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Quantum-Secure Hybrid Communication for Aviation Infrastructure

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Abstract. The rapid digitization of aviation communication and its dependent critical operations demand secure protocols that address domain-specific security requirements within the unique functional constraints of the aviation industry. These secure protocols must provide sufficient security against current and possible future attackers, given the inherent nature of the aviation community, that is highly complex and averse to frequent upgrades as well as its high safety and cost considerations. In this work we propose a pair of quantum-secure hybrid key exchange protocols (PQAG-KEM and PQAG-SIG) to secure communication between aircrafts in-flight and ground stations. PQAG-KEM leverages post-quantum and classical Key Encapsulation Mechanisms (KEMs) to ensure the hybrid security of the protocol against classical as well as future quantum adversaries. PQAG-SIG, alternatively, uses quantum-safe digital signatures to achieve authentication security. We provide an implementation of both PQAG-KEM and PQAG-SIG, and compare favourably with current state-of-the-art secure avionic protocols. Finally, we provide a formal analysis of our new PQAG protocols in a strong hybrid key exchange framework.

Keywords: Authenticated key exchange, hybrid key exchange, provable security, protocol analysis, avionics

1 Introduction

The state of the aviation communications (avionics) in the 1980s was characterised by channel isolation, and operated through the use of analog and legacy infrastructures of military origin. Over time this has gradually shifted towards commercial and digital communication technologies. This shift has been fairly slow owing to the lengthy standardization processes, but at the time of writing multiple state organizations in Europe (EUROCONTROL) and the US (Federal Aviation Administration-FAA) are actively involved in the standardisation and structuring of future communication infrastructures (FCI) for the aviation industry with *digital communication* as its central component. This transformation is in part due to drastic increases in air-traffic load, global ATC staff shortages, adverse environmental effects of the aviation industry, cost effectiveness and advanced surveillance and safety capabilities. However, by replacing

legacy military-grade infrastructures with digital products – both onboard aircrafts (ACs) as well as in ground systems (GSs) – the communication of the aviation industry is no longer isolated, and thus the threat of an eavesdropper or even an active attacker is no longer unrealistic. Thus, integrating digital technologies into critical communications without carefully considering their security will drastically undermine the safety of such avionics. Further exacerbating the situation is the highly-dependent nature of the aviation industry: attacks on a single component may cascade and cripple its entire infrastructure, ending in catastrophic failures. Such attacks are not unrealistic: EUROCONTROL reports that such cyberattacks against aviation systems are on the rise [8].

Controller-Pilot Data Link Communications (CPDLC) is a protocol that facilitates communication between the Air Traffic Control (ATC) stations and ACs over a datalink medium. CPDLC was designed to reduce communication loads on the Very High Frequency (VHF) band, in order to handle large numbers of ACs communicating with ATCs simultaneously in congested air traffic zones. CPDLC communication has multiple applications, ranging from route change and clearances to level assignments and crossing constraints [10]. At present all CPDLC communications are carried out over unencrypted and unauthenticated datalinks. Thus, there is a need to develop a future-proof secure communication protocol that both ensures confidentiality and authenticity of communication, while withstanding the evolving threat landscape.

At the time of writing, NIST's Post Quantum Cryptography (PQC) Standardization Process has concluded and chosen its winners for key encapsulation and digital signatures categories. This process standardizes post-quantum public key cryptographic primitives to potentially replace existing quantum-vulnerable primitives currently in widespread use. In addition, the White House recently published a memorandum [25], introducing a plan to increase resources and collaborative efforts for the multi-year process of migrating vulnerable computer systems to quantum-resistant cryptography.

However, almost all PQC increases computation and key sizes compared to quantum-vulnerable counterparts, complicating their practical adoption, especially in resource-constrained environments. For instance, the novel LDACS media for GS-to-AC communication provides a throughput of 303.33 kbps (forward-link) and 199.73 kbps (reverse-link) [2], and the average throughput of data-links used in avionic communication is 31.5kbps across the globe. Moreover, security analysis of PQC is relatively immature and constantly evolving compared to their classical counterparts [7]—corroborated by recent attacks undermining the security of digital signature scheme Rainbow [3] and the KEM SIKE [4]—both candidates previously shortlisted as NIST PQC finalists. Within this context, adopting a hybrid approach [7] combining classical and PQC primitives presents a realistic equilibrium, providing security guarantees against potentially undiscovered vulnerabilities (both algorithmic and in-code) in novel PQC and security against future quantum adversaries.

In this paper we propose two hybrid key exchange protocols to secure CPDLC communication between ground stations (GS) and aircrafts (AC). We provide a

formal proof of security for the proposed protocols against both quantum and classical adversaries. We instantiate our proposed protocols with different post-quantum algorithms to understand the real-world applicability of our protocols to the resource-constrained ecosystem of avionic communication. We compare our proposals to existing work and demonstrate that we provide satisfactory performance with the added benefit of heightened (hybrid) security.
Organisation: In Section 3 we describe our protocols, followed by details of implementation in Section 4. Sections 5 and 6 explain our security framework in detail and formally prove our protocols' security, respectively. The paper closes with conclusions and directions for future work in Section 7.

2 Related Work

Here we discuss pre-existing literature related to classical and post-quantum avionic communication protocols.

The Automatic Dependent Surveillance-Broadcast is a mandatory broadcast system for all aircrafts, where an aircraft periodically broadcasts its position, allowing it to be tracked. Wesson et. al. [27] discuss the security of the ADS-B protocol and lays out specific considerations required when designing the security of aviation communication; interoperability with existing policies and laws, bandwidth and interference constraints and how ADS-B operates in a *cryptographically untrusted environment*. Their work primarily evaluates symmetric and asymmetric settings, concluding that maintaining the secrecy of symmetric keys across multiple untrusted global domains is unsuitable for the purpose of secure communication in aviation.

Given the safety-critical nature of the aviation industry, complete encryption of critical communication is problematic, due to the non-negligible likelihood of system or human error resulting in decryption failures in a prompt real-time capacity. In fact, the federal aviation administration (FAA) has strongly recommended maintaining clear datalinks for aviation communication in order to keep critical surveillance data, such as positional information of aircraft, openly accessible [28]. Moreover, the use of shared keys also complicates key revocation scenarios in the event of key compromise, since it requires replacing keys across all parties involved. The novel ADS-B proposal [28] is based on symmetric-key primitives, specifically Format-Preserving Encryption for obscuring flight identity and the TESLA protocol [17] for authentication. While less computationally expensive, as previously discussed it is not a scalable solution in the global airspace. In addition, their work heavily relies on a Trusted Third Party for key management and distribution on a frequent basis. Furthermore, the use of TESLA protocol introduces additional latency to the scheme given how decryption keys are sent after sending the encrypted messages. Our protocols (presented in Section 3) avoid these issues, as they are constructed from asymmetric-key primitives, require only a single round for key establishment, and secure CPDLC communications specifically, not addressing ADS-B.

Other approaches to securing communication in aviation rely on asymmetric encryption schemes, introducing communication overhead compared to symmetric encryption, due to large public-key and ciphertext sizes. For instance, to achieve the symmetric-key equivalent strength of 112 bits (ADS-B packet size), which NIST claims is cryptographically secure until 2030, the ECDSA signature length is 448 bits, four times greater than an ADS-B message [27]. Since ECDSA generates the shortest signature for a given key length it has been proposed as a suitable scheme for securing ACARS messages by the ACARS message security standard [1] but is rarely adopted by any major airlines.

Khan et. al. [10] propose a lightweight protocol for securing CPDLC using elliptic-curve cryptography and Schnorr signature schemes. Their method provides authenticated communication while preserving confidentiality and non-repudiation properties against a classical adversary, but fails to achieve security against a quantum adversary. Mäurer et. al. [15] also propose an alternative to CPDLC, an LDACS-based protocol using different flavours of Diffie-Hellman key exchange (DHKE) to establish keys, and evaluate the practicality of their proposals. They modify the Station-to-Station (STS) protocol with distinct DHKE instantiations, comparing Elliptic Curve DHKE (ECDH) and Supersingular Isogeny DHKE (SIDH) as the underlying key exchange. Their work concludes while STS-ECDH provides the most resource efficient performance, STS-SIDH (believed at the time to provide post-quantum security) was the better option for long-term security. We implement our solution in Section 4, and compare with Mäurer et. al.'s STS-(C)SIDH protocol [15], demonstrating our protocol's relative practicality while also achieving stronger notions of security.

Finally, Bellido-Mangenell et. al. [2] and Mielke et. al. [16] propose a secure protocol for CPDLC over the LDACS medium, and simulate their protocol's communication between aircrafts and ground stations. Their proposal combines an asymmetric post-quantum public-key encryption scheme (McEliece, using code-based cryptography) and symmetric encryption (AES-256-GCM). The authors [16] highlight that the protocol does not guarantee security against potential man-in-the-middle attackers since the exchanged (ephemeral) public keys are not authenticated. Finally, neither provide any formal proof of security.

2.1 KEMTLS

KEMTLS [21] is a key exchange protocol, proposed to transition TLS 1.3 handshakes to a post-quantum setting. KEMTLS proposes the use of KEMs instead of digital signatures for server authentication, as post-quantum signature public keys and signatures tend to be larger than their post-quantum KEM counterparts. While TLS 1.3 commonly runs in unilateral (server-only) authentication, there are many scenarios which require mutual client-server authentication, which includes our air-to-ground communication setting. Schwabe et al.[22] propose a mutually-authenticated variant of KEMTLS, called KEMTLS-PDK, relying on pre-distributed server public keys prior to the protocol initiation.

While TLS 1.3's setting and requirements are similar to our own, the avionic communication media places significant restraints on both bandwidth and com-

putation, as it abhors congestion and communication must complete within a restricted timeframe, necessitating the introduction of a custom protocol. Our two proposed protocols, PQAG-SIG and PQAG-KEM, superficially resemble TLS 1.3 and KEMTLS-PDK respectively, but our design has been specifically tailored to the domain constraints of avionic communication. Our key schedule is significantly simplified, avoiding the complexity introduced in TLS 1.3. In addition, we achieve 1.5 round-trip times for both variants. The PQAG-SIG protocol, introduced to take advantage of existing certificate-based infrastructures, mutually authenticates aircrafts and ground-control stations using certificates (classical or post-quantum) and derives a post-quantum hybrid shared key. Concurrently, the PQAG-KEM variant of our protocol provides post-quantum hybrid implicit mutual authentication and derives a post-quantum hybrid shared key.

3 PQAG Key Exchange Protocols

In this section we describe two variants of our proposed post-quantum air-to-ground communications protocol PQAG, PQAG-KEM and PQAG-SIG, executed between an Aircraft A and a Ground Station G. We give the detailed cryptographic operations in Figures 1 and 2. Note that PQAG-KEM and PQAG-SIG have slightly different infrastructure requirements, as well as different communication and computational overheads. In PQAG-SIG, we require no predistribution of public keys, but imposes higher communication costs due to large public-key sizes. Overheads added by PQAG-SIG are particularly high if the implementation utilizes post-quantum digital signatures (see Tables 6 and 5). In PQAG-KEM, we assume that A already knows G's public keys, reducing communication and computational overhead, but may be a realistic (and scalable) assumption in avionics, where travel paths (and thus, G partners) can be known ahead of time.

3.1 PQAG-KEM

Broadly, PQAG-KEM executes a series of post-quantum and classical KEMs between A and G, combining the outputs into symmetric keys mk and k. A and G maintain two long-term KEM key pairs, one classical and one post-quantum. We assume that G has pre-distributed their public KEM key pairs to A prior to the protocol execution, and that public keys can be validated by some PKI, outside the scope of our protocol. The MAC tags, computed using mk (itself computed using outputs of the long-term KEMs), provide mutual authentication, and the ephemeral KEMs provide forward secrecy for the derived session key k.

A begins by generating post-quantum and classical KEM ephemeral public keys ($pqpk_{\mathsf{E}}$, cpk_{E} respectively) and a random nonce r_A. Next, A encapsulates secrets under the long-term key pairs of G (cpk_{G} and $pqpk_{\mathsf{G}}$), computing ciphertexts $cctxt_{\mathsf{G}}$ and $pqctxt_{\mathsf{G}}$. Afterwards, A forms a message m_0 by concatenating some (arbitrary) header information $\mathsf{header}_{\mathsf{A}}$, with r_{A}, id_{A}, $cctxt_{\mathsf{G}}$, $pqctxt_{\mathsf{G}}$,cpk_{E}, $pqpk_{\mathsf{E}}$ and the long-term public-keys of A cpk_{A}, $pqpk_{\mathsf{A}}$, sending m_0 to G.

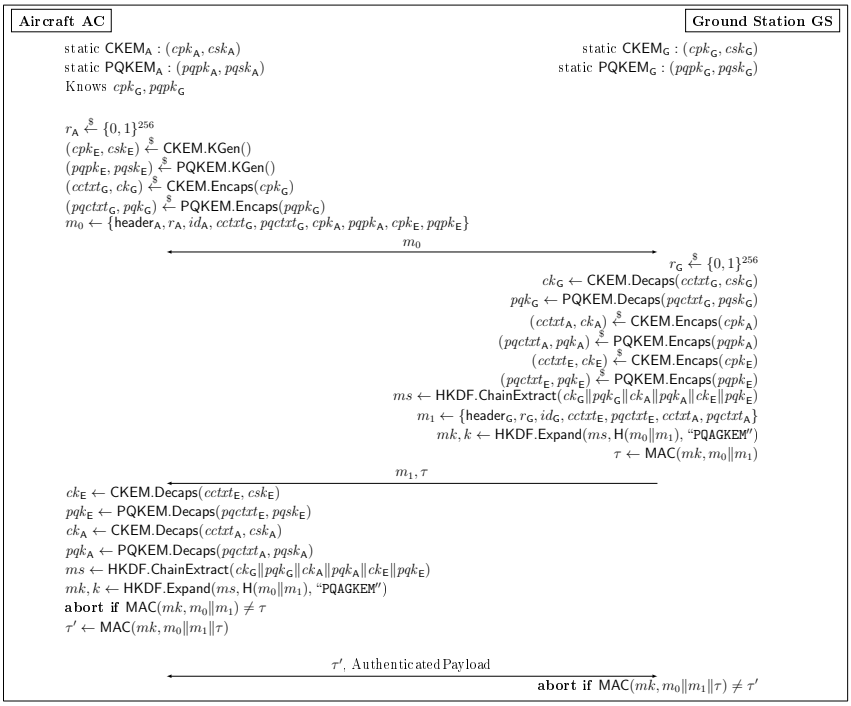

Fig. 1: The PQAG-KEM key exchange protocol. Note that $\mathsf{HKDF.ChainExtract}(a \| b \| \ldots \| n) = \mathsf{HKDF.Extract}(\ldots \mathsf{HKDF.Extract}(\mathsf{HKDF.Extract}(a, 0), b) \ldots, n)$.

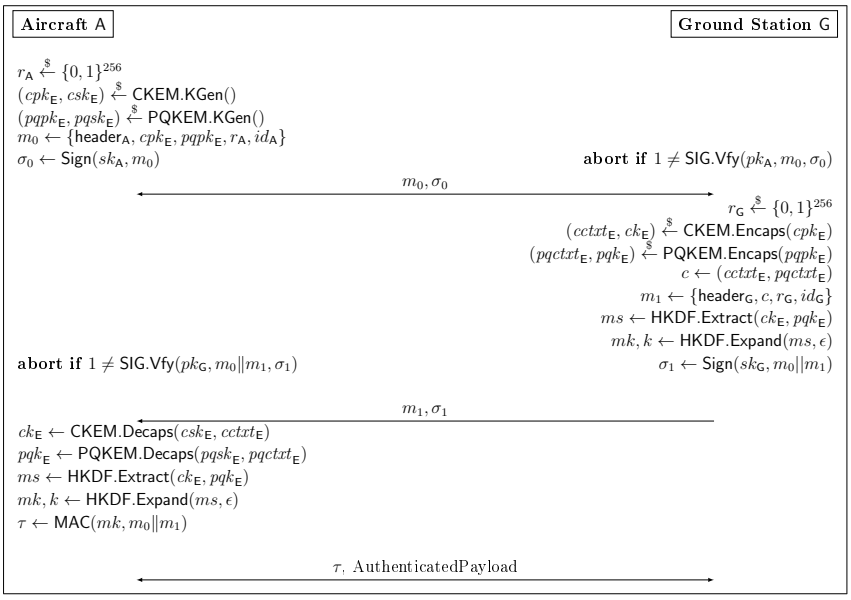

Fig. 2: The PQAG-SIG key exchange protocol.

Upon receiving m_0, G decapsulates post-quantum and classical ciphertexts $pqctxt_{\mathsf{G}}$ and $cctxt_{\mathsf{G}}$, deriving keys pqk_{G} and ck_{G}. Next, G encapsulates secrets under the ephemeral public keys $cpk_{\mathsf{E}}, pqpk_{\mathsf{E}}$, generating $cctxt_{\mathsf{E}}$ and $pqctxt_{\mathsf{E}}$ respectively. G further encapsulates secrets under A's long-term public keys $cpk_{\mathsf{A}}, pqpk_{\mathsf{A}}$, generating $cctxt_{\mathsf{A}}$ and $pqctxt_{\mathsf{A}}$. The key outputs of KEM encapsulations and decapsulations are used to derive a master key $ms = \mathsf{HKDF.ChainExtract}(ck_{\mathsf{G}} \| pqk_{\mathsf{G}} \| ck_{\mathsf{A}} \| pqk_{\mathsf{A}} \| ck_{\mathsf{E}} \| pqk_{\mathsf{E}})$. G forms message m_1 by concatenating some (arbtitrary) header $\mathsf{header}_{\mathsf{G}}$, with r_{G}, id_{G}, $cctxt_{\mathsf{E}}$, $pqctxt_{\mathsf{E}}$, $cctxt_{\mathsf{A}}$ with $pqctxt_{\mathsf{A}}$. G derives a final session key k and MAC key mk by computing $mk, k = \mathsf{HKDF.Expand}(ms, \mathsf{H}(m_0 \| m_1), \text{“}\texttt{PQAGKEM}''$). The G further computes $\tau \leftarrow \mathsf{MAC}(mk, m_0 \| m_1)$, sending m_1 and τ to A.

A decapsulates $cctxt_{\mathsf{E}}$, $pqctxt_{\mathsf{E}}$, $cctxt_{\mathsf{A}}$ and $pqctxt_{\mathsf{A}}$ and derives the shared keys mk, k. Next, A verifies τ under mk aborts the session if it fails. Finally, A computes $\tau' \leftarrow \mathsf{MAC}(mk, m_0 \| m_1 \| \tau)$ and returns τ' to G for authentication, and can immediately use k establish secure communications with G. G finally verifies τ', aborting the session if it fails.

3.2 PQAG-SIG

PQAG-SIG proceeds similarly to PQAG-KEM by combining ephemeral post-quantum and classical KEM outputs into a single session key k, achieving forward secrecy. However, authentication is achieved by post-quantum signature schemes SIG. Unlike PQAG-KEM, we do not assume any pre-distribution of public keys, but similarly assume that long-term public keys can be validated through some PKI outside the scope of our protocol.

A begins by generating post-quantum and classical KEM public keys $cpk_{\mathsf{E}}, pqpk_{\mathsf{E}}$ respectively and a random nonce r_{A}. A straightforwardly computes a message m_0 by concatenating header information $\mathsf{header}_{\mathsf{A}}$, cpk_{E}, $pqpk_{\mathsf{E}}$, r_{A} and its unique identifier id_{A}. A signs m_0 using its long-term secret key sk_{A} (outputting σ_0), sending m_0 and σ_0 to G.

G verifies σ_0, aborting the session if it fails. G then encapsulates post-quantum and classical secrets under $pqpk_{\mathsf{E}}, cpk_{\mathsf{E}}$, outputting $pqctxt_{\mathsf{E}}$ and $cctxt_{\mathsf{E}}$ respectively, which are concatenated into c. G forms message m_1 by concatenating header_B, ciphertext c, random nonce r_{G} and its unique identifier id_{G}. G derives an intermediate value $ms = \mathsf{HKDF.Extract}(ck, pqk)$, and derives the session key k and MAC key mk by computing $mk, k = \mathsf{HKDF.Expand}(ms, \epsilon)$. G generates its own signature $\sigma_1 \leftarrow \mathsf{Sign}(sk_{\mathsf{G}}, m_0 \| m_1)$ sending σ_1 and m_1 to A.

A verifies σ_1 using G's long-term public key sk_{G}, and upon failure will abort the session. A then decapsulates $cctxt_{\mathsf{E}}$ and $pqctxt_{\mathsf{E}}$, and derives the shared keys mk, k. Finally, A computes $\tau \leftarrow \mathsf{MAC}(mk, m_0 \| m_1)$ and returns τ to G, immediately using k to communicate securely with G.

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4 Implementation of PQAG

In this section we discuss our instantiation and reference implementation of the PQAG protocols. We implement PQAG-KEM and PQAG-SIG in Python, and benchmarked their performance on a Raspberry Pi to demonstrate practicality on constrained devices (and for uniform comparisons with previously existing protocols), and a standard desktop system. We compare our results with existing works [2,15]. We modified the STS-SIDH protocol [15] with CSIDH, due to SIDH weaknesses [4], using the post-quantum `sibc` library for CSIDH [20] and Ed25519 for digital signatures. It must be noted that [2] does not provide performance metrics, only network performance results. We begin by discussing our choices of instantiations for cryptographic primitives.

Instantiation PQAG aims for 128-bit post-quantum security against a quantum-equipped attacker. The efficiency of PQAG within resource-constrained environments was a critical consideration during instantiation. We instantiate PQAG-SIG with Kyber as the post-quantum KEM and Dilithium as the digital signature (which we denote PQAG-SIG-Ky-Di). To compare with classical signatures, we instantiate PQAG-SIG with either McEliece (PQAG-SIG-Mc) or Kyber (PQAG-SIG-Ky) as the underlying post-quantum KEM, but with classical signature scheme (EC)DSA. For all three variants, the final derived shared keys were 256-bits long. Thus our choices of cryptographic algorithms for PQAG-SIG are:

- Classic CKEM: Elliptic-curve DH key exchange using curve P384 [9].
- Post-Quantum PQKEM: McEliece with parameter set 348864f [24] and Kyber-512 [24], both achieving 128-bit quantum security.
- SIG: EdDSA using curve P-384 [9] For PQAG-SIG-Ky-Di with NIST level 2 security claimed [18,26].
- MAC: HMAC-SHA-256 [12] using 256-bit keys.
- KDF: HKDF-SHA-256 [11] using 256-bit keys.

We instantiate PQAG-KEM with Kyber as the post-quantum KEM (which we denote PQAG-KEM-Hy). To check how much our hybrid approach impacts performs, we also implemented a variant of PQAG-KEM that does not include any of the CKEM operations, instead simply performing PQKEM steps, which we denote PQAG-KEM-FQ. For both variants, the final derived shared keys were 256-bits long. Thus our choices of cryptographic algorithms for PQAG-KEM are:

- Classic CKEM: Elliptic-curve DH key exchange using curve P384 [9].
- Post-Quantum PQKEM: Kyber-512 [24], achieving 128-bit quantum security.
- MAC: HMAC-SHA-256 [12] using 256-bit keys.
- KDF: HKDF-SHA-256 [11] using 256-bit keys.

Implementation We require the use of the Python `cryptography` library [24] for implementing CKEM and KDF, and `PyNaCl` [23] libraries for SIG cryptographic primitives. For PQAG-SIG-Ky, PQAG-SIG-Mc and PQAG-SIG-Ky-Di we require the use of the Python `pqcrypto` [18] library.

4.1 PQAG-SIG Computational Costs

We now profile the performance of the underlying cryptographic functions in terms of average execution times (for 100 iterations per cryptographic functions). Our benchmarking experiments are designed to provide a comparative evaluation of the PQAG protocols among their different initiations as well as existing literature, and also evaluate the cost of different post-quantum algorithms used in PQAG-SIG and PQAG-KEM respectively. Table 1 compares the performance of the cryptographic components of PQAG-SIG-Mc, PQAG-SIG-Ky and PQAG-SIG-Ky-Di when run on two separate testbeds. Our experiments were performed on a Raspberry Pi 3 B+ running Raspberry Pi OS with a 1.4GHz quad core and 1GB RAM; and an Intel Core 1.80GHz i7-10510U CPU with 16GB RAM, running Windows 10 Home.

Operation (A)	PQAG McE **Pi**	PQAG Ky **Pi**	PQAG Di **Pi**	PQAG McE **Intel**	PQAG Ky **Intel**	PQAG Di **Intel**
PQKEM.KGen	1.1530	0.0015	0.0007	0.2814	0.0006	0.0002
CKEM.KGen	0.0178	0.0268	0.0201	0.0036	0.0027	0.0011
SIG.Sign	0.0051	0.0024	0.0072	0.0020	0.0003	0.0003
SIG.Vfy	0.0058	0.0014	0.0015	0.0021	0.0003	0.0001
KEM.Decaps	0.0216	0.0191	0.0190	0.0044	0.0028	0.0012
MAC	0.0053	0.0001	0.0002	0.0013	0.0001	0.0001
Operation (G)	PQAG McE **Pi**	PQAG Ky **Pi**	PQAG Di **Pi**	PQAG McE **Intel**	PQAG Ky **Intel**	PQAG Di **Intel**
CKEM.KGen	0.0177	0.0205	0.0194	0.0035	0.0027	0.0040
SIG.Sign	0.0056	0.0011	0.0056	0.0022	0.0003	0.0011
SIG.Vfy	0.0054	0.0019	0.0016	0.0022	0.0003	0.0004
KEM.Encaps	0.0194	0.0193	0.0192	0.0046	0.0031	0.0047
MAC	0.0047	0.0002	0.0002	0.0013	0.0001	0.0001

Table 1: Performance evaluation for cryptographic primitives (in seconds) on **Raspberry Pi 3 B+** (left-hand side) and on **Intel Core i7-10510U CPU @ 1.80GHz.** (right-hand side). KEM.Encaps and KEM.Decaps combines PQKEM, CKEM and KDF operations into a single function. Note that McE, Ky and Di refer to PQAG-SIG-Mc, PQAG-SIG-Ky and PQAG-SIG-Ky-Di, respectively, and A contains results for aircraft operations and G results for ground stations.

Both PQAG-SIG-Ky and PQAG-SIG-Ky-Di achieve significantly better performance than PQAG-SIG-Mc, and as expected the desktop testbed was better for all performance metrics than the Raspberry Pi testbed. In general, the McEliece key generation had the highest average time per operation for both the test environments. Kyber key generation, on the other hand, was faster even when

compared to the classical ECDH key generation. In addition, the SIG.Sign and SIG.Vfy operations of PQAG-SIG-Ky outperformed PQAG-SIG-Mc, due to the drastically smaller public keys. This also applies to the MAC execution on both A and G: due to the smaller key sizes the MAC operations were significantly faster for PQAG-SIG-Ky. Between PQAG-SIG-Ky's SIG.Sign operations performed faster than PQAG-SIG-Ky-Di's due to its smaller key sizes. However, for both the instantiations the difference in SIG.Vfy performance was negligible, confirming the faster verification times of Dilithium. For the desktop testbed, all cryptographic operations averaged well under a single second, whereas in the the Raspberry Pi 3 B+ testbed, all operations except the McEliece key generation averaged under a second. These results are promising for practical integration of PQAG-SIG-Ky into the constrained environments used in aviation infrastructure.

Variant	PQAG-SIG-Mc **Pi**	PQAG-SIG-Ky **Pi**	PQAG-SIG-Mc **Intel**	PQAG-SIG-Ky **Intel**
Online	1.17584	0.03074	0.28688	0.00355
Offline	0.04528	0.02557	0.00624	0.00286

Table 2: Aircraft walltime (in seconds) with online and offline post-quantum key generation on **Raspberry Pi 3 B+** (left-hand side) and **Intel Core i7-10510U CPU @ 1.80GHz** (right-hand side).

We also implemented optimised variants of both PQAG-SIG-Ky and PQAG-SIG-Mc that perform some offline computation to improve the online runtime of the protocol execution. Since aircrafts will have offline travel time between communication with different ground stations, the protocol can reduce online key generation by pre-computing and storing the post-quantum public keys prior to the session initialisation by A. For both the original "real-time" protocol and the "precomputed" protocols we profiled the wall-time performance of the average A execution times (for 100 executions of PQAG-SIG-Ky and PQAG-SIG-Mc respectively), and give the results in Table 2. For both test environments, offline key generation reduced PQAG-SIG-Mc's and PQAG-SIG-Mc's computational overhead, but it significantly decreased the total walltime of PQAG-SIG-Mc. This is because key generation in Kyber is significantly faster than McEliece, so the impact is not quite as high. We did not implement an offline version of PQAG-SIG-Ky-Di since it would not have impacted the signing operations.

4.2 PQAG-KEM Computational Costs

Tables 3 compares the computational overhead of each cryptographic primitive of PQAG-KEM-Hy, PQAG-KEM-FQ when run on our two testbeds.

The performance results of PQAG-KEM-Hy and PQAG-KEM-FQ are shown in Table 3. It is clear that PQAG-KEM-Hy has higher computational overhead, due to the additional CKEM operations and key derivation steps. However, it is

Operation (A)	PQAG Hy **Pi**	PQAG FQ **Pi**	PQAG Hy **Intel**	PQAG FQ **Intel**
$\mathsf{PQKEM.KGen}_{\mathsf{E}}$	0.00090	0.00115	0.00028	0.00028
$\mathsf{CKEM.KGen}_{\mathsf{E}}$	0.00301	NA	0.00068	NA
$\mathsf{PQKEM.Encaps}_{\mathsf{G}}$	0.00127	0.00243	0.00064	0.00029
$\mathsf{CKEM.Encaps}_{\mathsf{G}}$	0.00313	NA	0.00082	NA
$\mathsf{CKEM.Decaps}_{\mathsf{E}}$	0.00292	NA	0.00066	NA
$\mathsf{PQKEM.Decaps}_{\mathsf{A}}$	0.00133	0.00160	0.00021	0.000092
$\mathsf{PQKEM.Decaps}_{\mathsf{E}}$	0.00270	0.00135	0.00058	0.00011
MAC	0.00003	0.000061	0.00008	0.00007
Operation (G)	PQAG Hy **Pi**	PQAG FQ **Pi**	PQAG Hy **Intel**	PQAG FQ **Intel**
$\mathsf{CKEM.Encaps}_{\mathsf{E}}$	0.00579	NA	0.00129	NA
$\mathsf{PQKEM.Encaps}_{\mathsf{E}}$	0.00144	0.00090	0.00069	0.00019
$\mathsf{CKEM.Decaps}_{\mathsf{G}}$	0.00293	NA	0.00068	NA
$\mathsf{PQKEM.Encaps}_{\mathsf{A}}$	0.00124	0.00089	0.00065	0.00018
$\mathsf{PQKEM.Decaps}_{\mathsf{G}}$	0.00321	0.00190	0.00069	0.00017
MAC	0.00003	0.000061	0.00010	0.00007

Table 3: Performance evaluation for cryptographic primitives used in PQAG-SIG variants (in seconds) on **Raspberry Pi 3 B+** (left-hand side) and **Intel Core i7-10510U CPU @ 1.80GHz** (right-hand side). Note that Hy and FQ refers to the (hybrid secure) PQAG-SIG-Ky, and (purely post-quantum) PQAG-KEM-FQ, respectively, and A contains results for aircraft operations and G results for ground stations.

clear that the computational performance of the two instantiations is negligibly different, and we argue that the added hybrid layer of security in PQAG-KEM-Hy justifies the slight increase in computation time.

In Table 4 we compare the performance of all instantiations of PQAG against STS-SIDH [15]. For all protocols we profiled the walltime performance of the entire AC and GS execution (for 100 executions of PQAG-KEM, PQAG-SIG and STS-SIDH). Both the AC and the GS are running on the same machine, and communication is exchanged via `localhost`.

In general STS-CSIDH proved computationally most expensive, averaging at significantly higher performance times in both testbeds. Particularly, within the Raspberry Pi testbed STS-CSIDH struggled to perform, and with occasionally lengthy “hang" times between subsequent operations and periodic restarts. The original STS-SIDH [15] is instantiated with SIDH which is around 10x times faster compared to CSIDH. However, since the publication of [15], SIDH has been proven insecure [4] and thus we instantiated STS-CSIDH [15] with CSIDH to guarantee its post-quantum security while maintaining its SIDH basis. Fur-

Testbed	STS CSIDH	PQAG KEM FQ	PQAG KEM Hy	PQAG SIG McE	PQAG SIG Ky	PQAG SIG Di
Pi	312.8312	0.0103	0.0300	1.2513	0.0169	0.0947
Intel	57.5576	0.0013	0.0087	0.3059	0.0024	0.0133

Table 4: Comparison of walltime executions (in seconds).

thermore, STS-CSIDH requires an additional message from the ground station to the aircraft, adding to communication latency. In comparison PQAG-SIG-Mc fared significantly better in both testbeds, even when performance is affected by the large KEM public keys. Overall, among all the PQAG-SIG instantiations, PQAG-SIG-Ky offered the most competitive performance in the constrained Raspberry Pi testbed. Between PQAG-SIG-Ky and PQAG-SIG-Ky-Di, PQAG-SIG-Ky performed better due to the faster signing operations and smaller signature sizes of ECDSA compared to Dilithium.

Between the two implementations of PQAG-KEM, PQAG-KEM-FQ had a slight edge in performance, compared to PQAG-KEM-Hy. This was due to the fact that PQAG-KEM-FQ only uses post-quantum Kyber for all KEMs, whereas for PQAG-KEM-Hy we use hybrid KEMs, combining Kyber with ECDH to provide hybrid security. We conclude that the negligible increase in the computation times of PQAG-KEM-Hy is offset by the additional layer of security.

Protocol / Datalink	STS CSIDH	PQAG-KEM-FQ	PQAG-KEM-Hy
VDLm2/31.5kbps	$0.660+\Delta$	$0.998+\Delta$	$1.170+\Delta$
AeroMACS/1.8 – 9.2Mbps	$0.001\text{-}0.0002+\Delta$	$0.018\text{-}0.003+\Delta$	$0.0201\text{-}0.004+\Delta$
LDACS/0.6 – 2.8Mbps	$0.003\text{-}0.0007+\Delta$	$0.052\text{-}0.011+\Delta$	$0.062\text{-}0.013+\Delta$
InmarsatSB/432kbps	$0.005+\Delta$	$0.072+\Delta$	$0.085+\Delta$
IridiusmCertus/22 – 704kbps	$0.094\text{-}0.003+\Delta$	$1.42\text{-}0.044+\Delta$	$1.665\text{-}0.0521+\Delta$
Protocol / Datalink	**PQAG-SIG-Mc**	**PQAG-SIG-Ky**	**PQAG-SIG-Ky-Di**
VDLm2/31.5kbps	$66.940+\Delta$	$0.560+\Delta$	$1.554+\Delta$
AeroMACS/1.8 – 9.2Mbps	$1.164\text{-}0.2274+\Delta$	$0.01\text{-}0.0019+\Delta$	$0.028\text{-}0.006+\Delta$
LDACS/0.6 – 2.8Mbps	$3.492\text{-}0.7483+\Delta$	$0.029\text{-}0.0062+\Delta$	$0.082\text{-}0.018+\Delta$
InmarsatSB/432kbps	$4.85+\Delta$	$0.041+\Delta$	$0.113+\Delta$
IridiusmCertus/22 – 704kbps	$95.237\text{-}2.976+\Delta$	$0.797\text{-}0.025+\Delta$	$2.225\text{-}0.07+\Delta$

Table 5: Comparison of transmission times (in seconds) per round-trip.

Communication Cost For all our instantiations, Table 5 compares the transmission times per round-trip, for all frequently used data-links in ground-to-air

Implementation	Hello	Response
STS-CSIDH	128	128
PQAG-KEM-FQ	2384	1520
PQAG-KEM-Hy	2740	1844
PQAG-SIG-Mc	261447	455
PQAG-SIG-Ky	1090	1026
PQAG-SIG-Ky-Di	3075	3043

Table 6: Hello and Response message sizes (in bytes) for each implementation.

avionic communication. This was calculated by dividing the packet size of each instantiation (Table 6) by the expected bandwidth for each medium. For each calculated transmission time we have an additional Δ value to account for various factors affecting the round-trip time, such as latency. This Δ value is controlled by the specifics constraints of the communication links and the setting in which its used. For instance, the expected Δ of STS-CSIDH is extremely high due to the long computational times (10× slower than the now defunct SIDH). For PQAG-McEliece, the Δ unacceptably increases due to the large public keys (261,120 bytes). Satellite data-links have a longer RTT due to the distance between nodes, which in turn will affect the Δ.

Component	PQAG-SIG McE	PQAG-SIG Ky	PQAG-SIG Di	PQAG-KEM FQ	PQAG-KEM Ky
PQKEM *pqpk*	261120	800	800	800	800
PQKEM *pqctxt*	112	736	736	736	736
CKEM *cpk*	215	215	215	N/A	215
SIG σ	96	96	2044	N/A	N/A
MAC τ	32	32	32	32	32

Table 7: Communication cost of the underlying cryptographic components of PQAG-KEM and PQAG-KEM respectively (in bytes).

Table 7 compares the communication complexity of all our PQAG instantiations. We consider the length of the cryptographic components of each message, in particular we do not capture header sizes, nor nonce or *id* values, as they are either constant or setting-specific. The McEliece public key is the most bandwidth-consuming component of PQAG-SIG-Mc at 261Kb. However, the PQKEM ciphertext size of McEliece was smaller than that of Kyber, which may prove advantageous in certain circumstances, such as in the case of data-links with asymmetric bandwidths for forward-and-return links. Moreover, the signature size of Dilithium (2044B) is significantly larger compared to the classical counterpart EdDSA (96B), which will add to the communication overhead.

Additionally, it must be noted that although the output of ECC-P384R1 is 384-bits, due to additional encodings of the Python library used, the resulting final output produces ECC keys that are 215-bytes long, which can be optimised.

5 Hybrid Security Framework

In this section we detail the hybrid authenticated key exchange (HAKE) security framework HAKE (mostly verbatim [7]) for the analysis of the PQAG key exchange protocols. As in the original model [7], the modified HAKE model is based on Bellare-Rogaway-based AKE models, and captures adversaries of differing strength (quantum and classical) via a detailed key compromise interface. Specifically, we model quantum adversaries by allowing them to compromise non-post-quantum key establishment primitives (for instance, elliptic curve-based algorithms to establish a shared secret key). The majority of our modifications from the original HAKE model are simplifications – since HAKE explicitly captures the use of preshared symmetric keys, not used within our new PQAG protocols. We give a high-level description of the modified HAKE framework in Section 5.2 (and detail the differences between our variant and the original HAKE framework). We then describe cleanness and partnering definitions in Section 5.4 as well as Section 5.5.

5.1 Secret Key Generation

Recall that HAKE addresses secret key generation (the output of a "KGen" algorithm) of individual key establishment primitives explicitly, and categorises them into *long-term* (i.e. generated once and used in every execution of the protocol), and *ephemeral* (i.e. generated on a per-stage basis) secret generation. We simplify the HAKE model by only including the following sub-categories:

- *Post-quantum asymmetric secret generation* – long-term post-quantum asymmetric secrets (for example, signature secret keys), are generated by LQKeyGen, whereas ephemeral post-quantum asymmetric secrets (such as KEM secret keys) are generated by EQKeyGen.
- *Classical asymmetric secrets* – long-term classical asymmetric secrets (for example, ECDSA secret keys) are generated by LCKeyGen, whereas ephemeral classical asymmetric secrets (for example, ECDH secret keys) are generated by ECKeyGen.

In addition, our proposed PQAG protocols are not a multi-stage key exchange protocols, which establishes multiple keys throughout protocol execution. Thus, we remove all multi-stage specific state and indexing in the HAKE execution environment. With this context, we now formally define the HAKE execution environment, capturing how an adversary can interact with a hybrid AKE protocol.

5.2 Execution Environment

Consider an experiment $\mathsf{Exp}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda)$ played between a challenger $\mathcal{C}$ and an adversary $\mathcal{A}$. $\mathcal{C}$ maintains a set of n_P parties $P_1,\ldots,P_{n_P}$ (representing users interacting with each other in protocol executions), each capable of running up to (potentially parallel) n_S sessions of a probabilistic key exchange protocol Π. Each session is an execution of the key exchange protocol Π, represented as a tuple of algorithms $\Pi = (f, \mathsf{EQKeyGen}, \mathsf{ECKeyGen}, \mathsf{LQKeyGen}, \mathsf{LCKeyGen})$. We use π_i^s to refer to both the identifier of the s-th instance of the Π being run by party P_i and the collection of per-session variables maintained for the s-th instance of Π run by P_i, and f is a algorithm capturing the honest execution of the protocol Π by protocol participants. We describe generically these algorithms below:

$\Pi.f(\lambda, \vec{pk}_i, \vec{sk}_i, ps\vec{k}id_i, p\vec{s}k_i, \pi, m) \xrightarrow{\$} (m', \pi')$ is a (potentially) probabilistic algorithm that takes a security parameter λ, the set of long-term asymmetric key pairs $\vec{pk}_i, \vec{sk}_i$ of the party P_i, a collection of per-session variables π and an arbitrary bit string $m \in \{0,1\}^* \cup \{\emptyset\}$. f outputs a response $m' \in \{0,1\}^* \cup \{\emptyset\}$ and an updated per-session state π', behaving as an honest protocol implementation.

We describe a set of algorithms $\Pi.\mathsf{XYKGen}(\lambda) \xrightarrow{\$} (pk, sk)$, where $X \in \{\mathsf{E},\mathsf{L}\}$ and $Y \in \{\mathsf{C},\mathsf{Q}\}$. $\Pi.\mathsf{XYKGen}$ is a probabilistic *post-quantum ephemeral* (if $\mathsf{XY} = \mathsf{EQ}$), *post-quantum long-term* (if $\mathsf{XY} = \mathsf{LQ}$), *classic ephemeral* (if $\mathsf{XY} = \mathsf{EC}$), or *classic long-term* (if $\mathsf{XY} = \mathsf{LC}$) asymmetric key generation algorithms, taking a security parameter λ and outputting a public-key/secret-key pair (pk, sk).

$\mathcal{C}$ runs $\Pi.\mathsf{LQKeyGen}(\lambda)$ and $\Pi.\mathsf{LCKeyGen}(\lambda)$ n_P times to generate long-term post-quantum and long-term classical asymmetric key pairs (which we denote with $\vec{pk}_i, \vec{sk}_i$) for each party $P_i \in \{P_1,\ldots,P_{n_P}\}$, and delivers all public-keys $\vec{pk}_i$ for $i \in \{1,\ldots,n_P\}$ to $\mathcal{A}$. The challenger $\mathcal{C}$ then randomly samples a bit $b \xleftarrow{\$} \{0,1\}$ and interacts with $\mathcal{A}$ via the queries listed in Section 5.3, also maintaining a set of corruption registers, representing a list of ephemeral and long-term secrets that have been compromised by $\mathcal{A}$ via Reveal, $\mathsf{Corrupt}$ and $\mathsf{Compromise}$ queries. Eventually, $\mathcal{A}$ issues a Test query, to which $\mathcal{C}$ responds with k_b, either the real session key generated by the Test session (when $b = 0$), or a random key from the same distribution (when $b = 1$). $\mathcal{C}$ now interacts with $\mathcal{A}$ via the queries listed in Section 5.3 (except the Test query), and eventually terminates and outputs a guess d of the challenger bit b. The adversary $\mathcal{A}$ wins the HAKE key-indistinguishability experiment if $d = b$, and additionally if the test session π satisfies a cleanness predicate clean, which we discuss in more detail in Section 5.5. We give an algorithmic description of this experiment in Figure 3 in Section B. Each session maintains a set of per-session variables:

$\rho \in \{\texttt{init}, \texttt{resp}\}$**:** The role of the party in the current session. Note that parties can be directed to act as `init` or `resp` in concurrent or subsequent sessions.

$pid \in \{1,\ldots,n_P,\star\}$**:** The intended communication partner, represented with $\star$ if unspecified. Note that the identity of the partner session may be set during the protocol execution, in which case pid can be updated once.

$\alpha \in \{\texttt{active}, \texttt{accept}, \texttt{reject}, \perp\}$**:** The status of the session, initialised with $\perp$.

$\mathbf{m}_i \in \{0,1\}^* \cup \{\perp\}$, *where* $i \in \{\mathbf{s},\mathbf{r}\}$: The concatenation of messages sent (if $i = \mathbf{s}$) or received (if $i = \mathbf{r}$) by the session in each stage, initialised by $\perp$.

$\mathbf{k} \in \{0,1\}^* \cup \{\perp\}$: The session key, or $\perp$ if no session key has yet been computed.

$\mathbf{exk} \in \{0,1\}^* \cup \{\perp\}$, where $\mathbf{x} \in \{\mathbf{q},\mathbf{c}\}$: The *post-quantum ephemeral asymmetric* (if $\mathbf{x} = \mathbf{q}$), or *classic ephemeral asymmetric* (if $\mathbf{x} = \mathbf{c}$) secret key generated by the session, initialised by $\perp$.

$\mathbf{st} \in \{0,1\}^*$: Any additional state used by the session in each stage.

5.3 Adversarial Interaction

Our HAKE framework considers a traditional AKE adversary, in complete control of the communication network, able to modify, inject, delete or delay messages. They are able to compromise several layers of secrets: (a) long-term private keys, allowing our model to capture forward-secrecy notions and quantum adversaries. (b) ephemeral private keys, modelling the leakage of secrets due to the use of bad randomness generators, or potentially bad cryptographic primitives or quantum adversaries. (c) session keys, modelling the leakage of keys by their use in bad cryptographic algorithms. The adversary interacts with the challenger $\mathcal{C}$ via the queries below:

$\mathsf{Create}(i,j,role) \to \{(s),\perp\}$: Allows the adversary $\mathcal{A}$ to initialise a new session owned by party P_i, where the role of the new session is $role$, and intended communication partner party P_j. If a session π_i^s has already been created, $\mathcal{C}$ returns $\perp$. Otherwise, $\mathcal{C}$ returns (s) to $\mathcal{A}$.

$\mathsf{Send}(i,s,m) \to \{m',\perp\}$: Allows $\mathcal{A}$ to send messages to sessions for protocol execution and receive the output. If the session $\pi_i^s.\alpha \neq \texttt{active}$, then $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$. Otherwise, $\mathcal{C}$ computes $\Pi.f(\lambda, \vec{pk}_i, \vec{sk}_i, \pi_i^s, m) \to (m', \pi_i^{s\prime})$, sets $\pi_i^s \leftarrow \pi_i^{s\prime}$, updates transcripts $\pi_i^s.\mathbf{m}_r$, $\pi_i^s.\mathbf{m}_s$ and returns m' to $\mathcal{A}/\mathcal{Q}$.

$\mathsf{Reveal}(i,s)$: Allows $\mathcal{A}$ access to the session keys computed by a session. $\mathcal{C}$ checks if $\pi_i^s.\alpha = \texttt{accept}$ and if so, returns $\pi_i^s.\mathbf{k}$ to $\mathcal{A}$. In addition, the challenger checks if there exists another session π_j^t that *matches* with π_i^s, and also sets $\vec{\mathsf{SK}}_j^r \leftarrow \texttt{corrupt}$. Otherwise, $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$.

$\mathsf{Test}(i,s) \to \{k_b,\perp\}$: Allows $\mathcal{A}$ access to a real-or-random session key k_b used in determining the success of $\mathcal{A}$ in the key-indistinguishability game. If a session π_i^s exists such that $\pi_i^s.\alpha = \texttt{accept}$, then the challenger $\mathcal{C}$ samples a key $k_0 \xleftarrow{\$} \mathcal{D}$ where $\mathcal{D}$ is the distribution of the session key, and sets $k_1 \leftarrow \pi_i^s.\mathbf{k}$. $\mathcal{C}$ then returns k_b (where b is the random bit sampled during set-up) to $\mathcal{A}$. Otherwise $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$.

$\mathsf{CorruptXK}(\{i,j\}) \to \{k_i,\perp\}$: Allows $\mathcal{A}$ access to the secret post-quantum long-term key $pqsk_i$ (if $\mathsf{X} = \mathsf{Q}$) or the secret classical long-term key csk_i (if $\mathsf{X} = \mathsf{C}$), generated for the party P_i (and P_j, in the preshared case) prior to protocol execution. If the secret long-term key has already been corrupted previously, then $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$.

$\mathsf{CompromiseYK}(i,s) \to \{\mathbf{eqk},\mathbf{eck},\perp\}$: Allows $\mathcal{A}$ access to the secret ephemeral post-quantum key $\pi_i^s.\mathbf{eqk}$ (if $\mathsf{Y} = \mathsf{Q}$), or the secret ephemeral classical key

π_i^s.**eck** (if $\mathsf{Y} = \mathsf{C}$) generated for the session π_i^s prior to protocol execution. If π_i^s.**eqk**/π_i^s.**eck** has already been corrupted previously, then $\mathcal{C}$ returns $\perp$ to $\mathcal{A}$.

5.4 Partnering Definition

To determine which secrets $\mathcal{A}$ can reveal without trivially breaking the security of a given session, our model must define how sessions are *partnered*. In our work, we use the notion of *matching sessions* [13], and *origin sessions* [6]. On a high level, π_i^s is an origin session of π_j^t if π_i^s has received the messages that π_j^t sent without modification, even if the reply that π_i^s sent back has not been received by π_j^t. If all messages sent and received by π_i^s and π_j^t are identical, then the sessions *match*. We give detailed definitions and a precise pseudocode description of these functions in Appendix B. We give detailed definitions and a precise pseudocode description of these functions in Appendix B.

5.5 Cleanness Predicates

Cleanness predicates in authenticated key exchange protocols detail the exact restrictions on adversarial powers. For instance, in protocols that are not post-compromise secure, the leakage of the long-term key of a party trivially allows the adversary to impersonate that party. Thus, it follows that sessions established after that corruption (with that party as the communicating peer) cannot be secure. We note that the cleanness predicates defined below are specific to PQAG-KEM and PQAG-SIG.

The PQAG protocols defend against a quantum adversary. Thus, a successful adversary is allowed to compromise the long-term and ephemeral classical asymmetric secrets (via CorruptCK and CompromiseCK, respectively) without penalty. Since the PQAG protocols authenticate with post-quantum primitives, and aim to achieve perfect forward secrecy, we allow a successful adversary to issue a CorruptQK(j) query (where the $\pi_i^s.pid = j$ and Test(i, s) was queried), as long as π_i^s was completed before the CorruptQK(j) query was issued.

Thus, a "clean" session has not had $\mathcal{A}$ compromise: (a) the ephemeral post-quantum secrets of the Test session and its matching partner in the tested stage, (b) the long-term post-quantum secrets of the Test session's partner before the Test session completes. We formalise this intuition as $\mathsf{clean}_{\mathsf{PQAG}}$ in Definition 8, in Appendix B.

It may also be desirable to determine the security guarantees that PQAG provides against classical adversaries in the event of a new vulnerability discovered in the underlying post-quantum key establishment primitive, as demonstrated by the recent attacks against Rainbow [3], or SIDH [5,14,19].

In such a case, a "clean" session has not had the adversary compromise: Thus, a "clean" session has not had $\mathcal{A}$ compromise: (a) *either* the ephemeral classic secrets of the Test session and its matching partner in the tested stage, *or* (b) the ephemeral post-quantum secrets of the Test session and its matching partner in the tested stage, *and* (c) the long-term post-quantum secrets of the

Test session's partner before the Test session completes. In order to capture this scenario, we formalise this intuition as $\mathsf{clean}_{c\mathsf{HAKE}}$ in Definition 8, in Appendix B, and provide a second analysis against a PPT adversary $\mathcal{A}$.

Finally, we formalise the advantage of an adversary $\mathcal{A}$ in winning the HAKE key indistinguishability experiment in the following way:

Definition 1 (HAKE Key Indistinguishability). *Let Π be a key-exchange protocol, and n_P, $n_S \in \mathbb{N}$. For a particular given predicate* clean*, a QPT algorithm $\mathcal{A}$, we define the advantage of $\mathcal{A}$ in the* HAKE *key-indistinguishability game to be* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda) = 2 \cdot \left| \Pr\left[\mathsf{Exp}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda) = 1\right] - \frac{1}{2}\right|$*. We say that Π is post-quantum* HAKE*-secure if, for all QPT algorithms $\mathcal{A}$,* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda)$ *is negligible in the security parameter λ. We say that Π is classically* HAKE*-secure if, for all PPT algorithms $\mathcal{A}$,* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda)$ *is negligible in the security parameter λ.*

6 Security Analysis

In this section we analyse our proposed PQAG-SIG and PQAG-KEM protocols, by utilising the simplified HAKE model presented in Section 5. We begin by presenting our first result, the security of PQAG-SIG, in Theorem 1.

Theorem 1 (PQAG-SIG Security). *The* PQAG-SIG *protocol presented in Section 3 is post-quantum secure under cleanness predicate* $\mathsf{clean}_{q\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against $\mathcal{A}$). That is, for any QPT algorithm $\mathcal{A}$ against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$ *is negligible under the* prf*,* dual-prf*,* ind-cpa*,* eufcma *and* eufcma *security of the* PRF*,* PRF*,* KEM*,* MAC *and* SIG *primitives respectively.*

Proof. We now turn to proving our result. We split our analysis into three mutually-exclusive cases:

1. **Case 1** assumes that the test session π^s_i (such that $\mathcal{A}$ issued $\mathsf{Test}(i,s)$) is an initiator session, and that π^s_i has no *matching partner* (as in Figure 4). We define the QPT algorithm $\mathcal{A}$'s advantage in **Case 1** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.
2. **Case 2** assumes that the test session π^s_i is a responder , and that π^s_i has no *matching partner*. We define $\mathcal{A}$'s advantage in **Case 2** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.
3. **Case 3** assumes that the test session π^s_i has a *matching partner*. We define $\mathcal{A}$'s advantage in **Case 3** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.

It is clear that: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) \leq \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$, thus we bound $\mathcal{A}$'s advantage in each case separately.

In **Case 1** and **Case 2** we show that $\mathcal{A}$'s advantage in causing the test session π^s_i to accept without a matching partner is negligible, and thus the $\mathcal{A}$'s

advantage in winning the key-indistinguishability game is negligible (since the experiment does not differ based on the challenge bit b, as the π_i^s does not compute a real-or-random session key).

In **Case 3** we replace the computation of the real session key by the test session π_i^s with a uniformly random key. Thus, the distribution of the keys returned by π_i^s are identical, regardless of the value of the challenge bit b, and we can show that $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible. We now begin with the first case.

Case 1: Test `init` session without origin session We begin by showing that $\mathcal{A}$ has negligible change in causing π_i^s to reach an `accept` state without a matching session. We do so via the following sequence of game hops:

Game 0 This is the HAKE security game and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the intended partner j and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi_{i'}^{s'}$ such that $\pi_{i'}^{s'}.pid = j'$ and $(i,s,j) \neq (i',s',j')$. Thus: $\Pr(break_0) \leq {n_P}^2 \cdot n_S \cdot \Pr(break_1)$.

Game 2 In this game we abort if the test session π_i^s sets the status $\pi_i^s.\alpha \leftarrow$ `reject`. Note that by the previous game we abort if the Test query is issued to a session that is not π_i^s, and if $\pi_i^s.pid \neq j$. If the session π_i^s ever reaches the status $\pi_i^s.\alpha \leftarrow$ `reject`, then the challenger will respond to the $\mathsf{Test}(i,s)$ query with $\perp$, and thus the difference in $\mathcal{A}$'s advantage between **Game 2** and **Game 3** is 0. Thus: $\Pr(break_1) \leq \Pr(break_2)$.

Game 3 In this game we define an abort event $\mathbf{abort}_\alpha$ that triggers if the test session π_i^s sets the status $\pi_i^s.\alpha \leftarrow$ `accept`. We note that the response to $\mathsf{Test}(i,s)$ issued by $\mathcal{A}$ is always $\perp$, (since the challenger aborts the game if π_i^s accepts, and $\mathsf{Test}(i,s) = \perp$ when π_i^s rejects the protocol execution), and thus $\Pr(break_3) = 0$. In **Game 4** and **Game 5** we prove that the probability of $\mathcal{A}$ in causing $\mathbf{abort}_\alpha$ to trigger is negligible. Thus: $\Pr(break_2) \leq \Pr(\mathbf{abort}_\alpha)$.

Game 4 In this game we abort if the test session π_i^s receives a signature σ_1 (signed over $m_0 \| m_1$) that verifies correctly but there exists no honest session π_j^t that has output σ_1. Specifically, in **Game 4** we define a reduction $\mathcal{B}_1$ against the eufcma security of the signature scheme SIG. At the beginning of the experiment, when $\mathcal{B}_1$ receives the list of public-keys $(pk_1, \ldots, pk_{n_P})$ from $\mathcal{C}$, $\mathcal{B}_1$ initialises a eufcma challenger $\mathcal{C}_{\mathsf{eufcma}}$, and replaces pk_j with pk output by $\mathcal{C}_{\mathsf{eufcma}}$. Whenever $\mathcal{A}$ initialises a session owned by P_j, then $\mathcal{B}_1$ generates m_0 as usual, but queries $\mathcal{C}_{\mathsf{eufcma}}$ with m_0 to get a signature σ_0 over m_0. Similarly, whenever $\mathcal{A}$ issues $\mathsf{Send}(j,t,m)$ to a session π_j^t owned by P_j, then $\mathcal{B}_1$ verifies m and computes m_1 as usual, but queries $\mathcal{C}_{\mathsf{eufcma}}$ with $m \| m_1$ to receive σ_1 over $m \| m_1$. These changes are indistinguishable to $\mathcal{A}$, as $\mathcal{C}_{\mathsf{eufcma}}$ generates public-keys and signatures identically to $\mathcal{C}$, so $\mathcal{A}$ cannot detect this replacement. Note that if $\mathcal{A}$ has issued a $\mathsf{CorruptQK}(j)$ query before π_i^s receives σ_1, then $\mathsf{clean}_{q\mathsf{HAKE}} =$ `false` for π_i^s, and $\mathcal{C}$ returns $b^* \xleftarrow{\$} \{0,1\}$

regardless of the challenge bit b. Thus, any $\mathcal{A}$ with non-negligible advantage must not yet have issued $\mathsf{CorruptQK}(j)$. Also, as the result of **Game 2** and **Game 3**, the game aborts after π_i^s receives m_1, σ_1, so $\mathcal{A}$ cannot later issue a $\mathsf{CorruptQK}(j)$ query.
By the definition of **Case 1**, π_i^s sets the status $\pi_i^s.\alpha \leftarrow \texttt{accept}$ despite there being no honest session that outputs m_1, σ_1. Thus, $\mathcal{B}_1$ never queried $m_0 \| m_1$ to $\mathcal{C}_{\mathsf{eufcma}}$, and it follows that m_1, σ_1 is a forged message. Thus, if π_i^s receives a signature σ_1 (signed over $m_0 \| m_1$) that verifies correctly but there exists no honest session π_j^t that has output σ_1, then $\mathcal{B}_1$ wins the eufcma security game against the signature scheme SIG, and $\Pr(\mathbf{abort}_\alpha) = \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{B}_1}(\lambda)$.
Since π_i^s now aborts when verifying σ_1, it cannot trigger $\mathbf{abort}_\alpha$ and thus we have: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) \leq {n_P}^2 n_S \cdot \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{B}_1}(\lambda)$.

Case 2: Test responder session without origin session We now show that $\mathcal{A}$ has negligible change in causing π_i^s (with $\pi_i^s.\rho = \mathsf{responder}$) to reach an $\texttt{accept}$ state without an origin session. As the proof of **Case 2** follows analogously to **Case 1** with a minor change in notation up to **Game 3**, we omit these game hops and proceed from **Game 5**.

Game 4 In this game we abort if the test session π_i^s receives a signature σ_0 (signed over m_0) that verifies correctly but there exists no honest session π_j^t that has output σ_0. Specifically, in **Game 4** we define a reduction $\mathcal{B}_2$ against the eufcma security of the signature scheme SIG. At the beginning of the experiment, when $\mathcal{B}_2$ receives the list of public-keys $(pk_1, \ldots, pk_{n_P})$ from $\mathcal{C}$, $\mathcal{B}_1$ initialises a eufcma challenger $\mathcal{C}_{\mathsf{eufcma}}$, and replaces pk_j with pk output by $\mathcal{C}_{\mathsf{eufcma}}$. Whenever $\mathcal{A}$ initialises a session owned by P_j, then $\mathcal{B}_2$ generates m_0 as usual, but queries $\mathcal{C}_{\mathsf{eufcma}}$ with m_0 to get a signature σ_0 over m_0. Similarly, whenever $\mathcal{A}$ issues $\mathsf{Send}(j, t, m)$ to a session π_j^t owned by P_j, then $\mathcal{B}_2$ verifies m and computes m_1 as usual, but queries $\mathcal{C}_{\mathsf{eufcma}}$ with $m \| m_1$ to receive σ_1 over $m \| m_1$. These changes are indistinguishable to $\mathcal{A}$, as $\mathcal{C}_{\mathsf{eufcma}}$ generates public-keys and signatures identically to $\mathcal{C}$, so $\mathcal{A}$ cannot detect this replacement. Note that if $\mathcal{A}$ has issued a $\mathsf{CorruptQK}(j)$ query before π_i^s receives σ_0, then $\mathsf{clean}_{q\mathsf{HAKE}} = \texttt{false}$ for π_i^s, and $\mathcal{C}$ returns $b^* \xleftarrow{\$} \{0,1\}$ regardless of the challenge bit b. Thus, any $\mathcal{A}$ with non-negligible advantage must not yet have issued $\mathsf{CorruptQK}(j)$. Also, as the result of **Game 2** and **Game 3**, the game aborts after π_i^s receives τ, so $\mathcal{A}$ cannot later issue a $\mathsf{CorruptQK}(j)$ query.
By the definition of the abort event, $\mathcal{B}_2$ never queried m_0 to $\mathcal{C}_{\mathsf{eufcma}}$, and it follows that m_0, σ_0 is a forged message. Thus, if π_i^s receives a signature σ_0 that verifies correctly but there exists no honest session π_j^t that has output σ_0, then $\mathcal{B}_2$ wins the eufcma security game against the signature scheme SIG, and $\Pr(\mathbf{abort}_\alpha) = \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{B}_2}(\lambda) + \Pr(break_4)$.

Game 5 In this game, we replace the key pqk_{E} derived in the test session π_i^s with the uniformly random and independent value $\widetilde{pqk}$. We define a reduction $\mathcal{B}_3$ that interacts with a $\mathsf{ind\text{-}cpa}$ KEM challenger (as described in Definition

4) and replaces the $pqpk_{\mathsf{E}}$ value sent in m_0, and the ciphertext $pqctxt_{\mathsf{E}}$ sent in m_1 with the public-key pk and the ciphertext c received from the ind-cpa KEM challenger. By **Game 4**, we know that $pqpk_{\mathsf{E}}$ sent in m_0 must have been sent from an honest session π_j^t owned by P_j without modification. Any adversary that can detect the replacement of pqk_{E} with a uniformly random value $\widetilde{pqk}$ implies an efficient distinguishing algorithm $\mathcal{B}_3$ against the ind-cpa security of KEM. Thus: $\Pr(break_4) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_3}(\lambda) + \Pr(break_5)$.

Game 6 In this game we replace the computation of the extracted key $ms = \mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ with a uniformly random and independent value $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_4$ that initialises a dual-prf challenger $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ when π_i^s needs to compute $\mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ and instead queries ck_{E} to $\mathcal{C}_{\mathsf{dual\text{-}prf}}$. $\mathcal{B}_4$ uses the output of the query $\widetilde{ms}$ to replace the computation of ms. Since $\widetilde{pqk}$ is uniformly random and independent by **Game 5**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i,s)$ or $\mathsf{CompromiseQK}(j,t)$ (since the communicating partner has sent a message m_0 that was received without modification by $\mathcal{A}$), this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 0, then $\widetilde{ms} = \mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ and we are in **Game 5**. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 1, then $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 6**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_4$ against the dual-prf security of PRF, and we find: $\Pr(break_5) \leq \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_4}(\lambda) + \Pr(break_6)$.

Game 7 In this game we replace the computation of the expanded keys $mk, k = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ with a uniformly random and independent values $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_5$ that initialises a prf challenger $\mathcal{C}_{\mathsf{prf}}$ when π_i^s needs to compute $\mathsf{PRF}(\widetilde{ms}, \epsilon)$ and instead queries ϵ to $\mathcal{C}_{\mathsf{prf}}$. $\mathcal{B}_5$ uses the output $\widetilde{mk}, \widetilde{k}$ to replace the computation of mk, k. Since $\widetilde{ms}$ is already uniformly random and independent by **Game 6**, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ and we are in **Game 6**. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 7**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful distinguishing adversary $\mathcal{B}_5$ against the prf security of PRF, and we find $\Pr(break_6) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_5}(\lambda) + \Pr(break_7)$.

Game 8 In this game we abort if the test session π_i^s receives a message τ (computed over $m_0 \| m_1$) that verifies correctly but there exists no honest session π_j^t that has output τ. Specifically, in **Game 8** we define a reduction $\mathcal{B}_6$ against the eufcma security of the Message Authentication Code MAC. When $\mathcal{B}_6$ needs to compute a MAC over $m_0 \| m_1$ using $\widetilde{mk}$, $\mathcal{B}_6$ computes the MAC by initialising a eufcma challenger $\mathcal{C}_{\mathsf{eufcma}}$ and querying $m_0 \| m_1$. No changes to the experiment occur, as $\mathcal{C}_{\mathsf{eufcma}}$ computes MACs identically to

$\mathcal{C}$, so $\mathcal{A}$ cannot detect this replacement. Also, as the result of **Game 7**, $\widetilde{mk}$ is a uniformly random and independent value, so this replacement is sound. By the definition of **Case 2**, π_i^s sets the status $\pi_i^s.\alpha \leftarrow$ `accept` despite there being no honest session that matches π_i^s. Thus, $\mathcal{B}_6$ never queried $m_0 \| m_1$ to $\mathcal{C}_{\mathsf{eufcma}}$, and it follows that τ is a forged message. Thus, if π_i^s receives a MAC tag τ (over $m_0 \| m_1$) that verifies correctly but there exists no honest session π_j^t that matches π_i^s, then $(m_0 \| m_1, \tau)$ represents a valid forgery and $\mathcal{B}_6$ wins the eufcma security game against MAC, and $\Pr(break_8) = \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{B}_6}(\lambda)$. Since π_i^s now aborts when verifying τ, it cannot trigger $\mathbf{abort}_\alpha$ and thus:

$$\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) \leq {n_P}^2 n_S \cdot \big(\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{B}_2}(\lambda) + \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_3}(\lambda) + \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_4}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_5}(\lambda) + \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{B}_6}(\lambda)\big).$$

We now complete our proof by bounding $\mathcal{A}$'s advantage in **Case 3**.

Case 3: Test session with matching session

In **Case 3**, we show that if $\mathcal{A}$ that has issued a $\mathsf{Test}(i,s)$ query to a clean session π_i^s, then $\mathcal{A}$ has negligible advantage in guessing the test bit b. In what follows, we note that for the cleanness predicate $\mathsf{clean}_{q\mathsf{HAKE}}$ to be upheld by π_i^s, then $\mathsf{CompromiseQK}(i,s)$, $\mathsf{CompromiseQK}(j,t)$ cannot be queried (where π_j^t *matches* π_i^s). Thus, we can assume in what follows that $\mathcal{A}$ has not compromised the *post-quantum ephemeral* KEM secrets. We now show that $\mathcal{A}$ has negligible advantage in guessing the test bit b, via the following series of game hops:

Game 0 This is the HAKE security game, and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the matching session (j,t) and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi_{i'}^{s'}$ such that $\pi_{t'}^{j'}$ matches $\pi_{i'}^{s'}$ and $(i,s),(j,t) \neq (i',s'),(j',t')$. Thus $\Pr(break_0) \leq n_P^2 \cdot n_S^2 \cdot \Pr(break_1)$.

Game 2 In this game, we replace the key pqk_{E} derived in the test session π_i^s with the uniformly random and independent value $\widetilde{pqk}$. We define a reduction $\mathcal{B}_7$ that interacts with a $\mathsf{ind\text{-}cpa}$ KEM challenger (as described in Definition 4) and replaces the $pqpk_{\mathsf{E}}$ value sent in m_0, and the ciphertext $pqctxt_{\mathsf{E}}$ sent in m_1 with the public-key pk and the ciphertext c received from the $\mathsf{ind\text{-}cpa}$ KEM challenger. By the definition of **Case 3**, we know that $pqpk_{\mathsf{E}}$ (or $pqctxt_{\mathsf{E}}$, respectively) sent in m_0 (resp. m_1) must have been sent from an honest session π_j^t owned by P_j without modification if $\pi_i^s.\rho =$ `resp` (resp. `init`). Any adversary that can detect the replacement of pqk_{E} with a uniformly random value $\widetilde{pqk}$ implies an efficient distinguishing algorithm $\mathcal{B}_7$ against the $\mathsf{ind\text{-}cpa}$ security of KEM. Thus: $\Pr(break_1) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_7}(\lambda) + \Pr(break_2)$.

Game 3 In this game we replace the computation of the extracted key $ms = \mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ with a uniformly random and independent value $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the

protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_8$ that initialises a dual-prf challenger $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ when π_i^s needs to compute $\mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ and instead queries ck_{E} to $\mathcal{C}_{\mathsf{dual\text{-}prf}}$. $\mathcal{B}_8$ uses the output of the query $\widetilde{ms}$ to replace the computation of ms. Since $\widetilde{pqk}$ is uniformly random and independent by **Game 2**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i,s)$ or $\mathsf{CompromiseQK}(j,t)$, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 0, then $\widetilde{ms} = \mathsf{PRF}(ck_{\mathsf{E}}, \widetilde{pqk})$ and we are in **Game 2**. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 1, then $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 3**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_8$ against the dual-prf security of PRF, and we find: $\Pr(break_2) \leq \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_8}(\lambda) + \Pr(break_3)$.

Game 4 In this game we replace the computation of the expanded keys $mk, k = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ with a uniformly random and independent values $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and its matching session π_j^t. We define a reduction $\mathcal{B}_9$ that initialises a prf challenger $\mathcal{C}_{\mathsf{prf}}$ when π_i^s needs to compute $\mathsf{PRF}(\widetilde{ms}, \epsilon)$ and instead queries ϵ to $\mathcal{C}_{\mathsf{prf}}$. $\mathcal{B}_9$ uses the output $\widetilde{mk}, \widetilde{k}$ to replace the computation of mk, k. Since $\widetilde{ms}$ is already uniformly random and independent by **Game 3**, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ and we are in **Game 3**. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 4**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful distinguishing adversary $\mathcal{B}_9$ against the prf security of PRF, and we find $\Pr(break_3) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_9}(\lambda) + \Pr(break_4)$. Since $\widetilde{k}$ is now uniformly random and independent of the protocol flow regardless of the test bit b, $\mathcal{A}$ has no advantage in guessing the test bit and thus:
$\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) \leq {n_P}^2 {n_S}^2 \cdot \big(\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_7}(\lambda) + \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_8}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_9}(\lambda)\big)$.

Next we present the security of PQAG-SIG against purely classical adversaries, i.e. $\mathcal{A}$ is a PPT algorithm, with cleanness predicate $\mathsf{clean}_{c\mathsf{HAKE}}$ in Theorem 2.

Theorem 2 (PQAG-SIG Classical Security). *The* PQAG-SIG *protocol presented in Section 3 is secure under cleanness predicate* $\mathsf{clean}_{c\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against a classical adversary $\mathcal{A}$). That is, for any PPT algorithm $\mathcal{A}$ against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$ *is negligible under the* dual-prf, prf, ind-cpa, eufcma *and* eufcma *security of the* PRF, PRF, KEM, MAC *and* SIG *primitives respectively.*

Due to space restrictions, we provide the full proof (which closely follows the proof of Theorem 1) in Appendix C.

Now we turn to proving the security of PQAG-KEM. We note that the proof of PQAG-KEM follows closely the proof of Theorem 1, with minor changes in Case 1 and Case 2, where we demonstrate that a session will not accept without

a matching partner due to the use of post-quantum KEMs. As such, we mainly detail the proof of Case 1, and leave the full proof details to the appendix.

Theorem 3 (PQAG-KEM Security). *The* PQAG-KEM *protocol presented in Section 3 is post-quantum secure under cleanness predicate* $\mathsf{clean}_{q\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against* $\mathcal{A}$*). That is, for any QPT algorithm* $\mathcal{A}$ *against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$ *is negligible under the* dual-prf*,* prf*,* ind-cpa*,* ind-cca *and* eufcma *security of the* PRF*,* PRF*,* KEM*,* KEM *and* MAC *primitives respectively.*

Proof. We now turn to proving our result. We split our analysis into three mutually-exclusive cases:

1. **Case 1** assumes that the test session π^s_i (such that $\mathcal{A}$ issued $\mathsf{Test}(i,s)$) is an initiator session, and that π^s_i has no *matching partner* (as in Figure 4). We define the QPT algorithm $\mathcal{A}$'s advantage in **Case 1** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.
2. **Case 2** assumes that the test session π^s_i is a responder , and that π^s_i has no *matching partner*. We define $\mathcal{A}$'s advantage in **Case 2** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.
3. **Case 3** assumes that the test session π^s_i has a *matching partner*. We define $\mathcal{A}$'s advantage in **Case 3** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.

It is clear that: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) \leq \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$, thus we bound $\mathcal{A}$'s advantage in each case separately.

In **Case 1** and **Case 2** we show that $\mathcal{A}$'s advantage in causing the test session π^s_i to accept without a matching partner is negligible, and thus the $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible (since the experiment does not differ based on the challenge bit b, as the π^s_i does not compute a real-or-random session key).

In **Case 3** we replace the computation of the real session key by the test session π^s_i with a uniformly random key. Thus, the distribution of the keys returned by π^s_i are identical, regardless of the value of the challenge bit b, and we can show that $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible. We now begin with the first case.

Case 1: Test `init` session without origin session We begin by showing that $\mathcal{A}$ has negligible advantage in causing π^s_i to reach an `accept` state without a matching session. We do so via the following sequence of game hops:

Game 0 This is the HAKE security game and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the intended partner j and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi^{s'}_{i'}$ such that $\pi^{s'}_{i'}.pid = j'$ and $(i,s,j) \neq (i',s',j')$. Thus: $\Pr(break_0) \leq {n_P}^2 \cdot n_S \cdot \Pr(break_1)$.

Game 2 In this game we abort if the test session π_i^s sets the status $\pi_i^s.\alpha \leftarrow$ `reject`. Note that by the previous game we abort if the Test query is issued to a session that is not π_i^s, and if $\pi_i^s.pid \neq j$. If the session π_i^s ever reaches the status $\pi_i^s.\alpha \leftarrow$ `reject`, then the challenger will respond to the $\mathsf{Test}(i,s)$ query with $\perp$, and thus the difference in $\mathcal{A}$'s advantage between **Game 2** and **Game 3** is 0. Thus: $\Pr(break_1) \leq \Pr(break_2)$.

Game 3 In this game we define an abort event $\mathbf{abort}_\alpha$ that triggers if the test session π_i^s sets the status $\pi_i^s.\alpha \leftarrow$ `accept`. We note that the response to $\mathsf{Test}(i,s)$ issued by $\mathcal{A}$ is always $\perp$, (since the challenger aborts the game if π_i^s accepts, and $\mathsf{Test}(i,s) = \perp$ when π_i^s rejects the protocol execution), and thus $\Pr(break_3) = 0$. In **Game 4** and **Game 5** we prove that the probability of $\mathcal{A}$ in causing $\mathbf{abort}_\alpha$ to trigger is negligible. Thus: $\Pr(break_2) \leq \Pr(\mathbf{abort}_\alpha)$.

Game 4 In this game, we replace the key pqk_{G} derived in the test session π_i^s with the uniformly random and independent value $\widetilde{pqk_{\mathsf{G}}}$. We define a reduction $\mathcal{B}_1$ that interacts with a post-quantum $\mathsf{ind\text{-}cca}$ KEM challenger $\mathcal{C}_{\mathsf{ind\text{-}cca}}$ (as described in Definition 4), and replaces G's long-term KEM public key $pqpk_{\mathsf{G}}$ with the public key output from $\mathcal{C}_{\mathsf{ind\text{-}cca}}$. Note that $\mathcal{B}_1$ can do so at the beginning of the game, by guessing the identity of the partner session of Test in **Game 2**, A knows $pqpk_{\mathsf{G}}$ prior to protocol execution. When π_i^s should compute $pqctxt_{\mathsf{G}}$, $\mathcal{B}_1$ instead replaces the computation of $pqctxt_{\mathsf{G}}$ and pqk_{G} with the outputs of $\mathcal{C}_{\mathsf{ind\text{-}cca}}$, $\widetilde{pqctxt}$ and $\widetilde{pqk}$ respectively (and similarly for the (potential) partner session π_j^t). Whenever party j requires the use of the secret key to decapsulate a ciphertext $pqctxt' \neq \widetilde{pqctxt}$, $\mathcal{B}_1$ simply queries $pqctxt'$ to $\mathcal{C}_{\mathsf{ind\text{-}cca}}$, and replaces the computation of pqk' with the output of $\mathcal{C}_{\mathsf{ind\text{-}cca}}$. Detecting the replacement of pqk_{G} with a uniformly random value $\widetilde{pqk}$ implies an efficient distinguishing QPT algorithm $\mathcal{B}_1$ against the post-quantum $\mathsf{ind\text{-}cca}$ security of KEM. Thus: $\Pr(\mathbf{abort}_\alpha) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cca}}_{\mathsf{KEM},\mathcal{B}_1}(\lambda) + \Pr(break_4)$.

Game 5-9 Due to space constraints, we consolidate many PRF replacement hops into a single game. In this game we replace the computation of intermediate ms values $ms_1 = \mathsf{PRF}(ms_0, \widetilde{pqk})$ (where $ms_0 = \mathsf{HKDF.Extract}(ck_{\mathsf{E}}, \epsilon)$), $ms_2 = \mathsf{PRF}(\widetilde{ms_1}, ck_{\mathsf{A}})$, $ms_3 = \mathsf{PRF}(\widetilde{ms_2}, pqk_{\mathsf{A}})$, $ms_4 = \mathsf{PRF}(\widetilde{ms_3}, ck_{\mathsf{E}})$, $ms_5 = \mathsf{PRF}(\widetilde{ms_4}, pqk_{\mathsf{E}})$ with a uniformly random and independent values $\widetilde{ms_1}, \widetilde{ms_2}, \widetilde{ms_3}, \widetilde{ms_4}, \widetilde{ms_5} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define reductions $\mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5, \mathcal{B}_6$ that initialises a $\mathsf{dual\text{-}prf/prf}$ challenger $\mathcal{C}_{\mathsf{dual\text{-}prf/prf}}$ when π_i^s needs to compute $\mathsf{PRF}(\widetilde{ms_i}, k_X)$ and instead queries k_X to $\mathcal{C}_{\mathsf{dual\text{-}prf/prf}}$. $\mathcal{B}_i$ uses the output of the query $\widetilde{ms_i}$ to replace the computation of ms_i. Since $\widetilde{ms_{i-1}}$ is uniformly random and independent by **Game 4**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i,s)$ or $\mathsf{CompromiseQK}(j,t)$, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf/prf}}$ is 0, then $\widetilde{ms_i} = \mathsf{PRF}(\widetilde{ms_{i-1}}, k_X)$ and we are in **Game 4**. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf/prf}}$ is 1, then $\widetilde{ms_i} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 4 + i**. Thus any adversary $\mathcal{A}$ ca-

pable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_i$ against the dual-prf/prf security of PRF, and we find: $\Pr(break_4) \leq \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_2}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_3,\mathcal{B}_4,\mathcal{B}_5,\mathcal{B}_6}(\lambda) + \Pr(break_9)$.

Game 10 In this game we replace the computation of the MAC and session key $mk, k = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0\|m_1), \text{“}\mathtt{PQAGKEM}''\text{)}$ with a uniformly random and independent value $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We do so by initialising a prf challenger and querying the hash value $\mathsf{H}(m_0\|m_1)$, and use the output $\widetilde{mk}, \widetilde{k}$ from the prf challenger to replace the computation of mk, k. Since $\widetilde{ms_5}$ is uniformly random and independent by **Game 9**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i,s)$ or $\mathsf{CompromiseQK}(j,t)$, this is a sound replacement. If the test bit sampled by the prf challenger is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0\|m_1), \text{“}\mathtt{PQAGKEM}''\text{)}$ and we are in **Game 9**. If the test bit sampled by the prf challenger is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 10**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful QPT adversary $\mathcal{B}_7$ against the post-quantum prf security of PRF, and we find: $\Pr(break_9) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_7}(\lambda) + \Pr(break_{10})$.

Game 11 In this game we abort if the test session π_i^s receives a message τ (computed over $m_0\|m_1$) that verifies correctly but there exists no honest session π_j^t that has output τ. Specifically, in **Game 11** we define a reduction $\mathcal{B}_8$ against the eufcma security of the Message Authentication Code MAC. When $\mathcal{B}_8$ needs to compute a MAC over $m_0\|m_1$ using $\widetilde{mk}$, $\mathcal{B}_8$ computes the MAC by initialising a eufcma challenger $\mathcal{C}_{\mathsf{eufcma}}$ and querying $m_0\|m_1$. No changes to the experiment occur, as $\mathcal{C}_{\mathsf{eufcma}}$ computes MACs identically to $\mathcal{C}$, so $\mathcal{A}$ cannot detect this replacement. In addition, $\widetilde{mk}$ is a uniformly random and independent value, by **Game 10**.

By the definition of **Case 1**, π_i^s sets the status $\pi_i^s.\alpha \leftarrow \mathtt{accept}$ despite there being no honest session that matches π_i^s. Thus, $\mathcal{B}_8$ never queried $m_0\|m_1$ to $\mathcal{C}_{\mathsf{eufcma}}$, and it follows that τ is a forged message. Thus, if π_i^s receives a MAC tag τ (over $m_0\|m_1$) that verifies correctly but there exists no honest session π_j^t that matches π_i^s, then $(m_0\|m_1, \tau)$ represents a valid forgery and $\mathcal{B}_8$ wins the eufcma security game against MAC, and $\Pr(break_{10}) = \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{B}_8}(\lambda)$.

Since π_i^s now aborts when verifying τ, it cannot trigger $\mathbf{abort}_\alpha$ and thus we have: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) \leq {n_P}^2 n_S \cdot \big(\mathsf{Adv}^{\mathsf{ind\text{-}cca}}_{\mathsf{KEM},\mathcal{B}_1}(\lambda) + \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_2}(\lambda) + 5 \cdot \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_3,\mathcal{B}_4,\mathcal{B}_5,\mathcal{B}_6,\mathcal{B}_7}(\lambda) + \mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{B}_8}(\lambda) + \big)$.

Case 2: Test responder session without origin session We now show that $\mathcal{A}$ has negligible change in causing π_i^s (with $\pi_i^s.\rho = \mathsf{responder}$) to reach an accept state without an origin session. As the proof of **Case 2** follows analogously to **Case 1** with minor changes (notation to account for changes in role, and minor changes in **Game 11** to account for authenticating $m_0\|m_1\|\tau$) instead of $m_0\|m_1$), we omit these game hops and proceed to **Case 3**.

Case 3: Test session with matching session

In **Case 3**, we show that if $\mathcal{A}$ that has issued a $\mathsf{Test}(i,s)$ query to a clean session π_i^s, then $\mathcal{A}$ has negligible advantage in guessing the test bit b. In what follows, we note that for the cleanness predicate $\mathsf{clean}_{q\mathsf{HAKE}}$ to be upheld by π_i^s, then $\mathsf{CompromiseQK}(i,s)$, $\mathsf{CompromiseQK}(j,t)$ cannot be queried (where π_j^t *matches* π_i^s). Thus, we can assume in what follows that $\mathcal{A}$ has not compromised the *post-quantum ephemeral* KEM secrets. We now show that $\mathcal{A}$ has negligible advantage in guessing the test bit b, via the following series of game hops:

Game 0 This is the HAKE security game, and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the matching session (j,t) and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi_{i'}^{s'}$ such that $\pi_{t'}^{j'}$ matches $\pi_{i'}^{s'}$ and $(i,s),(j,t) \neq (i',s'),(j',t')$. Thus $\Pr(break_0) \leq n_P^2 \cdot n_S^2 \cdot \Pr(break_1)$.

Game 2 In this game, we replace the key pqk_{E} derived in the test session π_i^s with the uniformly random and independent value $\widetilde{pqk}$. We do so by interacting with a post-quantum $\mathsf{ind\text{-}cpa}$ KEM challenger (as described in Definition 4) and replace the $pqpk_{\mathsf{E}}$ value sent in m_0, and the ciphertext $pqctxt$ sent in m_1 with the public-key $\widetilde{pqpk}$ and the ciphertext $\widetilde{pqctxt}$ received from the post-quantum $\mathsf{ind\text{-}cpa}$ KEM challenger. Since π_i^s matches π_j^t, we know that the public-key and ciphertext sent in m_0 and m_1 respectively were received by the sessions without modification. Detecting the replacement of pqk_{E} with a uniformly random value $\widetilde{pqk}$ implies an efficient distinguishing QPT algorithm $\mathcal{B}_9$ against the post-quantum $\mathsf{ind\text{-}cpa}$ security of KEM. Thus: $\Pr(break_1) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_9}(\lambda) + \Pr(break_2)$.

Game 3 In this game we replace the computation of intermediate ms $ms_5 = \mathsf{PRF}(ms_4, \widetilde{pqk})$ with a uniformly random and independent value $\widetilde{ms_5} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_{10}$ that initialises a $\mathsf{dual\text{-}prf}$ challenger $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ when π_i^s needs to compute $\mathsf{PRF}(ms_4, \widetilde{pqk})$ and instead queries ms_4 to $\mathcal{C}_{\mathsf{dual\text{-}prf}}$. $\mathcal{B}_{10}$ uses the output of the query $\widetilde{ms_5}$ to replace the computation of ms_5. Since $\widetilde{pqk}$ is uniformly random and independent by **Game 2**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i,s)$ or $\mathsf{CompromiseQK}(j,t)$, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 0, then $\widetilde{ms_5} = \mathsf{PRF}(ms_4, \widetilde{pqk})$ and we are in **Game 2**. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 1, then $\widetilde{ms_5} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 3**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_{10}$ against the $\mathsf{dual\text{-}prf}$ security of PRF, and we find: $\Pr(break_2) \leq \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_{10}}(\lambda) + \Pr(break_3)$.

Game 4 In this game we replace the computation of the MAC and session key $mk, k = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0 \| m_1), \text{“}\mathtt{PQAGKEM}''\text{)}$ with a uniformly random and independent value $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We

do so by initialising a prf challenger and querying the hash value $\mathsf{H}(m_0 \| m_1)$, and use the output $\widetilde{mk}, \widetilde{k}$ from the prf challenger to replace the computation of mk, k. Since $\widetilde{ms_5}$ is uniformly random and independent by **Game 3**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseQK}(i, s)$ or $\mathsf{CompromiseQK}(j, t)$, this is a sound replacement. If the test bit sampled by the prf challenger is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0 \| m_1), \text{“}\texttt{PQAGKEM}''\text{)}$ and we are in **Game 3**. If the test bit sampled by the prf challenger is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 4**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful QPT adversary $\mathcal{B}_{11}$ against the post-quantum prf security of PRF, and we find: $\Pr(break_3) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{11}}(\lambda) + \Pr(break_4)$. Since $\widetilde{k}$ is now uniformly random and independent value of the protocol flow regardless of the value of the test bit b, $\mathcal{A}$ has no advantage in guessing the test bit and thus:
$\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{q\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) \leq {n_P}^2 {n_S}^2 \cdot \big(\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_9}(\lambda) + \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_{10}}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{11}}(\lambda)\big)$.

Next we present the security of PQAG-KEM against purely classical adversaries, i.e. $\mathcal{A}$ is a probabilistic polynomial-time algorithm, in Theorem 4.

Theorem 4 (PQAG-KEM Classical Security). *The* PQAG-KEM *protocol presented in Section 3 is secure under cleanness predicate* $\mathsf{clean}_{c\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against classical adversaries). That is, for any PPT algorithm* $\mathcal{A}$ *against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$ *is negligible under the* dual-prf*,* prf*,* ind-cpa*,* ind-cca *and* eufcma *security of the* PRF*,* PRF*,* KEM*,* KEM*, and* MAC *primitives respectively.*

Due to space restrictions, we provide the full proof (which closely follows the proof of Theorem 3) in the Supplementary Materials.

7 Conclusion and Future Work

In this work we have proposed a pair of quantum-secure hybrid key exchange protocols PQAG-KEM and PQAG-SIG for securing communication in avionic infrastructure. We formally verify the security of both protocols against future-quantum as well as classical attackers, highlighting its suitability to provide long-term security within critical national infrastructures. We benchmarked our protocols with different post-quantum algorithms and compared their performance against other state-of-the-art avionic communication protocols and illustrated that our protocols combines tight security with fast performance even within resource-constrained environments.

PQAG opens up many avenues for future work. First, instantiating PQAG with other post-quantum and classical cryptographic schemes to explore even more efficient combinations suitable for resource-constrained real-world application. For instance, combining Kyber with gap-Diffie-Hellman is likely to produce significantly faster performance even with real-time key generation. This

combination will also aid in reducing the communication cost by further shortening the size of the classical key exchange components. Moreover, comparing PQAG-KEM and PQAG-SIG with STS-SIDH by simulating all protocols within a realistic network environment, capturing the constraints of data links deployed within aviation infrastructures would further demonstrate the practicality of post-quantum cryptography within aviation infrastructure. In addition, PQAG needs to be extended so that it can also facilitate the verification of transcript consistency between AC and GS as the flight moves from one GS to another. The primary purpose of this extension would be to assist the hand-over of communication between ACs and GSs as an aircraft moves from one geographical location to another.

References

1. ARINC. DATALINK SECURITY PART 1 – ACARS MESSAGE SECURITY, 2007. https://standards.globalspec.com/std/1039315/ARINC%20823P1.
2. M. A. Bellido-Manganell, T. Gräupl, O. Heirich, N. Mäurer, A. Filip-Dhaubhadel, D. M. Mielke, L. M. Schalk, D. Becker, N. Schneckenburger, and M. Schnell. Ldacs flight trials: Demonstration and performance analysis of the future aeronautical communications system. *IEEE Transactions on Aerospace and Electronic Systems*, 2021.
3. W. Beullens. Breaking rainbow takes a weekend on a laptop. *Cryptology ePrint Archive*.
4. W. Castryck and T. Decru. An efficient key recovery attack on sidh (preliminary version). *Cryptology ePrint Archive*, 2022.
5. W. Castryck and T. Decru. An efficient key recovery attack on sidh (preliminary version). *Cryptology ePrint Archive*, 2022.
6. C. Cremers and M. Feltz. Beyond eck: Perfect forward secrecy under actor compromise and ephemeral-key reveal. In *European Symposium on Research in Computer Security*, pages 734–751. Springer, 2012.
7. B. Dowling, T. B. Hansen, and K. G. Paterson. Many a mickle makes a muckle: A framework for provably quantum-secure hybrid key exchange. In *International Conference on Post-Quantum Cryptography*, pages 483–502. Springer, 2020.
8. EUROCONTROL. Cybersecurity in aviation. *Think Paper #3*, 2019. https://www.eurocontrol.int/sites/default/files/2020-01/eurocontrol-think-paper-3-cybersecurity-aviation.pdf.
9. C. F. Kerry and P. D. Gallagher. Digital signature standard (dss). *FIPS PUB*, pages 186–4, 2013.
10. S. Khan, A. Gurtov, A. Braeken, and P. Kumar. A Security Model for Controller-Pilot Data. In *Integrated Communications Navigation and Surveillance Conference (ICNS)*, pages 1–10, 2021.
11. H. Krawczyk. Cryptographic extraction and key derivation: The hkdf scheme. In *Annual Cryptology Conference*, pages 631–648. Springer, 2010.
12. H. Krawczyk, M. Bellare, and R. Canetti. Hmac: Keyed-hashing for message authentication, 1997.
13. B. LaMacchia, K. Lauter, and A. Mityagin. Stronger security of authenticated key exchange. In *International conference on provable security*, pages 1–16. Springer, 2007.

14. L. Maino and C. Martindale. An attack on sidh with arbitrary starting curve. *Cryptology ePrint Archive*, 2022.
15. N. Mäurer, T. Gräupl, C. Gentsch, and C. Schmitt. Comparing different diffie-hellman key exchange flavors for ldacs. In *2020 AIAA/IEEE 39th Digital Avionics Systems Conference (DASC)*, pages 1–10. IEEE, 2020.
16. D. M. Mielke, N. Mäurer, T. Gräupl, and M. A. Bellido-Manganell. 1. quantum applications-fachbeitrag: Getting civil aviation ready for the post quantum age with ldacs. *Digitale Welt*, 5(4):28, 2021.
17. A. Perrig, R. Canetti, J. D. Tygar, and D. Song. The TESLA Broadcast Authentication Protocol. *CryptoBytes*, 5(2):2–13, 2002.
18. Phil Demetriou. pqcrypto 0.1.3, 2020. `https://pypi.org/project/pqcrypto/`.
19. D. Robert. Breaking sidh in polynomial time. *Cryptology ePrint Archive*, 2022.
20. F. Rodríguez-Henríquez. Sibc: A python-3 library for designing and implementing efficient isogeny-based protocols. 2021.
21. P. Schwabe, D. Stebila, and T. Wiggers. Post-quantum tls without handshake signatures. In *Proceedings of the 2020 ACM SIGSAC Conference on Computer and Communications Security*, pages 1461–1480, 2020.
22. P. Schwabe, D. Stebila, and T. Wiggers. More efficient post-quantum kemtls with pre-distributed public keys. In *European Symposium on Research in Computer Security*, pages 3–22. Springer, 2021.
23. The PyNaCl developers. PyNaCl 1.5.0, 2022. `https://pypi.org/project/PyNaCl/`.
24. The Python Cryptographic Authority. cryptography 37.0.2. 2022. `https://pypi.org/project/cryptography/`.
25. The White House. National security memorandum on promoting united states leadership in quantum computing while mitigating risks to vulnerable cryptographic systems. 2022. `https://www.whitehouse.gov/briefing-room/statements-releases/2022/05/04/national-security-memorandum-on-promoting-united-states-\leadership-in-quantum-computing-while-mitigating-risks-to-\vulnerable-cryptographic-systems/`.
26. L. D. E. K. T. L. P. S. G. S. D. S. Vadim Lyubashevsky, Shi Bai. Crystals-dilithium. 2022. `https://openquantumsafe.org/liboqs/algorithms/sig/dilithium`.
27. K. D. Wesson, T. E. Humphreys, and B. L. Evans. Can Cryptography Secure Next Generation Air Traffic Surveillance. *IEEE Security & Privacy*, 2014.
28. H. Yang, Q. Zhou, M. Yao, R. Lu, H. Li, and X. Zhang. A practical and compatible cryptographic solution to ADS-B security. *IEEE Internet of Things Journal*, 2019.

Supplementary Materials

A Preliminaries

In this section we introduce the cryptographic primitives and assumptions that we used to build our PQAG construction and prove its security, respectively. We describe a pseudorandom function and its prf security game as well as the dual-prf security game against both classical and post-quantum algorithms. Next, we discuss digital signatures and eufcma notions of security, as well as key encapsulation mechanisms and ind-cpa notions of security. Finally, we detail key derivation functions and *kdf* security. We begin by introducing PRF and prf security.

Definition 2 (prf Security). *A pseudo-random function family is a collection of deterministic functions* $\mathsf{PRF} = \{\mathsf{PRF}_\lambda : \mathcal{K} \times \mathcal{I} \to \mathcal{O} : \lambda \in \mathbb{N}\}$*, one function for each value of* λ*. Here,* $\mathcal{K}$*,* $\mathcal{I}$*,* $\mathcal{O}$ *all depend on* λ*, but we suppress this for ease of notation. Given a key* k *in the keyspace* $\mathcal{K}$ *and a bit string* $m \in \mathcal{M}$*,* PRF_λ *outputs a value* y *in the output space* $\mathcal{O} = \{0,1\}^\lambda$*. We define the security of a pseudo-random function family in the following game between a challenger* $\mathcal{C}$ *and a PPT adversary* $\mathcal{A}$*, with* λ *as an implicit input to both algorithms:*

1. $\mathcal{C}$ *samples a key* $k \xleftarrow{\$} \mathcal{K}$ *and a bit* b *uniformly at random.*
2. $\mathcal{A}$ *can now query* $\mathcal{C}$ *with polynomially-many distinct* m_i *values, and receives either the output* $y_i \leftarrow \mathsf{PRF}_\lambda(k, m_i)$ *(when* $b = 0$*) or* $y_i \xleftarrow{\$} \{0,1\}^\lambda$ *(when* $b = 1$*).*
3. $\mathcal{A}$ *terminates and outputs a bit* b'*.*

We say that $\mathcal{A}$ *wins the* PRF *security game if* $b' = b$ *and define the advantage of a algorithm* $\mathcal{A}$ *in breaking the* pseudo-random function security *of a PRF family* PRF *as* $\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda) = |2 \cdot \Pr(b' = b) - 1|$*. We say that* PRF *is post-quantum* prf*-secure if for all QPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*. We say that* PRF *is secure if for all PPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*.*

We now turn to describe dual-prf security for pseudorandom functions. On a high-level, a PRF achieves dual-prf security if the PRF retains its prf security when keyed with either the k input or the m.

Definition 3 (dual-prf Security). *Let* PRF *be a PRF family. We define a second PRF family* $\mathsf{PRF}^{\mathsf{dual}} = \{\mathsf{PRF}^{\mathsf{dual}}_\lambda : \mathcal{I} \times \mathcal{K} \to \mathcal{O} : \lambda \in \mathbb{N}\}$ *by setting* $\mathsf{PRF}^{\mathsf{dual}}_\lambda(m,k) = \mathsf{PRF}_\lambda(k,m)$*. We define the advantage of* $\mathcal{A}$ *in breaking the* dual-prf *security of* PRF *as* $\mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda) = \max\{\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda), \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF}^{\mathsf{dual}},\mathcal{A}}(\lambda)\}$*. We say that* PRF *is post-quantum* dual-prf*-secure PRF family if, for all QPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*, and* PRF *is* dual-prf*-secure PRF family if, for all PPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*.*

Next we define key encapsulation mechanisms and ind-cpa-security against both classical and post-quantum algorithms.

Definition 4 (Key Encapsulation Mechanism). *A key encapsulation mechanism (KEM) is a triple of algorithms* $\mathsf{KEM} = \{\mathsf{KGen}, \mathsf{Encaps}, \mathsf{Decaps}\}$ *with an associated keyspace* $\mathcal{K}$*. We describe the algorithms below:*

- $\mathsf{KGen}(\lambda) \xrightarrow{\$} (pk, sk)$: KGen *is a probabilistic algorithm that takes as input the security parameter* λ *and returns a public/secret key pair* (pk, sk)*.*
- $\mathsf{Encaps}(pk) \xrightarrow{\$} (c, k)$: Encaps *is a probabilistic algorithm that takes as input a public key* pk *and outputs a ciphertext* c *as well as a key* $k \in \mathcal{K}$*.*
- $\mathsf{Decaps}(sk, c) \rightarrow (k)$: Decaps *is a deterministic algorithm that takes as input a secret key* sk *and a ciphertext* c *and outputs a key* $k \in \mathcal{K}$*, or a failure symbol* $\perp$*.*

KEM *is correct if* $\forall (pk, sk)$ *such that* $\mathsf{KGen}(\lambda) \xrightarrow{\$} (pk, sk)$*, and* (c, k) *such that* $\mathsf{Encaps}(pk) \xrightarrow{\$} (c, k)$*, it holds that* $\mathsf{Decaps}(sk, c) = k$*. We define the* ind-cpa *security of a key encapsulation mechanism in the following game played between a challenger* $\mathcal{C}$ *and an adversary* $\mathcal{A}$*.*

1. $\mathcal{C}$ *generates a public-key pair* $\mathsf{KGen}(\lambda) \xrightarrow{\$} (pk, sk)$
2. $\mathcal{C}$ *generates a ciphertext and key* $\mathsf{Encaps}(pk) \xrightarrow{\$} (c, k_0)$
3. $\mathcal{C}$ *samples a key* $k_1 \xleftarrow{\$} \mathcal{K}$ *and a bit* b *uniformly at random.*
4. $\mathcal{A}$ *is given* (pk, c, k_b) *and outputs a guess bit* b'

We say that $\mathcal{A}$ *wins the* ind-cpa *security game if* $b' = b$ *and define the advantage of an algorithm* $\mathcal{A}$ *in breaking the* ind-cpa *security of a key encapsulation mechanism* KEM *as* $\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{A}}(\lambda) = |2 \cdot \Pr(b' = b) - 1|$*. We say that* KEM *is post-quantum* ind-cpa*-secure if for all QPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*. We say that* KEM *is* ind-cpa*-secure if for all PPT algorithms* $\mathcal{A}$*,* $\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ*.*

Now we strengthen our assumptions by defining ind-cca security for KEMa:

Definition 5 (Key Encapsulation Mechanism). *A key encapsulation mechanism (KEM) is a triple of algorithms* $\mathsf{KEM} = \{\mathsf{KGen}, \mathsf{Encaps}, \mathsf{Decaps}\}$ *with an associated keyspace* $\mathcal{K}$*, as described above.*

We define the ind-cca *security of a key encapsulation mechanism in the following game played between a challenger* $\mathcal{C}$ *and an adversary* $\mathcal{A}$*.*

1. $\mathcal{C}$ *generates a public-key pair* $\mathsf{KGen}(\lambda) \xrightarrow{\$} (pk, sk)$
2. $\mathcal{C}$ *generates a ciphertext and key* $\mathsf{Encaps}(pk) \xrightarrow{\$} (c, k_0)$
3. $\mathcal{C}$ *samples a key* $k_1 \xleftarrow{\$} \mathcal{K}$ *and a bit* b *uniformly at random.*
4. $\mathcal{A}$ *is given* (pk, c, k_b)
5. *The adversary may adaptively query the challenger; for each query value* $ctxt_i$ *the challenger responds with* $k_i = \mathsf{Decaps}(sk, ctxt_i)$

6. *The adversary outputs a guess bit* b'

We say that $\mathcal{A}$ *wins the* ind-cca *security game if* $b' = b$ *and define the advantage of an algorithm* $\mathcal{A}$ *in breaking the* ind-cca *security of a key encapsulation mechanism* KEM *as* $\mathsf{Adv}^{\mathsf{ind\text{-}cca}}_{\mathsf{KEM},\mathcal{A}}(\lambda) = |2 \cdot \Pr(b' = b) - 1|$. *We say that* KEM *is post-quantum* ind-cca*-secure if for all QPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{ind\text{-}cca}}_{\mathsf{KEM},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ. *We say that* KEM *is classically* ind-cca*-secure if for all PPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{ind\text{-}cca}}_{\mathsf{KEM},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ.

Next, we turn to defining classical and post-quantum eufcma security for message authentication codes (MACs).

Definition 6 (Message Authentication Code (MAC) security). *A message authentication code (*MAC*) scheme is a tuple of algorithms* $\mathsf{MAC} = \{\mathsf{KGen}, \mathsf{Tag}\}$ *where:*

- KGen *is a probabilistic key generation algorithm taking input a security parameter* λ *and returning a symmetric key* k.
- Tag *is a deterministic algorithm that takes as input a symmetric key* k *and an arbitrary message* m *from the message space* $\mathcal{M}$ *and returns a tag* τ.

Security is formulated via the following game that is played between a challenger $\mathcal{C}$ *and an algorithm* $\mathcal{A}$*:*

1. *The challenger samples* $k \xleftarrow{\$} \mathcal{K}$
2. *The adversary may adaptively query the challenger; for each query value* m_i *the challenger responds with* $\tau_i = \mathsf{Tag}(k, m_i)$
3. *The adversary outputs a pair of values* (m^*, τ^*) *such that* $(m^*, \tau^*) \notin \{(m_0, \sigma_0), \ldots (m_i, \sigma_i)\}$

The adversary $\mathcal{A}$ *wins the game if* $\mathsf{Tag}(k, m^*) = \tau^*$, *producing a tag forgery. We define the advantage of* $\mathcal{A}$ *in breaking the existential unforgeability property of a MAC* MAC *under chosen-message attack to be:*

$$\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{A}}(\lambda) = \Pr\left(\mathsf{Tag}\left(k, m^*\right) = \tau^*\right)$$

We say that MAC *is* post-quantum eufcma*-secure if, for all QPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ, *and is classically* eufcma*-secure if, for all PPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ.

Finally, we turn to defining classical and post-quantum eufcma security for digital signatures.

Definition 7 (Digital Signature eufcma**-signature).** *A digital signature (*SIG*) scheme is a tuple of algorithms* $\mathsf{SIG} = \{\mathsf{KGen}, \mathsf{Sign}, \mathsf{Vfy}\}$ *where:*

- KGen *is a probabilistic key generation algorithm taking input a security parameter* λ *and returning a public key* pk *and a secret key* sk.

- Sign *is a probabilistic algorithm that takes as input a secret key* sk *and an arbitrary message* m *from the message space* $\mathcal{M}$ *and returns a signature* σ.
- Vfy *is a deterministic algorithm that takes as input a public key* pk, *an message* m *and a signature* σ *and returns bit* $b \in \{0,1\}$.

We require correctness of a digital signature scheme SIG. *Specifically, for all* $(pk, sk) \xleftarrow{\$} \mathsf{SIG.KGen}$, *we have* $\mathsf{SIG.Vfy}(pk, m, \mathsf{SIG.Sign}(sk, m)) = 1$. *Security is formulated via the following game that is played between a challenger* $\mathcal{C}$ *and an algorithm* $\mathcal{A}$*:*

1. *The challenger samples* $pk, sk \xleftarrow{\$} \mathcal{K}$
2. *The adversary may adaptively query the challenger; for each query value* m_i *the challenger responds with* $\sigma_i = \mathsf{Sign}(sk, m_i)$
3. *The adversary outputs a pair of values* (m^*, σ^*) *such that* $(m^*, \sigma^*) \notin \{(m_0, \sigma_0), \ldots (m_i, \sigma_i)\}$

The adversary $\mathcal{A}$ *wins the game if* $\mathsf{Vfy}(pk, m^*, \sigma) = 1$, *producing a signature forgery. We define the advantage of* $\mathcal{A}$ *in breaking the existential unforgeability property of a digital signature scheme* SIG *under chosen-message attack to be:*

$$\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{A}}(\lambda) = \Pr\left(\mathsf{Vfy}\left(pk, m^*, \sigma^*\right) = 1\right)$$

We say that SIG *is* post-quantum eufcma*-secure if, for all QPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{SIG},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ, *and is* eufcma*-secure if, for all PPT algorithms* $\mathcal{A}$, $\mathsf{Adv}^{\mathsf{eufcma}}_{\mathsf{MAC},\mathcal{A}}(\lambda)$ *is negligible in the security parameter* λ.

B Detailed HAKE Security Experiment

In this section we give the algorithmic description of the HAKE security experiment. We also provide an exact pseudocode definition for matching sessions [13] and origin sessions [6], as well as our future-quantum cleanness predicate $\mathsf{clean}_{q\mathsf{HAKE}}$ and classical cleanness predicate $\mathsf{clean}_{c\mathsf{HAKE}}$.

Finally, we provide our formal cleanness predicate against future quantum adversaries in Definition 8, and our formal cleanness predicate against classical adversaries in Definition 9.

Definition 8 ($\mathsf{clean}_{q\mathsf{HAKE}}$)**.** *A session* π^s_i *such that* $\pi^s_i.\alpha = \mathtt{accept}$ *and* $\pi^s_i.pid = j$ *in the security experiment defined in Figure 3 is* $\mathsf{clean}_{q\mathsf{HAKE}}$ *if all of the following conditions hold:*

1. *The query* $\mathsf{Reveal}(i, s)$ *has not been issued.*
2. *For all* $(j, t) \in n_P \times n_S$ *such that* π^s_i matches π^t_j, *the query* $\mathsf{Reveal}(j, t)$ *has not been issued.*
3. *If there exists a session* π^t_j *such that* π^t_j *matches* π^s_i, *then the following sets of queries has not been issued:*

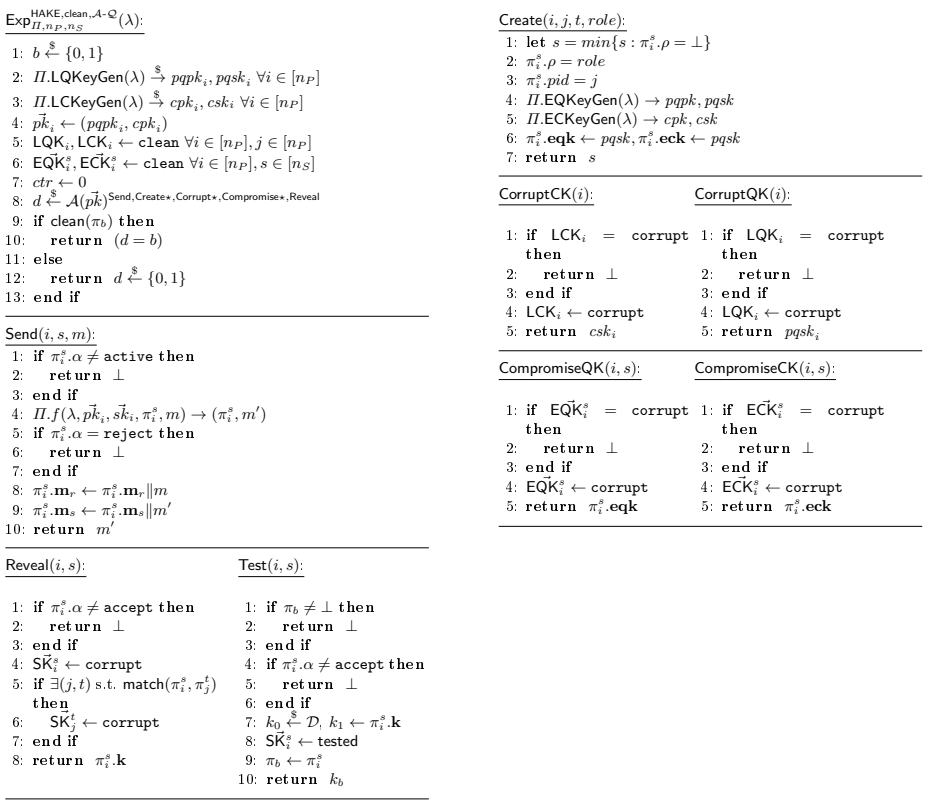

Fig. 3: HAKE experiment for an adversary $\mathcal{A}$ against the key-indistinguishability security of protocol Π. Note that the values $\vec{pk}$ given as input to $\mathcal{A}$ and $\mathcal{Q}$ represent the vector $\vec{pk}_i$ for all n_P parties. The function match takes as input two sessions π^s_i and π^t_j and determines if they are *matching* according to some matching definition. For the definition of the matching sessions function used in our HAKE experiment, see Section 5.4.

- $\mathsf{CompromiseQK}(i,s)$, $\mathsf{CompromiseQK}(j,t)$ *have not been issued, where* π^t_j matches π^s_i.

4. *If there exists no* $(j,t) \in n_P \times n_S$ *such that* π^t_j *is an origin session of* π^s_i, *then* $\mathsf{CorruptQK}(j)$ *has not been issued before* $\pi^s_i.\alpha \leftarrow \mathtt{accept}$.

Definition 9 ($\mathsf{clean}_{c\mathsf{HAKE}}$)**.** *A session* π^s_i *such that* $\pi^s_i.\alpha = \mathtt{accept}$ *and* $\pi^s_i.pid = j$ *in the security experiment defined in Figure 3 is* $\mathsf{clean}_{q\mathsf{HAKE}}$ *if all of the following conditions hold:*

1. *The query* $\mathsf{Reveal}(i,s)$ *has not been issued.*
2. *For all* $(j,t) \in n_P \times n_S$ *such that* π^s_i matches π^t_j, *the query* $\mathsf{Reveal}(j,t)$ *has not been issued.*
3. *If there exists a session* π^t_j *such that* π^t_j *matches* π^s_i, *then at least one of the following sets of queries has not been issued:*

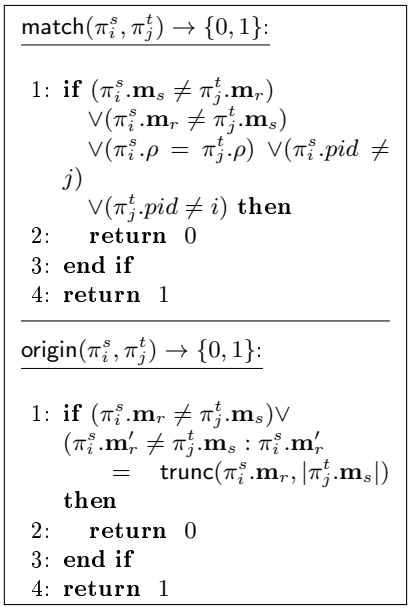

Fig. 4: A pseudocode description of the *matching session* and *origin session* functions.

- CompromiseQK(i,s), CompromiseQK(j,t) *have not been issued, where* π_j^t matches π_i^s.
- CompromiseCK(i,s), CompromiseCK(j,t) *have not been issued, where* π_j^t matches π_i^s.

4. *If there exists no* $(j,t) \in n_P \times n_S$ *such that* π_j^t *is an origin session of* π_i^s, *then* CorruptQK(j) *has not been issued before* $\pi_i^s.\alpha \leftarrow$ `accept`.

C Full Classical Proofs

Next we prove the security of PQAG-SIG against purely classical adversaries, i.e. $\mathcal{A}$ is a PPT algorithm.

Theorem 5 (PQAG-SIG Classical Security). *The* PQAG-SIG *protocol presented in Section 3 is secure under cleanness predicate* $\mathsf{clean}_{c\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against a classical adversary* $\mathcal{A}$*). That is, for any PPT algorithm* $\mathcal{A}$ *against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$ *is negligible under the* dual-prf, prf, ind-cpa, eufcma *and* eufcma *security of the* PRF, PRF, KEM, MAC *and* SIG *primitives respectively.*

Proof. We now turn to proving our result. We split our analysis into three mutually-exclusive cases:

1. **Case 1** assumes that the test session π_i^s (such that $\mathcal{A}$ issued $\mathsf{Test}(i,s)$) is an $\mathsf{initiator}$ session, and that π_i^s has no *matching partner* (as in Figure 4). We define the PPT algorithm $\mathcal{A}$'s advantage in **Case 1** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.
2. **Case 2** assumes that the test session π_i^s is a $\mathsf{responder}$, and that π_i^s has no *matching partner*. We define $\mathcal{A}$'s advantage in **Case 2** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.
3. **Case 3** assumes that the test session π_i^s has a *matching partner*. We define the PPT algorithm $\mathcal{A}$'s advantage in **Case 3** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$.

It is clear that: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\Pi,n_P,n_S}(\lambda) \leq \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda)$, thus we bound $\mathcal{A}$'s advantage in each case separately.

In **Case 1** and **Case 2** we show that $\mathcal{A}$'s advantage in causing the test session π_i^s to accept without a matching partner is negligible, and thus the $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible (since the experiment does not differ based on the challenge bit b, as the π_i^s does not compute a real-or-random session key). As the analysis of **Case 1** and **Case 2** follows identically the corresponding analysis of Theorem 1, we omit the details here, and point the reader to Section 6.

In **Case 3** we replace the computation of the real session key by the test session π_i^s with a uniformly random key. Thus, the distribution of the keys returned by π_i^s are identical, regardless of the value of the challenge bit b, and we can show that $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible. We now begin with the third case.

Case 3: Test session with matching session In **Case 3**, we show that if $\mathcal{A}$ that has issued a $\mathsf{Test}(i,s)$ query to a clean session π_i^s, then $\mathcal{A}$ has negligible advantage in guessing the test bit b. In what follows, we split our analysis of Case 3 into the following sub-cases, each corresponding to a condition necessary for the cleanness predicate $\mathsf{clean}_{c\mathsf{HAKE}}$ to be upheld by π_i^s. These are the subcases (where π_j^t *matches* π_i^s):

- **Subcase 3.1**: $\mathsf{CompromiseQK}(i,s)$, $\mathsf{CompromiseQK}(j,t)$ were not queried.
- **Subcase 3.2**: $\mathsf{CompromiseCK}(i,s)$, $\mathsf{CompromiseCK}(j,t)$ were not queried.

It is straightforward to see that the advantage of $\mathcal{A}$ in Case 3 is bound by the sum of the advantages of $\mathcal{A}$ in all subcases. It is also straightforward to see that the proof of **Subcase 3.1** follows identically the corresponding analysis of **Case 3** of Theorem 1, and so we omit the details here, and point the reader to Section 6. We now treat the second subcase (**Subcase 3.2**), where $\mathcal{A}$ has not compromised the *classic ephemeral* KEM secrets.

3.2: $\mathsf{CompromiseCK}(i,s)$, $\mathsf{CompromiseCK}(j,t)$ have not been issued, where π_j^t *matches* π_i^s.

Game 0 This is the HAKE security game, and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{\mathsf{cHAKE}},\mathcal{A},\mathbf{C3.2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the matching session (j,t) and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi^{s'}_{i'}$ such that $\pi^{j'}_{t'}$ matches $\pi^{s'}_{i'}$ and $(i,s),(j,t) \neq (i',s'),(j',t')$. Thus $\Pr(break_0) \leq n_P^2 \cdot n_S^2 \cdot \Pr(break_1)$.

Game 2 In this game, we replace the key ck_{E} derived in the test session π^s_i with the uniformly random and independent value $\widetilde{ck}$. We define a reduction $\mathcal{B}_{10}$ that interacts with a ind-cpa KEM challenger (as described in Definition 4) and replaces the cpk_{E} value sent in m_0, and the ciphertext $cctxt_{\mathsf{E}}$ sent in m_1 with the public-key pk and the ciphertext c received from the ind-cpa KEM challenger. By the definition of **Case 3**, we know that cpk_{E} (or $cctxt_{\mathsf{E}}$, respectively) sent in m_0 (resp. m_1) must have been sent from an honest session π^t_j owned by P_j without modification if $\pi^s_i.\rho = \texttt{resp}$ (resp. $\texttt{init}$). Any adversary that can detect the replacement of ck_{E} with a uniformly random value $\widetilde{ck}$ implies an efficient distinguishing algorithm $\mathcal{B}_{10}$ against the ind-cpa security of KEM. Thus: $\Pr(break_1) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_{10}}(\lambda) + \Pr(break_2)$.

Game 3 In this game we replace the computation of the extracted key $ms = \mathsf{PRF}(\widetilde{ck}, pqk_{\mathsf{E}})$ with a uniformly random and independent value $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π^s_i, and (potentially) its matching session π^t_j. We define a reduction $\mathcal{B}_{11}$ that initialises a prf challenger $\mathcal{C}_{\mathsf{prf}}$ when π^s_i needs to compute $\mathsf{PRF}(\widetilde{ck}, pqk_{\mathsf{E}})$ and instead queries pqk_{E} to $\mathcal{C}_{\mathsf{prf}}$. $\mathcal{B}_{11}$ uses the output of the query $\widetilde{ms}$ to replace the computation of ms. Since $\widetilde{ck}$ is uniformly random and independent by **Game 2**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseCK}(i,s)$ or $\mathsf{CompromiseCK}(j,t)$, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 0, then $\widetilde{ms} = \mathsf{PRF}(\widetilde{ck}, pqk_{EP})$ and we are in **Game 2**. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 1, then $\widetilde{ms} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 3**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_{11}$ against the prf security of PRF, and we find: $\Pr(break_2) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{11}}(\lambda) + \Pr(break_3)$.

Game 4 In this game we replace the computation of the expanded keys $mk, k = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ with a uniformly random and independent values $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π^s_i, and its matching session π^t_j. We define a reduction $\mathcal{B}_{12}$ that initialises a prf challenger $\mathcal{C}_{\mathsf{prf}}$ when π^s_i needs to compute $\mathsf{PRF}(\widetilde{ms}, \epsilon)$ and instead queries ϵ to $\mathcal{C}_{\mathsf{prf}}$. $\mathcal{B}_{12}$ uses the output $\widetilde{mk}, \widetilde{k}$ to replace the computation of mk, k. Since $\widetilde{ms}$ is already uniformly random and independent by **Game 3**, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{PRF}(\widetilde{ms}, \epsilon)$ and we are in **Game 3**. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 4**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful distinguishing adversary $\mathcal{B}_{12}$ against the prf security

of PRF, and we find $\Pr(break_3) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{12}}(\lambda) + \Pr(break_4)$. Since $\widetilde{k}$ is now uniformly random and independent of the protocol flow regardless of the test bit b, $\mathcal{A}$ has no advantage in guessing the test bit and thus: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3.2}}_{\mathsf{PQAG\text{-}SIG},n_P,n_S}(\lambda) \leq {n_P}^2{n_S}^2 \cdot \big(\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_{10}}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{11}}(\lambda) + \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{12}}(\lambda)\big)$.

Theorem 6 (PQAG-KEM Classical Security). *The* PQAG-KEM *protocol presented in Section 3 is secure under cleanness predicate* $\mathsf{clean}_{c\mathsf{HAKE}}$ *(capturing perfect forward security and resilience to KCI attacks against classical adversaries). That is, for any PPT algorithm* $\mathcal{A}$ *against the key-indistinguishability game (defined in Figure 3),* $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$ *is negligible under the* prf, ind-cpa, ind-cca *and* eufcma *security of the* PRF, KEM, KEM, *and* MAC *primitives respectively.*

Proof. We now turn to proving our result. We split our analysis into three mutually-exclusive cases:

1. **Case 1** assumes that the test session π_i^s (such that $\mathcal{A}$ issued $\mathsf{Test}(i,s)$) is an initiator session, and that π_i^s has no *matching partner* (as in Figure 4). We define the PPT algorithm $\mathcal{A}$'s advantage in **Case 1** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.
2. **Case 2** assumes that the test session π_i^s is a responder , and that π_i^s has no *matching partner*. We define $\mathcal{A}$'s advantage in **Case 2** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.
3. **Case 3** assumes that the test session π_i^s has a *matching partner*. We define the PPT algorithm $\mathcal{A}$'s advantage in **Case 3** as $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$.

It is clear that: $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) \leq \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C1}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) + \mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda)$, thus we bound $\mathcal{A}$'s advantage in each case separately.

In **Case 1** and **Case 2** we show that $\mathcal{A}$'s advantage in causing the test session π_i^s to accept without a matching partner is negligible, and thus the $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible (since the experiment does not differ based on the challenge bit b, as the π_i^s does not compute a real-or-random session key). As the analysis of **Case 1** and **Case 2** follows identically the corresponding analysis of Theorem 3, we omit the details here, and point the reader to Section 6.

In **Case 3** we replace the computation of the real session key by the test session π_i^s with a uniformly random key. Thus, the distribution of the keys returned by π_i^s are identical, regardless of the value of the challenge bit b, and we can show that $\mathcal{A}$'s advantage in winning the key-indistinguishability game is negligible. We now begin with the third case.

Case 3: Test session with matching session In **Case 3**, we show that if $\mathcal{A}$ that has issued a $\mathsf{Test}(i,s)$ query to a clean session π_i^s, then $\mathcal{A}$ has negligible

advantage in guessing the test bit b. In what follows, we split our analysis of Case 3 into the following sub-cases, each corresponding to a condition necessary for the cleanness predicate $\mathsf{clean}_{c\mathsf{HAKE}}$ to be upheld by π_i^s. These are the subcases (where π_j^t *matches* π_i^s):

- $\mathsf{CompromiseQK}(i,s)$, $\mathsf{CompromiseQK}(j,t)$ were not queried.
- $\mathsf{CompromiseCK}(i,s)$, $\mathsf{CompromiseCK}(j,t)$ were not queried.

It is straightforward to see that the advantage of $\mathcal{A}$ in Case 3 is bound by the sum of the advantages of $\mathcal{A}$ in all subcases. It is also straightforward to see that the proof of **Subcase 3.1** follows identically the corresponding analysis of **Case 3** of Theorem 3, and so we omit the details here, and point the reader to Section 6. We now treat the second subcase (**Subcase 3.2**), where $\mathcal{A}$ has not compromised the *classic ephemeral* KEM secrets.

3.2: $\mathsf{CompromiseCK}(i,s)$, $\mathsf{CompromiseCK}(j,t)$ have not been issued, where π_j^t *matches* π_i^s.

Game 0 This is the HAKE security game, and $\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{c\mathsf{HAKE}},\mathcal{A},\mathbf{C3.2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) = \Pr(break_0)$.

Game 1 In this game, we guess the index (i,s) and the matching session (j,t) and abort if, during the execution of the experiment, a query $\mathsf{Test}(i',s')$ is received to a session $\pi_{i'}^{s'}$ such that $\pi_{t'}^{j'}$ matches $\pi_{i'}^{s'}$ and $(i,s),(j,t) \neq (i',s'),(j',t')$. Thus $\Pr(break_0) \leq n_P^2 \cdot n_S^2 \cdot \Pr(break_1)$.

Game 2 In this game, we replace the key ck_{E} derived in the test session π_i^s with the uniformly random and independent value $\widetilde{ck}$. We do so by interacting with a classical $\mathsf{ind\text{-}cpa}$ KEM challenger (as described in Definition 4) and replace the cpk_{E} value sent in m_0, and the ciphertext $cctxt$ sent in m_1 with the public-key $\widetilde{cpk}$ and the ciphertext $\widetilde{cctxt}$ received from the classical $\mathsf{ind\text{-}cpa}$ KEM challenger. Since π_i^s matches π_j^t, we know that the public-key and ciphertext sent in m_0 and m_1 respectively were received by the sessions without modification. Detecting the replacement of ck_{E} with a uniformly random value $\widetilde{ck}$ implies an efficient distinguishing PPT algorithm $\mathcal{B}_{12}$ against the classical $\mathsf{ind\text{-}cpa}$ security of KEM. Thus: $\Pr(break_1) \leq \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_{12}}(\lambda) + \Pr(break_2)$.

Game 3 In this game we replace the computation of intermediate ms $ms_4 = \mathsf{PRF}(ms_3, \widetilde{ck})$ with a uniformly random and independent value $\widetilde{ms_4} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_{13}$ that initialises a $\mathsf{dual\text{-}prf}$ challenger $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ when π_i^s needs to compute $\mathsf{PRF}(ms_3, \widetilde{ck})$ and instead queries ms_3 to $\mathcal{C}_{\mathsf{dual\text{-}prf}}$. $\mathcal{B}_{13}$ uses the output of the query $\widetilde{ms_4}$ to replace the computation of ms_4. Since $\widetilde{ck}$ is uniformly random and independent by **Game 2**, and $\mathcal{A}$ cannot issue $\mathsf{CompromiseCK}(i,s)$ or $\mathsf{CompromiseCK}(j,t)$, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 0, then $\widetilde{ms_4} = \mathsf{PRF}(ms_3, \widetilde{ck})$ and we are

in **Game 2**. If the test bit sampled by $\mathcal{C}_{\mathsf{dual\text{-}prf}}$ is 1, then $\widetilde{ms_4} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 3**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_{13}$ against the dual-prf security of PRF, and we find: $\Pr(break_2) \leq \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_{13}}(\lambda) + \Pr(break_3)$.

Game 4 In this game we replace the computation of intermediate ms $ms_5 = \mathsf{PRF}(\widetilde{ms_4}, pqk_{\mathsf{E}})$ with a uniformly random and independent value $\widetilde{ms_5} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ (where $\{0,1\}^{\mathsf{PRF}}$ is the output space of the PRF) used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We define a reduction $\mathcal{B}_{14}$ that initialises a prf challenger $\mathcal{C}_{\mathsf{prf}}$ when π_i^s needs to compute $\mathsf{PRF}(\widetilde{ms_4}, pqk_{\mathsf{E}})$ and instead queries pqk_{E} to $\mathcal{C}_{\mathsf{prf}}$. $\mathcal{B}_{14}$ uses the output of the query $\widetilde{ms_5}$ to replace the computation of ms_5. Since $\widetilde{ms_4}$ is uniformly random and independent by **Game 3**, this is a sound replacement. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 0, then $\widetilde{ms_5} = \mathsf{PRF}(\widetilde{ms_4}, pqk_{\mathsf{E}})$ and we are in **Game 3**. If the test bit sampled by $\mathcal{C}_{\mathsf{prf}}$ is 1, then $\widetilde{ms_5} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 4**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful adversary $\mathcal{B}_{14}$ against the prf security of PRF, and we find: $\Pr(break_3) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{14}}(\lambda) + \Pr(break_4)$.

Game 5 In this game we replace the computation of the MAC and session key $mk, k = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0 \| m_1), \text{“}\mathtt{PQAGKEM}''\text{)}$ with a uniformly random and independent value $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ used in the protocol execution of the test session π_i^s, and (potentially) its matching session π_j^t. We do so by initialising a prf challenger and querying the hash value $\mathsf{H}(m_0 \| m_1)$, and use the output $\widetilde{mk}, \widetilde{k}$ from the prf challenger to replace the computation of mk, k. Since $\widetilde{ms_5}$ is uniformly random and independent by **Game 4**, this is a sound replacement. If the test bit sampled by the prf challenger is 0, then $\widetilde{mk}, \widetilde{k} = \mathsf{HKDF.Expand}(\widetilde{ms_5}, \mathsf{H}(m_0 \| m_1), \text{“}\mathtt{PQAGKEM}''\text{)}$ and we are in **Game 4**. If the test bit sampled by the prf challenger is 1, then $\widetilde{mk}, \widetilde{k} \xleftarrow{\$} \{0,1\}^{\mathsf{PRF}}$ and we are in **Game 5**. Thus any adversary $\mathcal{A}$ capable of distinguishing this change can be turned into a successful PPT adversary $\mathcal{B}_{15}$ against the post-quantum prf security of PRF, and we find: $\Pr(break_4) \leq \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{15}}(\lambda) + \Pr(break_5)$.

Since $\widetilde{k}$ is now uniformly random and independent value of the protocol flow regardless of the value of the test bit b, $\mathcal{A}$ has no advantage in guessing the test bit and thus:
$\mathsf{Adv}^{\mathsf{HAKE},\mathsf{clean}_{cHAKE},\mathcal{A},\mathbf{C3.2}}_{\mathsf{PQAG\text{-}KEM},n_P,n_S}(\lambda) \leq {n_P}^2 {n_S}^2 \cdot \big(\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{KEM},\mathcal{B}_{12}}(\lambda) + \mathsf{Adv}^{\mathsf{dual\text{-}prf}}_{\mathsf{PRF},\mathcal{B}_{13}}(\lambda) + 2 \cdot \mathsf{Adv}^{\mathsf{prf}}_{\mathsf{PRF},\mathcal{B}_{14},\mathcal{B}_{15}}(\lambda)\big)$.