Precision Fertilisation via Spatio-temporal Tensor Multi-task Learning and One-shot Learning

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Abstract—**Precision fertilisation is essential in agricultural systems for balancing soil nutrients, conserving fertiliser, decreasing emissions and increasing crop yields. Access to comprehensive and diverse agricultural data is problematic due to the lack of sophisticated sensor and network technologies on the majority of farms, and available agricultural data is generally unstructured and difficult to mine. The absence of agricultural data is consequently a significant impediment to the utilisation of machine learning approaches for precision fertilisation. In this research, we investigate newly gathered genuine agricultural dataset from nine real winter wheat farms in the United Kingdom, which encompass an extensive variety of agricultural variables including climate, soil nutrients and farming data. To deal with the spatio-temporal characteristics of agricultural dataset and to address the problem of scarcity in agricultural data, we propose a novel machine learning approach integrating multi-task learning (MTL) and one-shot learning, which utilises a multi-dimensional tensor constructed from original data combined with fertilisation temporal patterns extracted by contrasting with environmental information from existing real farms to accurately predict the amount and timing of base and top dressing fertilisation. Specifically, agricultural data are converted into a three-dimensional tensor and tensor decomposition technique is utilised to derive a set of comprehensible spatio-temporal latent factors from the original data. The latent factors are subsequently utilised to construct the spatio-temporal tensor prediction model as multi-task relationships. The proposed one-shot learning approach utilises the Mahalanobis distance to evaluate the similarity of environmental information between the target farm and existing real-world farms as a determinant of whether to transfer the fertilisation temporal pattern of existing farm to the target farm. Comprehensive experiments are conducted to compare the proposed approach with standard regression models utilising the real-world agricultural dataset. The experimental results demonstrate that our proposed approach presents superior accuracy and stability for fertilisation prediction.**

Index Terms—**Multi-task learning, One-shot learning, Precision fertilisation, Real-world agricultural data, Spatio-temporal tensor**

I. INTRODUCTION

THE challenges in the field of agriculture are crucial to mankind. Approximately 780 million of the 7.2 billion people on the globe are currently in threat of succumbing to hungry. We will require 60% more food calories by 2050 since there will be 9 billion individuals on the planet [1]. Due to overuse of fertilisers and pesticides, we are dealing with environmental and soil deterioration, which exposes human health in jeopardy [2]. Agriculture-related artificial intelligence research, application and development commenced in this century. These consist of intelligent machinery for sowing, ploughing, and harvesting [3][4], intelligent detection systems for pest and disease detection, soil testing and environmental disaster forecasting [5][6]. These applications are assisting humankind in improving yields, enhancing efficiency, and reducing the use of pesticides and fertilisers.

For fertiliser application, the soil can occasionally fail to deliver the ideal nutrients for crops; farmers must rotate the crops on a regular basis, and modern agricultural technology has resulted in numerous creative fertilisation alternatives. Sophisticated artificial intelligence approaches can ascertain the amount of fertiliser is required to avoid waste and contamination. Traditional approaches for estimating fertilisation rates primarily rely on the nutrient balance approach [7] and the fertiliser effect function approach [8]. The calculations performed for the nutrient balance approach are frequently imprecise and still demand professional knowledge because of the numerous factors required. The fertiliser effect function approach involves significant quantities of experimental data and an adequate fit of ternary quadratic equations, however its fitting has a poor effectiveness and failing data is typically eliminated, resulting in wasted time, materials and cost. Furthermore, obtaining fertilisation data is a significant challenge. Massive amounts of data are gathered by national and international agricultural research institutions, and that data can theoretically assist machine learning approaches, but these data are frequently non-recoverable, non-interpretable, or non-reusable [9]. Poor agricultural assistance system performances can be caused by incomplete, biased or irrelevant data. This can erode farmers' confidence in digital extension programs and specialist systems which would eventually jeopardise food security. To the best of our knowledge, there is no well-known public database on agricultural fertilisation.

The aim of this research is to utilise MTL and one-shot learning concepts combined with spatio-temporal data from diverse farms to predict the amount and timing of specific fertiliser applications. Integrating spatio-temporal agricultural information from diverse farms, multi-dimensional tensor, multi-task learning and one-shot learning as a novel fertilisation prediction solution faces a variety of challenges. Firstly, it is difficult to acquire and digitise agricultural data and fertiliser application records from various farms into a

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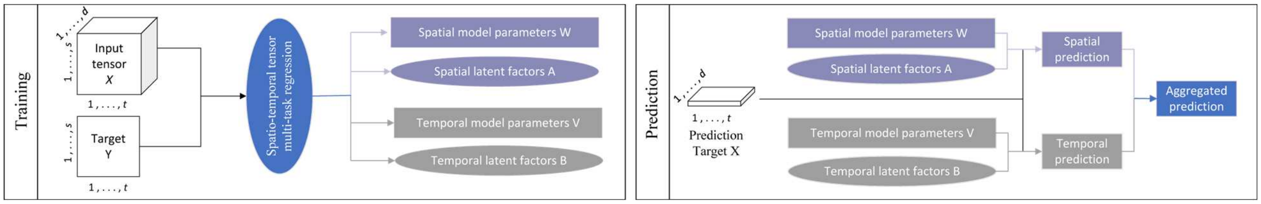

Fig. 1. Overview of the proposed spatio-temporal tensor multi-task regression approach. The left figure demonstrates the training phase of the proposed approach and the right figure demonstrates the prediction phase of the proposed approach in the application scenario. The approach simultaneously learns an aggregation of spatial and temporal multi-task learning models utilising latent factors obtained from agricultural spatio-temporal data, and then combines the aggregated outputs to achieve the final prediction.

dataset. Secondly, it is complicated to seamlessly integrate temporal and spatial information and knowledge into the algorithm. Thirdly, traditional MTL relationships are built on a number of assumptions made by machine learning task models, such as the low rank assumption and the temporal smoothing assumption, but for the combination of spatio-temporal multi-dimensional tensor and MTL, it is a challenge to define the task and task relationships in the model. Finally, it is challenging to incorporate the concept of one-shot learning into the algorithm to address the problem of limited agricultural data.

To solve above challenges, we obtained a real-world agricultural dataset from nine genuine farms planting winter wheat, which comprises various forms of agricultural factors including climate information, soil nutrients, crop yield, fertilisation records, etc. In terms of algorithms, this research proposes an MTL approach to predict and model the fertilisation process utilising agricultural spatio-temporal data and multi-dimensional tensor. First, we implement a third-order spatio-temporal tensor to describe real-world agricultural data from multiple farms. The tensor has three dimensions: space (numerous farms), time (from September to August over the next year) and features (diverse input agricultural factors). The proposed method utilises the CANDECOMP/PARAFAC (CP) approach [10] to decompose tensors and obtain a collection of rank-one latent factors from the original data. The agricultural spatio-temporal multi-dimensional tensor can be decomposed into a variety of rank-one tensors, and each rank-one tensor is obtained by computing the outer product of three rank-one latent factors. An interpretable approach is presented to explain the latent factors that control data variability since each latent factor can be characterised in terms of space, time and feature dimensions. The temporal latent factors represent the temporal patterns shared across various farms for the same crop fertilisation operation, whereas the spatial latent factors are the influence of different farm locations on the same crop fertilisation operation. The Fig. 1 illustrates the overview of the training and prediction phases for the proposed spatio-temporal tensor multi-task regression approach. Moreover, we incorporate the concept of one-shot learning into the proposed approach to further address the problem of limited agricultural data. The hypothesis of this concept is that farms with similar environmental information have similar fertilisation temporal patterns, therefore when the similarity calculation demonstrates that the target farm and the existent farm have similar environments, then the same fertilisation temporal patterns can be implemented to the target farm.

The main contributions of this research are as follows:

- We gathered and extracted real-world agricultural dataset from various farms and encoded the data into a multi-dimensional tensor to allow the spatio-temporal information of the data to be simultaneously applied to the prediction algorithm.
- We proposed a multi-dimensional tensor MTL approach for precision fertilisation prediction that utilises tensor decomposition techniques to learn task relationship from the original data and seamlessly integrates temporal and spatial latent factors in the algorithm to enhance the accuracy and stability of fertilisation prediction.
- We incorporated the concept of one-shot learning in the proposed approach to further address the problem of limited agricultural data.

The rest of this paper is organized as follows: The related work is introduced in Section II. Section III presents details of the spatio-temporal tensor multi-task regression model and the one-shot learning approach utilised for precision fertilisation research. Section IV presents specifics on our real-world agricultural dataset, preprocessing and experimental procedures. Section V presents the experimental results of our proposed approach on the agricultural dataset, which are utilised to validate the prediction performance of the proposed approach, along with an analysis of the significant agricultural features and findings of our proposed approach. Section VI presents the discussion for the existing challenges of precision fertilisation and our future research development trends. The paper is concluded in Section VII.

II. Related Work

Precision agriculture is a system for integrating a variety of contemporary agricultural business approaches and management into practice at a specified time, position, and quantity, based on geographical variability, and assisted by advanced information technologies [11]. The central concept is to alter agricultural inputs in accordance with soil elements, promote soil production, and generate the same or greater profits with a minimal amount of input while protecting the environment [12]. Precision fertilisation is a significant aspect of precision agricultural technology. Precision fertiliser application technology aims at enhancing fertiliser utilisation by modifying the amount of fertiliser implemented, the nitrogen, phosphorus, and potassium ratio, and the application cycle based on crop fertiliser demands, soil nutrient circumstances, and goal yields [13]. While protecting the natural resources and agroecological environment, the utilization of land resources should be improved to obtain the maximum yield and economic benefits with a minimum quantity of fertiliser supplies [14]. Utilising precision fertilisation approaches may lower fertiliser usage, balance soil nutrients, and improve crop yields. The primary technologies for precision fertilisation involve precise soil nutrient detection and crop nutrient evaluation according to the geographical distribution of soil nutrients within the fertilisation region [15], establishing feasible fertilisation models to accomplish sensible fertilisation decisions [16], and implementing adequate fertilisation approaches to attain precision fertilisation [17].

In the field of machine learning, various algorithms and models have been proposed in previous researches for applications in the domain of precision fertilisation. Neural network-based precision fertiliser application techniques [18][19] have been proposed for different type of crops. Machine learning algorithms implemented by support vector machines and random forests have been utilised [20][21] for crop yield predictions and recommended the appropriate fertiliser for each specific crop. Crop suitability and fertiliser recommendation systems [22][23] have been developed based on random forest algorithms and k-means clustering algorithms, with fertiliser recommendations based on the N, P and K content of the soil and accomplished with data from previous years' research stored in the ontology. The above-mentioned models and algorithms can provide acceptable results when predicting fertiliser application amounts. The above researches cannot integrate spatial and temporal information from the data into the algorithms to incorporate agricultural multi-dimensional knowledge into the calculation process to enhance the accuracy and stability of the predictions, and they cannot address the problem of limited agricultural data.

To overcome these challenges, we utilise the concept of multi-task learning to integrate both temporal and spatial information from the original data into the algorithm in order to enhance the generalization, prediction accuracy and stability for the model. The concept underlying multi-task learning is that there is an intrinsic relationship between information recordings from different individuals, and that capturing this intrinsic relationship enhances the generality of the prediction model [24]. MTL can enhance the ability of the model to summaries the initial task by sharing knowledge among related tasks. The MTL approach focuses on how to define tasks and their relationships. Present MTL approaches define task relationship mainly by novel regularization [25][26], priori assumptions [27], parameter sharing [28][29] and the incorporation of kernel approach allowing algorithms to fit non-linear relationships [30][31].

III. Methodology

A. Notation

For brevity, we represent tensors as italic capital letters, such as X or Y, and matrices by capital letters, such as A or B. Vectors are denoted by lowercase letters such as x whereas Scalars are denoted by italic lowercase letters such as a.

B. Spatio-temporal tensor multi-task regression

To calculate the precise quantity and timing for fertilisation across a 12-month period. Consider a tensor multi-task regression problem with t time points (months) and s training samples (farms) of d features (agricultural input data). Let $X \in \mathbb{R}^{s \times t \times d}$ be the input tensor from numerous farm samples, $\mathrm{Y} \in \mathbb{R}^{s \times t}$ be the targets (12-month period fertilisation plan). The targets are provided as a set of 12-month time series, enabling us to understand precisely the quantity of fertiliser required to be applied in each month. Fertilisation is a multilayered procedure requiring numerous processes (such as base fertiliser and top dressing). Fig. 2 illustrates the timing and frequency of fertilisation for winter wheat in our farm samples, known as the winter wheat fertilisation target.

The objective function of the proposed approach can be stated as follows:

$$\min_{\mathrm{W, V, A, B, C}} \frac{1}{2}\left\|\hat{\mathrm{Y}} - \mathrm{Y}\right\|_{\mathrm{F}}^{2} + \frac{\lambda}{2}\|X - [\![\mathrm{A, B, C}]\!]\|_{\mathrm{F}}^{2} + \Omega_m(\mathrm{W, V}) + \Omega_l(\mathrm{A, B, C})$$

$$\hat{y}_{st} = (\mathrm{A}_s \mathrm{W}^{\mathrm{T}} + \mathrm{B}_t \mathrm{V}^{\mathrm{T}})\mathrm{x}_{st}^{\mathrm{T}} \tag{1}$$

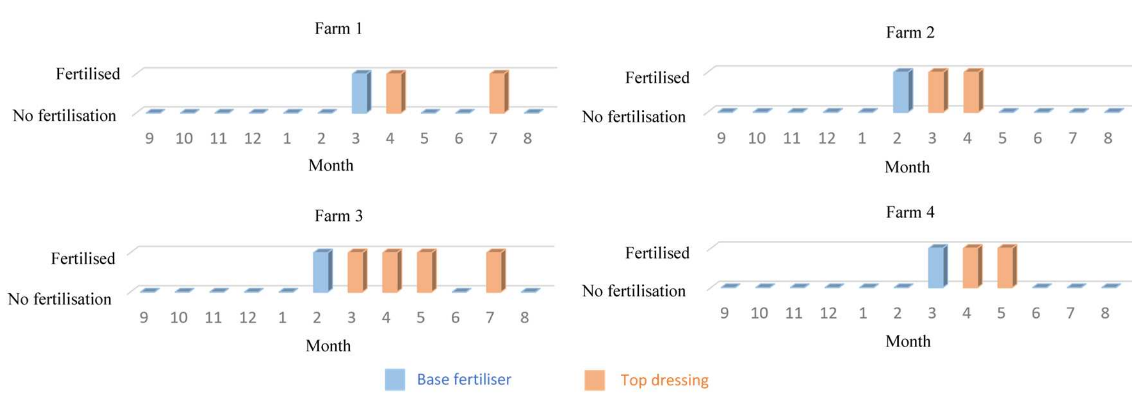

Fig. 2. An example of the target for winter wheat fertilisation.

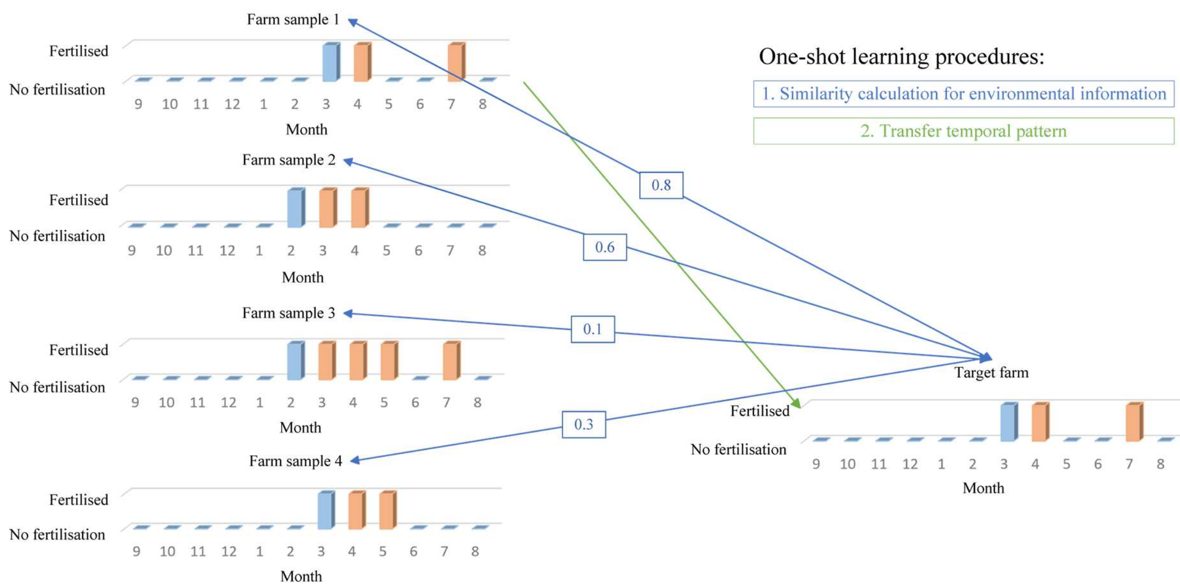

Fig. 3. Demonstration for procedure 1 and 2 of the proposed one-shot learning approach.

where the first term evaluates the training data's empirical error. $\widehat{\mathrm{Y}} \in \mathbb{R}^{s\times t}$ are prediction values, $\mathrm{W} \in \mathbb{R}^{d\times r}$ is the spatial model parameter matrix, $\mathrm{V} \in \mathbb{R}^{d\times r}$ is the temporal model parameter matrix, $\mathrm{A} \in \mathbb{R}^{s\times r}$ is the spatial latent factor matrix, $\mathrm{B} \in \mathbb{R}^{t\times r}$ is the temporal latent factor matrix with r latent factors. λ is the regularization parameter, Ω_m are regularization terms for model parameters W and V, Ω_l are regularization terms for latent factors A, B and C. Acquire latent factors by optimising CP tensor decomposition objective function $\|X - [\![\mathrm{A}, \mathrm{B}, \mathrm{C}]\!]\|_{\mathrm{F}}^2$, given a tensor X with the size $s \times t \times d$, the size of matrix A, B and C is $s \times r, t \times r$ and $d \times r$ respectively, where $X \approx [\![\mathrm{A}, \mathrm{B}, \mathrm{C}]\!] = \sum_{i=1}^{r} \mathrm{A}_i \circ \mathrm{B}_i \circ \mathrm{C}_i$, where $\circ$ denotes the outer product operation among two vectors, while $\mathrm{A}_i, \mathrm{B}_i$ and C_i correspond to the vectors associated with the i-th latent factor.

The model parameters W and V, the latent factors A, B and C can be acquired by repeatedly optimising the objective function for each collection of variables to which a solution needed. We utilise the proximal gradient descent approach to address each subproblem in this research since not all of the parts of the objective function are differentiable. For instance, components in the objective function that involve Frobenius norms are differentiable, whereas not these with the sparsity-inducing ℓ_1-norms. In the MTL model, the proximal approach is extensively utilised to construct the proximal problem for non-smooth objective function [32][33], which is the combination of the non-smooth and smooth functions, via altering the smooth function with the quadratic function. Consider the case of a non-differentiable objective function f(x), which can be decomposed into a non-smooth function n(x) and a smooth differentiable function d(x), i.e., f(x) = n(x) + d(x). To iteratively adjust the model parameters, we implement the proximal gradient descent method as follows:

$$\mathrm{x}^{(s)} = \mathbf{prox}_{z_s,\mathrm{n}}\left(\mathrm{x}^{(s-1)} - z_s \nabla \mathrm{d}\left(\mathrm{x}^{(s-1)}\right)\right) \quad (2)$$

where $\mathrm{x}^{(s)}$ is parameter to be assessed at step s. $\mathbf{prox}_{z_s,\mathrm{n}}$ is the proximal operator for non-differentiable function n, $\nabla \mathrm{d}\left(\mathrm{x}^{(s-1)}\right)$ is gradient for smooth function d, $\mathrm{x}^{(s-1)}$ and z_s is the step size for gradient descent update. Proximal operator for the ℓ1-norm function is soft-thresholding operator [34] as follows: $\mathbf{prox}_{\xi,\mathrm{n}}(\mathrm{v}) = (\mathrm{v} - \xi)_+ - (-\mathrm{v} - \xi)_+$, where ξ is threshold parameter. Iteratively updating the parameters involves calculating the gradient on the smooth component of the objective function and then employing the soft-thresholding operator (proximal mapping function for ℓ1-norm) to determine its next value. A line search technique can be utilised to determine the step size. The approach can simplify the formulation of distributed optimisation algorithms and accelerate the optimisation process's convergence.

C. One-shot learning

Limited agricultural data is a common problem in agricultural algorithm research, and while our proposed spatio-temporal tensor multi-task regression approach can accurately predict the total amount of fertiliser required, it is difficult to predict the exact time points. To address this problem, we present the concept of one-shot learning in our proposed approach, the hypothesis of this concept is that farms with similar environmental information will have similar fertiliser application temporal patterns. It is divided into the following three main procedures.

Firstly, when we want to make a prediction for a target farm, the model will perform a similarity calculation between the environmental information of the target farm and all our farm samples, if the similarity score is high, it means that the environmental information between the sample and the target farm is highly similar. The Mahalanobis distance is utilised to calculate the similarity of two vectors to reflect the similarity of the environmental information between the two farms. The Mahalanobis distance was utilised since it is scale-independent when the covariance matrix is divided [35]. The Mahalanobis

TABLE I

STRUCTURES OF THE WINTER WHEAT REAL-WORLD AGRICULTURAL DATASET

Winter Wheat Dataset	Agricultural feature			Range of target values (kg/ha)
	Climate data	Soil properties and nutrients data	Cropping data	
Nitrogen fertiliser	Mean daily temperature (°C)	Soil pH value	Potential grain yield (t/ha)	0 – 138.07
	Monthly rainfall (mm)	Soil water holding capacity (mm)	Seeds sown per m^2	
	Monthly solar radiation (TJ/ha)	Soil phosphorus content (mg/l)	Working ha	
		Soil potassium content (mg/l)		
		Soil magnesium content (mg/l)		

distance between the vectors x_i and x_j is defined as: $\mathrm{Ma}(x_i, x_j) = \sqrt{(x_i - x_j)^T S^{-1}(x_i - x_j)}$, where S is covariance matrix. The quantified Mahalanobis distance ranges between 1 and 0, with 1 being completely similar and 0 being completely dissimilar.

The second procedure is to transfer the fertilisation temporal pattern from the sample farm to the target farm, then the algorithm will perform fertilisation predictions based on that temporal pattern. If there is no high similarity score, indicating that this target farm has its own temporal pattern, then all time points and fertilisation amounts are given by the algorithm. Fig. 3 demonstrates procedure 1 and 2 of the proposed one-shot learning approach.

The third procedure is to add fertilisation amount from the wrong month to the nearest correct month. Most farms will concentrate on a number of specific months because of economic or cost reasons, rather than making a fixed monthly application of fertiliser [36][37]. Therefore, in order to adapt our predictions to real-world fertilisation situations, i.e., to maximise the economic benefits while improving fertiliser utilisation, the aim of this procedure is to enhance the accuracy of the time-point predictions of fertilisation months while maintaining the overall accurate fertilisation predictions.

IV. EXPERIMENTAL SETTINGS

A. Real-world agricultural dataset

We acquired a real-world agricultural dataset from nine genuine farms (samples) with winter wheat, and it covers a variety of agricultural factors. Farms comprise an extensive variety of information for agricultural data, and our chosen factors have to satisfy two standards. First, from an agronomic standpoint, it can affect crop growth and output. Second, it is a value that can be obtained prior to the fertilisation stage. Specifically, our dataset includes three categories of content. The first category is climate data, which can be gathered via a weather prediction tool, and it contain three factors: monthly rainfall, mean daily temperature and monthly solar radiation. The second category of content is soil properties and nutrients data, which can be acquired through the soil test. Soil properties consist of two factors: soil water holding capacity and soil pH value, soil nutrients consist of three factors: soil phosphorus, potassium and magnesium content. The third category of content is cropping data, which can be collected through cropping records, it has three factors: seeds sown per m^2, potential grain yield and working ha. Overall, there are 11 input agricultural features for fertilisation prediction. All agricultural features contain data for the entire crop duration (12 months, i.e., from the crop sowing to the crop harvesting), thus allowing us to integrate seasonal variation and climate change related information and knowledge in our proposed approach. For the prediction targets, nitrogen fertiliser is the only fertiliser applied on all farms after crop planting, it is the most widely manufactured and applied fertiliser worldwide, the right amount of nitrogen fertiliser contributes to improved crop yields and better quality of agricultural products. Therefore, this research conducted experiments for nitrogen fertilisers and the proposed approach can apply to the prediction of all type of fertilisers if the data permits. The structure of the winter wheat dataset is summarised in Table I.

B. Evaluation metrics

We designed and constructed a nitrogen fertilisation prediction model based on a multi-dimensional tensor of the input agricultural data. Due to the limited number of farms and the challenges of acquiring agricultural data, we randomly utilised data from eight of the farm samples for model training and one of the remaining farms for testing in each experiment. We perform 2-fold cross-validation for the training data to determine model parameters since the number of latent factors *r* and regularization parameters have to be selected during the training phase.

In this research, the root mean square error (rMSE) is utilised as a major evaluation metric to assess the accuracy of multiple prediction algorithms. And for overall regression performance, we utilise R squared (R^2), which evaluates the degree that the predicted value fits the actual value; the scale of the R^2 is from $-\infty$ to 1, with the closer the value to 1, the greater the prediction performance. The following are the definitions of rMSE and R^2:

$$\mathrm{rMSE}(y, \hat{y}) = \sqrt{\frac{\|y-\hat{y}\|_2^2}{n}} \tag{3}$$

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2} \tag{4}$$

where for the rMSE, y is the ground truth of target and $\hat{y}$ is the corresponding prediction by a model. For the R^2, y_i is the

TABLE II

COMPARISON OF THE RESULTS FROM OUR PROPOSED APPROACHES WITH STANDARD REGRESSION MODELS FOR NITROGEN FERTILISATION PREDICTION WITH REAL-WORLD WINTER WHEAT DATASET. THE BEST RESULTS ARE BOLDED

Regression models	rMSE	R^2
Ridge regression [38]	30.9204±4.8024	0.3253±0.1631
Lasso regression [39]	28.5121±4.8872	0.3860±0.1817
Elastic-Net [40]	31.0371±5.2460	0.2902±0.1607
Bayesian ridge regression [41]	32.0966±5.0307	0.2356±0.1826
Bayesian automatic relevance determination regression [42]	26.5936±4.2736	0.4120±0.2013
Linear support vector regression [43]	34.8790±6.2156	0.2049±0.1631
Sigmoid kernel support vector regression [43]	34.7391±5.1614	0.2087±0.1833
Regression based on k-nearest neighbors [44]	36.3881±4.5729	0.1937±0.1794
Decision tree regressor [45]	35.5407±5.4081	0.2038±0.1378
Multi-layer perceptron regressor [46]	32.3471±6.1906	0.2368±0.1136
TMTR-s	12.2049±1.3623	0.6271±0.1018
TMTR-t	10.7256±0.3269	0.7020±0.1280
TMTR-b	**8.3143±0.3319**	**0.7476±0.1069**

ground truth of target at number i, $\hat{y}_i$ is the corresponding prediction from a model and $\bar{y}$ is the mean of the true y values. The mean and standard deviation of 20 test iterations on various random data splits are reported.

C. Spatio-temporal tensor multi-task regression and its variants

In the research, we investigate the prediction performance of tensor multi-task regression incorporating both spatial and temporal latent factors, and with spatial latent factors or temporal latent factors solely supplemental with the ℓ_1-norm regularization term to validate model sparsity. Following are descriptions of specific tensor multi-tasking regressions and their variants.

i) Tensor multi-task regression comprehends both spatial and temporal latent factors and model parameters (TMTR-b):

$$\min_{\mathrm{W,V,A,B,C}} \frac{1}{2}\left\|\hat{\mathrm{Y}} - \mathrm{Y}\right\|_{\mathrm{F}}^{2} + \frac{\lambda}{2}\|\mathcal{X} - [\![\mathrm{A,\ B,\ C}]\!]\|_{\mathrm{F}}^{2} + \beta\|\mathrm{W,V,A,B,C}\|_1$$
$$\hat{y}_{ij} = \textstyle\sum_{i=1}^{s}\sum_{j=1}^{t}(\mathrm{A}_i\mathrm{W}^{\mathrm{T}} + \mathrm{B}_j\mathrm{V}^{\mathrm{T}})\,\mathrm{x}_{ij}^{\mathrm{T}} \quad (5)$$

ii) Tensor multi-task regression comprehends spatial latent factors and model parameters (TMTR-s):

$$\min_{\mathrm{W,A,B,C}} \frac{1}{2}\left\|\hat{\mathrm{Y}} - \mathrm{Y}\right\|_{\mathrm{F}}^{2} + \frac{\lambda}{2}\|\mathcal{X} - [\![\mathrm{A,\ B,\ C}]\!]\|_{\mathrm{F}}^{2} + \beta\|\mathrm{W,A,B,C}\|_1$$
$$\hat{y}_{ij} = \textstyle\sum_{i=1}^{s}\sum_{j=1}^{t}\mathrm{A}_i\mathrm{W}^{\mathrm{T}}\,\mathrm{x}_{ij}^{\mathrm{T}} \quad (6)$$

iii) Tensor multi-task regression comprehends temporal latent factors and model parameters (TMTR-t):

$$\min_{\mathrm{V,A,B,C}} \frac{1}{2}\left\|\hat{\mathrm{Y}} - \mathrm{Y}\right\|_{\mathrm{F}}^{2} + \frac{\lambda}{2}\|\mathcal{X} - [\![\mathrm{A,\ B,\ C}]\!]\|_{\mathrm{F}}^{2} + \beta\|\mathrm{V,A,B,C}\|_1$$
$$\hat{y}_{ij} = \textstyle\sum_{i=1}^{s}\sum_{j=1}^{t}\mathrm{B}_j\mathrm{V}^{\mathrm{T}}\,\mathrm{x}_{ij}^{\mathrm{T}} \quad (7)$$

D. Experimental parameter and computational infrastructure settings

The following are the number and range of values for each parameter attempted during the research development and experiments.

- Hyperparameters:
  λ [0.001 0.01 0.1 1 10 100 1000 10000]
  β [0.001 0.01 0.1 1 10 100 1000 10000]
- The number of latent factors:
  r [1 2 3 4 5 6 7 8 9 10]

We maintained the number of latent factors in a small range to save computation time for the model training phase. It is challenging to determine the precise rank of a tensor since it is an NP-hard problem in most instances. As a result, the rank is typically obtained in practice by fitting numerous CP decompositions with different ranks until a reasonably "good" rank is derived.

The following are the computational infrastructures were utilised to conduct experiments in this research.

- Utilised software: MATLAB
- GPU: NVIDIA GeForce RTX 2070 with Max-Q Design
- CPU: Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz
- Amount of memory: 16GB

V. EXPERIMENTAL RESULTS AND ANALYSIS

A. Comparison with the standard regression models

The precision fertilisation prediction performance of the proposed spatio-temporal tensor multi-task regression and its variants with the standard regression models (including Ridge regression, Lasso regression, Elastic-Net, Bayesian ridge regression, Bayesian automatic relevance determination regression, Linear support vector regression, Sigmoid kernel support vector regression, Regression based on k-nearest neighbors, Decision tree regressor and Multi-layer perceptron regressor) was evaluated utilising the real-world winter wheat dataset. To the best of our knowledge, there is no general agricultural fertilisation datasets or fertilisation prediction methods, thus current research on precision fertilisation is mainly focused on algorithmic research for specific agricultural datasets and they cannot be applied to datasets from other research. Therefore, we have chosen to compare our approach performance with highly generalisable regression models (i.e., the standard regression models). For our proposed spatio-temporal tensor multi-task regression

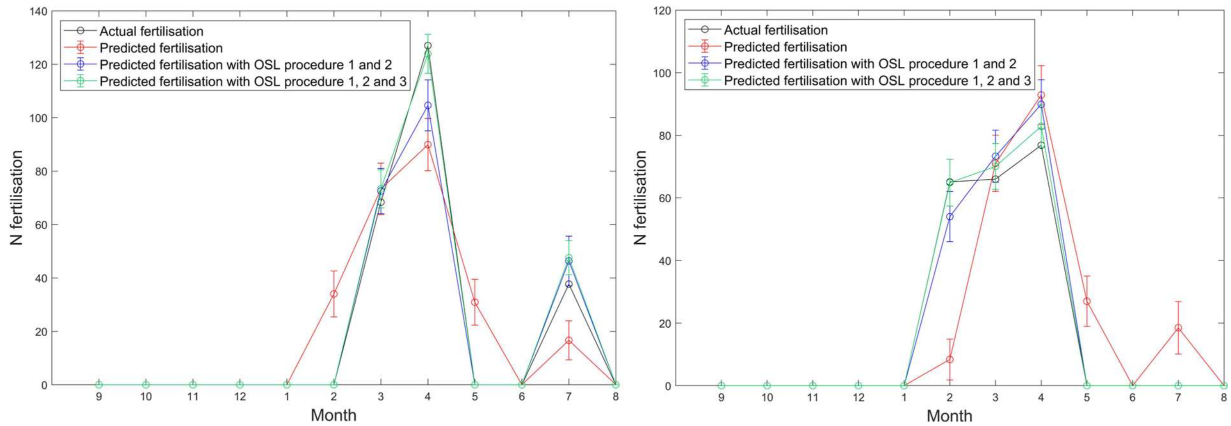

Fig. 4. Ablation studies for one-shot learning (OSL). The prediction performance comparison for procedures 2 and 3 of the proposed one-shot learning approach and the proposed MTL approach without the one-shot learning concept.

approaches, the dataset is encoded as a third-order tensor. The dataset is given as a matrix having dimensions of months × features for the standard regression models utilised for comparisons. Table II presents the experimental results for nitrogen fertilisation prediction.

The following are our primary observations:

1) The proposed spatio-temporal tensor multi-task regression approaches superior to standard regression models for the winter wheat dataset, demonstrating the effectiveness of the spatial and temporal latent factor hypothesis and multi-task learning concepts in our regression formulation.
2) The best performance was achieved by TMTR-b. This indicates that the prediction performance can be enhanced by having an integration of spatial and temporal models compared to modelling with spatial or temporal factors individually.
3) The proposed spatio-temporal tensor multi-task regression approach significantly improves prediction stability. The standard deviation of the 20 iterative experiments was smaller than that of the standard regression models of comparison. This may be due to the incorporation of spatial and temporal latent factors to the prediction model to enhance stability. In other words, all farms share a set of fertilisation temporal patterns by the proposed multi-task learning approach, while each farm has its own spatial distinctive properties, which enhances the stability and generalization of the proposed approach.

To sum up, the proposed spatio-temporal tensor multi-task regression approach can seamlessly integrate the spatial and temporal knowledge of agricultural multi-dimensional data to enhance the accuracy and stability of fertilisation prediction, while the proposed approach maintains a high level of interpretability, its computational process and results have a significant degree of transparency. However, the proposed approach has certain limitations. Firstly, our current real-world agricultural dataset only contains fertilisation data for winter wheat, so it is impossible to verify whether the proposed approach is precise in predicting fertilisation for other crops. Secondly, the current experiments are only for the prediction of nitrogen fertilisation, and more intensive data collection and experimental validation regarding the prediction performance for other types of fertilisers are required for future researches.

TABLE III

THE RANK OF AGRICULTURAL FEATURES ACCORDING TO THE WEIGHT PARAMETER VALUES FOR THE PROPOSED TMTR-B MODEL ON NITROGEN FERTILISATION

Rank	Agricultural feature	Weight parameter value
1	Monthly solar radiation (TJ/ha)	1.0143
2	Potential grain yield (t/ha)	0.9313
3	Monthly rainfall (mm)	0.8480
4	Soil pH value	0.7925
5	Mean daily temperature (°C)	0.6233
6	Working ha	0.5707
7	Soil phosphorus content (mg/l)	0.4517
8	Soil water holding capacity (mm)	0.1560
9	Soil potassium content (mg/l)	0.0839
10	Soil magnesium content (mg/l)	0.0454
11	Seeds sown per m^2	0.0159

B. Interpretability

Table III illustrates the rank of features in a descending sequence of weight parameter values of the proposed TMTR-b model (the best performing model). A higher ranking indicates a greater influence on the final prediction.

Table III shows that eleven agricultural factors can be classified into three levels based on their importance for winter wheat nitrogen fertilisation prediction. The first level is ranked 1 to 4, meaning it has a significant influence on winter wheat nitrogen fertilisation prediction. Two of these are meteorological factors, which are monthly solar radiation and rainfall, while the other two are potential grain yield and soil pH value. For solar radiation, higher amounts of nitrogen fertiliser can be applied in places that have optimal sunshine conditions to encourage vegetative and reproductive growth of crops, while fewer applications of nitrogen fertiliser ought to be applied in places with poor sunshine conditions to avoid crops from maturing late [47]. For potential grain yield, effective nitrogen fertiliser application enhances crop growth and development, resulting in greater yields and superior quality [48]. For rainfall, it has a significant impact on the rate of nitrogen loss. Rainfall-induced nitrogen loss is a major cause of agricultural pollution, and the larger the fertilisation rate, higher the nitrogen loss [49]. For soil pH value, the major direct influence to the fertilisation effects is on soil nutrient solubility. Furthermore, it will impact life activities of soil microorganisms, lowering the efficacy of soil nutrients indirectly [50].

Second level is ranked 5 to 7, their presence has a moderate influence on winter wheat nitrogen fertilisation prediction. Third level is ranked 8 to 11, indicated minimal or negligible influence on winter wheat nitrogen fertilisation prediction.

C. Ablation studies

In order to validate the effectiveness of the proposed one-shot learning approach on prediction performance, we present ablation studies in this section to compare the prediction performance between various procedures of the proposed one-shot learning approach and the proposed MTL approach without the one-shot learning concept.

Fig. 4 illustrates the prediction performance comparison for procedures 2 and 3 of the proposed one-shot learning approach and the proposed MTL approach without the one-shot learning concept. The experimental results demonstrate that the MTL model with the proposed one-shot learning approach has superior fertilisation prediction results than the MTL without the one-shot learning approach. Furthermore, for the proposed one-shot learning approach, procedure 3 better fits the actual fertiliser application curve than procedure 2.

VI. Discussion

Accompanied by the in-depth development of modern agriculture and the strengthening of national agricultural support, precision agriculture is receiving unprecedented attention in the world, and its emergence has generated tremendous impetus to agricultural production and sustainable agricultural management. Meanwhile, precision fertilisation as a direction in precision agriculture technology is receiving growing attention from agricultural production managers and agricultural scientists. The traditional experience-based fertiliser application with on-site guidance by agricultural experts is inefficient and difficult to adapt to the new situation of precision agriculture advocated by the contemporary world situation. The application of precision fertilisation to various types of crops is of far-reaching significance in solving the long-standing problem of fertilisation.

A. Challenges of precision fertilisation

There are inevitably challenges in the research and application of the current precision fertilisation technology. The first challenge is the data collection of fertiliser application for different crops and the construction of corresponding models. The fertilisation operations of different crops in the agricultural data domain vary widely, thus the fertilisation data of different crops need to be collected separately, which is a time-consuming and labour-intensive task, and there are no models that can accurately predict the precise fertilisation patterns of different crops.

The second challenge is the application data collection of diversified fertiliser types. The nitrogen fertiliser is the major fertiliser in crop duration and most farms apply nitrogen fertiliser to ensure their crop yields [51][52], thus in the agricultural data scarcity problem scenario, the nitrogen fertilisation data is the most abundant. The application of other types of fertilisers (e.g., phosphorus, potassium and sulphur) is primarily based on the health status, stress tolerance and disease resistance of the crop at the time. However, the combined application of various fertilisers is certainly important. Firstly, it can balance the nutrients, as a single fertiliser typically provides only one or two elements, whereas a combination of fertilisers can provide a well-balanced range of nutrients [53]. Secondly, it can enhance fertiliser efficiency, as the elements in different fertilisers can interact with each other, with certain elements assisting the plant to better absorb other elements. For instance, phosphorus promotes root development, which can help plants absorb other nutrients effectively [54]. Thirdly, it can reduce the burden on the environment, a rational fertiliser combination and application schedule can reduce over-reliance on a single fertiliser, which can contribute to reducing nutrient wastage and environmental pollution [55]. Finally, it can be adapted to soil conditions, as soil types and nutrient conditions vary in different regions, and by adapting fertiliser combinations, the specific requirements of soils and crops can be met precisely [56].

B. Future development trends of precision fertilisation

For the future development of precision fertiliser application, the priority is to further enhance real-world agricultural dataset. A well-developed and comprehensive dataset is the cornerstone of the development of prediction models, which requires the inclusion of various crop-specific climatic data, environmental data, soil nutrient data, soil property data, crop data, and application data of various fertilisers. The ultimate objective is to assist farmers and farm managers to effectively predict and manage the types of fertilisers to be applied to various crops, with the timing and amount of fertiliser applications. Therefore, the prediction model must have a comprehensive dataset, model library, and the ability to learn and update in order to provide precise and stable prediction results.

The second trend is the development of integrated precision fertiliser application systems. It can enhance the functions of the system and facilitate its application by integrating the precision fertiliser application prediction models with databases, model libraries, geographic information systems, remote sensing technologies, etc. The integrated precision fertiliser application system can provide various functions including fertilisation process prediction, monitoring, decision-making, training, etc. Furthermore, it can be networked to enable rapid transmission and information sharing.

VII. Conclusion

In this research, agricultural data from diverse farms were collected and merged into a real-world agricultural dataset, and then we proposed an precision fertilisation prediction approach based on the spatio-temporal multi-dimensional tensor multi-task learning integrated with one-shot learning approach, which constructs the prediction model based on the spatio-temporal input information of individual farms, conducts multi-task regression utilising the spatio-temporal latent factors obtained from the tensor decomposition approach as multi-task relationships, and then the proposed one-shot learning approach utilises Marxian distance similarity calculations to evaluate the similarity of environmental information between the target farm and existing real-world farms as the decision factor on whether to transfer the fertilisation temporal pattern of existing farm to the target farm, and then calculates the final prediction. The prediction model can be utilised to calculate the optimal amount and timing of various fertiliser applications in order to prevent environmental hazards caused by over-fertilisation while maintaining the crop yield. The experimental results demonstrate that the proposed spatio-temporal multi-dimensional tensor multi-task regression integrated with one-shot learning approach can enhance the prediction accuracy and stability of agricultural fertilisation. It can assist farmers and managers with more rational and effective fertiliser usage, decreasing fertiliser pollution while sustaining or improving agricultural production.

Acknowledgment

This research is supported by the Innovate UK (Project 107462: Farmer-centred Interoperable Mobile-Cloud System: Integrating Data from Farming Activities and Environmental Information for Sustainable Fertiliser Management). We would like to thank Innovate UK for funding this research.

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