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Congestion-Driven Positioning of Grid Enhancing Technologies

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Abstract—Transition towards low carbon electricity networks increases the bottlenecks in transmission networks which necessitates transmission network expansion. Yet, traditional transmission network expansion which relies on building new transmission lines is challenging due to the limited rights of way, long development lead times and capital intensive investment requirements. Grid enhancing technologies such as power flow control devices, and dynamic line rating are expected to play a crucial role in reducing transmission network congestion in low-carbon electricity networks. Nevertheless, there are limited decision-making tools which can assist the stakeholders to select the most appropriate grid enhancing technology for different locations of the network. In this paper, we examine the feasibility regions of bulk power systems to determine the influencing factors on congestion-driven positioning of grid enhancing technologies. Depth and duration of congestion are introduced as the determining factors for selecting the appropriate grid enhancing technology. It is demonstrated that these influencing factors can be monetized to justify the installation and deployment of the grid enhancing technology. The significance of the influencing factors are demonstrated by characterizing the feasibility region of a three-bus test system.

Index Terms—Low carbon electricity networks, congestiondriven transmission expansion planning, grid enhancing technologies, decision-making, non-wire technologies.

I. INTRODUCTION

T HE rapid integration of renewable energy resources and electrification of transport sector is expected to significantly increase congestion in transmission networks. Transmission congestion plays a crucial role in determining the flexibility and economics of bulk power systems [1], [2]. Transmission congestion occurs when there is not enough transmission capacity to support all requests to transfer power from low-cost generation to load centres [3]. Transmission line capacity limits may stem from thermal, voltage, or stability considerations. The transmission capacity limits may cause the dispatch of some higher cost generation instead of lowercost generation. The net cost of lower-cost generation replaced by higher cost generation is called congestion cost which forms the main driver of the congestion-driven transmission expansion planning [3].

The classic way of transmission network expansion includes building new transmission lines. Yet, building new transmission lines are challenging due to three main reasons. First, the rights of way for building transmission lines are limited due to environmental concerns and regulatory considerations. Second, building new transmission lines involves long lead time which does not match the lead time associated with installing renewable energy resources and electrification of transport sector. Moreover, building a new line is a capital intensive investment. As such, it is a viable option only when the value of alleviating congestion exceeds the cost of building a new transmission line.

Grid enhancing technologies are considered cost-effective solutions to address these challenges and reduce the need to build new transmission lines [4]. Nevertheless, it is imperative to determine which technology has the highest potential to unlock the capacity of transmission lines for different locations of the network. Grid-enhancing technologies are a family of technologies that assist system operators to maximize power transfer over the existing grid such as power flow control devices and dynamic line rating [4]. Power flow control devices include software and hardware that assist operators to decrease power flow on overloaded transmission lines and instead increase power flow on transmission lines with underutilized capacity. Dynamic line rating includes software and hardware that assist operators to accurately quantify the thermal limits of transmission lines in real-time based on weather conditions [4].

Congestion-driven transmission expansion planning has been examined extensively in the literature [5], [6]. Yet, to the best of authors' knowledge, no prior paper has systematically examined the influencing factors for congestion-driven positioning of grid enhancing technologies. In this paper, we first characterize the feasibility regions of the bulk power systems using generation and transmission capacity limits. Afterwards, depth and duration of congestion-driven positioning of grid enhancing technologies. A formulation is further provided to monetize these two factors. These concepts are demonstrated using a three-bus test system. The main contributions of this paper are as follows:

- The critical factors influencing congestion-driven positioning of grid enhancing technologies are introduced and discussed.
- The impact of grid enhancing technologies on reshaping the feasibility regions of bulk power systems is examined and compared illustratively with classic transmission expansion planning, i.e. building new transmission lines.

The remainder of the paper are organized as follows. In Section II, the feasibility regions of the bulk power systems are characterized by identifying the binding generation, and transmission line constraints. Moreover, the impact of grid enhancing technologies on reshaping the feasibility regions of the bulk power systems is discussed. The concepts discussed in Section II are demonstrated in Section III using a three-bus test system. The concluding remarks are provided in Section IV.

II. FEASIBILITY REGIONS OF TRANSMISSION CONSTRAINED BULK POWER SYSTEMS

In this section, the feasibility regions of the bulk power systems are first characterized by identifying the binding generation, and transmission line constraints. Afterwards, the impacts of various grid enhancing technologies on reshaping the feasibility regions of bulk power systems are discussed.

A. Characterizing The Feasibility Regions of The Bulk Power Systems

The optimization problem in (1)-(21) is employed to identify the constraints that shape the feasibility regions of the bulk power systems. The problem formulation is inspired by [7].

$$\min \sum_{n=1}^{N} v_n^+ + v_n^- + \hat{v}_n^+ + \hat{v}_n^- + \sum_{l=1}^{L} v_l^+ + v_l^- \qquad (1)$$

subject to

$$\sum_{n=1}^{N} g_n = \sum_{n=1}^{N} d_n$$
 (2)

$$g_n - \sum_{m \in \mathcal{M}_n} g_m^{\max} u_m \le 0 \quad \forall n \tag{3}$$

$$-g_n + \sum_{m \in \mathcal{M}_n} g_m^{\min} u_m \le 0 \quad \forall n \tag{4}$$

$$d_n - d_n^{\max} \le 0 \quad \forall n \tag{5}$$
$$-d_n + d_n^{\min} \le 0 \quad \forall n \tag{6}$$

$$-a_n + a_n \ge 0 \quad \forall n \quad (0)$$

$$\sum_{\substack{n=1\\ N}} \gamma_{n,s}^{l,w}(g_n - d_n) \le f_l^{\max} \quad \forall l \tag{7}$$

$$-\sum_{n=1}^{N} \gamma_{n,s}^{l,w}(g_n - d_n) \le f_l^{\max} \quad \forall l \qquad (8)$$

$$g_n - \sum_{m \in \mathcal{M}_n} g_m^{\max} u_m + z_n^+ \ge 0 \quad \forall n$$
(9)

Ν

N

$$-g_n + \sum_{m \in \mathcal{M}_n} g_m^{\min} u_m + z_n^- \ge 0 \quad \forall n \tag{10}$$

$$d_n - d_n^{\max} + \hat{z}_n^+ \ge 0 \quad \forall n \tag{11}$$

$$-d_n + d_n^{\min} + \hat{z}_n^- \ge 0 \quad \forall n \tag{12}$$

$$\sum_{n=1}^{l} \gamma_{n,s}^{l,w}(g_n - d_n) + z_l^+ \ge f_l^{\max} \quad \forall l \qquad (13)$$

$$-\sum_{n=1}^{N} \gamma_{n,s}^{l,w}(g_n - d_n) + z_l^- \ge f_l^{\max} \quad \forall l \qquad (14)$$

$$\frac{z_n^+}{\Omega} \le v_n^+, \frac{z_n^-}{\Omega} \le v_n^-, \frac{\hat{z}_n^+}{\Omega} \le \hat{v}_n^+, \frac{\hat{z}_n^-}{\Omega} \le \hat{v}_n^- \quad \forall n$$
(15)

$$\frac{z_l'}{\Omega} \le v_l^+, \frac{z_l}{\Omega} \le v_l^- \quad \forall l \tag{16}$$

$$u_m \in \{0, 1\} \quad \forall m \tag{17}$$

$$z_{n}^{+} \ge 0, z_{n}^{-} \ge 0, \hat{z}_{n}^{+} \ge 0, \hat{z}_{n}^{-} \ge 0 \quad \forall n$$

$$z_{l}^{+} \ge 0, z_{l}^{-} \ge 0 \quad \forall l$$
(18)
(19)

$$v_n^+, v_n^-, \hat{v}_n^+, \hat{v}_n^- \in \{0, 1\} \quad \forall n$$
(20)

$$v_l^+, v_l^- \in \{0, 1\} \quad \forall l$$
 (21)

where g_n , and d_n denote the total generation and demand at bus n, respectively. g_m^{\min} and g_m^{\max} denote the minimum and maximum capacity limits of the generator m at bus n, respectively. u_m denotes the commitment status of the generating unit m. d_n^{\min} , and d_n^{\max} denote the minimum and maximum demand limits at bus n, respectively. $\gamma_{n,s}^{l,w}$ denotes the shift factor of the transmission line l when bus s is considered to be the slack bus and the topology of the network is w. f_l^{\max} denotes the capacity limit of the transmission line l. z_n^{\pm} , \hat{z}_n^{\pm} , and z_l^{\pm} denote positive auxiliary variables associated with generation, demand and transmission line constraints, respectively. v_n^{\pm} , \hat{v}_n^{\pm} and v_l^{\pm} denote binary auxiliary variables associated with z_n^{\pm} , \hat{z}_n^{\pm} , and z_l^{\pm} , respectively. Ω denotes a large positive number.

The objective function in (1) minimizes the sum of the binary variables, v_n^{\pm} , \hat{v}_n^{\pm} and v_l^{\pm} , by finding the maximum number of constraints shaping the feasibility regions of a bulk power system considering network topology w. Constraint (2) enforces the system-wide power balance, while the blocks of constraints (3)-(4), (5)-(6) and (7)-(8) represent the generation capacity limits at each bus, the demand limits at each bus, and the transmission line capacity limits, respectively. The blocks of constraints (9)-(16) connects the auxiliary variables z_n^{\pm} , \hat{z}_n^{\pm} , z_l^{\pm} , v_n^{\pm} , \hat{v}_n^{\pm} and v_l^{\pm} to the blocks of constraints in (3)-(8) to identify the constraints that shape the feasibility regions of the bulk power system.

The formulation in (1)-(21) is solved iteratively to identify all the constraints that shape the feasibility regions of a bulk power system by allowing the generation and demand to freely change at each bus within their limits. The power balance constraint in (2) and transmission line constraints in (7)-(8) ensure that the power balance and transmission capacity limits are always respected while generation and demand at each bus are changing freely within their limits.

B. Impact of Grid Enhancing Technologies on Reshaping the Feasibility Regions of Bulk Power Systems

We define the depth of congestion as the amount of available low cost generation that cannot be transferred to the load centres due to insufficient transmission line capacity. Moreover, we define the duration of congestion as the number of hours that congestion occur over the horizon under study. These two factors are impacted differently by various transmission expansion options.

Dynamic line rating changes the capacity of a transmission line. As such, the constraints of the transmission line with dynamic line rating changes as given in (22)-(23). The capacity limit of the transmission line equipped with dynamic line rating is denoted by $\alpha^{dynamic} f_l^{max}$.

$$\sum_{n=1}^{N} \gamma_{n,s}^{l,w}(g_n - d_n) \le \alpha^{\text{dynamic}} f_l^{\max} \quad \text{for} \quad l = l_k \quad (22)$$
$$-\sum_{n=1}^{N} \gamma_{n,s}^{l,w}(g_n - d_n) \le \alpha^{\text{dynamic}} f_l^{\max} \quad \text{for} \quad l = l_k \quad (23)$$

The impact of dynamic line rating on reducing the depth of congestion depends on the weather conditions and may change hour by hour. Therefore, this technology may not be as beneficial as other grid enhancing technologies when the transmission line is heavily congested for long hours. Moreover, it is worth noting that dynamic line rating is only suitable for cases when transmission capacity constraints arise from thermal considerations. Yet, the major benefit of dynamic line rating is its low investment cost.

Installing power flow control devices changes the shift factor of the congested line as well as the shift factors of other transmission lines. As such, the constraints of the transmission lines change as given in (24)-(25). The shift factors of the transmission lines after installing power flow control devices are denoted by $\hat{\gamma}_{n,s}^{l,w}$ for the network topology w.

$$\sum_{n=1}^{N} \hat{\gamma}_{n,s}^{l,w}(g_n - d_n) \le f_l^{\max} \quad \forall l$$
(24)

$$-\sum_{n=1}^{N}\hat{\gamma}_{n,s}^{l,w}(g_n - d_n) \le f_l^{\max} \quad \forall l$$
(25)

The impact of power flow control devices on congestion depth and duration depends on several factors including the location of the power flow control device, the level of compensation and the reactance of other transmission lines. In contrast to dynamic line rating, this technology does not depend on factors such as time and weather. The compensation level provided by the power flow control device determines the decrease in the depth of congestion. It is worth noting that the installation of power flow control device on a transmission line not only impacts the power flow on that line but also impacts the power flow on other transmission lines.

Building new transmission lines may add/remove constraints to/from the feasibility regions of a bulk power system and modifies the shift factors of the transmission lines. As such, the constraints of the transmission lines change as given in (26)-(27) where \hat{l} denotes both the original transmission lines and the new transmission lines.

$$\sum_{n=1}^{N} \hat{\gamma}_{n,s}^{l,w}(g_n - d_n) \le f_l^{\max} \quad \forall \hat{l}$$

$$(26)$$

$$-\sum_{n=1}^{N}\hat{\gamma}_{n,s}^{l,w}(g_n-d_n) \le f_l^{\max} \quad \forall \hat{l}$$
(27)

The impact of building a new line on congestion depth and duration depends on several factors including the location of the new transmission line, the capacity and the reactance of the new transmission line and the original transmission lines.

III. CASE STUDIES

The studies are conducted using a three-bus test system for demonstration purposes, but the concepts are general and can be extended to large power systems. The three-bus test system under study consists of four transmission lines, two generators and a load point as illustrated in Fig. 1. All the transmission lines have the same per unit series reactance while the capacity of the transmission lines are 100 MW, 100 MW, 120 MW, and 80 MW, respectively. The minimum and maximum capacity of the generators are considered to be 0 MW and 500 MW, respectively. It is worth noting that the capacity of the generating units are intentionally considered to be large compared to the available transmission line capacities to better demonstrate the concepts. We assumed that the threebus test system does not have generation adequacy problem. In this paper, we focus on the transmission line congestion.



Fig. 2. Feasibility region of the three-bus test system.

The feasibility region of the three-bus test system is illustrated in Fig. 2. The constraints characterizing the feasibility region of the three-bus test system are given in (28)-(32).

$$g_1 + g_3 = d_2 \tag{28}$$

$$2g_1 + g_3 \le 500 \tag{29}$$

$$g_1 + 3g_3 \le 400 \tag{30}$$

$$-g_1 \le 0 \tag{31}$$

$$-g_3 \le 0 \tag{32}$$

The blue area illustrates the power balance plane given by (28). The green line depicts the constraint associated with the capacity of the transmission lines 1 and 2 given in (29). The red line depicts the constraint associated with the capacity of the transmission line 4 given in (30). The orange hatched area shows the feasibility region of the three-bus test system. The projection of the hatched area on the d_2 axis shows the demand levels at bus 2 that can be supplied by this power system. There is a congestion in the system when the operating point is on the green line or red line in Fig. 2. The following two cases are considered to explain the transmission lines congestion.

A. Case 1: Generator 1 is More Expensive

In this case, the feasibility region of the three-bus test system shown in Fig. 2 reduces to the edges of the hatched area given by (33)-(34). This is because the cheaper generator, i.e. g_3 , supplies demand as much as possible before the transmission line 4 becomes congested.

$$g_3 = d_2 \tag{33}$$

$$g_1 + 3g_3 = 400 \tag{34}$$

The congestion occurs when the demand goes above d_{c1} . For any demands larger than d_{c1} , the operating point is on the red line at the edge of the hatched area. The depth of congestion is determined by (35) which depends on the unused capacity of the cheaper generator and demand level above d_{c1} . The capacities of the generating units are intentionally considered large in this paper to put the focus on the congestion of the transmission lines.

Congestion^{depth} = max{0, min{
$$d_2 - d_{c1}, g_3^{max} - d_{c1}$$
}}
(35)

B. Case 2: Generator 3 is More Expensive

In this case, the feasibility region of the three-bus test system shown in Fig. 2 shrinks to the edges of the hatched area given by (36)-(37). This is because the cheaper generator, i.e. g_1 , supplies demand as much as possible before the transmission lines become congested.

$$g_1 = d_2 \tag{36}$$

$$2g_1 + g_3 = 500 \tag{37}$$

The congestion occurs when the demand goes above d_{c2} . For any demands larger than d_{c2} , the operating point is on the green line at the edge of the hatched area. The depth of congestion is determined by (38) which depends on the unused capacity of the cheaper generator and demand level above d_{c2} .

Congestion^{depth} = max{0, min{
$$d_2 - d_{c2}, g_1^{max} - d_{c2}$$
}}

(38)

C. Comparative Analysis of Case 1 and 2

Fig. 2 illustrates that the edge of the hatched area on the red line is much longer than the edge on the green line. This is due to the possibility of higher power transfer from bus 1 to bus 2 over the transmission lines connected to bus 1 compared to the possibility of power transfer from bus 3 to bus 2 over the transmission lines connected to bus 3. As a result, d_{c1} is much smaller than d_{c2} . This implies that higher congestion duration and depth may occur in case 1 compared to case 2. The duration of congestion experienced by a transmission line is a determining factor in choosing the grid enhancing technology as introduced in Section II.B. The depth of congestion is another crucial factor in choosing the grid enhancing technology. This factor was also introduced in Section II.B. The depth and duration of congestion can be monetized as given in (39) and (40).

$$\mathbf{B}^{\text{tech}}(t) = \begin{cases} 0 & d_2(t) \le d_{ci}(t) \\ \Delta c_g(d_2(t) - d_{ci}(t)) & d_{ci}(t) < d_2(t) \le d_{c'i}(t) \\ \Delta c_g(d_{c'i}(t) - d_{ci}(t)) & d_{c'i}(t) < d_2(t) \end{cases}$$
(39)

$$TB^{tech} = \sum_{t=1}^{T} B^{tech}(t)$$
(40)

Where $B^{\text{tech}}(t)$ and TB^{tech} denote the benefit gathered at hour t and total benefit, respectively. $d_{ci}(t)$ and $d_{c'i}(t)$ denote the demand level at time t which causes transmission congestion before and after implementing the grid enhancing technology, respectively. Subscript i is either 1 or 2 depending on which generator is more expensive. Δc_g denotes the difference between the cost of the cheap and expensive generating unit.



Fig. 3. Feasibility region of the three-bus test system after implementing dynamic line rating on the transmission lines 1 and 2.



Fig. 4. Feasibility region of the three-bus test system after implementing dynamic line rating on the transmission line 4.

D. Grid Enhancing Options

1) Dynamic Line Rating: Dynamic line rating may increase the capacity of a transmission line at certain hours. Fig. 3 illustrates the case when dynamic line rating increases the capacity of the transmission lines 1 and 2 by 10%. As illustrated in Fig. 3, the constraint associated with the transmission lines 1 and 2, i.e. the green line, moves upward in the power balance plane. The yellow hatched area in Fig. 3 shows the expansion of the feasibility region due to implementing dynamic line rating on the transmission lines 1 and 2. This is beneficial when the generator g_3 is more expensive than the generator g_1 . Fig. 4 illustrates the case when dynamic line rating increases the capacity of the transmission line 4 by 25%. The yellow hatched area in Fig. 4 shows the expansion of the feasibility region due to implementing dynamic line rating on the transmission line 4. This is beneficial when the generator g_1 is more expensive than the generator g_3 . Figs. 3 and 4 show that the depth and timing of congestion play a crucial role in determining the benefit of dynamic line rating.

2) Power Flow Control Devices: Fig. 5 illustrates the impact of installing the power flow control devices on the transmission lines 1 and 2. In Fig. 5, the reactance of the transmission lines 1 and 2 decreased by 50%. The yellow



Fig. 5. Feasibility region of the three-bus test system after installing the power flow control devices on the transmission lines 1 and 2.



Fig. 6. Feasibility region of the three-bus test system after installing the power flow control device on the transmission line 3.

and white hatched areas in Fig. 5 show the parts that have been added to and eliminated from the feasibility region due to installing power flow control devices on the transmission lines 1 and 2, respectively. This case is beneficial when the generator g_1 is more expensive. Fig. 6 illustrates the impact of installing the power flow control device on the transmission line 3. In Fig. 6, the reactance of the transmission lines 3 decreased by 50%. The yellow hatched area in Fig. 6 shows the expansion of the feasibility region. This case is beneficial to both cases when either generator g_1 or g_3 is more expensive. This is due to the location of the transmission line 3.

As illustrated in Figs. 5 and 6, the impact of installing power flow control devices depends on the location and reactance of the transmission lines. Moreover, the increase in the power transfer of cheaper generating units cannot be increased beyond certain limits.

3) Building a Transmission Line: Fig. 7 illustrates the impact of building a new transmission line in parallel with the transmission line 4. The capacity and reactance of this new transmission line is considered to be equal to the capacity and reactance of the transmission line 4. The yellow hatched area in Fig. 7 shows the expansion of the feasibility region due to building a new line in parallel with the transmission line 4.



Fig. 7. Feasibility region of the three-bus test system after building a new transmission line in parallel with the transmission line 4.

Building this new line is beneficial when the generator g_1 is more expensive than the generator g_3 . The maximum power that can be generated by the generator g_3 increased from 133.3 MW to 213.3 MW after building the new line. This shows that building a new transmission line is imperative when a transmission line is heavily congested for long hours.

IV. CONCLUSION

This paper examined congestion-driven positioning of grid enhancing technologies based on the feasibility regions of bulk power systems. The impact of various grid enhancing technologies such as dynamic line rating and power flow control devices on reshaping the feasibility regions of bulk power systems are demonstrated using a three-bus test system. It is shown that various grid enhancing technologies have different impacts on congestion relief. Furthermore, we introduced congestion depth and duration as the influencing factors for determining the appropriate grid enhancing technology. In addition, it is shown that these two factors can be monetized to justify the cost of implementing grid enhancing technologies. The future research direction is to analyse the proposed method for real-size power systems. Investigating the benefits of other grid enhancing technologies such as energy storage and transmission topology optimization for congestion management is another direction for future research.

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