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Developmental trajectories of symbolic magnitude and order processing and their relation with arithmetic development

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ABSTRACT

Efficient processing of absolute magnitude and relative order of numbers is key for arithmetic development. This longitudinal study tested 1) whether there is a developmental shift in the contribution of symbolic magnitude and order processing to arithmetic between Grades 1 and 2, and 2) whether the development of symbolic numerical abilities is characterized by reciprocal predictive relations. In two independent samples (UK: N = 195, Austria: N = 161), order processing did not predict early developmental change in arithmetic, but emerged as a predictor in Grade 2. Symbolic magnitude processing in Grade 1 predicted subsequent developmental change in arithmetic, but not later on. Moreover, we observed cross-lagged relations between symbolic magnitude and order processing. Our findings confirm that the contributions of symbolic magnitude and order processing to arithmetic development are interactive and change across the first years of schooling. This may be driven by a developmental shift from procedural strategies to retrieval of arithmetic facts.

1. Introduction

Every day, we encounter numerical information in a symbolic format such as Arabic digits, and our ability to process such symbolic number forms constitutes a core foundation of acquiring mathematical competences (De Smedt et al., 2013; Göbel et al., 2014; Goffin & Ansari, 2019; Lyons et al., 2014; Rodic et al., 2015). Number symbols can reflect different semantic dimensions: First, the numeral "2" informs us about the underlying quantity (e.g., two pencils), and we can easily judge that "2" is less than "3". This ability to represent the absolute size or magnitude of Arabic digits is commonly described as symbolic magnitude processing. Second, numerals are also characterized by their relative size or rank within a sequence. The dimension of ordinality allows us to efficiently judge, for example, that the integer "502" comes immediately after "501", but before "503". This efficient positional classification would be impossible if numerals only conveyed meaning in terms of their absolute quantities. An increasing number of behavioral and

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neuroimaging findings have demonstrated that symbolic magnitude and order processing constitute two separate cognitive dimensions (Franklin & Jonides, 2009; Lyons et al., 2016; Lyons & Beilock, 2013; Sommerauer et al., 2020; Vogel et al., 2015). Moreover, neuropsychological case studies have reported that processing of numerical magnitude and order can be selectively impaired (Delazer & Butterworth, 1997; Turconi & Seron, 2002).

While a large body of evidence suggests that children's symbolic magnitude processing skills serve as a key foundation for the acquisition of higher-order arithmetic skills (review: De Smedt et al., 2013; meta-analysis: Schneider et al., 2017), the precise role of order processing in arithmetic development is still being debated (Lyons et al., 2014, 2016; Lyons & Ansari, 2015; Orrantia et al., 2019; Sasanguie et al., 2017; Sasanguie & Vos, 2018; Vogel et al., 2015, 2021; Vos et al., 2017; Xu & Lefevre, 2021). Importantly, cross-sectional evidence is suggestive of a fundamental shift in the predictive pattern of arithmetic from dominance of magnitude to order processing between Grades 1 and 2 (Sasanguie & Vos, 2018; Xu & Lefevre, 2021). However, this account of a changing predictive contribution of symbolic numerical abilities during arithmetic development has not yet been examined longitudinally. In particular, it is largely unknown how processing of numerical magnitudes and order interact in the prediction of arithmetic during development (Lyons et al., 2016). Thus, the current longitudinal study set out to investigate the interactive patterns by tracking the developmental trajectories of symbolic magnitude and order processing, as well as arithmetic abilities across the first three years of primary school.

1.1. Symbolic magnitude processing and arithmetic performance

At the beginning of formal schooling, children are becoming increasingly familiar with the intricacies of the symbolic number system and acquire a lexicon of readily accessible arithmetic facts. Much of the research on the symbolic numerical predictors of arithmetic development in young children has focused on symbolic magnitude processing. Interindividual differences in symbolic magnitude processing are typically assessed by means of a number comparison task requiring to indicate the larger of two numbers. Individuals are typically slower to indicate the larger number in pairs with a small numerical distance (e.g., 1–2) compared to pairs with a large numerical distance (e.g., 1–9). This numerical distance effect is thought to be evoked by the fact that internal semantic magnitude representations are more overlapping for neighboring numbers compared to more distant number pairs, thus causing interference upon the retrieval of magnitude representations or during the decision process (Henik & Tzelgov, 1982). Interindividual differences in the size of this numerical distance effect as well as efficiency in performing symbolic magnitude comparisons were found to be related to arithmetic performance in children and adults (meta-analyses: Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017). Several aspects may play a role in explaining this relation: First, symbolic magnitude processing may serve as an indicator for familiarity with the exact symbolic number system, which is a key determinant for arithmetic (Göbel et al., 2014). The relation may also be driven by the ability to link visual-Arabic digits to their verbal counterparts (Habermann et al., 2020). Another suggestion is that intentional processing of the semantic meaning underlying the number symbols in terms of their concrete magnitudes may explain the relevance for arithmetic development (Bugden & Ansari, 2011; Vogel et al., 2015).

Symbolic magnitude processing seems to be mostly relevant during early arithmetic development: Whereas meta-analytical evidence suggests that zero-order correlations between symbolic comparison skills and arithmetic decrease only very slightly with age (Schneider et al., 2017), unique contributions over and above other numerical predictors were found to be more prominent at the beginning of primary school than in higher school grades (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & Lefevre, 2021). For instance, based on data from a large cross-sectional study, Lyons et al. (2014) reported that symbolic magnitude comparison was the most important predictor of arithmetic in Grade 1, but its contribution showed a steady decrease until Grade 6. There have been some theoretical explanations as to why this might be the case: First, magnitude processing may be less important for arithmetic development once the symbolic number system matures (Lyons et al., 2014). Once all children have acquired a certain level of proficiency with symbolic numbers, arithmetic development may increasingly rely on other skills. It has also been speculated that magnitude processing may be particularly relevant in Grade 1 because children still critically rely on procedural calculation skills (Sasanguie & Vos, 2018). At the onset of formal mathematics education, many children solve even basic problems such as 1 + 4 by counting on from the first addend (Fuson, 1988; Secada et al., 1983). Being able to easily access the absolute magnitude of numbers may enable using more efficient strategies such as counting on from the largest addend. Later, the relevance of magnitude processing may decrease once children are able to retrieve simple arithmetic facts such as 1 + 4 from long-term memory (Sasanguie & Vos, 2018), or apply more advanced mathematical concepts (Schneider et al., 2009).

1.2. Symbolic ordinal processing and arithmetic performance

A main goal of early math education is that children build up complex and multiple associations between numbers and number symbols (Lyons et al., 2016; Sasanguie & Vos, 2018; Schneider et al., 2017). Basic associations include the order of numbers in the counting sequence, whereas more advanced associations comprise an understanding of non-count-list associations, e.g., multiples such as 2–4–6 (Lyons & Beilock, 2011). Generally, performance on ordinal processing tasks may provide an index of the efficiency with which associations between number symbols stored in long-term memory can be retrieved when arithmetic calculations are performed (Lyons et al., 2016; Lyons & Beilock, 2013; Vogel et al., 2019).

In ordinal processing tasks, individuals are typically asked to indicate whether a triplet of digits is in order (e.g., 1–2–3) or not in order (e.g., 1–3–2). In contrast to the classical distance effect observed in magnitude comparison tasks, reaction time measures for ordinal processing tasks show a reverse distance effect (Goffin & Ansari, 2016; Sella et al., 2020; Turconi et al., 2006; Vogel et al., 2019): Individuals are generally faster to make order judgments on sequences with an inter-item distance of one (e.g., 1–2–3) than sequences with an inter-item distance of two (e.g., 1–3–5). It has been proposed that adjacent ordered sequences such as 1–2–3 are solved particularly fast because they can be directly retrieved from the verbal count-list stored in long-term memory (Lyons et al., 2016; Sasanguie & Vos, 2018; Vogel et al., 2017, 2019, 2021). The reversal of the distance effect suggests that cognitive mechanisms involved in order processing extend beyond access to magnitude representations and highlights the importance of inter-item associations between symbolic numbers. In particular, recent evidence supports the notion that efficient retrieval of the count list stored in long-term memory is one of the key cognitive mechanisms supporting symbolic ordering at the beginning of primary school (Finke et al., 2021). Across development, children's ordering skills may serve as indicators of an increasingly rich semantic network of inter-item associations between number symbols.

Indeed, several cross-sectional studies point into the direction that symbolic ordering emerges as an important predictor of arithmetic during primary school. First, a large cross-sectional study (Lyons et al., 2014) found only small zero-order correlations between symbolic ordering and arithmetic in Grade 1 ($r = 0.16$), but a much higher correlation in Grade 2 ($r = 0.44$), with further minor increases until Grade 6 ($r = 0.55$). Second, Lyons et al. (2014) reported that ordinal processing skills did not uniquely contribute to the prediction of arithmetic over and above other numerical skills in Grade 1; however, its predictive contribution steadily increased across grades, overtaking all other predictors in Grade 6. Two studies examined the unique contributions of both symbolic order and magnitude processing to arithmetic, contrasting samples of children in Grade 1 and 2 (Sasanguie & Vos, 2018; Xu & Lefevre, 2021): In both studies, symbolic ordering did not uniquely contribute to arithmetic in Grade 1 over and above symbolic magnitude processing. However, in Grade 2, symbolic ordering was a significant predictor of arithmetic, whereas magnitude processing did not exert any direct influence on arithmetic (Sasanguie & Vos, 2018; Xu & Lefevre, 2021). Similarly, studies assessing both symbolic magnitude processing and ordering in slightly older children aged 7–10 years (Sommerauer et al., 2020), as well as young adults (Sasanguie et al., 2017) observed that only order processing uniquely contributes to arithmetic.

Explanations have been put forward as to why order processing emerges as an increasingly important predictor of arithmetic. First, it has been proposed that this change may be driven by a shift in the focus of math instruction from procedural to retrieval-based strategies (Sasanguie & Vos, 2018). Order processing may help promote the acquisition and retrieval of arithmetic facts by providing a sophisticated network of associative links between symbolic numbers. More recently, authors have provided an intriguing explanation for the changing predictive contribution of symbolic numerical skills for arithmetic based on a conceptual shift in the notion of ordinality during the first years of formal schooling: Notably, Xu and Lefevre (2021) argued that only symbolic magnitude processing was uniquely associated with arithmetic in their study in Grade 1, because young children may not yet have fully developed a readily accessible network of ordinal associations between number symbols in Grade 1. Recent evidence supports this idea: Even at the end of Grade 1, a large proportion of children seem to struggle in generalizing the concept of ordinality to non-count-list associations, and accordingly judge non-adjacent sequences such as 2–4–6 to be not in order (Hutchison et al., 2022). Especially the notion that ordered sequences can "skip" numbers may serve as an important prerequisite for grasping and memorizing multiplication tables. According to Xu and Lefevre (2021), the dominance of ordinal processing as a predictor of arithmetic in Grade 2 may reflect that children have come to develop a refined network of complex associations between symbolic numbers. Thus, despite the bivariate relation between symbolic magnitude processing and arithmetic staying relatively stable across development, its relative contribution to the acquisition of arithmetic skills may drop once children have gained access to a readily accessible network of ordinal associations.

To sum up, cross-sectional studies are suggestive of a developmental shift in the contribution of symbolic number skills to arithmetic between Grades 1 and 2: Whereas symbolic magnitude processing is a unique predictor of arithmetic at the very beginning of primary school, its predictive value is superseded by symbolic ordering in Grade 2. Children appear to progress from accessing the relative magnitude (i.e., cardinality) of number symbols to accessing a rich network of semantic associations between number symbols when doing arithmetic. However, the empirical support of these theoretical assumptions provided by previous cross-sectional studies remains arguably limited. Thus, the current study explicitly tests the assumption of a developmental shift from magnitude to ordinal processing of number symbols in a longitudinal design.

1.3. Inter-relation between processing of symbolic magnitude and order

Another key question relates to the inter-relation between symbolic magnitude and order processing across development. Specifically, it is still unclear whether magnitude processing predicts growth in order processing at the beginning of formal education, or the other way around. One view is that cardinal associations serve as a foundation for the acquisition of ordinal associations, and that children's processing of number symbols progresses from linking them to their underlying quantities to forming abstract connections between the number symbols themselves (Nieder, 2009). Indeed, numerous cross-sectional studies have shown that magnitude processing skills are predictive of order processing in children and adults (e.g., Lyons et al., 2014; Lyons & Beilock, 2011; Morsanyi et al., 2017; Vogel et al., 2015). Moreover, several cross-sectional studies with older children and adults proposed mediation models in which the influence of magnitude processing on arithmetic could be fully explained (i.e., mediated) by symbolic ordering (Lyons & Beilock, 2011; Sasanguie & Vos, 2018; Sommerauer et al., 2020; Vos et al., 2017; Xu & Lefevre, 2021).

Alternatively, once children have acquired some understanding of ordinal associations between numbers, they can access a more complex network of associations between number symbols, which may foster growth in symbolic magnitude processing. In other words, acquiring higher-order representations of symbolic numbers may help refine more basic numerical representations (Vogel, 2019). In turn, refined symbolic magnitude processing skills might again promote growth in ordering skills. Thus, there is reason to assume a reciprocal predictive relation between developmental trajectories of symbolic magnitude and order processing. Indeed, preliminary cross-sectional evidence suggests that the relation between magnitude and order processing is highly interactive: Whereas in Grade 1, symbolic magnitude processing was found to serve as a mediator for the relation between symbolic ordering and arithmetic, the opposite pattern was observed in Grade 2 with symbolic ordering mediating the relation between symbolic magnitude processing and arithmetic (Sasanguie & Vos, 2018).

Overall, theoretical accounts and first cross-sectional evidence suggest that representations of numerical magnitude and order may closely interact when children form precise representations of symbolic number at the beginning of primary school (see also Sury & Rubinsten, 2012). However, a stronger test of these accounts requires a large-scale longitudinal study tracking symbolic magnitude processing and ordering across multiple timepoints during the relevant period at the beginning of primary school. Such a study design allows testing cross-lagged path models, which estimate the longitudinal relations from one variable to another, and across different time points.

1.4. The present study

We set out to empirically investigate the developmental trajectories and cross-lagged relations of symbolic magnitude and order processing, as well as arithmetic abilities across the first three years of primary school in two independent samples. We focused on this period because children are expected to gain increasing proficiency with the symbolic number system and at the same time acquire a lexicon of readily-accessible arithmetic facts between first and third grade.

We addressed two questions: 1) Is there a shift in the predictors of arithmetic development between Grades 1 and 2? and 2) Is the development of symbolic numerical abilities characterized by reciprocal predictive relations between symbolic magnitude and order processing? We tested a model (Fig. 1) in which paths are based on previous cross-sectional evidence (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & Lefevre, 2021). In this model, symbolic magnitude processing in Grade 1 predicts arithmetic skills in Grade 2 while symbolic ordering does not. In Grade 2, a reversed pattern is assumed: Symbolic ordinal processing predicts arithmetic in Grade 3, whereas symbolic magnitude processing does not uniquely contribute to the prediction. Moreover, this model allows for a reciprocal predictive relation between symbolic magnitude processing and ordering. In case we would fail to confirm this model, we planned to specify the best-fitting model separately for the two samples, allowing for cross-lagged paths from arithmetic abilities to its predictors.

2. Method

2.1. Participants

Data were collected in the context of a longitudinal study on the relation between symbolic number processing and arithmetic abilities spanning a period from the end of Grade 1 to the end of Grade 3. The data analyzed in this study, as well as detailed project and task descriptions are available at [link to repository removed for blinding purposes].

Sample 1 (UK) consisted of 195 children from 11 primary schools across [region removed for blinding purposes] (95 females and 100 males). Sample 2 (Austria) consisted of 161 children from 5 primary schools in [region removed for blinding purposes] (77 females and 84 males). Most children were monolingual native English- or German-speakers (UK: 97%, Austria: 88%). Groups were matched on duration of formal education, but children in the UK were approximately one year younger than their peers in Austria as schooling starts one year earlier in the UK (mean age: 6;3 years 74.88 months at first assessment; *SD* = 4 months) than in Austria (mean age: 7;2 years; *SD* = 3 months). Initially, 309 children from the UK and 177 children from Austria were recruited at the first assessment time point in Grade 1. At the following assessment time points, several students were not available due to relocation and school changes or temporary absences. Higher attrition rates in the sample from the UK are due to the fact that one school (n = 56) decided to discontinue participation during the study period. By Grade 3, 199 children from the UK and 167 children from Austria had fully completed all three assessments of the current study variables. We excluded four children from the UK sample and six children from the Austrian sample who showed a clearly biased answering pattern in the ordinal processing tasks (e.g., ticking or crossing out 10 or more items in a row). We ran a test for data completely missing at random (MCAR) with the R package MissMech (Jamshidian et al., 2014). There was no sufficient evidence to reject the assumption that data were completely missing at random at an alpha level of.05. Children in the UK came from urban, town and rural schools across [region removed for blinding purposes]. Children in Austria came from a

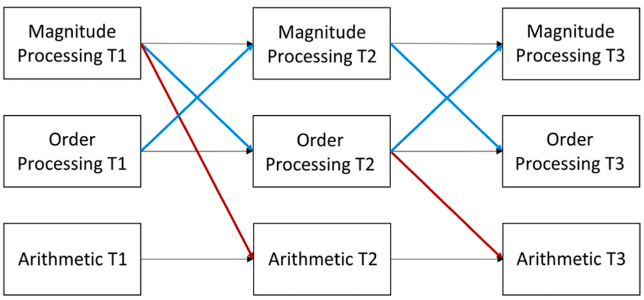

Fig. 1. A Priori Model Based on Previous Cross-Sectional Evidence Tested in the Study. Note. This model assumes 1) a developmental shift in the predictors of arithmetic development: Symbolic magnitude processing in Grade 1 uniquely predicts arithmetic performance in Grade 2, whereas symbolic order processing in Grade 2 uniquely predicts arithmetic in Grade 3 (red paths), and 2) cross-lagged relations between symbolic magnitude and order processing (blue paths). Autoregressive paths are depicted in black.

middle-income urban school district.

According to national curricula, children are first introduced to arithmetic by using procedural calculation strategies before progressing to fact retrieval in both countries. In the UK, children are expected to add and subtract single-digit and two-digit numbers in the number range up to 20 and solve one-step problems using concrete objects and pictorial representations by the end of Grade 1 (Department for Education, 2013). By the end of Grade 2, children in the UK should be able to use place value and arithmetic facts to solve problems in the number range up to 100. The Austrian mathematics curriculum (Bundesministerium für Bildung, Wissenschaft und Forschung, 2012) does not specify separate learning goals for Grades 1 and Grade 2, but states that children should first gain experience with arithmetic operations in a smaller number range by concrete manipulations and pictorial representations before progressing to more difficult problems. By the end of Grade 2, children in Austria are expected to have automatized basic arithmetic problems, especially multiplication facts. By the end of Grade 3, the number range is extended to 1000 in both countries.

Post hoc RMSEA- and χ^2-based power analyses were conducted with the Shiny app power4SEM (Jak et al., 2021), following criteria for closeness of fit proposed by Browne and Cudeck (1992) and MacCallum et al. (2006). Firstly, we conducted a power analysis for the RMSEA test of close fit, which showed that the final sample of 356 participants had a power of 90.2% to reject the hypothesis of close fit (H_0: RMSEA ≤ 0.05) with an alpha of 0.05 and 36 degrees of freedom when in the population there is not close fit (H_1: RMSEA $= 0.08$). Secondly, we calculated a χ^2-based power analysis with H_1 based on the RMSEA, which revealed that the final sample of 356 participants achieved 100% power to reject an H_0 RMSEA of zero when the H_1 RMSEA is 0.08 with 36 degrees of freedom. To achieve a power of 0.80, power4SEM indicated a minimum sample size of 116.

The study was approved by the ethics committees of [institution removed for blinding purposes] and [institution removed for blinding purposes], and written informed consent was granted by the parents or legal guardians.

2.2. Tasks

2.2.1. Arithmetic

At each assessment time point, children completed a selection of items of the Numerical Operations subtest of the Wechsler Individual Achievement Test (WIAT-II; Wechsler, 2005) adapted for group use and modified to account for language- or culture-dependent notation differences in the Austrian sample (e.g., multiplication sign UK: 7×6 vs. Austria: $7 \cdot 6$). In Grade 1, children received nine items involving arithmetic calculations with varying degrees of difficulty (addition, subtraction and multiplication with one-, two- or three-digit numbers) to be solved within 15 min. Arithmetic ability was scored as the total number of correct calculations (maximum score 9). At the start of this task, children received six items involving number identification, writing Arabic digits to dictation, and dot enumeration. Responses to these six items were excluded because they provide a measure of basic symbolic number processing rather than arithmetic performance. In Grade 2, children received an extended task version involving 8 additional arithmetic calculations (including division), resulting in a maximum score of 17. In Grade 3, the task was adapted by dropping the first six items (due to expected ceiling effects) and extended by advanced arithmetic calculations involving three-digit and decimal numbers. Children worked through 26 items within a time limit of 15 min.

2.2.2. Symbolic magnitude processing

Children completed two digit comparison tasks as reported in a previous study, each comprising 60 pairs of single-digit numbers from 1 to 9 (Göbel et al., 2014). We included two task versions with different numerical distances between number pairs: While one task consisted of pairs with a small numerical distance (one or two), the other task included number pairs with a large distance (five, six, or seven) between both digits. Six pairs of items were presented on each page of a 14.8×21.0 cm booklet. Children were instructed to process as many pairs as possible in 30 s by ticking the numerically larger quantity in each pair. The symbolic magnitude comparison score was calculated as the sum of total correct scores across both booklets. For the present samples, Cronbach's alpha for

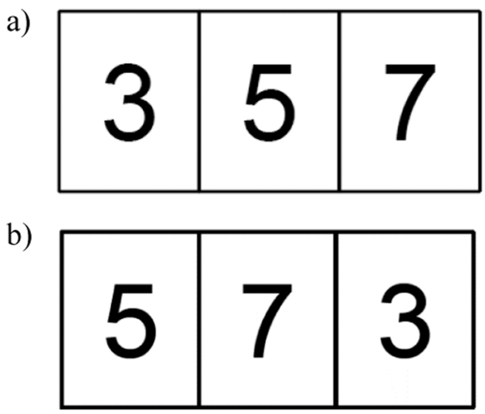

Fig. 2. Example Items of the Symbolic Order Processing Task: a) Ascending and b) Not-in-Order. *Note.* Children were asked to indicate whether sequences of digits were presented in an ascending order or not.

the symbolic magnitude comparison task ranged between.77 and.87.

2.2.3. Symbolic order processing

Children were asked to decide whether sequences of three single-digit Arabic numbers were presented in ascending numerical order (e.g., 2–4–6) or not (e.g., 2–6–4) (Fig. 2). Children were presented with a 21.0 × 29.7 cm booklet with eight pages, each containing two columns with five items, resulting in a total of 80 items. Numerical distance between the three digits in the ordered condition was either one (e.g., 1–2–3), two (e.g., 1–3–5) or three (e.g., 1–4–7). Children were instructed to tick items in ascending order and to cross out items in which the number sequence was not in order. They were specifically instructed to work as fast as possible, without making mistakes. Before starting the actual tasks, the experimenter presented and discussed six practice items and made sure that children understood the instructions. The number of correct responses given within a time limit of 90 s was scored. Test-retest reliability for this task was.74 (Finke et al., 2021).

2.3. Procedure

All study variables were assessed at three assessment points: at the end of Grade 1 (T1), at the end of Grade 2 (T2), and at the end of Grade 3 (T3). Tasks were administered in a classroom setting by trained experimenters. In both samples (UK and Austria), all tasks were given in the same order, and administration and scoring procedures were standardized.

2.4. Analyzing cross-lagged relations

To examine the cross-lagged relations between symbolic magnitude processing, ordering and mathematic abilities, we performed path analyses with maximum likelihood estimation in lavaan (Rosseel, 2012) in R version 4.0.0 (R Core Team, 2020). We started out with a multigroup model including both samples based on previous cross-sectional evidence suggesting a developmental shift in the relative contribution of symbolic magnitude and order processing to arithmetic development (Fig. 1): In this model, symbolic magnitude processing in Grade 1 predicts arithmetic abilities in Grade 2, and symbolic ordering in Grade 2 predicts arithmetic abilities in Grade 3. Additionally, cross-lagged relations between symbolic magnitude and order processing are included. Thus, only theory-driven paths were estimated; all other paths were fixed to zero. In case we would fail to find clear evidence for this model, we planned to identify the best-fitting model separately for each sample (UK vs. Austria) to inspect age-related similarities and differences.

To evaluate the fit of the obtained models, we examined fit indices following recommendations by Hu and Bentler (1999) and Beauducel and Wittmann (2005), suggesting that the comparative fit index (CFI) should be higher than.95, the root square error of estimation (RMSEA) and the standardized root mean square residual (SRMR) should be less than.05. As advised by Greiff and Heene (2017), we also considered overall model fit as indicated by the chi-square model test.

3. Results

3.1. Descriptive statistics

Table 1 presents the descriptive statistics for symbolic magnitude processing, symbolic order processing and arithmetic abilities. Performance in the symbolic magnitude and symbolic order processing tasks showed a steady increase across the study period. Note that arithmetic abilities were assessed by items of increasing levels of difficulty throughout the assessment points, thus avoiding ceiling or floor effects. All measures were sensitive to age: They increased across assessment point and at each time point, performance was significantly lower in the younger UK sample compared to their older counterparts from Austria.

3.2. Correlational analysis

Concurrent and longitudinal correlations between all study variables are depicted in Table 2. As expected, both symbolic numerical tasks (i.e., symbolic magnitude comparison, as well as ordinal processing) were significantly related to arithmetic in both samples. Importantly, the correlation analyses showed that symbolic magnitude and ordinal processing are distinct constructs: We only found moderate concurrent correlations between these variables. Note that stability of arithmetic performance and symbolic magnitude comparison was relatively high in both samples. Stability of order processing was somewhat weaker in the younger children from the UK than in the Austrian sample, and Fisher's z tests confirmed that this difference was statistically significant (T1-T2: $r = 0.45$ vs $r = 0.58$, $z = -1.66$, $p = .049$; T2-T3: $r = 0.62$ vs $r = 0.72$, $z = -1.70$, $p = .045$).

Table 1
Means and Standard Deviations for Arithmetic Abilities, Symbolic Magnitude and Order Processing across Samples and Assessment Time Points.

	Sample 1 UK (N = 195)		Sample 2 Austria (N = 161)		Group Differences		
Variable	*M*	*SD*	*M*	*SD*	*t*	*p*	*d*
Arithmetic							
T1	4.14	1.94	5.52	1.42	-7.48	< 0.001	0.81
T2	8.87	3.22	11.36	2.59	-8.12	< 0.001	0.85
T3	13.75	5.33	16.70	3.37	-6.10	< 0.001	0.66
Magnitude Processing							
T1	31.05	10.41	37.06	9.36	-5.75	< 0.001	0.61
T2	43.87	9.99	51.39	10.02	-7.53	< 0.001	0.75
T3	52.60	11.52	60.47	10.79	-6.77	< 0.001	0.71
Order Processing							
T1	14.34	6.14	19.25	7.00	-7.04	< 0.001	0.75
T2	21.24	7.11	25.82	8.10	-5.73	< 0.001	0.60
T3	26.18	8.65	29.74	8.81	-3.93	< 0.001	0.41

Table 2
Correlations between Study Variables.

Variable	1	2	3	4	5	6	7	8	9
1. Arithmetic T1		0.56	0.61	0.43	0.48	0.42	0.40	0.44	0.37
2. Arithmetic T2	0.59		0.71	0.45	0.50	0.47	0.34	0.46	0.32
3. Arithmetic T3	0.56	0.63		0.44	0.51	0.48	0.35	0.51	0.52
4. Magnitude Processing T1	0.22	0.24	0.28		0.63	0.56	0.51	0.43	0.47
5. Magnitude Processing T2	0.21	0.18	0.23	0.67		0.68	0.42	0.64	0.59
6. Magnitude Processing T3	0.22	0.12	0.23	0.53	0.75		0.49	0.57	0.55
7. Order Processing T1	0.27	0.24	0.26	0.42	0.43	0.44		0.45	0.44
8. Order Processing T2	0.36	0.35	0.39	0.41	0.57	0.54	0.58		0.62
9. Order Processing T3	0.31	0.30	0.35	0.42	0.53	0.60	0.51	0.72	

Note. The correlation coefficients for the UK sample are depicted above the diagonal. The correlation coefficients for the Austrian sample are below the diagonal. All correlations (except for the correlation between symbolic comparison T3 and arithmetic T2 for the Austrian sample, $r = 0.12$) were significant, $p \leq .02$.

3.3. Predicting longitudinal development

We first tested the multigroup a priori developmental model (cf. Fig. 2). Fit indices of the model were not satisfactory, $\chi^2(36) = 204.69$, $p < .001$, CFI = 0.894, SRMR = 0.105 and RSMEA = 0.162.[1] Testing the a priori developmental model separately for each sample did not yield satisfactory fit indices either (Table 3). Thus, the a priori model was not supported by the present data.

Because model fit was not satisfactory, we applied the following model modification strategy starting from the a priori model separately for each sample (Saris et al., 2009): a) Nonsignificant paths were omitted, and b) a misspecification search was conducted in lavaan according to the following procedure:

a) Standardized path coefficients ≥ 0.10 and residual correlations ≥ 0.10 were considered as critical misspecifications. Following recommendations by Saris et al. (2009), power to detect these misspecifications was set to.75.
b) If a parameter was pointed out as misspecified and if it was causally possible (i.e., only directional paths from consecutive measurement occasions besides residual correlations were allowed), this parameter was freely estimated and the model parameters were reestimated.
c) Step b was repeated until a satisfactory model fit was obtained or no reasonable model modifications could be made according to the rationale outlined in Step b.
d) Possibly misspecified path coefficients were preferred over residual correlations in the misspecification search because the former directly indicates causal misspecifications among the study variables, whereas the latter may point to unknown third variables omitted in the model.

The best-fitting models obtained from the misspecification search are shown in Fig. 3, and fit indices are reported in Table 3. The resulting modified model for the UK sample was in many ways similar to the postulated a priori model, particularly in terms of the

[1] We also examined a multigroup model with age in months as a predictor of cognitive abilities (symbolic magnitude and order processing, and arithmetic) at T1, yielding unsatisfactory fit indices as well, $\chi^2(48) = 212.57$, $p < .001$, CFI = 0.898, SRMR = 0.098 and RSMEA = 0.139. Notably, in the sample from Austria, age in months did not significantly predict any of the cognitive abilities at T1 (*p*-values of regression weights.060 −0.674), whereas age was a significant predictor of all cognitive abilities at T1 in the sample from the UK (*p*-values of regression weights <0.001 −0.004).

Table 3
FIT Indices for the A Priori Developmental Shift Model, and the Empirically Modified Model by Country.

	χ^2 (df)	RMSEA	CFI	SRMR
Developmental				
Sample 1: UK	141.81(18), $p < .001$	0.188	0.862	0.126
Sample 2: Austria	62.89(18), $p < .001$	0.124	0.935	0.080
Modified				
Sample 1: UK	17.05(11), $p = .136$	0.048	0.994	0.030
Sample 2: Austria	19.14(16), $p = .262$	0.035	0.995	0.047

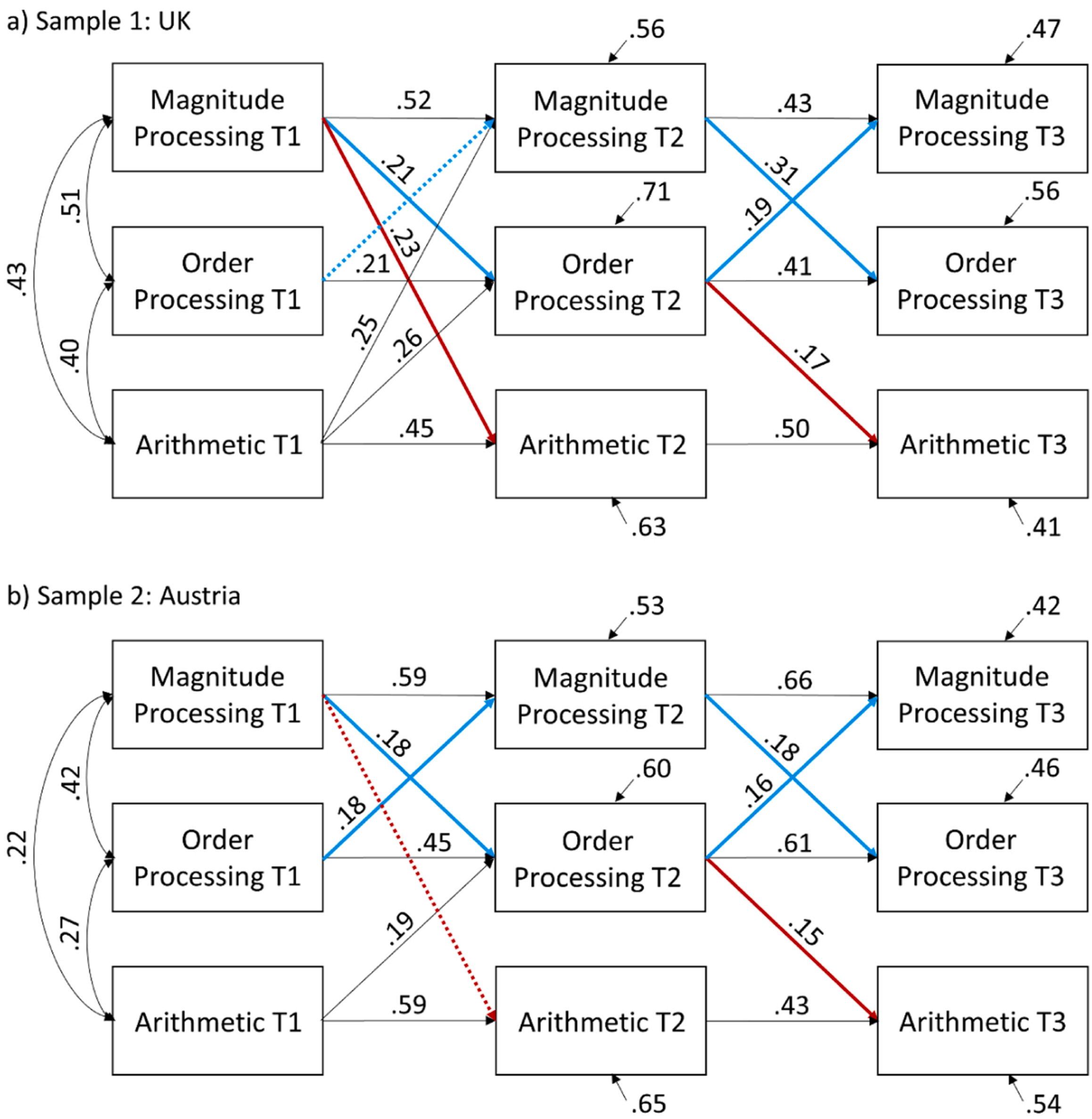

Fig. 3. Best-Fitting Models for the Sample from a) the UK and b) Austria. Note. We had assumed 1) a developmental shift in the predictors of arithmetic development: Symbolic magnitude processing in Grade 1 uniquely predicts arithmetic performance in Grade 2, whereas symbolic order processing in Grade 2 uniquely predicts arithmetic in Grade 3 (red paths), and 2) cross-lagged relations between symbolic magnitude and order processing (blue paths). Paths that were confirmed by the present data are depicted in solid and paths that were not confirmed are dotted.

predictors of arithmetic abilities: Whereas symbolic magnitude processing in Grade 1 predicted later arithmetic skills, this pattern later changed with symbolic ordering in Grade 2 predicting subsequent arithmetic performance (solid red paths). Moreover, there were cross-lagged predictive relations between symbolic magnitude and order processing (solid blue paths). There are, however, two crucial deviations from the a priori model: First, there was no cross-lagged relation between T1 and T2 symbolic magnitude and order processing (dotted blue path), as T1 order processing did not significantly predict T2 magnitude processing, Interestingly, T1 order processing was only a rather weak autoregressive predictor of subsequent order processing. Indeed, order processing in Grade 2 was predicted to a similar extent by magnitude processing. The second important contrast to the predictions of the a priori model was that the modified UK model revealed a predictive contribution of arithmetic skills at T1 to symbolic numerical processing (both magnitude and order processing) at T2.

For the sample from Austria, the predictive path from symbolic order processing at T2 to T3 arithmetic abilities was significant (solid red path), consistent with the UK sample. However, neither symbolic magnitude nor order processing contributed significantly to the prediction of arithmetic during the earlier time window between Grades 1 and 2 (dotted red path). The modified Austrian model revealed cross-lagged correlations between symbolic magnitude and order processing throughout the study period (solid blue paths). Interestingly, and similar to the UK sample, arithmetic abilities in Grade 1 predicted symbolic order processing in Grade 2.

4. Discussion

The current project investigated the developmental trajectories and cross-lagged relations of symbolic numerical abilities and arithmetic across the first three years of primary school. First, we wanted to test whether there is a developmental shift in the predictive contribution of symbolic magnitude and order processing to arithmetic performance between Grades 1 and 2. Second, we examined whether the development of symbolic numerical abilities is characterized by reciprocal predictive relations between symbolic magnitude and order processing. To date, the present study constitutes the most comprehensive examination of the direction of the predictive relations between symbolic magnitude and order processing in early mathematic development in which symbolic number skills and arithmetic abilities were assessed three times in two independent samples over a two-year period. Thus, we could determine the predictors of developmental change in symbolic magnitude and order processing, as well as arithmetic. Notably, children in the UK start school one year earlier than in Austria. As there were only few differences in the predictive patterns between the two samples with different ages of school entry, changes in the cross-lagged relations between symbolic number skills and mathematic abilities seem to be driven by formal instruction rather than children's age.

4.1. Symbolic numerical predictors of arithmetic performance

The present study critically extends the research literature on the predictors of arithmetic performance by confirming a developmental shift from symbolic magnitude to ordinal processing of number symbols in a longitudinal design.

A major finding of our study was that order processing only emerged as a unique predictor of developmental change in arithmetic in Grade 2, whereas it was not yet predictive in Grade 1. This pattern was evident in two independent samples from the UK and Austria, and our results are consistent with previous cross-sectional evidence (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & Lefevre, 2021). Our longitudinal design advances the literature by showing that symbolic ordering is not only related to concurrent arithmetic performance, but also contributes to developmental change in mathematical skills.

Explanations have been proposed as to why ordinal processing does not yet act as a unique predictor of arithmetic at the very beginning of primary school. On the one hand, ordinal processing has been connected to retrieval of higher-order association between numbers from long-term memory and thus our findings may indicate an increasing importance of retrieval-based strategies when solving calculations (Sasanguie & Vos, 2018). This seems plausible given that the acquisition of a readily available lexicon of arithmetic facts in long-term memory is a main focus of mathematics instruction in early primary school.

More recently, Xu and Lefevre (2021) proposed that ordinal processing is unrelated to arithmetic in Grade 1 because young children have not yet fully developed a network of ordinal associations between number symbols. Indeed, the development of ordinal processing still seems to be highly dynamic at the beginning of primary school: Most children in Grade 1 do not succeed in generalizing the concept of ordinality to include non-adjacent relations among numbers with an inter-item distance of more than 1, which shows that children are still in the process of extending notions of ordinality beyond count-list associations such as 1–2–3 (Hutchison et al., 2022). Evidence from atypical development further suggests that an understanding of ordinality beyond the count-list may be key for arithmetic development: Morsanyi et al. (2018) compared a group of children with dyscalculia aged 9–10 years with a typically achieving control group on a variety of numerical and non-numerical tasks. Performance on an ordinal processing task was an excellent predictor of group differences. Although not explicitly tested by the authors, the group difference in order processing performance appeared to increase with numerical distance between the constituents of the sequences (Fig. 1; Morsanyi et al., 2018). Note that we did not distinguish between sequences with different numerical distances between the numbers in this study either. Thus, future research should address the question whether order processing is especially useful for arithmetic once children have acquired an understanding of more advanced associative links between numbers such as multiples (e.g., 3–6–9). If this is indeed the case, teaching practices aimed at extending notions of ordinality beyond adjacent items should be already introduced at an early point in formal education (e.g., counting in multiples such as skip-counting in steps of three). Additionally, it is possible that children who struggle with this developmental hurdle are at an increased risk for atypical mathematical development, providing promising opportunities for diagnostic and intervention. However, more research is clearly needed to understand exactly how formal and informal learning opportunities might support the acquisition of non-adjacent ordinal associations between numbers (Hutchison et al., 2022).

In addition to learning opportunities, Hutchison et al. (2022) pointed out that interindividual differences in cognitive variables might play a role in explaining why some children are more successful than others in acquiring a more mature concept of ordinality including non-adjacent associations at the beginning of primary school. The authors proposed that children with higher inhibitory control should hypothetically be at an advantage when dealing with non-adjacent ordered sequences such as 3–6–9 by inhibiting their initial impulse that these are not in order as they do not belong to the count-list. Indeed, Xu and Lefevre (2021) recently reported a predictive contribution of inhibitory control to ordering skills in a sample of children from Grade 2. While the precise role of inhibitory control for order processing still has to be unraveled in future research, it may provide an additional theoretical explanation as to why children with good inhibitory control are at an advantage in arithmetic development.

Symbolic magnitude processing seems to play an important role for arithmetic only before ordinal processing emerges as a predictor starting in Grade 2: Extending cross-sectional results by Sasanguie and Vos (2018), we found a predictive contribution of symbolic magnitude processing in Grade 1 to future arithmetic performance in our younger UK sample. Surprisingly, this path was not significant in the older sample from Austria. To follow up on this unexpected finding, we conducted a post hoc analysis testing whether the size of the regression coefficients differed between samples, revealing no significant difference.[2] Power issues may offer an explanation as to why we did not find a significant contribution of symbolic magnitude processing at T1 to future arithmetic in the Austrian sample. However, further longitudinal studies investigating unique contributions of symbolic magnitude and order processing to the development of arithmetic at the beginning of primary school are clearly needed.

In Grade 2, symbolic magnitude processing did not act as a unique predictor of arithmetic development in either of the samples. Thus, our results support the view that children progress from accessing the relative magnitude (i.e., cardinality) of number symbols to accessing a rich network of semantic associations (i.e., ordinality) when doing arithmetic. Although these correlational findings do not inform us about the precise mechanisms, children may fundamentally rely on procedural calculation skills at the very beginning of primary school before later progressing to retrieval-based strategies (Sasanguie & Vos, 2018).

4.2. Predictors of symbolic magnitude and order processing

Another important open question concerns the interplay between symbolic magnitude and order processing in the prediction of mathematical development. Our longitudinal design allowed us to examine cross-lagged pathways between all study variables in two independent samples. We could thus test whether the development of symbolic numerical abilities is characterized by reciprocal predictive relations between symbolic magnitude and order processing.

In line with our expectations, we observed cross-lagged predictive relations between symbolic magnitude and order processing across the entire study period and in both independent samples. Thus, our results extend previous theoretical conceptions proposing a unidirectional relation, with symbolic magnitude processing acting as a developmental foundation for the acquisition of ordinal associations (e.g., Nieder, 2009). Instead, our results suggest that developing an understanding of the ordinal associations between numbers provides access to a more complex network of associations between number symbols, which in turn promotes growth in symbolic magnitude processing. In other words, more basic numerical representations are refined as children acquire higher-order representations of symbolic numbers (Vogel, 2019).

Past theoretical explanations have afforded considerable attention to the viewpoint that children's symbolic magnitude processing acts as a developmental foundation for the acquisition of more abstract ordinal associations between number symbols (Lyons & Beilock, 2011; Sommerauer et al., 2020; Vos et al., 2017). Recently, Hutchison et al. (2022) proposed that children's understanding of magnitudes in their symbolic form may help children when they learn to extend their understanding to higher-order associations beyond the count-list. However, our results similarly point towards an impact of ordinal processing on the refinement of symbolic magnitude processing. The present longitudinal study goes beyond previous cross-sectional evidence: Although previous studies have consistently documented significant associations between symbolic magnitude and order processing, the direction of this relation was unclear due to the cross-sectional nature of these studies. The current longitudinal study clearly revealed bidirectional cross-lagged predictive relations between symbolic magnitude and order processing. For instance, in both samples, symbolic ordering at T2 predicted symbolic magnitude processing at T3 over and above the contributions of symbolic magnitude processing at T2, and vice-versa. This suggests that the developmental trajectories of symbolic magnitude and order processing are strongly intertwined during early primary school. As previously proposed, both dimensions appear to closely interact when precise symbolic number representations are formed (e.g., Sury & Rubinsten, 2012). It is entirely possible that the predictive patterns may change across development. However, during a critical developmental period at the beginning of primary school in which symbolic number skills are substantially refined (e.g., Lyons et al., 2014), our results strongly suggest a reciprocal interrelation between symbolic magnitude and order processing.

Our finding of a prediction from arithmetic in Grade 1 to symbolic magnitude and order processing in Grade 2 seems particularly surprising and interesting. This finding raises the possibility that the dynamic refinement of symbolic number skills at the very beginning of primary school may be partly driven by the higher-order mathematical skills children are starting to acquire (e.g., procedural and retrieval-based calculation skills). In the current literature, symbolic magnitude and order processing are mostly considered as predictors of mathematic abilities. However, we additionally observed a contribution of arithmetic skills to the development of symbolic number skills: Grade 1 arithmetic abilities explained variance in subsequent symbolic ordering in both samples, and additionally contributed to symbolic magnitude processing in the younger UK sample. As pointed out by Sasanguie and

[2] A moderated regression conducted with the PROCESS macro (Hayes, 2013) revealed no significant interaction term between symbolic magnitude processing at T1 and sample (UK, Austria) in the prediction of arithmetic at T2, $\Delta R^2 = .002$, $b = -0.03$, $F(1351) = 1.233$, $p = .268$.

Vos (2018), arithmetic skills rely on both procedural and retrieval-based strategies at the beginning of primary school. It thus appears plausible that the dramatic increase in children's symbolic magnitude and order processing skills during this period may be partly driven by arithmetic development. Whereas the ability to compare the magnitude of two numbers may particularly benefit from procedural skills, the ability to access information about the relative position of numbers is probably scaffolded by retrieval-based mechanisms.

Vanbinst et al. (2019) proposed another explanation as to why arithmetic abilities might contribute to the acquisition of symbolic numerical skills. The authors hypothesized that formal learning of arithmetic might have an impact on the acquisition of symbolic magnitude processing: For instance, by gaining experience with arithmetic operations such as addition of natural numbers (e.g., $3 + 4 = 7$), children learn that both summands are smaller than the sum. In their longitudinal study with a small sample of 56 children, Vanbinst et al. (2019) indeed observed that arithmetic performance in Grade 1 predicted symbolic magnitude processing skills in Grade 6.

Nevertheless, note that the findings of the current study should be treated as exploratory as they only emerged as data-driven modifications of existing theoretical models. While the predictive contribution of mathematical abilities to the development of order processing was evident in both independent samples, we are not aware of other longitudinal studies investigating this relation. Thus, further longitudinal studies are needed to confirm that arithmetic skills contribute to the refinement of symbolic number processing skills during this period.

4.3. Conclusion

The current longitudinal study showed that the predictive contributions of symbolic magnitude and order processing to mathematical development change substantially across the first years of primary school. Across two independent samples, order processing emerged as a significant predictor of developmental change in mathematical abilities in Grade 2. This may be driven by a shift in the focus of mathematic instruction from procedural strategies to retrieval of arithmetic facts (Sasanguie & Vos, 2018), or by a conceptual shift enabling children to generalize the concept of ordinality beyond the count-list to include non-adjacent relations among numbers (Hutchison et al., 2022; Xu & Lefevre, 2021). Further studies are clearly needed to distinguish between the contribution of count-list associations to arithmetic development, as well as notions of numerical order beyond the count list. Results also showed a highly interactive relation between symbolic magnitude and order processing. This is consistent with emerging evidence suggesting that the development of numerical skills and arithmetic is not unidirectional, but a highly dynamic process in which different dimensions interact to shape our understanding of numbers and their associations. Overall, the present findings may help to refine theories of arithmetic development at the beginning of primary school.

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CRediT authorship contribution statement

Sabrina Finke: Data curation, Formal analysis, Investigation, Visualization, Writing – original draft. **Stephan E. Vogel**: Methodology, Writing – review & editing. **H. Harald Freudenthaler**: Formal analysis, Writing – review & editing. **Chiara Banfi**: Data curation, Investigation, Writing – review & editing. **Anna F. Steiner**: Data curation, Investigation, Writing – review & editing. **Ferenc Kemény**: Writing – review & editing. **Silke M. Göbel**: Conceptualization, Funding acquisition, Methodology, Project administration, Supervision, Writing – review & editing. **Karin Landerl**: Conceptualization, Funding acquisition, Methodology, Project administration, Supervision, Writing – original draft.

Declarations of interest

None.

Data availability

The data analyzed in this study, as well as detailed project and task descriptions are available at https://reshare.ukdataservice.ac.uk/854398/.

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