each direction of the global coordinates system, $\mathbb{U}$ is the feasible set of the UAV acceleration control input, $\mathbf{y}=[x_a,\, y_a]^T$ is the position of the UAV in the global coordinates system, $\mathbf{z}=[x_t,\, y_t]^T$ is the position of the ground vehicle in the global coordinates system, t_0 is the current time, and t_f is the final time of the cost function horizon. The cost function is the integral of the squared 2-norm distance difference between the pursuer and the target vehicles over the finite time interval.

Simplified dynamics of the UAV and the ground vehicle are given by

$$\dot{\mathbf{x}}_a = A_a\mathbf{x}_a + B_a\mathbf{u} \tag{2a}$$

$$\mathbf{y} = C_a\mathbf{x}_a \tag{2b}$$

and

$$\dot{\mathbf{x}}_t = B_t\mathbf{w} \tag{3a}$$

$$\mathbf{z} = C_t\mathbf{x}_t \tag{3b}$$

respectively, where $\mathbf{x}_a = \begin{bmatrix}\mathbf{y}^T & \mathbf{v}^T\end{bmatrix}^T$, $\mathbf{v}=\dot{\mathbf{y}}=\begin{bmatrix}v_x & v_y\end{bmatrix}^T$, $\dot{\mathbf{y}}=d\mathbf{y}/dt$, v_x and v_y are the velocity components to the x and y directions, respectively, $\mathbf{x}_t=\mathbf{z}$,

$$A_a = \begin{bmatrix}0_2 & I_2\\ 0_2 & 0_2\end{bmatrix},\ B_a = \begin{bmatrix}0_2\\ I_2\end{bmatrix},\ C_a = \begin{bmatrix}I_2 & 0_2\end{bmatrix},$$

$B_t = C_t = I_2$, I_2 is the $2{\times}2$ identity matrix, and 0_2 is the $2{\times}2$ zero matrix.

The ground vehicle can stop and reverse, assuming an isotropic speed capability, i.e., the speed control input $\mathbf{w}$ belongs to $\mathbb{W}$ given by

$$\mathbb{W} = \{\mathbf{w}|\ 0 \le \|\mathbf{w}\| \le w_{\max}\} \tag{4}$$

where $\|\cdot\|$ is the 2-norm and $w_{\max}$ is the maximum forward velocity of the ground vehicle. The acceleration control input space of the fixed wing UAV is given by

$$\begin{aligned}\mathbb{U} = \{\mathbf{u}|\ & u_{x_{\min}} \le u_x\cos\phi + u_y\sin\phi \le u_{x_{\max}}\\ & u_{y_{\min}} \le -u_x\sin\phi + u_y\cos\phi \le u_{y_{\max}}\}\end{aligned} \tag{5}$$

where ϕ is the UAV heading angle measured with respect to the x-axis in the global coordinates system. In addition to the control input constraints, the dynamics of UAV introduces several state constraints. The velocity is constrained by

$$0 < v_{\min} \le \|\mathbf{v}\| \le v_{\max} \tag{6}$$

indicating the inability of the fixed wing aircraft hovering or flying backwards direction. One of the critical characteristics of fixed-wing aircraft is the restriction on how much sharp it can change its flight path given by the minimum radius of turn, $r_{\min}$, as follows:

$$-\frac{\|\mathbf{v}\|^3}{r_{\min}} \le v_x u_y - v_y u_x \le \frac{\|\mathbf{v}\|^3}{r_{\min}} \tag{7}$$

B. Worst-Case Tracking Algorithm

Approximate the cost function, (1), with a finite sum, by discretizing the dynamics, (2) and (3), as follows [11]:

$$J \approx \frac{\Delta t^5}{2}\left\{\|\mathbf{u}(t_0)\|^2 + \mathbf{c}^T\mathbf{u}(t_0)\right\} + d \tag{8}$$

where $\mathbf{c}$ and d are functions of the initial conditions and the target current and the future input, $\mathbf{w}(t_0)$ and $\mathbf{w}(t_1)$, $t_1 = t_0 + \Delta t$, $\Delta t = (t_f - t_0)/2$. The integration interval is divided into two sub-intervals, which is the minimum number of intervals that the UAV control input, $\mathbf{u}$ at $t=t_0$, appears, i.e., the relative degrees of the system.

$\mathbf{c}$ and d are given by

$$\begin{aligned}\mathbf{c} = 2\Delta t^2\{&\mathbf{y}(t_0) - \mathbf{z}(t_0)\\ &-\Delta t\left[2\mathbf{v}(t_0) - \mathbf{w}(t_0) - \mathbf{w}(t_1)\right]\}\end{aligned} \tag{9}$$

and

$$\begin{aligned}d = \Delta t^2 &\left\{5\|\mathbf{v}(t_0)\|^2 - \mathbf{v}^T(t_0)\left[6\mathbf{w}(t_0) + 4\mathbf{w}(t_1)\right]\right.\\ &\left. + 2\|\mathbf{w}(t_0)\|^2 + \|\mathbf{w}(t_1)\|^2 + 2\mathbf{w}^T(t_0)\mathbf{w}(t_1)\right\}\\ &+ \Delta t\left\{\left[6\mathbf{v}^T(t_0) - 4\mathbf{w}^T(t_0) - 2\mathbf{w}^T(t_1)\right]\left[\mathbf{y}(t_0) - \mathbf{z}(t_0)\right]\right\}\\ &+ 2\left[\|\mathbf{y}(t_0)\|^2 + \|\mathbf{z}(t_0)\|^2 f - 2\mathbf{y}^T(t_0)\mathbf{z}(t_0)\right]\end{aligned} \tag{10}$$

where the current and the next step velocity control inputs of the ground vehicle, $\mathbf{w}(t_0)$ and $\mathbf{w}(t_1)$, are unknown to the UAV.

To design an optimal tracking algorithm, consider the worst-case scenario. The cost function can be viewed as proportional to the sum of the distance differences as follows:

$$J \propto \Delta\ell(t_0) + \Delta\ell(t_1) + \Delta\ell(t_f) \tag{11}$$

where $\Delta\ell(t_i)$ is the distance norm between the UAV and the ground target for $i = 0, 1$ and f.

The distance difference at t_f is given by

$$\Delta\ell(t_f) = \|\mathbf{y}(t_f) - \mathbf{z}(t_f)\| \tag{12}$$

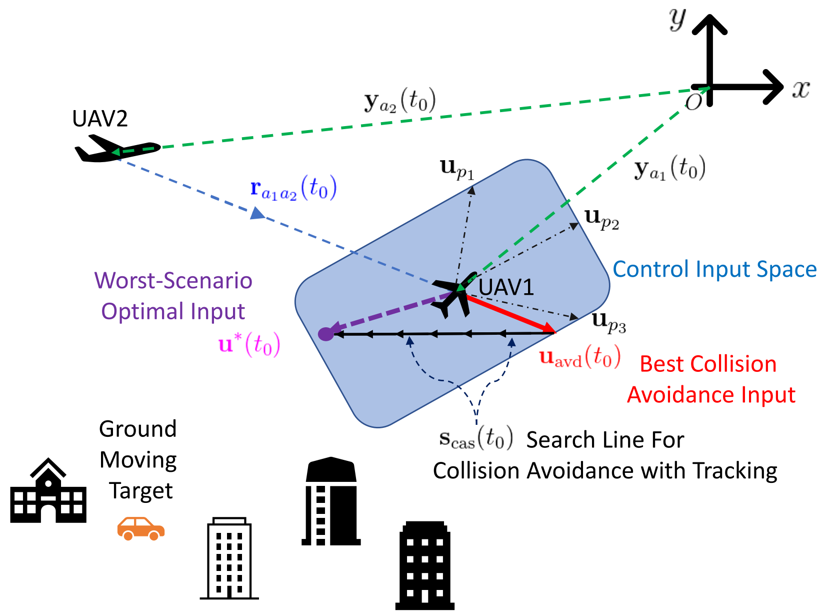

Fig. 1. Two UAVs track the ground-moving vehicle while avoiding collision with each other. The best linear combination of the optimal worst-case tracking acceleration command, $\mathbf{u}^*(t_0)$, and the best collision avoidance acceleration command, $\mathbf{u}_{\text{avd}}(t_0)$, provides the collision-avoidance target-tracking acceleration command.

As $\mathbf{z}(t_f) = \mathbf{z}(t_1) + \Delta t \ \mathbf{w}(t_1)$, the ground vehicle can maximize $\Delta\ell(t_f)$ by driving away from the direction given by

$$\mathbf{d}_{\text{worst}}(t_1) = \frac{\mathbf{z}(t_1) - \mathbf{y}(t_f)}{\|\mathbf{z}(t_1) - \mathbf{y}(t_f)\|} \tag{13}$$

with the maximum velocity, i.e., $w_{\max}$. Hence, the best velocity input for maximizing the distance is given by

$$\mathbf{w}^*(t_1) = w_{\max}\mathbf{d}_{\text{worst}}(t_1) \tag{14}$$

Similarly, the distanced difference at t_1 is given by

$$\Delta\ell(t_1) = \|\mathbf{y}(t_1) - \mathbf{z}(t_1)\| \tag{15}$$

where $\mathbf{z}(t_1) = \mathbf{z}(t_0) + \Delta t \ \mathbf{w}(t_0)$. The best evasion direction of the ground vehicle at t_0 is not trivial as one at t_1. However, there is no reason that the vehicle uses the slower velocity than the maximum velocity. Hence, the best velocity input for the ground vehicle to maximize $\Delta\ell(t_1)$ is given by

$$\mathbf{w}^*(t_0) = w_{\max} \begin{bmatrix} \cos\theta^* & \sin\theta^* \end{bmatrix}^T \tag{16}$$

where the optimal direction θ^* to be determined is the angle measured from the x-axis.

Substitute (14) and (16) into (8), take the derivative with respect to θ^* as follows:

$$\frac{\partial J}{\partial \theta^*} = -a\cos\theta^* + b\sin\theta^* = 0 \tag{17}$$

where

$$a = u_y(t_0)\Delta t^2 + 3v_y(t_0)\Delta t + 2[y_a(t_0) - y_t(t_0)]$$
$$b = u_x(t_0)\Delta t^2 + 3v_x(t_0)\Delta t + 2[x_a(t_0) - x_t(t_0)]$$

Hence,

$$\cos\theta^* = \frac{b}{\sqrt{a^2+b^2}}, \ \sin\theta^* = \frac{a}{\sqrt{a^2+b^2}} \tag{18}$$

Finally, substituting (14) and (16) into (8) results in the minimization problem for a quadratic convex cost function with the constraints given by (5), (6) and (7).

We call the cost function with the worst-case $\mathbf{w}(t_0)$ and $\mathbf{w}(t_1)$ the worst-case cost function, i.e., $J_{\text{worst}}[\mathbf{u}(t_0)]$. The minimization solution of J_{worst} occurs either inside the constraint set, which satisfies all the constraints, where $\partial J_{\text{worst}}/\partial u_x^*(0) = 0$ and $\partial J_{\text{worst}}/\partial u_y^*(0) = 0$, or at the boundary of the constraint set.

Details of the algorithm for one UAV to track a single target can be found in [11] and its implementation in Matlab or Python can be found in [12].

III. Collision Avoidance Algorithm

We extend the tracking algorithm to multiple UAVs tracking a single target scenario, where collision avoidance between UAVs is vital.

Assumption 1: (UAV Positions) Each UAV knows the location of other UAVs in the operation area, but no speed information is available. The UAV broadcasts its location to other UAVs, or UAVs have radar or vision sensors to detect the location of other UAVs.

Figure 1 shows two UAVs, UAV1 and UAV2, tracking a ground-moving target, where each UAV avoids collision with the others. From the UAV1 point of view, avoiding collisions is exerting the maximum magnitude acceleration in the relative direction of the current UAV1 position, $\mathbf{y}_{a1}(t_0)$, to the UAV2 position, $\mathbf{y}_{a2}(t_0)$. $\mathbf{r}_{a_1a_2}(t_0)$ indicates the ideal input direction to apply the evasion acceleration command for UAV1.

Assumption 2: (Best Control Input To Avoid Collision) $\mathbf{u}_{\text{avd}}(t_0)$ in Figure 1 achieves the ideal collision evasion manoeuvre.

The constraint bound illustrated in Figure 1 changes dynamically depending on the current velocity of the aircraft. To find $\mathbf{u}_{\text{avd}}(t_0)$, we calculate s_p to measure the closeness of each point at the constraint boundary to $\mathbf{u}_{\text{avd}}(t_0)$:

$$s_p = \frac{\mathbf{r}_{a_1 a_2}(t_0) \cdot \mathbf{u}_p}{\|\mathbf{u}_{a_1 a_2}(t_0)\| \cdot \|\mathbf{u}_p\|} \tag{19}$$

where $\mathbf{u}_p$ is a vector from the centre of the control input space to a point at the constraint boundary. Several examples of $\mathbf{u}_p$ are shown in Figure 1. The $\mathbf{u}_p$ having the maximum s_p, i.e., closest to 1 within a tolerance, is found to be $\mathbf{u}_{\text{avd}}(t_0)$.

The worst-case optimal tracking command described in Section II-B is $\mathbf{u}^*(t_0)$ shown in Figure 1. Depending on the current state of the UAVs and the ground target, the worst-case optimal input may occur within the control input constraint space or at the boundary.

Before introducing collision avoidance control actions, we identify the worst-cost function being convex, which is useful in designing the control actions.

The worst cost function is convex over $u_x(t_0)$ and $u_y(t_0)$ if and only if the determinants of two leading principle matrices are positive as follows:

$$\frac{\partial^2 J_{\text{worst}}}{\partial u_x(t_0)^2} > 0 \tag{20}$$

$$\frac{\partial^2 J_{\text{worst}}}{\partial u_x(t_0)^2}\frac{\partial^2 J_{\text{worst}}}{\partial u_y(t_0)^2} - \left[\frac{\partial^2 J_{\text{worst}}}{\partial u_x(t_0)\partial u_y(t_0)}\right]^2 > 0 \tag{21}$$

As we check the conditions for ranges of initial conditions, the Hessian matrices are all positive definite.

Performing collision avoidance and tracking the ground target, the control input acceleration is a linear combination given by

$$\mathbf{u}(t_0) = \mathbf{u}_{\text{avd}}(t_0) + \gamma\ \mathbf{s}_{\text{cas}}(t_0) \tag{22}$$

where $\gamma \in [0, \gamma_{\max}]$,

$$\mathbf{s}_{\text{cas}}(t_0) = \frac{\mathbf{u}^*(t_0) - \mathbf{u}_{\text{avd}}(t_0)}{\gamma_{\max}} \tag{23}$$

and

$$\gamma_{\max} = \|\mathbf{u}^*(t_0) - \mathbf{u}_{\text{avd}}(t_0)\| \tag{24}$$

We want to avoid collisions and minimize the increase in the cost function at $J_{\text{worst}}[\mathbf{u}^*(t_0)]$ at the same time. Let J_{th} be the maximum allowed cost function, where $J_{\text{th}} \geq J_{\text{worst}}[\mathbf{u}^*(t_0)]$.

Searching the optimal collision avoidance tracking input becomes the one-dimensional maximization problem of the worst-cost function over γ as follows:

Problem 1: (Collision Avoidance Tracking Maximization)

$$\underset{\gamma \in [0, \gamma_{\max}]}{\text{Maximize}}\ J(\gamma) = J_{\text{worst}}[\mathbf{u}_{\text{avd}}(t_0) + \gamma\ \mathbf{s}_{\text{cas}}(t_0)] \tag{25}$$

subject to

$$J(\gamma) \leq J_{\text{th}} \tag{26}$$

where J_{th} is the maximum threshold of the cost function allowed to sacrifice how much $J(\gamma_{\max}) = J_{\text{worst}}[\mathbf{u}^*(t_0)]$ will be sacrificed.

To solve Problem 1, we search for the minimum γ satisfying the constraint so that the resulting $\mathbf{u}(t_0)$ provides the maximum chance of collision avoidance, i.e., make it as close as possible to $\mathbf{u}_{\text{avd}}(t_0)$.

Given $J_{\text{worst}}[\mathbf{u}(t_0)]$ satisfying the convexity conditions in (20) $J(\gamma)$ is also convex over γ in $[0, \gamma_{\max}]$ whose minimum occurs at $\gamma = \gamma_{\max}$. $J(\gamma)$ is a non-increasing function as γ increase from 0 to $\gamma_{\max}$. Therefore, searching the first $J(\gamma)$ equal to J_{th} starting from γ equal to zero provides the best collision avoidance for the given cost function constraint. Any line search methods, e.g., bisection, could be used to solve Problem 1. However, the constraint control space is not necessarily convex because of the velocity constraint, (6), and the line might be separated into multiple segments We use a random sampling algorithm to find the collision avoidance acceleration command. In addition, the condition to activate the collision avoidance manoeuvre is given by

Assumption 3: (Collision Avoidance Manoeuvre) Each UAV activates the collision avoidance manoeuvre only if the distance to the closest UAV is less than r_{th}.

Algorithm 1 summarizes how the collision avoidance target-tracking acceleration is generated.

Algorithm 1 Collision Avoidance Tracking Command

Input: $\mathbf{J}_{th}, \mathbf{u}^*(t_0)$, and $\mathbf{u}_{\text{avd}}(t_0)$

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Sample N-random points along (22) for γ ∈ [0, γ_max]
if ‖r_a1a2(t0)‖ ≤ r_th then
    Find all γ satisfying J(γ) < J_th
    return u(t0) of the minimum γ in the samples
else
    return u*(t0)
end
```

We test the algorithm with a simple case, where two UAVs fly towards each other, and a ground vehicle at the centre is stationary at the beginning as shown in Figure 2.

The initial velocities of the two UAVs are 90km/h towards each other, the initial distance between the two UAVs is 1km apart, $r_{\text{th}} = 1$km and $J_{\text{th}} = (1+\alpha)J_{\text{worst}}$, $\alpha = 0.02$, i.e., allowed 2% increment in the cost function. The simulation values for the UAVs are as follows: $r_{\min} = 400$m, $v_{\max} = 40$m/s, $v_{\min} = 20$m/s, $u_{x_{\max}} = 10$m/s^2, $u_{x_{\min}} = -1$m/s^2, $u_{y_{\max}} = 2$m/s^2, $u_{y_{\min}} = -2$m/s^2, $v_{\min} = 20$m/s and $v_{\max} = 40$m/s. The maximum target speed, $w_{\max}$, is equal to 60km/h.

Figure 2 shows the closest distance between the UAVs is about 200m. The UAVs keep tracking the ground target while stationary at the beginning and moving towards the positive x-axis with the maximum speed from the middle of the simulation.

Now, we extend the algorithm for the number of UAVs greater than or equal to two inside the r_{th} radius circle centred

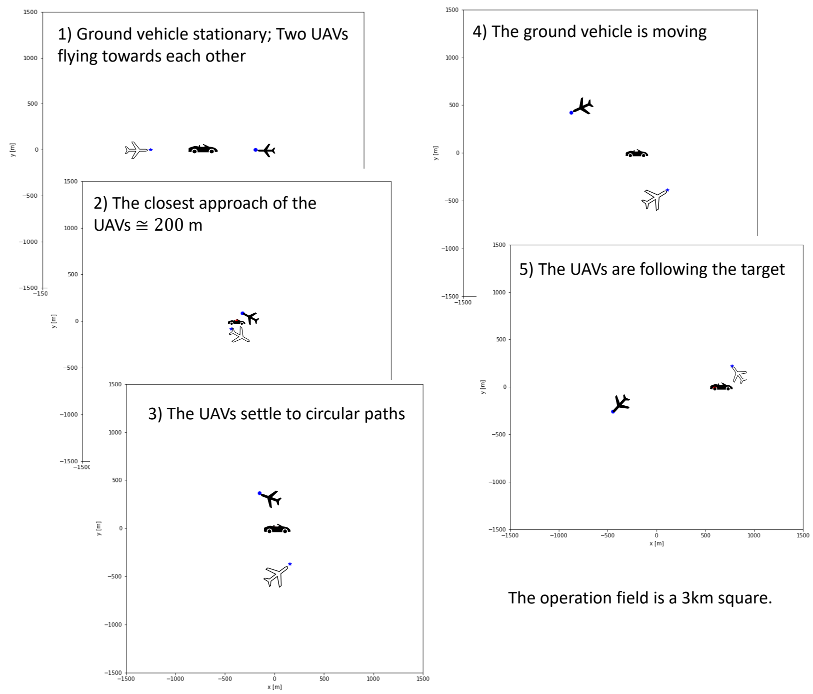

Fig. 2. Two UAVs tracking a vehicle. The animation can be found in https://youtu.be/NYngODQp0-g

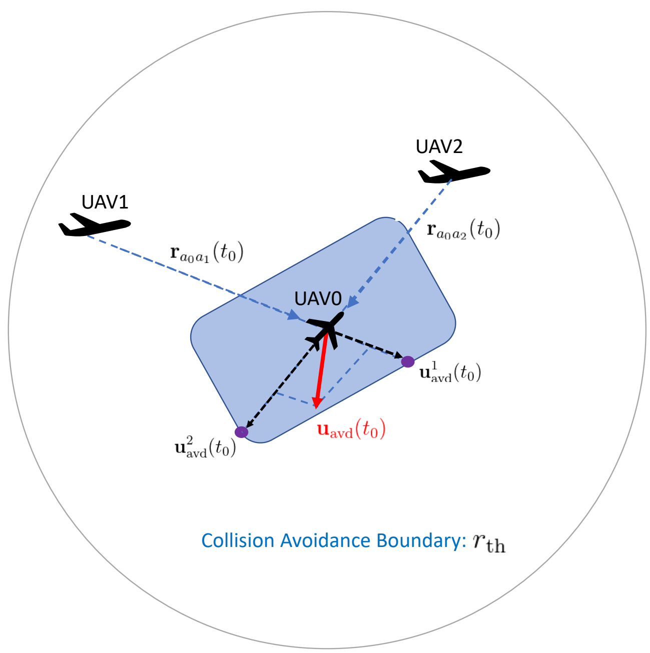

Fig. 3. Optimal acceleration command combining the tracking and the collision avoidance command

at a UAV. The UAV must avoid collisions with multiple UAVs. The best collision avoidance acceleration command to avoid the i-th UAV in the circle, $\mathbf{u}^i_{\text{avd}}(t_0)$, is scaled inversely proportional to the relative distance and its magnitude, and the sum of the scaled acceleration becomes the collision avoidance acceleration, $\mathbf{u}_{\text{avd}}(t_0)$ as follows:

$$\mathbf{u}_{\text{avd}}(t_0) = \left(\frac{1}{\sum_i s_i}\right) \sum_{i=1}^{N_{\text{ac}}} s_i \mathbf{u}^i_{\text{avd}}(t_0) \tag{27}$$

where N_{ac} is the total number of UAVs inside the collision avoidance threshold, r_{th}, and s_i is the scaling factor given by

$$s_i = \frac{\|\mathbf{u}^i_{\text{avd}}(t_0)\|}{\|\mathbf{r}_{a_0a_i}(t_0)\|} \tag{28}$$

where $\mathbf{r}_{a_0a_i}(t_0)$ is the position vector from the UAV to the i-th UAV. An illustration of the scaling for a two UAVs case is shown in Figure 3.

Substitute the combined collision avoidance acceleration, (27), into (22), solve Problem 1, and the collision avoidance target tracking acceleration command for multiple UAV cases is obtained.

Monte-Carlo Simulations

We find the minimum distance between two aircraft for each simulation to determine the effect of α in the cost function on collision avoidance ability. The bigger α provides a safer flight path than the flight path of the smaller α, while it increases the tracking cost function more than the smaller α.

Figure 5 shows the average minimum distances and the 1σ error bars for the range of $\alpha \in [0, 0.095]$, where $\alpha = 0$ is without the collision avoidance algorithm. From $\alpha = 0.01$ to higher α values, the performance remains at a similar level. The mean minimum distances for $\alpha \neq 0$ are always bigger than one for $\alpha = 0$. Some of the 1σ lower bounds,

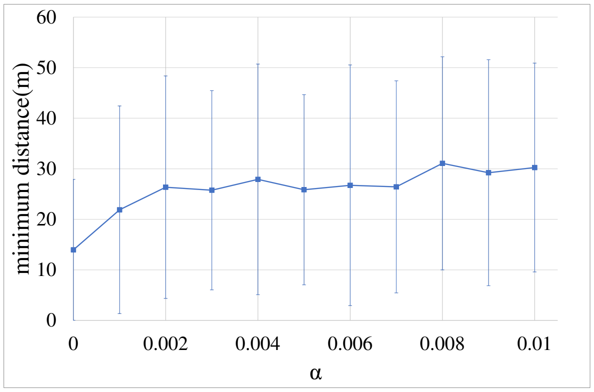

Fig. 4. Average minimum distance and 1σ bound for various α with five UAVs & one target

i.e., the worst-case bounds, are smaller than one for $\alpha = 0$. Increasing α or r_{th} will improve these bounds.

Figure 4 shows the Monte-Carlo simulation results for the 5 UAVs tracking one target. Similar to the previous case, the average minimum distances between the 5 UAVs for $\alpha > 0$ are always larger than the one of $\alpha = 0$.

IV. Conclusions & Future Works

We extend the single UAV-single target tracking algorithm to the multiple UAVs-single target tracking algorithm by introducing the collision avoidance algorithm. The collision avoidance algorithm provides the optimal way to prevent collisions by adding additional acceleration commands, which sacrifices the minimal amount of the original optimal cost for the given tolerance bound. The effectiveness of the collision avoidance tracking algorithm has been verified by two Monte-Carlo simulations for 2 UAVs and 5 UAVs cases. For both cases, they show the collision algorithm improves the mean distance between the UAVs.

Currently, we focus on developing design procedures for turning optimal parameters in the algorithm and the sensitivity of the performance with respect to the sensor noises. In addition, the extension of the algorithm for 3-dimensional cases is investigated.

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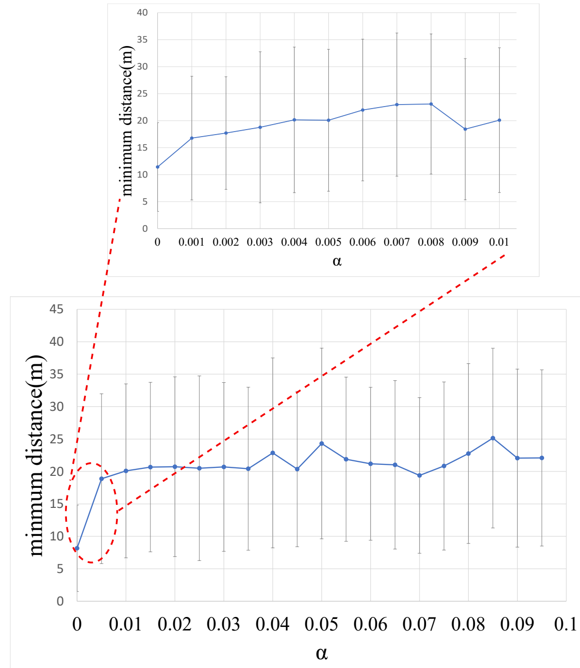

Fig. 5. Average minimum distance and 1σ bound for various α with two UAVs & one target

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