A Low-Cost Cross-Correlation Residual Phase Noise Measurement System & Efficient Digital Signal Processing Techniques

Luke Dummott
School of Physics, Engineering and Technology
University of York
York, United Kingdom
luke.dummott@york.ac.uk

Simon Bale
School of Physics, Engineering and Technology
University of York

Jeremy Everard
School of Physics, Engineering and Technology
University of York

***Abstract*— This paper describes a residual phase noise measurement system using cross-correlation techniques that is built from relatively low-cost and readily available components and exhibits a system noise floor of around -177 dBc/Hz when measuring a device operating at 0 dBm output power. A software library for the digital signal processing of residual noise measurements has been produced, which will be provided open-source.**

Keywords*— *cross correlation, residual phase noise, phase noise measurement

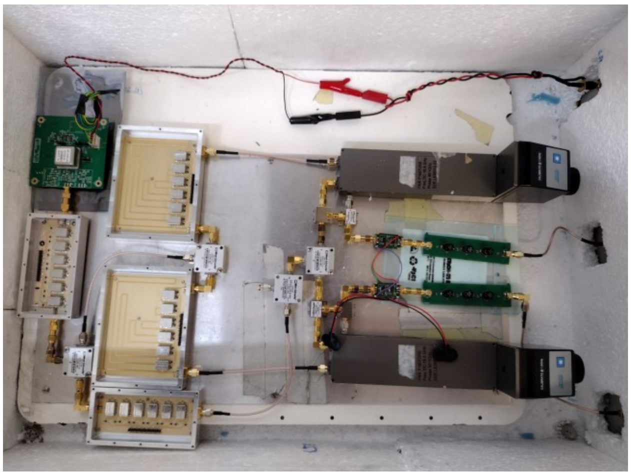

Fig. 1. Photo of the cross-correlation residual noise measurement system

I. Introduction

When designing electronic devices to be used in the production of oscillators with exceptional phase noise performance, it is important to be able accurately measure their residual phase noise characteristics. This becomes increasingly difficult, especially at lower operating power levels, due to the internal noise limitations of residual phase noise measurement systems

Cross-correlation techniques can be used as a way to suppress the internal noise floor of residual phase noise measurement systems [1]. When combined with efficient digital signal processing algorithms, it becomes possible to perform accurate residual phase noise measurements of very low noise devices in a reasonable amount of time.

This research is based on previous work done at York to build a cross-correlation residual noise measurement system [2], but here the system is designed to be low-cost and operate at 100 MHz with a DUT output power of around 0 dBm for the purpose of measuring amplifiers to be used in an ultra-low phase noise 100 MHz crystal oscillator. The digital signal processing aspects of the system have also been greatly expanded upon to reduce measurement times.

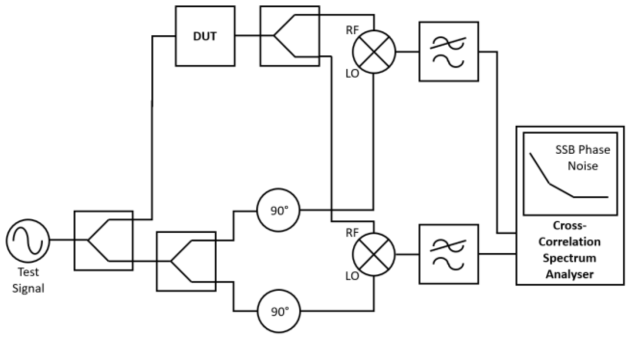

Fig. 2. Simplified block diagram of a cross-correlation residual phase noise measurement

II. The Cross-Correlation Method

A simplified block diagram of a cross-correlation residual phase noise measurement system is shown in Fig.2. Residual phase noise of a DUT (device under test) can be determined by measuring the spectrum of phase variation between a reference signal and the same signal after it has been passed through the DUT.

To measure the phase variation, a frequency mixer with RF and LO ports in quadrature is used as a phase detector. The baseband part of the output of the mixer corresponds to the variation in phase between the RF and LO ports and the high frequency part is removed using a suitable low pass filter.

A cross-correlation system splits the signals from the reference signal and the DUT output so that the measurement is performed in two separate channels. The phase noise of the DUT will be present as correlated noise between both channels. A cross-correlation spectrum analyser can then be used to determine then single-sideband residual phase noise.

A. Derivation

In the model described by Rubiola & Vernotte [3] the time domain signals captured at the input to the cross-correlation spectrum analyser, $x(t)$ and $y(t)$, can be considered to consist of a correlated component, representing the noise due to the DUT; and an uncorrelated component, representing the system noise in each channel. This be can written as:

$$x(t) = a(t) + c(t)$$
$$y(t) = b(t) + c(t) \quad (1)$$

Where $x(t)$ and $y(t)$ represent the uncorrelated system noise and $x(t)$ represents the correlated DUT noise.

The Fourier transform can be applied to give the frequency spectrum of each channel:

$$X(f) = A(f) + C(f)$$
$$Y(f) = B(f) + C(f) \quad (2)$$

The cross-spectrum of the two channels can then be calculated by performing the cross product of the frequency spectrum of one channel with the complex conjugate of the other and taking the average over M measurements. This is converted to power spectral density by multiplying by the measurement time, T, and dividing by the sampling frequency, f_s:

$$\begin{aligned} S_{XY} &= \frac{T}{f_s}\cdot\frac{1}{M}\sum_{m=1}^{M}[X_m Y_m^*] \\ &= \frac{T}{f_s}\cdot\frac{1}{M}\sum_{m=1}^{M}[(A_m + C_m)\times(B_m + C_m)^*] \\ &= \frac{T}{f_s}\cdot\frac{1}{M}\sum_{m=1}^{M}[(A_m B_m^*) + (A_m C_m^*) \\ &\qquad + (C_m B_m^*) + (C_m C_m^*)] \end{aligned} \quad (3)$$

The cross-spectrum of two perfectly correlated signals will be unchanged, while the cross-spectrum of two uncorrelated signals will be reduced in magnitude. In (3) it can be seen that the uncorrelated terms will decrease as the number of measurements increases, leaving just the $(C_m C_m^*)$ term, i.e. the equation becomes approximately equal to the power spectral density of the DUT noise.

It is shown in [3] that the uncorrelated terms which represent the system noise are suppressed by a ratio of $\sqrt{m}$. In other words, the cross-correlation method can be used to suppress the noise floor of the system along with the number of correlations performed at a rate of 5 dB per multiple of 10 correlations.

III. System Overview

A block diagram of the hardware part of the cross-correlation residual phase noise measurement system is shown in Fig. 2. and the various aspects of the system are described in detail in the following sections.

A. Reference Oscillator

It is important that the reference oscillator has as low AM ans PM noise as possible, so as not to limit the internal noise floor of the system. A KVG O-40-ULPN-100M oven-controlled crystal oscillator [4] was selected for the reference oscillator as it exhibits very low phase noise: around -138 dBc/Hz at a 100 Hz offset with a far-out noise floor of around -185 dBc/Hz. It is also capable of providing enough output power to saturate the LO ports of the mixers.

B. Quadrature Phase Shift

The required 90° phase shift between the RF and LO ports is achieved using a combination of 6-bit 180° digitally-controlled phase shifter and a 9° mechanical phase shifter.

The digitally-controlled phase shifter was originally built for use in ultra-low phase noise 100 MHz crystal oscillators. It uses latching RF relays to switch different lengths of microstrip transmission line into the signal path to create a phase shifter that can be adjusted linearly according to a digital control code. It is a completely passive device that was specifically designed for very low residual noise.

Once the digitally-controlled phase shifter has been used to set the phase shift to roughly 90°, the mechanical phase shifter is used to fine tune to the phase shift so that DC output of the mixers is as close to 0 V as possible.

C. DUT Power Level

A digitally-controlled attenuator with a range of 31.5 dB and a resolution of 0.5 dB is used to set the source power to the DUT, so that residual phase noise of the device can be measured at the desired operating conditions. This is a similar design to the digitally-controlled phase shifter, but here the microstrip delay line elements are replaced with resistive attenuator networks.

D. Mixers

To optimise the system for the desired DUT output level of 0 dBm, a low-power mixer was selected: the Mini Circuits ZX05-1L-S+ [5]. This mixer is designed to operate with an LO power of 3 dBm. Using such a low power mixer allows the RF and LO ports to be as close to saturation as possible to achieve the best performance even when measuring low-power device and also removes the need for additional amplifiers at the inputs to the mixer.

E. Signal Capture

A 16-bit PicoScope 4262 USB oscilloscope [6] is used to capture the time-domain signals so that they can be transferred to a PC for processing using the techniques described in later sections. It has two input channels that can capture samples simultaneously and is operated at a sampling frequency of 1 MHz.

F. Signal Conditioning

To provide the required baseband phase noise signals to the PicoScope, the high frequency part of the mixer output signals, i.e. the sum of the RF and LO frequencies, must be removed. This is achieved using a 10 MHz diplexer with high frequency output terminated in 50 Ω, so that it can be removed from the signal without unwanted reflections, and low frequency output connected to the rest of the baseband circuitry.

The amplitude of the output from each mixer is too low to be precisely measured directly with the PicoScope, so two-stage LT6018 operational amplifier circuits with a total voltage gain of 900 (around 59 dB) are used to increase the signal level.

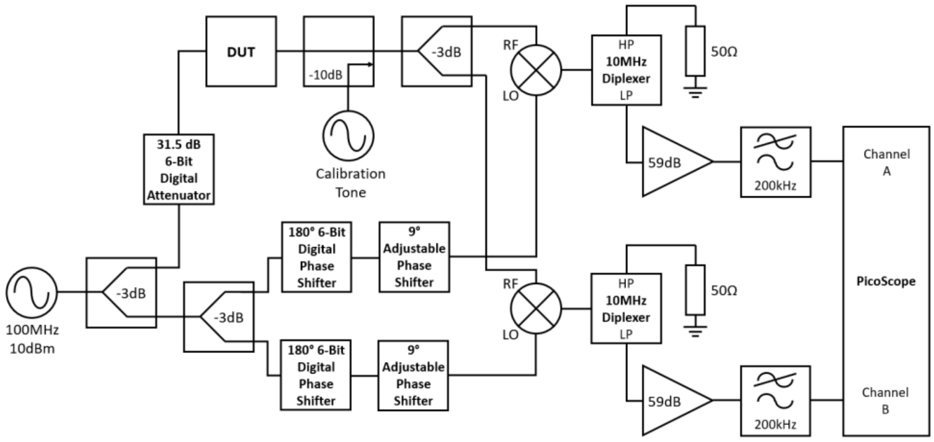

Fig. 3. Block diagram of the cross-correlation residual phase noise measurement system

An anti-aliasing filter is used at the input to the PicoScope to filter out any frequencies above the Nyquist frequency (half the sampling rate) prior to be being sampled by the PicoScope, as these frequencies will be aliased into the measured noise spectral density. It is a 5th-order Butterworth low-pass LC filter with a cutoff frequency of around 250 kHz and was designed as part of previous work at York on phase noise measurement instrumentation [7].

G. Calibration

It is necessary to calibrate the system so that the noise power spectral density measured at the input to the PicoScope can be referenced to the carrier signal power to produce a residual phase noise measurement in dBc/Hz. This is achieved by injecting a tone into the system through a 10 dB coupler at the output of the DUT of a known offset frequency and power level in relation to the carrier signal. By observing the amplitude of the peak due to the calibration tone on the PicoScope, a reference can be obtained to convert the absolute noise spectral density measured on the spectrum analyser in dBV/Hz to dBc/Hz. As shown in [2], the calibrated single-sideband phase noise, $L(f)$, can be calculated from:

$$L(f) = S_\varphi + K_1 - K_2 - 6\ dB \tag{4}$$

Where S_φ is the measured double-sideband power spectral density, K_1 is the power ratio of the carrier signal to the injected spur and K_2 is the amplitude of the injected spur.

IV. Digital Signal Processing Techniques

The primary purpose of the software that handles the DSP (digital signal processing) for the system is to convert captured time-domain domain voltage samples at the inputs to the capture device to a cross-correlated power spectral density of the DUT residual phase noise. This involves performing a FFT (fast Fourier transform) for each channel and calculating the cross spectrum.

The secondary purpose of the software is to reduce the amount of time it takes to obtain useful measurements by utilising a frequency-banding algorithm that adjusts the resolution bandwidth versus number of correlations for different ranges, which allows for a higher degree of internal noise floor suppression at high offset frequencies whilst maintaining a narrow resolution bandwidth close-to-carrier. Frequency banding techniques are used in commercially-available residual phase noise measurement systems, such as the R&S FSWP, although their implementation is not described in detail.

To allow residual phase noise measurements to be performed in real-time and on inexpensive hardware, a C++ library of efficient DSP algorithms has been produced, which will be provided open-source and explained in detail in further work. A brief description of the techniques used and their effect on processing time is given in the following sections.

A. Frequency Banding Using Variable FFT Window Size

Frequency banding is performed by splitting a block of captured samples into sets of different sizes for each frequency band. An FFT is performed on each set and the number of correlations performed at each frequency of a measurement is equal to the number of sets in the sample block. At lower offset frequencies, the data is split into fewer sets with a larger FFT window, so the results can have a narrow resolution bandwidth at the expense of reduced noise floor suppression due to fewer correlations. For higher offset frequencies, the data is split into more sets of a smaller window size, so a lower noise floor can be achieved at the expense of resolution bandwidth. Fig.4. shows a residual phase noise measurement of the baseband front end part of the system (amplifier, filter & PicoScope). The effect of the frequency banding can be seen in the cross-correlation measurement; the resolution bandwidth decreases with each subsequent band, but the noise floor suppression increases.

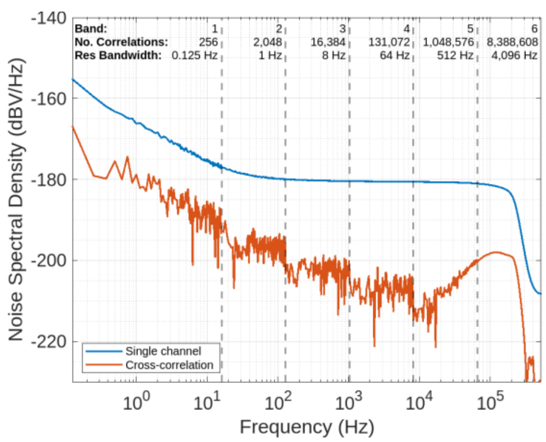

Fig. 4. Comparison of system front end noise floor (only amplifier, filter & PicoScope) for single channel and cross-correlation measurements

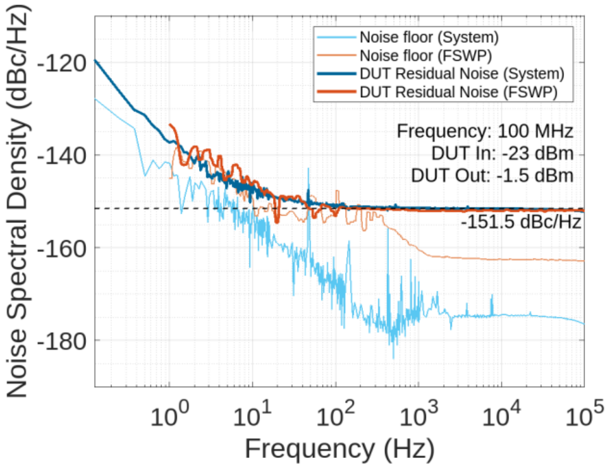

Fig. 5. Comparison the residual noise of a MAR-6+ amplifier when measured using this system and an R&S FSWP

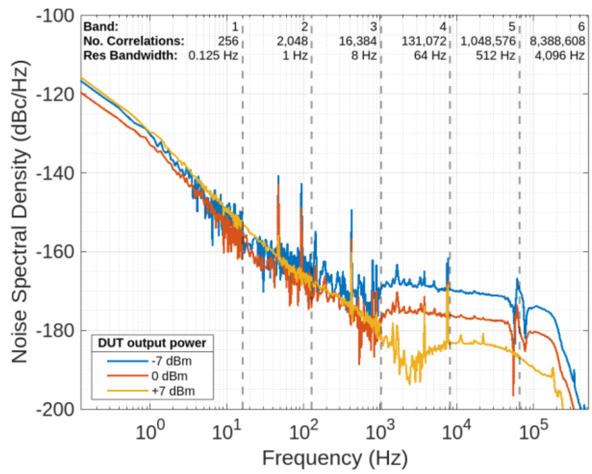

Fig. 6. Comparison of the system noise floor for various DUT output power levels

B. Optimising the FFT for Frequency Banded Data

The implementation of the frequency banding means that much of the output data from each Fourier transform performed in a block when using a standard FFT algorithm remains unused, i.e. only a certain frequency range of frequency points are required for each band, which could be much less than the size of the FFT. An optimised FFT algorithm is implemented that is much more efficient when the required output data size is less than the input data size.

C. Processing Time

It has been shown that using the described techniques, the processing time of frequency banded residual noise measurements is comparable to conventional fixed resolution bandwidth techniques even on inexpensive hardware. These techniques have been used to perform a measurement with sub-Hz close-to-carrier resolution bandwidth and over 3,000,000 cross-correlations at high offset frequencies in less than 15 minutes.

V. Measurements

A. Verification and Comparison to Existing Systems

To confirm the accuracy of the system, a measurement was made using the Mini Circuits MAR-6+ amplifier as the DUT. This amplifier has a gain of around 22 dB at 100 MHz and it was operated at an input power of -23 dBm. The same measurement was performed using a commercially-available R&S FSWP [8] measurement system and the results are compared in Fig.5. The internal noise floor of each system was also measured by replacing the amplifier with a straight through connection and setting the power level at this point to the output power of the amplifier. Each system was set to run for around 40 minutes with similar resolution bandwidths, though the difference in implementation of frequency banding means data points are not directly comparable.

It can be seen the measured residual phase noise was almost identical for each system, though comparisons cannot easily be made at frequencies below 1 kHz as the amplifier noise is not sufficiently above the noise floor of the FSWP. Despite its comparatively low cost, at these operating conditions the internal noise floor of this system is significantly lower than that of the FSWP; around 12 dB at high offset frequencies.

B. System Noise Floor

The internal noise floor of the system was measured for various DUT output powers as shown in Fig.6. The minimum noise floor was measured for a DUT output power of 7 dBm, when both ports of the mixers were saturated at around 3 dBm. Here, the far-out noise floor was less than -180 dBc/Hz.

VI. Conclusions

A low-cost cross-correlation residual noise measure operating at low power levels has been produced that has an internal noise floor lower than that of the commercially-available R&S FSWP. A set of digital signal processing algorithms for residual phase noise measurements has been produced that allows for measurements to be performed efficiently, in real-time and on inexpensive hardware.

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