A Statistical Damage Constitutive Model for Graded Gravels Incorporating Degree of Compaction and Damage-Softening Index

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Abstract

Strength characteristics of graded gravels are essential in the construction of roadway and railway substructures. Traditional constitutive models, primarily nonlinear elastic and plastic types, fall short in accurately capturing the strain-softening properties of such materials. To address this limitation, the current study introduces a statistical damage model designed to outline the stress-strain behavior of densely-compacted graded gravels in transport infrastructures. Utilizing medium-sized triaxial tests, the model examines variations in strength and deformation parameters in relation to compaction levels and incorporates a unique Damage-Softening Index (DSI) along with a threshold axial strain to improve accuracy. The study establishes that the DSI and threshold axial strain effectively regulate stress-strain relations in the post-peak segment, the model's statistical parameters and threshold axial strain can be precisely determined through the introduction of DSI, and the model closely aligns with experimental data across multiple compaction levels. These findings are especially relevant for engineering design in the context of roadway and railway construction and indicate potential for further refinement, such as the incorporation of loading rate considerations.

Keywords: Statistical damage-softening model; graded gravel; degree of compaction; damage variable; damage-softening index; stress-strain relation

1. Introduction

Graded gravels serve as a foundational material in the construction of pavement base and subbase layers, as well as in high-speed railway subgrades (Hicks et al. 1971; Rahman et al. 2015; NRAPRC 2014; Gobel and Lieberenz 2009; Luo et al. 2020; Luo et al. 2019; Wang et al. 2021a). These layers are essential for supporting transportation pathways. With rising transportation demands, these gravel layers face increasing stress from heavier traffic volumes. Consequently, controlling the degree of compaction (DoC) during construction becomes imperative. Engineering design often relies on static tests due to their relative simplicity and empirical relationships between static and dynamic strength (TB10001-2016). Also, the static strength or stiffness is usually adopted to predict dynamic behavior (Lentz and Baladi, 1981). As a pivotal parameter for graded gravels in geotechnical engineering, DoC can significantly impact the strength, stiffness, as well as dynamic responses. Thus, exploring the quantitative mechanisms by which DoC influences the static properties is essential.

Barksdale and Itani (1989) observed a significant increase in the elastic modulus of gravel samples under low-stress conditions when initial density was high; this effect diminished under high confining pressure. Chen and Zhang (2016) performed large-scale triaxial tests on gravel samples at varying relative densities under low confining pressures. Their findings showed that higher relative density led to increased volume dilatancy, shear strength, elastic modulus, and dilatancy angle. Similarly, Yang et al. (2022) suggested that greater relative density in granular materials enhances the interlocking effects between gravel particles. They found that peak shear strength correlates positively with the degree of compaction (DoC), whereas residual shear strength shows little sensitivity to DoC variations. Multiple tests indicate that densely compacted gravel exhibits strain-softening characteristics at low confining pressures. Given these findings, establishing the stress-strain relationship for graded gravels at different DoC levels is essential, particularly in understanding their softening behavior.

Constitutive models for soils and rocks are often macroscopic or phenomenological, establishing direct relationships between macroscopic variables like stress and strain. These models typically rely on elastoplastic theory, with nonlinear elastic and plastic models as key examples

(Duncan and Chang 1970; Saboya and Byrne 1993; Daouadji and Hicher 2010; Liu and Zou 2013; Liu and Gao 2017; Tennakoon et al. 2015). While the nonlinear elastic approach effectively portrays stress-strain relationships in soils, it falls short in capturing softening properties. Plastic models, evolving from strain hardening theories, also inadequately address soil structure damage and softening. To simulate strain softening in densely-compacted soils, the introduction of damage mechanics theory proves beneficial. This approach led to the development of the statistical damage model, initially employed for rock and concrete softening behaviors (Li et al. 2012; Cheng et al. 2021; Hou et al. 2022). The model, adapted to Weibull distribution, effectively characterizes sandstone with varying porosity (Pan et al., 2020). It also captures macroscopic and mesoscopic flaws in rock masses (Liu et al., 2015). Further, the model's applicability extends to high-strength concrete, validated by test results for C60 and C70 concrete (Zhang et al., 2021). Beyond rock and concrete, the model proves effective for frozen soils and coarse-grained materials subjected to freeze-thaw cycles.

Lai et al. (2009) performed triaxial tests to show that frozen sandy soil exhibits cross-anisotropic damage. To capture this, researchers investigated cross-anisotropic damage variables and developed an elastoplastic damage constitutive model. This model simulates the softening that occurs in the post-peak segment of the stress-strain relationship. Similarly, Sun et al. (2020) introduced a damage variable and bond strength parameter into an elastoplastic damage constitutive model for frozen sandy soil. This inclusion accounts for the effects of micro-cracking on the soil, enabling predictions of stress-strain relations under negative temperatures (Li et al. 2019).

Further studies have extended the statistical damage model to examine the stress-strain behavior of coarse-grained soils under freeze-thaw cycles (Ling et al. 2020; Li et al. 2021). Researchers contend that defects in these soils result from both loading and freeze-thaw cycles. To quantify this, they employed the Weibull distribution to characterize the strength of mesoscopic elements, thereby determining the damage variable. Consequently, they developed a damage-based constitutive model that is applicable to coarse-grained soils subjected to freeze-thaw cycles.

Based on the foregoing discussion, statistical damage constitutive models have proven effective not only for rock and concrete but also for fine-grained and coarse-grained soils. Despite this, limited research has addressed the applicability of such models to graded gravels. Traditional nonlinear and plastic models reveal considerable limitations, especially in capturing strain-softening properties and correlating these with soil structure damage. These shortcomings necessitate the development of a statistical damage constitutive model tailored for graded gravels with varying Degrees of Compaction (DoC).

To create this model, the present study conducts medium-sized triaxial tests on graded gravels featuring different DoC levels. Shearing velocity is maintained at a low rate of 0.01 mm/min to minimize the development of pore pressure, thus preventing its impact on the softening properties of the gravels. Subsequently, the deformation mechanisms occurring throughout the loading process of graded gravels are analyzed. This is accomplished by plotting the damage variable against axial strain.

The stress-strain relation in graded gravels is bifurcated into two stages: densification strengthening and shear damage. A statistical constitutive model is then developed to predict this relationship across different DoC levels. Model parameters are determined based on the Damage-Softening Index (DSI), which effectively captures the strain-softening characteristics of graded gravels. The newly developed model aims to accurately forecast the strength and deformation properties of graded gravels, thereby offering theoretical guidance for the construction of roadways and railways.

2. Materials and Methods

A series of triaxial tests were performed on the gravel samples with different DoC under varied σ_3. The laboratory tests were intended to investigate the critical physical properties of gravels in terms of DoC and σ_3, which will provide evidence for the determination of the physical parameters in the damage-softening constitutive model.

2.1 Graded Gravel

Graded gravels were created by crushing boulders and pebbles and washing the resulting particles with water to remove fine particles. They were then oven-dried and divided into 8 groups based on their particle sizes, as shown in Fig. 1. The eight groups were mixed in appropriate proportions to produce the graded gravel materials with the required gradation for constructing subgrades of high-speed railways. Their grain size distributions are depicted in Fig. 2.

2.2 Sample Preparation

The Modified Proctor compaction test (ASTM D1557-12) provided the maximum dry density and optimum water content (OWC) for the gravel materials. A series of triaxial tests examined the softening properties of these materials at different Degrees of Compaction (DoC). To enhance test precision, the diameter and height of triaxial samples were set at 150 mm and 300 mm, respectively. Gravel materials were oven-dried for 24 hours and weighed to prepare samples at DoCs of 0.9, 0.95, and 1.0. Subsequently, de-aired water mixed with the materials achieved the OWC state. Samples were then stored in a humidor for 24 hours to equalize moisture content.

For sample preparation, gravel materials at OWC were partitioned into five equal parts, ensuring uniform layer height and weight. Samples underwent compaction in a three-way split casing. During this process, each partition was compacted using a Proctor hammer, with consistent initial height set for each hammer drop. The top surface of each layer was scraped to a 2-mm depth to promote interlocking with adjacent layers (Cao et al. 2017; Cao et al. 2018). Although this method could potentially induce depth-wise nonuniformity in samples—due to varying compaction energy between lower and upper layers—the resulting error is considered negligible as it equally affects each sample. Following compaction, each sample was enclosed in a rubber membrane, and silicone grease was applied to the top and bottom surfaces to reduce friction against the apparatus caps.

2.3 Testing Protocol

The triaxial test apparatus (Fig. 3) was designed and manufactured by GDS Instruments, Ltd. Following the recommendations of AASHTO T307-99 and GB T 50123-2019, the triaxial gravel samples had a diameter of 150 mm and a height of 300 mm to ensure a sample-to-particle size ratio of at least 5 (AASHTO 2007; MHUDPRC 2019). Consequently, the confining pressure cell had to be large enough to accommodate such a triaxial sample. Additionally, an infinite volume controller was connected to two confining pressures so that sufficient de-aired water can be alternatively provided to maintain a constant confining pressure in the large pressure cell throughout the test process.

The consolidated-drained monotonic triaxial test was performed to obtain the stress-strain relations of the graded gravel samples. The shearing velocity was set as 0.01 mm/min to minimize the pore pressure in the loading stage. Such a low shearing velocity was selected to avoid the inhibiting effect of negative pore pressures on the softening properties of the graded gravel. The shearing process was stopped once ε_1 of the gravel samples reached 10%. Since gravel is usually adopted to construct functional layers that are buried relatively shallow, the stress-strain relations of the granular materials were measured on the condition that σ_3 was 20 kPa, 40 kPa and 60 kPa respectively.

3. Experimental Results

3.1 Stress-Strain Behavior

The stress-strain relations for the graded gravel samples with different DoC and σ_3 are presented in Figure 4. All samples exhibited strain softening and volume expansion. q reached a peak value before declining rapidly with increasing ε_1, indicating significant strain softening properties. Although the triaxial samples contracted mildly during the initial loading stage, they expanded gradually until reaching a residual state.

In Fig. 4(a), the stress-strain relations and volume change properties of the graded gravel samples with DoC of 0.90, 0.95, and 1.00 are shown under a σ_3 of 20 kPa. Initially, an increase in q is observed, indicating densification strengthening. However, as ε_1 increases, the increase in q slows down, and a decreasing trend for q can be observed, indicating that the sample enters the shear damage stage. ε_1 corresponding to q_p is relatively small, ranging from 0.9% to 1.6%. The subsequent shearing process exhibits clear strain-softening behavior for all samples. q_p of the graded gravel samples significantly increases from 215 kPa to 680 kPa as DoC increases from 0.90 to 1.00, and # q_r also increases from 108 kPa to 156 kPa with an increase in DoC. Figure 4(a) also shows the relationship between ε_v and ε_1. For samples with different DoC, the transformation phase from contraction to expansion is obtained when ε_1 approximately equals 1%. Gradual increases in both strain softening and volume expansion properties are also observed with an increase in DoC.

Figures 4(b) and 4(c) display the stress-strain relations and volume change properties of the gravel samples under σ_3 of 40 kPa and 60 kPa, respectively. The samples demonstrate similar strain softening and volume expansion properties under all three confining pressures. q_p and q_r of the gravel samples with a given DoC increase significantly with σ_3. For instance, for gravel samples with DoC=1.0, q_p rises from 680 kPa to 995 kPa as σ_3 increases from 20 kPa to 60 kPa, and q_r also increases from 156 kPa to 277 kPa. Additionally, the lower σ_3 results in more noticeable strain softening and volume expansion behaviors for the same DoC. The volume expansion of the gravel samples is positively correlated with DoC but negatively correlated with σ_3, which may be due to the tightening effect of confining pressure.

3.2 Secant Modulus of Elasticity

Earlier research (e.g. Byrne et al. 1987; Sawangsuriya et al. 2003) frequently utilized the initial elastic modulus and reloading modulus to assess the deformation resistance of soils. In contrast, this study employs the secant modulus of elasticity (E_e) to examine the deformation resistance of gravel samples and to formulate a predictive model for their softening properties. E_e for soils is commonly assessed by the secant slope of the stress-strain curve within 1.0% of axial strain (ε_1) (Tang et al. 2018; Li et al. 2015). For the graded gravel samples examined here, the axial strain at peak stress ($\varepsilon_{1,p}$) occurs around 1.0%, indicating that these samples transition to a plastic state before ε_1 reaches 1.0%. Given that the linear elasticity portion of the stress-strain curve should be observed prior to the peak shear stress, E_e for the gravel samples is depicted in Fig. 5 and defined by the subsequent equation:

$$E_e = \frac{\Delta q}{\Delta \varepsilon_1} = \frac{q_{0.5\varepsilon_{1,p}} - q_{int.}}{0.5\varepsilon_{1,p} - \varepsilon_{int.}} \qquad (1)$$

In Fig. 6, the relationship between E_e of the graded gravels versus DoC and σ_3 is presented. The results indicate that E_e of the gravel samples increases with both DoC and σ_3. When σ_3 is held at 20 kPa, increasing the DoC from 0.9 to 1.0 results in an increase in E_e from 33.2 MPa to 71.6 MPa. Additionally, increasing σ_3 from 20 kPa to 60 kPa results in a 35.5% increase in E_e for gravel samples with a DoC of 0.9, from 33.2 MPa to 45.0 MPa. These findings highlight the positive influence of increasing both DoC and σ_3 on E_e of the graded gravels.

3.3 Shear Strength

Fig. 7 displays q_p and q_r of the graded gravel samples in response to varying DoC and σ_3. In Fig. 7(a), q_p of the gravel samples under σ_3 = 20 kPa increase with DoC, from 215 kPa to 680 kPa.

Similarly, when σ_3 increases to 40 kPa and 60 kPa, q_p increases to 295 kPa, 605 kPa, 855 kPa, 395 kPa, 713 kPa, and 995 kPa, respectively. Compared to those under σ_3 = 20 kPa, q_p increases by 37.2%, 57.6%, and 25.7%, respectively. Additionally, Fig. 7(b) presents the variation of q_r against DoC and σ_3. The results demonstrate a significant increase in q_r with increasing DoC, with values of 108.4 kPa, 116.4 kPa, and 144.0 kPa for DoC values of 0.9, 0.95, and 1.0, respectively, under σ_3 = 20 kPa. When σ_3 = 60 kPa, q_r rises to 226.8 kPa, 267.3 kPa, and 277 kPa, showing an increase of 109.2%, 129.6%, and 92.4%, respectively, compared to those under σ_3 = 20 kPa. These findings highlight the significant influence of both DoC and σ_3 on q_p and q_r for the graded gravel samples.

In granular materials, strain softening refers to a reduction in resistance that occurs during continuous shearing after reaching the peak resistance (Chu et al. 2012; Chu et al. 1997). To investigate this behavior in the gravel samples, we used I_s, which can be expressed by the following equation (Consoli et al. 1998; Wang et al. 2021b):

$$I_s = \frac{q_p - q_r}{q_p} \tag{2}$$

Figure 8 displays how I_s changes with DoC and σ_3. The gravel sample with DoC=1.0 exhibits the highest I_s value of 0.79 for σ_3 = 20 kPa, indicating that the most densely compacted sample exhibits the most significant strain-softening behavior at the lowest σ_3. In contrast, the sample with DoC = 0.9 exhibits the lowest I_s value at a σ_3 of 60 kPa. Notably, σ_3 can suppress the softening behavior of gravel samples.

4. Development of Statistical Damage Constitutive Model

In this section, a statistical damage constitutive model is established for the gravels with differing DoC in terms of evolution of damage. The method to determine model parameters are also proposed with DSI taken into consideration.

4.1 General Concepts of Damage

Kachanov's damage concept was originally proposed to study the creep properties of certain metals under one-dimensional conditions (Kachanov 1967). Later, this theory was extended to many other materials. It has been verified that the damage concepts can describe the strain-softening behavior of materials such as rock and concrete (Li et al. 2012; Dragon and Mroz 1979; Lemaitre 1985; Frantziskonis and Desai 1987; Mazars and Pijaudier-Cabot 1989). Additionally, damage concepts were applied to explain the stress-strain properties of coarse-grained material samples subjected to freeze-thaw cycles (Ling et al. 2020). Therefore, this paper uses damage mechanics concepts to investigate the damage-softening behaviors of gravel materials.

The concepts of D and σ_i^* (i = 1,2,3) in damage theory are necessary to be illustrated in detail to conveniently carry out the work in this paper. The initial cross-section of a material body is assumed to be A. During the loading period, internal defects (e.g. cracks, voids and joints) emerge gradually and develop. The area of cross-section with defects after occurrence of damage is denoted as A'', which is unable to bear external load. Therefore the net area excluding the area with defects is $A'=A-A''$. Then, the state of damage can be characterized by a measure of defects in a whole cross-sectional area and expressed as $D=A''/A$, where D ranges from 0 to 1. D is determined by the intact area and the damaged part of a material body. If D=0, it represents that the material body does not have any damage, while D = 1.0 indicates that the sample is completely damaged. Suppose that the applied stress (or apparent stress) that acts on the surface of a material body is σ_i (i=1,2,3), and the net stress (or true stress) acting on the net area of the undamaged part is σ_i^*. Thus, the external load that acts on the material body can be expressed as $T=\sigma_i A$. Since T can only be supported by the net area of the undamaged portion, it can also be given as $T=\sigma_i^* A'$, yielding:

$$\sigma_i A = \sigma_i^* A' \tag{3}$$

or

$$\sigma_i A = \sigma_i^* (A - A'') \tag{4}$$

By dividing with A on both sides of Eq. (4), we obtain

$$\sigma_i = \sigma_i^* (1 - D)(i = 1,2,3). \tag{5}$$

The subscript of the notation σ_i or σ_i^* taken as 1, 2, 3 represents the major, intermediate and minor principal stress, which are signified as σ_1, σ_2 and σ_3. Eq. (5), as a widely recognized expression of damage relationship, demonstrates the relationship between σ_i and σ_i^*. This implies that σ_i and σ_i^* can be transformed to each other through Eq. (5), which characterizes the damage evolution.

In a triaxial test, the apparent q can be expressed by $\sigma_1-\sigma_3$. Similarly, the net q can be denoted as $\sigma_1^*-\sigma_3^*$. Therefore, the relationship between apparent and true q can be expressed by:

$$\sigma_1 - \sigma_3 = (\sigma_1^* - \sigma_3^*)(1 - D) \tag{6}$$

The initial closure of voids and cracks of gravel samples is not considered (Daouadji and Hicher 2010). The relationship between q and ε_1 of gravel samples is linear before yielding (as shown in Fig. 4). From the definition of E_e, we can obtain:

$$\sigma_1^* - \sigma_3^* = E_e \varepsilon_1^* \tag{7}$$

Substituting Eq. (7) into Eq. (6) yields

$$\sigma_1 - \sigma_3 = E_e \varepsilon_1^* (1 - D) \tag{8}$$

Based on the hypothesis of strain equivalence, which states that ε_1 of the material body induced by the apparent stress equals ε_1^* of the undamaged part of the material body (Li et al. 2012; Lemaitre 1985; Lemaitre and Chaboche 1990), yields

$$\varepsilon_1 = \varepsilon_1^* \tag{9}$$

Substituting Eq. (9) into Eq. (8) yields:

$$\sigma_1 - \sigma_3 = E_e \varepsilon_1 (1 - D) \tag{10}$$

The residual state of the gravel samples cannot be reflected by Eq. (10) as $D = 1$ leads to $(\sigma_1 - \sigma_3) = 0$. However, there is an obvious residual stress demonstrated in the stress-strain relations of gravel samples, as shown in Fig. 4. To depict the whole stress-strain relations more precisely, Eq. (10) is modified to characterize the residual state of gravel samples, expressed by (Wang et al. 2018):

$$\sigma_1 - \sigma_3 = E_e \varepsilon_1 (1 - D) + (\sigma_1 - \sigma_3)_r D \tag{11}$$

Replacing $\sigma_1 - \sigma_3$ and $(\sigma_1 - \sigma_3)_r$ into q and q_r, an expression that can demonstrate the stress-strain relationship is:

$$q = E_e \varepsilon_1 (1 - D) + q_r D \tag{12}$$

In Eq. (12), $D = 0$ represents a gravel sample that does not have any damage, whereas $D = 1.0$ indicates gravel sample that is completely damaged. D can describe the microstructural changes of gravel samples induced by an external load.

4.2 Damage Evolution

The expression of D, $D = A''/A$ (or $D = (A - A')/A$), illustrates the evolution of damage to some extent. However, application of such an equation to determine D is troublesome due to the difficulty in measuring A' and A'' directly for a graded gravel sample. To establish an equation that can better calculate D, a deep understanding for the evolution of D is essential. Eq. (12) can be transformed into the following form:

$$D = \frac{q - E_e \varepsilon_1}{q_r - E_e \varepsilon_1} \tag{13}$$

With known q and q_r, the relationship between D and ε_1 can be calculated and presented in Fig. 9.

As shown in Fig. 9, D demonstrates significant irregularity as ε_1 stays at a relatively small magnitude. It may sometimes decrease with an increase of ε_1. Moreover, D may even become negative in the initial loading stage. This is because damage doesn't occur in the gravel sample initially, and the sample remains in the densification strengthening stage. As ε_1 goes beyond the critical value $\varepsilon_{1,d}$, D increases with ε_1. It should be noting that D varies from 0 to 1 indicating that damage to the gravel sample initiates once ε_1 reaches a certain extent. At the same time, the gravel sample enters into the shear damage stage. D increases continuously with applied load and ε_1 increases. As the sample collapses, D is close to unity. The critical strain $\varepsilon_{1,d}$ is the so-called damage threshold mentioned in previous literature (Sidoroff 1981; Martin and Chandler 1994; Aubertin and Simon 1997). Theoretically, D equals to zero when $\varepsilon_1 < \varepsilon_{1,d}$. Damage to the gravel sample begins only in the case that the threshold $\varepsilon_{1,d}$ is acquired. D will increase towards unity with ε_1 as $\varepsilon_1 \geqslant \varepsilon_{1,d}$.

4.3 Constitutive Model Development

It is assumed that the gravel samples are composed of numerous mesoscopic elements that can be regarded as basic failure units. The internal defects induced by external loading on gravel samples is determined by the strength of it's mesoscopic elements. The defects in the gravel samples are randomly distributed, which implies that the damage or failure of individual mesoscopic element is also random. Therefore, a statistical method can be employed to illustrate the strength of mesoscopic elements existing in the sample. The probability distribution type to depict the strength levels of mesoscopic elements includes Weibull distribution, normal distribution and lognormal distribution (Wang et al. 2018). Considering the Weibull distribution has been widely adopted to feature the strength levels of mesoscopic elements for geomaterials (Li et al. 2012; Ling et al. 2020), it is also selected in the current study to investigate the strength properties of mesoscopic elements.

The strength of mesoscopic element for gravel materials is denoted as F and obeys Weibull distribution. The probability density function $P(F)$ can then be presented as:

$$P(F) = \frac{m}{F_0}\left(\frac{F}{F_0}\right)^{m-1} exp\left[-\left(\frac{F}{F_0}\right)^{m}\right] \tag{14}$$

This allows the damage evolution to be represented using a statistical expression for strength. The damage process of a gravel sample originates from the accumulation of failed mesoscopic elements. Based on N and N_f, D can be then measured using:

$$D = \frac{N_f}{N} \tag{15}$$

N_f can be expressed on the basis of the Weibull distribution and in a differential form, the number of failed elements is denoted $NP(F)\cdot \mathrm{d}F$. As the strength levels of mesoscopic elements range from 0 to F, the quantities of failed elements in a gravel sample can be demonstrated as:

$$N_f = \int_0^F NP(y)dy = N\left\{1 - exp\left[-\left(\frac{F}{F_0}\right)^{m}\right]\right\} \tag{16}$$

Substituting Eq. (16) into Eq. (15), D can be obtained in a statistical form:

$$D = 1 - \exp\left[-\left(\frac{F}{F_0}\right)^{m}\right] \tag{17}$$

Further, Eq. (17) can be presented in a strain form as in previous studies (Ling et al. 2020):

$$D = 1 - \exp\left[-\left(\frac{\varepsilon_1^{'}}{\varepsilon_0}\right)^{m}\right] \tag{18}$$

Since D of the graded gravel samples suffers from irregularity in the densification strengthening stage, D should be zero theoretically. As ε_1 goes beyond $\varepsilon_{1,d}$, D can be calculated from Eq. (18). Therefore, $\varepsilon_1^{'}$ in Eq. (18) should be the net value in the shear damage stage with $\varepsilon_{1,d}$ deducted from ε_1. Thereby D based on Weibull distribution is determined as:

$$D = \begin{cases} 0, \ 0 \le \varepsilon_1 < \varepsilon_{1,d} \\ 1 - \exp\left[-\left(\frac{\varepsilon_1 - \varepsilon_{1,d}}{\varepsilon_0}\right)^{m}\right], \varepsilon_1 \ge \varepsilon_{1,d} \end{cases} \tag{19}$$

Combining Eq. (19) with Eq. (12), a constitutive model of stress-strain relations of the graded gravel samples can be determined:

$$q = \begin{cases} E_e\varepsilon_1, \ 0 \le \varepsilon_1 < \varepsilon_{1,d} \\ E_e\varepsilon_1 \exp\left[-\left(\frac{\varepsilon_1 - \varepsilon_{1,d}}{\varepsilon_0}\right)^{m}\right] + q_r\left\{1 - \exp\left[-\left(\frac{\varepsilon_1 - \varepsilon_{1,d}}{\varepsilon_0}\right)^{m}\right]\right\}, \ \varepsilon_1 \ge \varepsilon_{1,d} \end{cases} \tag{20}$$

4.4 Parameter Determination

Three kinds of parameters are involved in the proposed constitutive model. One kind describes the statistical parameters, m and ε_0, of Weill distribution; the second is $\varepsilon_{1,d}$; while the third kind is the physical parameters of gravel samples, including E_e and q_r. $\varepsilon_{1,d}$ is not included into the physical parameters due to the fact that the dividing line between densification strengthening and shear damage stages of gravel materials is usually ambiguous. Therefore, the determination of $\varepsilon_{1,d}$ is different from other physical parameters.

The statistical parameters m and ε_0 can be formulated in relation to $\varepsilon_{1,d}$ using the "Extremum Method" at the peak point, as described by Wang et al. (2018) and Huang et al. (2018). In this approach, q equates to q_p when ε_1=$\varepsilon_{1,p}$. This relationship can be articulated as follows:

$$q\left|\varepsilon_1 = \varepsilon_{1,p} = q_p\right. \tag{21}$$

Moreover, at the peak of the stress-strain curve, the derivative of q with respect to strain should be equal to zero:

$$\frac{\partial q}{\partial \varepsilon_1}\left|\varepsilon_1 = \varepsilon_{1,p} = 0\right. \tag{22}$$

Combining Eq. (21) and Eq. (22) together, m and ε_0 can be expressed using:

$$m = \frac{E_e(\varepsilon_{1,p} - \varepsilon_{1,d})}{\left(ln\frac{E_e\varepsilon_{1,p} - q_r}{q_p - q_r}\right)(E_e\varepsilon_{1,p} - q_r)} \tag{23}$$

$$\varepsilon_0 = \frac{\varepsilon_{1,p} - \varepsilon_{1,d}}{\left(ln\frac{E_e\varepsilon_{1,p} - q_r}{q_p - q_r}\right)^{\frac{\left(ln\frac{E_e\varepsilon_{1,p} - q_r}{q_p - q_r}\right)(E_e\varepsilon_{1,p} - q_r)}{E_e(\varepsilon_{1,p} - \varepsilon_{1,d})}}} \tag{24}$$

In these equations, the physical parameters possess distinct meanings and can be ascertained either through experimental testing or parametric analysis. However, pinpointing $\varepsilon_{1,p}$ for graded gravel materials proves challenging due to the unclear boundary between densification strengthening and shear damage. To address this, the Damage-Softening Index (DSI) serves as an indicator for the gravel sample's critical damage threshold. DSI represents the maximal slope of the stress–strain curve following peak strength and correlates closely with both the subsequent peak state and the transition to the residual state. To express DSI mathematically, one derives the slope of the stress-strain curve as follows:

$$-\frac{\partial q}{\partial \varepsilon_1} = e^{-\left(\frac{\varepsilon_1-\varepsilon_{1,d}}{\varepsilon_0}\right)^m}\left[\frac{m(E_e\varepsilon_1-q_r)}{\varepsilon_0}\left(\frac{\varepsilon_1-\varepsilon_{1,d}}{\varepsilon_0}\right)^{m-1} - E_e\right] \tag{25}$$

Since *DSI* is the maximum slope, let the second derivative of Eq. (20) equals to zero:

$$\frac{\partial^2 q}{\partial {\varepsilon_1}^2} = 0 \tag{26}$$

The following equation is then acquired:

$$\left(\frac{\varepsilon_{1,s}-\varepsilon_{1,d}}{\varepsilon_0}\right)^m = \frac{(E_e\varepsilon_{1,s}-q_r)(m-1)+2E_e(\varepsilon_{1,s}-\varepsilon_{1,d})}{m(E_e\varepsilon_{1,s}-q_r)} \tag{27}$$

Combining Eq. (27) with Eq. (25), the relationship between m, ε_0 and $\varepsilon_{1,d}$ can be established in accordance with *DSI* and $\varepsilon_{1,s}$:

$$-\frac{\partial q}{\partial \varepsilon_1}\Big|\varepsilon_1 = \varepsilon_{1,s} = e^{-\left(\frac{\varepsilon_{1,s}-\varepsilon_{1,d}}{\varepsilon_0}\right)^m}\left[E_e + \frac{(E_e\varepsilon_{1,s}-q_r)(m-1)}{\varepsilon_{1,s}-\varepsilon_{1,d}}\right] = \mathrm{DSI} \tag{28}$$

Equation (28) enables the calculation of m, ε_0 and $\varepsilon_{1,d}$ when considering Equations (23) and (24). Consequently, these statistical parameters and $\varepsilon_{1,d}$ are determinable through the integrated use of these equations. To assess the accuracy of these calculated values, Equation (13) provides a measured value of $\varepsilon_{1,d}$ in terms of D for comparison. As previously highlighted, the damage process in graded gravel initiates when $\varepsilon_{1,d}$ surpasses the threshold $\varepsilon_{1,d}$. Therefore, $\varepsilon_{1,d}$ signifies the axial strain at which D begins its regular increase. This measured value aligns closely with data illustrated in Figure 9.

For further validation, the calculated $\varepsilon_{1,d}$ is compared with both the measured value for gravel samples and values reported for other granular materials, such as aggregates and a specific type of granitic soil (Byun et al. 2020; Zhao et al. 2013). Figure 10 presents these comparisons. Be noted that these granular materials exhibit varying grain size distributions, identified as source gradation and engineering gradation ("SG" and "EG" respectively) in literature.

Fig. 10 indicates that both calculated and measured values of $\varepsilon_{1,d}$ for the graded gravels in this study and granular materials reported previously align along the 1:1 line. This suggests that the proposed method can accurately determine $\varepsilon_{1,d}$, as well as m and ε_0.

Some physical parameters, including E_e, q_r, q_p and *DSI*, are necessary to be correlated with DoC and σ_3. However, it is challenging to evaluate the maximum slope of post-peak section of stress-strain curves, meaning it is also challenging to directly determine *DSI*. Nevertheless, it is found that *DSI* of gravel samples is insensitive to the variation of σ_3 for low confining pressure conditions. Therefore, the average values of *DSI* under different σ_3 can be calculated for gravel samples with a given DoC. The gravel samples with different DoC of 0.9, 0.95 and 1.0 demonstrate quite different average values of *DSI*, indicating that *DSI* is an excellent indicator to describe strain-softening properties of gravel samples with different DoC. Besides, the average values of *DSI* under different σ_3 is found to be positively correlated with $q_p - q_r$, as presented by Fig. 11. A regression analysis is conducted to give the following linear equation a correlation coefficient of 0.97:

$$DSI = 0.05(q_p - q_r) - 2.68 \tag{29}$$

Using Eq. (29), *DSI* can be calculated when $q_p - q_r$ is known. Since E_e, q_r, q_p and $q_p - q_r$ is closely related with DoC and σ_3, they can be determined by parametric analysis in terms of DoC and σ_3. The detail process to determine the physical parameters is presented in Appendix A.

5. Model Validation

With the proposed model and determination method of model parameters, the predicted stress-strain relations of graded gravel samples can be obtained and compared with the experimental data to validate the proposed damage-softening model. These are shown, along with curves from an alternative approach (Ling et al., 2020) in Fig. 12.

Based on the proposed statistical damage-softening model, the predicted stress-strain relations of the gravel samples closely match the experimental data. In contrast, the predicted relations from Ling (2020) exhibit some divergence from the experimental data in the post-peak region. The comparisons show that the proposed method is capable of representing the damage-softening properties of graded gravels. Additionally, the statistical parameters (i.e. m and ε_0) obtained from the *DSI* condition are more precise than those calculated by using a derivative function at the peak state. As a result, the proposed constitutive model and statistical parameter determination method exhibit advantages.

To demonstrate the applicability of the proposed constitutive model and the determination method of model parameters, triaxial test results from a previous study on a granite gravel were used (Chen and Zhang 2016). The gravel sample, which has a particle size ranging from 10 mm to 40 mm, as commonly used to fill roadbed, has particles with varying shapes and high angularity.

The optimum moisture content and maximum dry density of the gravel samples were determined to be 8.3% and 2.11 g/cm^3, respectively. Drained triaxial tests were performed on the gravel samples, and the resulting q against ε_1 plotted in Fig. 13. Model parameters for the granite sample were calculated using the method proposed in Section 4.4 and Appendix A, and the resulting stress-strain relations for the granular materials were also obtained. The calculated stress-strain relations agree reasonably well with test results, indicating the damage-softening model and the method proposed in this study can effectively characterize the strain softening properties of gravel samples and can be applied to solve relevant problems.

6. Conclusions

In the present study, triaxial tests on graded gravels under varying conditions of compaction and confining pressures were executed. Damage evolution in these gravels was closely analyzed, leading to the introduction of a statistical damage-softening model augmented by a novel damage-softening index. Key findings are as follows:

1. Analysis of the damage variable against axial strain uncovers the gravel samples' damage evolution. The constitutive model should incorporate threshold axial strain, as damage only initiates when axial strain exceeds a specific value.

2. The newly proposed damage-softening index serves as a tool for determining model parameters that govern stress-strain relations in the post-peak region. This index aids in the precise calculation of threshold axial strain and other vital parameters.

3. Triaxial tests provided invaluable insights into the stress-strain behavior of graded gravels. Based on these insights, a statistical damage constitutive model was developed to address strain softening in gravels with different degrees of compaction.

4. Model predictions closely align with experimental data across varying degrees of compaction, validating the constitutive model's efficacy in accurately capturing stress-strain relations.

The proposed statistical damage-softening model effectively predicts graded gravel behavior under drained triaxial tests. In practical application, the stress-strain relationship of graded gravels can be predicted by using the proposed statistical damage constitutive model, with the statistical parameters (i.e., ε_0, m) and threshold axial strain $\varepsilon_{1,d}$ determined by physical parameters. The predicted stress-strain relationship and strength properties of graded gravels can provide guidance in engineering design of functional layers in roadway and railway. In addition, the proposed model can also be adopted in numerical simulation to highlight the strain softening features of graded gravels.

It is worth noting that the model proposed is limited to conventional triaxial conditions. Despite this, it provides a valuable method for predicting stress-strain relations for graded gravel with differing DoC, for conventional triaxial tests are still widely employed in engineering practice due to their simplicity and ease of operation. In addition, the model's accuracy hinges on precise physical parameters such as E_e, q_r and q_p, which can be affected by factors like loading rate. While particle shape and fabric have minimal impact in transportation geotechnics, loading rate significantly influences the regression coefficients. Thus, for improved model performance, additional data is advised for function and coefficient refinement.

Data Availability Statement

All of the data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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Appendix A

The average values of E_e, q_r, q_p and q_p−q_r are firstly calculated for the samples with different DoC under a given σ_3. Then, the average values can be linearly correlated with σ_3:

$$E_{e,avg} = \frac{1}{n}\sum_{i=1}^{n} E_{e,i} = \alpha_1\sigma_3 + \beta_1 \quad \text{(A1)}$$

$$q_{r,avg} = \frac{1}{n}\sum_{i=1}^{n} q_{r,i} = \alpha_2\sigma_3 + \beta_2 \quad \text{(A2)}$$

$$q_{p,avg} = \frac{1}{n}\sum_{i=1}^{n} q_{p,i} = \alpha_3\sigma_3 + \beta_3 \quad \text{(A3)}$$

$$\left(q_p - q_r\right)_{avg} = \frac{1}{n}\sum_{i=1}^{n}\left(q_p - q_r\right)_i = \alpha_4\sigma_3 + \beta_4 \quad \text{(A4)}$$

where n=3. Therefore, $E_{e,avg}$ under the specified σ_3 can be calculated using Eq. (A1). Similarly, $q_{r,avg}$, $q_{p,avg}$ and $(q_p-q_r)_{avg}$ can be obtained using the same method. α_1, α_2, α_3, α_4, β_1, β_2, β_3 and β_4 are coefficients that can be evaluated using regression analysis on the test data of the gravel samples in this study, as presented in Fig. A1. The values of the coefficients are presented in Table A1.

Then, E_e, q_r, q_p and q_p-q_r can be normalized with the average values known. The normalized parameters can be correlated with DoC by fitting the following equations:

$$\frac{E_{e,i}}{E_{e,avg}} = A_1 \cdot (DoC) + B_1 \quad \text{(A5)}$$

$$\frac{q_{r,i}}{q_{r,avg}} = A_2 \cdot (DoC) + B_2 \quad \text{(A6)}$$

$$\frac{q_{p,i}}{q_{p,avg}} = A_3 \cdot (DoC) + B_3 \quad \text{(A7)}$$

$$\frac{(q_p-q_r)_i}{(q_p-q_r)_{avg}} = A_4 \cdot (DoC) + B_4 \quad \text{(A8)}$$

where A_1, A_2, A_3, A_4, B_1, B_2, B_3 and B_4 are the coefficients with values presented in Table A1. The regression analysis to determine the coefficients is performed and presented in Fig. A2. Since the coefficients in Table A1 can be determined with DoC and σ_3 known, the four physical parameters, E_e, q_r, q_p and q_p-q_r (*DSI*), can be determined.

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Table A1. Regression Coefficients of Model Parameters

Fitting equations	Regression coefficients	Values	R^2
$E_{e,avg} = \alpha_1\sigma_3 + \beta_1$	α_1	0.34	0.96
	β_1	44.38	
$q_{r,avg} = \alpha_2\sigma_3 + \beta_2$	α_2	3.35	0.99
	β_2	57.11	
$q_{p,avg} = \alpha_3\sigma_3 + \beta_3$	α_3	6.57	0.97
	β_3	304.11	
$(q_p - q_r)_{avg} = \frac{1}{n}\sum_{i=1}^{n}(q_p - q_r)_i = \alpha_4\sigma_3 + \beta_4$	α_4	3.22	0.90
	β_4	248.00	
$\frac{E_{e,i}}{E_{e,avg}} = A_1 \cdot (DoC) + B_1$	A_1	6.79	0.97
	B_1	−5.45	
$\frac{q_{r,i}}{q_{r,avg}} = A_2 \cdot (DoC) + B_2$	A_2	2.40	0.89
	B_2	−1.28	
$\frac{q_{p,i}}{q_{p,avg}} = A_3 \cdot (DoC) + B_3$	A_3	9.91	0.98
	B_3	−8.41	
$\frac{(q_p - q_r)_i}{(q_p - q_r)_{avg}} = A_4 \cdot (DoC) + B_4$	A_4	13.66	0.99
	B_4	−11.98	

Nomenclature

A	Initial area of the cross-section	$q_p - q_r$	Difference of peak and residual strength
A'	Area of undamaged portion of the cross-section	$(q_p - q_r)_{\mathrm{avg}}$	Average difference of peak and residual strength under different confining pressures
A''	Area of damaged portion of the cross-section	q_r	Residual strength
D	Damage variable	$q_{r,\mathrm{avg}}$	Average residual strength under different confining pressures
DoC	Degree of compaction	T	External load acted on a material body
DSI	Damage softening index	ε_0	Scale parameter in strain form
E_e	Secant modulus of elasticity	ε_1	Axial strain
$E_{e,\mathrm{avg}}$	Average secant modulus of elasticity under different confining pressures	ε_1'	Axial strain from the initiation of damage evolution
F	Strength level of the mesoscopic elements	ε_1^*	Net axial strain
F_0	Scale parameter in strength form	$\varepsilon_{1,d}$	Damage threshold value of axial strain
I_s	Softening coefficient	$\varepsilon_{1,\mathrm{int}}$	Initial axial strain
m	Shape parameter	$\varepsilon_{1,p}$	Axial strain at the peak state
N	Quantities of mesoscopic elements	$\varepsilon_{1,s}$	Axial strain corresponding to the maximum slope of stress-strain curve
N_f	Quantities of failed mesoscopic elements	ε_v	Volumetric strain
n	Sample quantities with different DoC under the same confining pressure	σ_1	Major principal stress (Axial stress in triaxial test)
q	Deviatoric stress	σ_3	Minor principal stress (Confining pressure in triaxial test)
$q_{0.5\varepsilon1,p}$	Deviatoric stress corresponding to half value of axial strain at the peak state	σ_1^*	Net major principal stress
q_{int}	Initial deviatoric stress	σ_3^*	Net minor principal stress
q_p	Peak strength	Δq	Increment of deviatoric stress
$q_{p,\mathrm{avg}}$	Average peak strength under different confining pressures	$\Delta\varepsilon_1$	Increment of the axial strain