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Abstraction, truth, and free logic

Bahram Assadian

University of Leeds, UK

Abstractionism is the view that Fregean abstraction principles underlie our knowledge of the existence of mathematical objects. It is often assumed that the abstractionist proof for the existence of such objects requires 'negative free logic' in which all atomic sentences with empty terms are false. I argue that while negative free logic is not indispensably needed for the proof of abstract existence, there is a motivation for it—along broadly Fregean lines. The standard motivation for negative semantics rests on the explanation of truth in terms of reference. This line of reasoning, however, is not available in a context in which the reference of abstract terms must be proved, and not presupposed. I reverse the direction of explanation, thereby offering a novel motivation, Truth Priority, for the use of negative semantics. Some of the implications of Truth Priority for the abstractionist conception of ontology and reference will also be explored.

Keywords: abstractionism; neo-Fregeanism; free logic; negative free logic; Frege's Context Principle; truth priority; abstract objects.

I. Abstractionism and free logic

Abstractionism in the philosophy of mathematics is the thesis that Fregean abstraction principles play an essential role in our knowledge of mathematical truths, the existence of mathematical objects, and our capacity to effect singular reference to these objects. The general form of an abstraction principle can be symbolized as follows:

(AP) $\quad \S\alpha = \S\beta \leftrightarrow \alpha \sim \beta$

where '§', which is intended to mean 'the abstract of', is a term-forming operator that applies to variables α and β, and $\sim$ stands for an equivalence

Correspondence to: Bahram Assadian, bahram.assadian@gmail.com

relation on the entities over which α and β range. AP states that the abstract of α —i.e. the value of the abstraction function §—is identical to the abstract of β if and only if α and β are related by $\sim$. (Here and below, I will omit the initial universal quantifiers $\forall\alpha$ and $\forall\beta$).[1]

An important example of an abstraction principle is known as Hume's Principle (HP), which defines the cardinality operator # by specifying that the cardinal number of the Fs is identical to the cardinal number of the Gs precisely when there is a one-to-one correspondence between the Fs and the Gs:

$$\text{(HP)} \qquad \#F = \#G \leftrightarrow F \approx G$$

where '$\approx$' stands for the equivalence relation of equinumerosity between the Fs and the Gs.[2]

Since Frege and his neo-Fregean abstractionists—notably Hale and Wright (2001), Heck (2011), and Linnebo (2018)—allow for semantically complex singular terms such as 'the abstract of α' or 'the number of Fs', they must leave the possibility open that a singular term may have no reference. As Tennant writes, the neo-Fregean abstractionists cannot assume,

> that all singular terms denote just by virtue of being grammatically well-formed. Instead, we have to take seriously the possibility of 'empty', or non-denoting singular terms, even when they are grammatically well-formed. What we need, in short, is a *free* logic. (Tennant, 1997: 311)

MacFarlane (2009: 446) also stresses on free logic for abstractionism: If all abstract expressions of the form '§α' are taken to be non-empty, such as the individual constants of the standard non-free logic, there would be a trivial proof for the existence of abstracts, with no reliance on the associated abstraction principle: the law of identity, $\forall x(x = x)$, entails, by Universal Elimination, $\S\alpha = \S\alpha$, which entails, by Existential Introduction, $\exists x(x = \S\alpha)$. However, this way of establishing the existence of abstracts seems to be epistemologically entirely pointless. Abstractionism is, after all, an attempt to show that our knowledge of the existence of abstracts rests on a particular form of definition, which ties the existence of abstracts to the truth of identity sentences whose truth-conditions are specified in terms of the equivalence relations given on the right-hand sides. Therefore, any attempt to prove the existence of abstracts

[1] Various forms of abstractionism have been defended by Wright (1983), Hale (1987), Tennant (1987, 1997, 2004), Hale and Wright (2001), Fine (2002), Heck (2011), Rayo (2013), and Linnebo (2018).

[2] Given suitable definitions, every true sentence in the language of the second-order Dedekind–Peano Arithmetic is a logical consequence of HP. This result has come to be known as Frege's Theorem. That the result could be established without using Frege's ill-fated Basic Law V was hinted in Parsons (1965) and then proved by Wright (1983). See also Boolos (1990).

without relying on abstraction principles will undercut one of the main motivations behind abstractionism.[3]

The abstractionist proof for the existence of abstracts—let us call it the Master Argument—employs abstraction principles as follows:

(1) $\S\alpha = \S\alpha \leftrightarrow \alpha \sim \alpha$	An instance of AP
(2) $\alpha \sim \alpha$	Equivalence relation $\sim$
(3) $\S\alpha = \S\alpha$	2, 3
(4) $\exists x(x = \S\alpha)$	3, Existential Introduction

The abstractionists have always emphasized that the truth-value of the left- or the right-hand side of any instance of AP is no part of the stipulation. What is stipulated to be true, rather, is that the matching sides of AP have the same truth conditions. In this sense, (3) does not follow from the law of identity; rather, it rests on (1) and on the satisfaction of the condition given on its right-hand side, which is independent from the stipulation of the truth of the biconditional. Thus, in accordance with the demands of free logic, the stipulation of the truth of AP, as such, leaves entirely open the question of whether expressions of the form '$\S\alpha$' have reference or not.[4]

The goal of this paper is to explore the role of free logic in abstractionism. In Section II, I introduce the main variations of free logic—positive free logic, negative free logic, and neutral free logic—and their role in the Master Argument. It is often said that the logic needed for the Master Argument cannot be a positive free logic. For in this branch of free logic, (3) could be true even if '$\S\alpha$' is empty—just as 'Pegasus = Pegasus' is true in positive semantics, even though 'Pegasus' is empty. Accordingly, Existential Introduction cannot be applied to identity sentences linking empty terms. Thus, (4) does not follow from (3). In negative semantics, by contrast, an atomic sentence cannot be true unless its ingredient singular terms are all non-empty. Thus, (4) does follow from (3). As I argue in Section III, however, there is a system of positive free logic in which the proof of abstract existence goes through. (I do not claim that the particular version of positive free logic that I will consider is the *only* way to modify positive free logic so as to be usable by the abstractionists in the present context.) There is, therefore, no indispensable need to use

[3]It would be a bit quick to recommend—as MacFarlane does—free logic to the abstractionists solely on the basis of this observation. The abstractionists would not include '§' in their language unless they antecedently gave a meaning to it. And in accordance with their inferentialism—see, for instance, Hale and Wright (2000)—'§' is to have the reference (if there is such a reference) that will make its associated abstraction principle true. All the same, as said above, the key motivation for using free logic stems from a more general need for handling empty terms. Linnebo (2018), for instance, works with abstraction principles with *partial* equivalence relations. So, '$\S\alpha = \S\beta$' will be false for any argument for which the §-function is undefined, which is equivalent to the non-reflexivity of $\sim$. That is, $\alpha \sim \alpha \leftrightarrow \exists x(x = \S\alpha)$.

[4]For more on this proof, see Wright (1983: 147), Hale and Wright (2001: 10; 146, n. 48; 309–10; 2008; 2009), Linnebo (2009: 61–2), and MacFarlane (2009: 446–7).

negative free logic. All the same, as I show, the proof in that particular variant of positive free logic has some limitations that do not arise for its counterpart in negative free logic. This places greater weight on reasoning in accordance with negative systems within abstractionism.[5]

The question of Section IV is whether the abstractionists' use of negative semantics is philosophically motivated. In his classic paper, 'Truth and singular terms', Burge (1974) motivates negative semantics by arguing that it retains the standard explanation of truth in terms of sub-sentential reference: Since 'Pegasus' is empty, 'Pegasus = Pegasus' is false. In Burge's view, the positive free logician cannot explain the truth of 'Pegasus = Pegasus' along this line. Plausible as it may be, in the present dialectical situation, the standard motivation for the use of free logic, which rests on the explanation of truth in terms of sub-sentential reference, is not available to the abstractionists who need to give an explanation in the opposite direction; they must prove, and not presuppose, that abstract terms of the form '$\S\alpha$' have referents.

To fill this gap, I provide a different motivation for the use of negative semantics. The key suggestion, initially hinted in Payne (2013a: 26), is to reverse the order of the explanation of truth and reference in terms of what I call Truth Priority: The reference of sub-sentential expressions is to be explained in terms of the truth of whole sentences in which they feature. My plan is to fill out in detail Payne's suggestion (though I do not claim that he would endorse my position). As I discuss in Section IV, Truth Priority is a *metasemantic* thesis, which seeks to *explain* or *ground* the referentiality of singular terms: What explains the fact that '$\S\alpha$' refers to an object is that it occurs in true atomic sentences embedded in extensional contexts. There is thus a tight connection between Truth Priority and Frege's famous Context Principle, which states that 'only in the context of a sentence does a word stand for anything'. The Context Principle can be taken to sum up this metasemantic thesis that semantic relations are to be explained at the level of sentences, rather than at the level of sub-sentential expressions and their references.

Along similar lines, Wright writes that the 'possession of reference is imposed on a singular term by its occurrence in true statements of an appropriate type' (Wright, 1983: 53). If Wright's passage is to be read as a thesis concerning the *explanation* of the reference of a singular term in terms of its figuring in true atomic sentences of an appropriate type, then Truth Priority will correspond to this aspect of the Context Principle. Nonetheless, the thesis behind Wright's passage can be construed as a weaker claim, which merely states the sufficient condition for a term to be referential or non-empty. I will call this weaker thesis Referentiality, which states that if a singular term features in a

[5]For more discussion on negative free logic in the context of abstractionism, see Hale and Wright (2003: 260; 2008: Section V, and 2009: 464–5), Tennant (1997, 2004), Payne (2013a: §2.2; 2013b: §1), Rayo (2013: 15, n.5), Linnebo (2018: 49–50), and deRosset and Linnebo (2023: 6). For objections to the use of free logic within abstractionism, see Shapiro and Weir (2000: 188), Potter and Smiley (2001: 336–7), Rumfitt (2003: 208–9), and MacFarlane (2009: 447–9).

true atomic sentence in an extensional context, then it refers to an object. The difference between Truth Priority and Referentiality will be discussed in detail in Section IV.[6]

In Section V, I locate Truth Priority in a broader context. The first question concerns the abstractionist conception of ontological claims: Can one establish the existence of abstracts solely on the basis of Truth Priority, with no reliance on abstraction principles? If so, abstraction principles, contrary to what the abstractionists hold, do not have any special place in establishing ontological claims. I argue that Truth Priority is what sanctions the Master Argument in which abstraction principles play an essential role. On the other hand, Truth Priority without abstraction principles will be left completely idle in determining the semantic features of the sentences in which such abstract expressions figure.

The second question concerns reference: given the role of Truth Priority in the explanation of the referentiality of singular terms, can it contribute towards picking out uniquely determined referents for the putative singular terms, and thereby towards dispelling the threat of the indeterminacy of reference? The answer is negative: Truth Priority is a minimal thesis that merely specifies what it is for a singular term to refer to a particular object—whatever that object might be. Its task is not to identify certain features concerning the use of a singular term that would determine a particular object as its referent.

II. Varieties of free logic

Free logics are traditionally divided into the following three groups, depending on their treatment of the truth-value of atomic sentences containing empty singular terms:

Positive free logic
Some atomic sentences containing empty singular terms are true; the rest is false.

Neutral free logic
All atomic sentences containing empty singular terms are undefined.

Negative free logic
All atomic sentences containing empty singular terms are false.[7]

Two points are in order. First, this taxonomy is not exhaustive. For example, as we shall see in the next section, the above characterization of positive free logic allows for a system in which some atomic sentences containing empty terms

[6] For similar passages to Wright's, see Hale (1987: 12–14), Hale and Wright (2001: 8), and Dummett (1981: 497, 504 and 509). Some authors—for instance, Ebert (2015: 24)—have read the Context Principle along the lines of Referentiality.

[7] For introductions to systems of free logic, see Lambert (2001) and Bencivenga (2002).

are true; some of the remaining sentences are false, and some others are undefined: they are truth-valueless. All the same, in this section, we will focus on the above standard taxonomy. Second, I will not discuss neutral free logic in this paper, for two reasons. First, because of its unacceptable logical consequences, most free logicians have rejected this strategy for handling empty terms. Second, as we shall discuss below, neutral free logic, in the context of the Master Argument, will have the same consequences as negative free logic.[8]

One of the main motivations that is often given in favour of positive free logic is that it is only this version of free logic that can account for true identity sentences involving empty names. For example, in his defence of positive free logic, Lambert points out that 'Pegasus = Pegasus' "follows from the unexceptionable identity principle '$x = x$'" (Lambert 1991: 25). Thus, since $t = t$ holds for any term t, the rule of Identity Introduction in the standard positive free logic will be as follows:

$$(=I^+)\ \frac{}{t\ =\ t}$$

But the positive free-logician does not want to infer 'Pegasus exists' from 'Pegasus = Pegasus'. From the atomic sentence $\varphi(t)$, they infer $\exists x\varphi x$ only if there is the additional premise that $\exists x(x = t)$. Existential Introduction $(\exists I^+)$ and Universal Elimination $(\forall E^+)$ must accordingly be restricted: They must be weakened by a premise that the terms involved are non-empty. The resulting rules are as follows:

$$(\forall E^+)\ \frac{\forall x\varphi(x) \qquad \exists x(x\ =\ t)}{\varphi(t)} \qquad\qquad (\exists I^+)\ \frac{\varphi(t) \qquad \exists x(x\ =\ t)}{\exists x\varphi(x)}$$

In negative free logic, on the other hand, all atomic sentences in which an empty singular term features are false. In particular, even $t = t$ is false when t is empty. The rule of Identity Introduction must thus be restricted:

$$(=I^-)\ \frac{\exists x(x = t)}{t = t}$$

In the negative system, Existential Introduction and Universal Elimination are exactly like $(\exists I^+)$ and $(\forall E^+)$. In particular, if Existential Introduction in negative free logic is unrestricted, then from $\neg\exists x(x = t)$, where t is empty, we can derive $\exists y\neg\exists x(x = y)$, which is logically false in this system. Thus, Existential Introduction in negative free logic must be restricted:

$$(\exists I^-)_1\ \frac{\varphi(t) \qquad \exists x(x\ =\ t)}{\exists x\varphi(x)}$$

There is, however, a subtlety here. Since an atomic sentence in negative semantics can be true only if all of its constituent terms are non-empty, then Existential Introduction can be restricted to atomic sentences:

[8]Some of the logical and philosophical problems of neutral systems have been discussed by Evans (1982: 24), Lehman (2002: 233–237), and Sainsbury (2005: §2.3).

$(\exists I^-)_2 \quad \dfrac{\varphi(t)}{\exists x(x = t)}$ where $\varphi(t)$ is an atomic formula containing the closed term t

$(\exists I^-)_2$ is what Tennant (2004: 110) calls the Rule of Atomic Denotation.[9] Part of the motivation behind invoking this rule is pragmatic. As Tennant (2007: 1061) observes, in order to infer $\exists x\varphi(x)$ by an application of $(\exists I^-)_1$, we need to warrant one of its premisses, viz. $\exists x(x = t)$, which is of the same form as the sought conclusion. One way—or perhaps, as Tennant holds, the only way—to terminate this regress is to allow $\exists x(x = t)$ to be inferred directly from any atomic sentence of the form $\varphi(t)$—and this is what $(\exists I^-)_2$ captures.

In the previous section, we said that the logic of the abstractionists' Master Argument must be free. But which free logic? As Payne (2013a: §2.2.1; 2013b: §1) points out, positive free logic is of no use. For in order to apply $(\exists I^+)$ in line (3), i.e. $\S\alpha = \S\alpha$, we must already have (4), i.e. $\exists x(x = \S\alpha)$, as an additional premise. The reason is that according to Existential Introduction in positive free logic, the atomic sentence $\varphi(t)$ entails $\exists x\varphi x$ only if there is the additional premise that $\exists x(x = t)$. However, in the context of the Master Argument, having (4) as an additional premise would clearly beg the question: The existence of abstracts must be proved, and not assumed. In negative free logic, however, this issue does not arise. An atomic sentence is true only if all of its constituent singular terms are non-empty; so, the truth of (3) already ensures that its ingredient term is non empty. There is, therefore, no need to assume an additional existential premise—as $(\exists I^-)_2$ makes plain.

In neutral free logic, all atomic sentences containing empty singular terms are undefined. This sanctions an inference from the truth of an atomic sentence featuring a term t to the existence of the referent of t. The truth of (3), therefore, already ensures that the abstract term is non empty. In this context, the behaviour of neutral free logic is exactly like that of negative free logic: In both cases, the move from (3) to (4) is permitted.

III. A variation of positive free logic

As we saw in the previous section, for the Master Argument to go through, it would appear that the background logic *must* be a negative free logic. The abstractionists have emphasized this point.[10] For instance, Hale and Wright write:

> These remarks are, of course, based on the assumption that the relevant kind of free logic is one which restricts the introduction of identities–the availability of statements

[9] See also Tennant (1987: 276; 2007: 1061). For broadly similar restrictions of Existential Introduction to atomic formulas in negative free logic, see Burge (1974: 312) and Evans (1982: 37).

[10] See also the works cited in footnote 5.

of the form 'a = a' without further assumption as premises in proofs. (Hale & Wright, 2003: 260, n. 9)

They suggest that Identity Introduction must be restricted, which goes against the central component of positive free logic. In this section, first I present a slightly modified version of positive free logic, and then show how it can accommodate the proof of the existence of abstracts. After that, I discuss some of the limitations of the proof.

Traditionally, in positive semantics, some atomic sentences containing empty singular terms are true—for instance, identity sentences of the form $t = t$. However, since only some sentences with empty terms are true, this allows that some of the remaining sentences are false, and some others are undefined. In particular, consider a variation of positive semantics in which $t = t$ is true, for any term t; whereas $s = t$ is undefined if s and t are distinct empty terms—for example, 'Santa = Pegasus'. More generally:

(i) $s = t$ is true iff s and t are non-empty terms referring to the same object;
(ii) $t = t$ is true, whether or not t is empty;
(iii) $s = t$ is undefined iff s and t are distinct empty terms;
(iv) $s = t$ is false iff s is empty and t is not.

(The rules for quantifiers correspond to those of the standard positive free logic we sketched in the previous section. It should also be noted that (iv) does not play any role in the following argument.) The above clauses allow us to see to what extent this variant of positive free logic provides the abstractionists with their Master Argument. For two distinct variables α and β, let $\alpha \sim \beta$ be a logical truth. Then consider the following proof:

(5) $\S\alpha = \S\beta \leftrightarrow \alpha \sim \beta$
(6) $\alpha \sim \beta$
(7) $\S\alpha = \S\beta$
(8) $\exists x(x = \S\alpha)$

Since α and β are taken to be distinct, the ingredient singular terms $\S\alpha$ and $\S\beta$ must also be distinct. But these terms cannot be empty: If the terms flanking the identity sign in (7) were distinct and empty, the identity sentence would be, by (iii), undefined; but it is true. Thus, $\S\alpha$ and $\S\beta$ must be distinct and *non-empty*. By (i), they refer to the same object. Hence, (8) follows from (7).

That the existence of abstracts can be proved within this version of positive free logic shows that there is no *indispensable* need to use negative free logic. However, the proof has obvious limitations. First, it works only when α and β are distinct variables. For otherwise, we would have $\S\alpha = \S\alpha$ which is, by (ii), true even if the term $\S\alpha$ is empty. But in that case, we would not be able to existentially generalize to derive the existence of the abstract in (8).

In addition, (6) does not follow from the equivalence relation $\sim$ of the right-hand side of the associated abstraction principle; and so, it needs an independent justification. This contrasts with the (1)–(4) proof in which (2), i.e. $\alpha \sim \alpha$, immediately follows from the equivalence relation. In some cases, of course, one might be able to provide a justification for (6) in a *logical* way. For example, in the case of HP, we can use two instances of the second-order comprehension principle (i.e. the principle that any condition defines some property, namely the property whose instances satisfy the condition), with provably equinumerous extensions, to get equinumerous concepts F and G, for distinct predicates 'F' and 'G'. It is not clear, though, if this method can be applied to all abstraction principles.[11]

Furthermore, there is an explanatory challenge for the positive free logician to account for the truth of identity sentences of the form $t = t$, for any term t. As mentioned in the previous section, one reply is to say that $t = t$ follows from the principle of identity, $\forall x(x = x)$. But it is far from clear how this inference is to be justified given that the unrestricted Universal Elimination is banned in any system of free logic.[12] The positive free logician thus owes us an account of the truth of $t = t$. I do not have any *general* reply to offer in favour of them. However, in the particular context of abstractionism, one line of reply is to restrict (ii) to abstract terms; i.e. $t = t$ is true if and only if the term t has the form $\S\alpha$, for some abstraction operator §. The idea is to account for the truth of $\S\alpha = \S\alpha$ in terms of $\alpha \sim \alpha$, and not in terms of the principle of identity.[13]

None of these worries arises in negative free logic. The Master Argument goes through regardless of whether the identity sentence of the left-hand side of the abstraction principle links the same term or two distinct terms. In addition, since in negative free logic, Identity Introduction ($=I^-$) is restricted to non-empty terms, there is no need to justify the truth of identity sentences linking the same empty term.

Although negative free logic is not indispensably needed for the proof of abstract existence, in the next section, I will articulate and defend a philosophical motivation to adopt negative semantics in reasoning with abstraction principles.

[11] The reasoning behind the above example relies on second-order comprehension principle; i.e. $\exists F \forall x(Fx \leftrightarrow \varphi(x))$, where F is a second-order variable and φ is any formula of the language in which F does not occur free. However, some abstraction principles, such as Frege's direction principle, are *first-order*, in the sense that 'α' and 'β' range over first-order entities such as lines. It is, therefore, less clear how one can get $\alpha \sim \beta$ for distinct terms 'α' and 'β'. It seems that the situation is more promising for *second-order* abstraction principles. Thanks to Salvatore Florio for this point.

[12] For more on this challenge, see Burge (1974: 318) and Sainsbury (2005: 66–7).

[13] Thanks to an anonymous referee for this suggestion.

IV. Motivating negative free logic

The abstractionists have advanced various considerations in favour of laying down abstraction principles—for example, to implicitly define the abstraction function; to specify the truth-conditions of identity sentences in which abstract terms feature; or to explain a 'sortal concept' with a suitable criterion of identity and a criterion of application.[14] However, these considerations, as such, do not tell in favour of or against a negative or positive free logic in reasoning with abstraction principles.

If the abstractionists reply that it is *only* in negative free logic that abstraction principles entail the existence of abstracts, their claim is either false or question-begging. It is false, because, as we discussed in the previous section, there is at least a variant of the positive system in which the proof of the existence of abstracts goes through. On the other hand, even if there is no existence-importing positive system, then by preferring negative over positive free logic, the abstractionists would make a question-begging move in favour of the existence of abstracts. For the use of an existentially importing notion of identity would beg the question in a context in which what needs to be proved is the very existence of a class of objects. As Shapiro and Weir write:

> [I]f the neo-logicist [the abstractionist] assumes the innocence of standard non-free first-order logic then he or she begs the question against opponents of neo-logicism. If not [that is, if they use free logic], then if identity does not have existential import [as in positive free logic], Frege's Theorem fails whereas if it does have existential import [as in negative free logic], then Frege's Theorem holds but the interpretation of the required abstraction principles...will beg the question in much the same way. (Shapiro and Weir, 2000: 188)[15]

Following a suggestion by Payne (2013a: 26), I argue, against this line of objection, that the abstractionists are in a position to offer an independent justification for the use of negative semantics.

Burge (1974) and Sainsbury (2005) argue that negative free logic is the appropriate free logic to use in the framework of a semantic theory with languages containing empty terms. In their view, the problem with positive free logic is that it allows for true identities involving empty terms, but as we discussed in the previous section, it is far from clear how the positive free logician

[14] Understanding any general (first-level) predicate F requires grasping what is often called a *criterion of application*—knowing what is required for F to apply to a given object. This goes both for adjectival predicates, such as 'red' and 'smooth', as well as *sortal* predicates such as 'horse' and 'number'. Sortal predicates are distinguished from merely adjectival ones by their being associated with not only a criterion of application, but also with a *criterion of identity.* For this definition of sortal predicates, see Dummett (1981: 73–6 and 546–50), Hale and Wright (2001: 367), and Linnebo (2018: 164).

[15] See also Potter and Smiley (2001: 336–7) and MacFarlane (2009: 448–9) for objections along similar lines.

can account for such truths. Burge formulates the fundamental reason for rejecting true identities involving empty singular terms as follows:

> An extremely intuitive feature of Tarski's theory of truth is that it explicates what it is for a sentence to be true in terms of a relation (satisfaction) between language (open sentences) and the world (sequences of objects)...It is difficult to see how the purported truth of, say, ['Pegasus = Pegasus'] can be explicated in terms of a correspondence relation...sentences expressing identities are true or false *by virtue of* the relation that the identity predicate and its flanking singular terms bear to the world–never merely by virtue of the identity of the singular terms. (Burge, 1974: 322–3; emphasis added)

There are two different theses here that must be kept apart. The first is what we see in a standard truth-conditional *semantic* theory—with clauses such as $F(t)$ is true if and only if the referent of the term t has the property expressed by the predicate F. The second thesis concerns *metasemantics*, which seeks to provide an explanation of semantic facts—with clauses such as $F(t)$ is true in virtue of the fact that t refers to an object that has the property expressed by F; or, conversely, t refers to an object that has the property expressed by F in virtue of the fact that $F(t)$ is true. The locution 'in virtue of' in these clauses and also in Burge's passage quoted above, suggests that the central explanation in metasemantics is *grounding*, where the relata of the grounding relation are semantic facts.[16]

Thus construed, I present Burge's motivation for negative semantics in terms of the following metasemantic thesis:

> **Reference Priority**. What it is, partially, for atomic sentences to be true is for the ingredient singular terms to be referential or non-empty.

Reference Priority tells us that the fact that the atomic sentence $F(t)$ is true is to be grounded in the fact that t refers to an object that has the property expressed by F. Reference Priority thus captures the standard order of the explanation of truth and reference. In Burge's view, the negative free logician is in a position to explain or ground the falsity of 'Pegasus = Pegasus' in terms of the fact that 'Pegasus' does not refer to anything. The positive free logician, by contrast, cannot appeal to Reference Priority: 'Pegasus = Pegasus' is true, but its truth cannot be explained in terms of a reference relation between 'Pegasus' and what, if anything, the term refers to.

All the same, in the present dialectical situation, this kind of motivation is not available to the abstractionists. For according to Reference Priority, subsentential singular terms featuring in a true atomic sentence stand in some reference relations to individual objects. But what the abstractionists must account for by means of the Master Argument is precisely the obtaining of such

[16]For more on this construal of metasemantics, see Burgess and Sherman (2014). See Rosen (2010) for a more general discussion on the logic of ground and some of its philosophical applications.

relations for abstract expressions. I maintain, however, that they can motivate the use of negative semantics in terms of the following metasemantic thesis, which runs in the opposite direction of Reference Priority:

> **Truth Priority**. What it is for a singular term to refer to an object is for it to feature in true atomic sentences embedded in extensional contexts.

Truth Priority, as I advertise it here, is a thesis emerging from a broadly holistic picture in which reference is not identified with some antecedently given relation, but is instead constituted by, or is explained in terms of, the truth of whole sentences. This approach sharply contrasts with the Reference Priority metasemantic strategy once mocked by Davidson (1977) as the 'building block' account of reference, which gives priority to sub-sentential reference.

Truth Priority tells us that the fact that singular terms refer is to be explained in terms of the fact that they feature in true atomic sentences. And herein lies its significance for the use of negative free logic. The referentiality of singular terms is to be explained in terms of the truth of atomic sentences in which they occur. This also points to the fact that Truth Priority is not available to the positive free logician. For her, the truth of $\S\alpha = \S\alpha$ does not ensure that the sub-sentential expression is non-empty. She thus presupposes a prior assurance for sub-sentential reference independently of the truth of whole sentences.

To appreciate the fundamental message behind TruthPriority, we must distinguish it from the following weaker thesis:

> **Referentiality**. If a singular term features in a true atomic sentence embedded in an extensional context, then it refers to an object.

Referentiality merely states a sufficient condition for a singular term to be non-empty, whereas Truth Priority explains a singular term's being non-empty by its occurrence in a true atomic sentence. Nevertheless, Truth Priority entails, but is not entailed by, Referentiality. According to Truth Priority, what it is for a singular term to refer to an object is for it to feature in a true atomic sentence. So, given the assumption that the term features in a true atomic sentence, it follows that it refers to an object; hence, Referentiality. But from Referentiality alone, we cannot infer the stronger thesis concerning what it takes for a singular term to refer to an object. To put the point more precisely: Truth Priority is a thesis in metasemantics, and, as said above, one way to understand it is in terms of grounding between semantic facts. In the grounding literature, the common assumption is that a grounding claim entails, but is not entailed by, the corresponding conditional; i.e. from 'p grounds q', we can infer that 'if p, then q'. So is the case with the move from Truth Priority to Referentiality: The fact that the atomic sentence $F(t)$ is true grounds the fact that t is non-empty; hence, if $F(t)$ is true, then t is non-empty.

The question we have been discussing concerns the abstractionists' justification for the use of negative free logic in their argument for the existence of abstracts. From this standpoint, it must be clear that Referentiality is of no use here: It merely restates the characteristic feature of negative semantics. It is, in effect, a metalinguistic reading of Existential Introduction in negative free logic, $(\exists I^{-})_2$, and so fails to provide the abstractionists with a motivation for employing negative semantics.

This task, however, can be undertaken by Truth Priority, which grounds reference in terms of truth, and thereby does not bestow upon reference an independent life that would make it possible to determine that a class of expressions have (or have no) reference prior to, and independently from, the truth of the atomic sentences in which they figure. In the next section, we pursue some of the variations on this theme, and locate them in a broader context.[17]

V. Truth priority in a broader context

The aim of this section is to locate Truth Priority in a broader context. First, I address its status in the abstractionist conception of ontological questions, and then I explore its role in the explanation of reference.

Eklund (2006, 2016) argues that there are 'two separate strands of thought' in the abstractionist metaontology. The first is essentially encapsulated in the Master Argument presented in Section I: The truth of an abstraction principle allows us to move from the truth of a sentence of the form $\alpha \sim \beta$ to the truth of a matching sentence of the form $\S\alpha = \S\beta$, which entails the existence of an abstract. The second ontological argument, however, does not rest on abstraction principles; Truth Priority alone does the job. The idea is, roughly, as follows. Suppose that there are true sentences that purport to refer to (abstract) objects of a certain kind K that can consistently exist. Then, by Truth

[17] See Eklund (2006, 2016) for more discussion on what he calls 'the constitutive priority of truth over reference', though not in the context of free logic. He claims to find this thesis in Dummett (1956) and Wright (1983; 1992). The distinction between Truth Priority and Referentiality corresponds, respectively, to the distinction Taylor (2021) makes between what he calls (TR) and (tr) – again, not in connection with free logic. Taylor's paper came to my attention when this paper was under review. Let me just add one potential source of difference between our views: Taylor formulates (TR) as follows: 'What it is for a singular term t to refer to an object is for t to feature in some true sentence of an appropriate type' (Taylor, 2021: 11506). In Taylor's view, the abstractionists' commitment to (TR) comes from their commitment to the Master Argument and also to the status of abstraction principles as implicit definitions. Although in the next section, I lend some support to the entanglement between (TR) and the Master Argument, in my view, the abstractionists' commitment to (TR) emerges from more general principles concerning semantic holism and Frege's Context Principle. See Davidson (1977) for more discussion. For a generalization (with respect to other types of linguistic expressions) of the priority of truth over reference, see Warren (2020: ch. 9).

Priority, reflection on the structure of such sentences—that, for example, they contain singular terms purporting to refer to Ks—yields that the K-terms refer, and that Ks exist. For instance, if p is a sentence in the language of pure arithmetic —e.g. '$2 + 3 = 5$'—and if ordinary arithmetical criteria qualify p as true, then we conclude that the singular term '2' featuring in p refers to an object. Given that the question of truth can be settled prior to the question of reference, if Ks can consistently exist, then Ks do exist. Since this latter type of argument leads to, as Eklund puts it, 'maximal ontological promiscuity', let us call it the Maximalism Argument.

Eklund holds that the Maximalism Argument is, in effect, an ontological argument that 'proceeds independently of abstraction principles' (Eklund, 2016: 87), and does not accord 'any special place' for them. It is thus unclear, or so Eklund points out, why abstraction principles should play such a central role in establishing ontological claims in the abstractionist programme. After distinguishing the above allegedly separate metaontological strands, Eklund writes:

> HP is a central plank in the neo-Fregean's philosophy of arithmetic. But given the neo-Fregean's ontological outlook as it has been laid out here [i.e. the priority of truth over reference], it is sufficient for the existence of numbers that the hypothesis that there are numbers is consistent (given that the question of the existence of numbers is not hostage to contingent empirical fact). But what then is the relevance of HP? (Can one not then equally well rely on the consistency of the axioms of Peano Arithmetic?) (Eklund, 2006: 105)

If Eklund's point is that HP, or any other admissible abstraction principle, does not have any direct ontological outputs, there is a sense in which he is right: Any abstraction principle is, after all, nothing more than a stipulatively true implicit definition serving to fix the meaning of the abstraction function. In fact, part of the attraction of abstraction principles is that they are merely meaning-constitutive stipulations, which do not 'arrogantly' stipulate the existence of abstract objects. The existence of such objects follows from the truth of an abstraction principle only given the truth of the right-hand sides. Thus, an abstraction principle, as such, does not stipulate abstract objects into existence.[18] If Eklund's point is that an abstraction principle is merely a conceptual truth or a meaning-constitutive stipulation with no direct ontological consequences, he is indeed recording his full agreement with (Hale and Wright's) abstractionism.

However, Eklund's observation seems to be entirely different. He maintains that the existence of numbers and other abstract objects can be made defensible merely in terms of Truth Priority, 'without appeal to abstraction

[18]For more on this crucial point, see Hale (2001: 43) and Hale and Wright (2000: 146–50; 2003: 262; 2008: 16–19.)

principles' (Eklund, 2016: 88). In his view, abstraction principles, contrary to what the abstractionists claim, do not have any special place in establishing ontological claims. So, Eklund must show how the abstractionists could establish the existence of abstracts independently of abstraction principles. It would be enough, it might be responded, to find a sentence that purports to refer to $\S\alpha$, and which is such that the only obstacle to its truth is the failure of some of its singular terms to refer. The relevant sentential context, however, cannot be $\S\alpha = \S\alpha$ or any other atomic sentence in which an abstract term features: For the reasons, we have discussed in the previous sections, the argument from $\S\alpha = \S\alpha$ to $\exists x(x = \S\alpha)$ is precisely what the abstractionists would not recourse to. In fact, the main point of the abstractionist proposal is that we do not have any epistemic and semantic handle on the truth of identity sentences linking abstract terms—and hence, on the existence of the corresponding abstracts—independently of, or prior to, abstraction principles, which specify the truth-conditions of such identities.

Truth Priority and abstraction principles are thus more entangled than Eklund envisages. On the one hand, without Truth Priority, abstraction principles cannot be used to prove the existence of abstracts. As we saw in the previous section, Truth Priority is what sanctions the Master Argument in which abstraction principles have an indispensable role. On the other hand, precisely because of the role that abstraction principles play in specifying the truth-conditions of the identity sentences linking abstract terms, Truth Priority without abstraction principles will be left completely idle in determining the semantic features of the sentences in which such expressions appear.[19]

The second point that I wanted to discuss about Truth Priority concerns its metasemantic role. As is explained in the previous section, the task of Truth Priority is to explain what it takes for a singular term to have a particular referent. This way of putting the point might present Truth Priority as a thesis about the *determinacy of reference*. This is an understandable temptation: Since a singular term, if it refers at all, refers to a particular object, there must be some explanation as to what it takes for it to have a particular referent. One may thus expect Truth Priority to provide us with the required explanation in terms of the truth(-conditions) of the sentences in which the relevant expressions figure.[20]

This is, however, an unjustifiably strong reading of Truth Priority. Suppose that an expression is referentially indeterminate, in the sense that there is a range of objects, none of which is the unique referent of the expression: No

[19]Some authors—for instance, Divers and Miller (1995)—have argued that Truth Priority would suffice to commit the abstractionists to a plenitudinous ontology of fictional characters. My argument for the abstractionists' commitment to an entanglement between Truth Priority and abstraction principles can be used to undermine such arguments.

[20]See Button and Walsh (2018: 46–7) for this reading. See also Linnebo's (2018: 92–3) objection concerning the 'inexplicability' of reference.

amount of reflection on our use of it—in a broad sense of 'use'—could determine a unique referent for it. There is no 'fact of the matter' as to what particular object the expression refers to. The task of Truth Priority is not to dispel the threat of referential indeterminacy, and thereby to bestow a uniquely determined referent upon the term. Truth Priority is not supposed to specify what features concerning the use of a singular term would pick out a particular object as its referent. Instead, it merely tells us that what it is for a singular term to refer to a particular object—whatever that object might be—is for it to feature in a true atomic sentence of the appropriate type.

Thus, Truth Priority, as such, is compatible with the indeterminacy of reference. If the abstractionists wish to alleviate this kind of indeterminacy, they must enrich their theory of reference by further facts concerning, for example, criteria of identity, applications of abstracts (in collecting, counting, measuring, etc.), Lewisian naturalness, a framework of 'real definitions', or whatever else that one must add to their metasemantics in order to secure determinate reference. Whether such constraints can give the abstractionists the determinacy of reference is a further question that goes beyond the scope of this paper.[21]

VI. Conclusion

To sum up, I have defended four main claims. First, the abstractionist's Master Argument for the existence of abstracts can be captured in a system of positive free logic. The proof, however, has obvious limitations that do not arise for its counterpart in negative free logic. Second, the abstractionists' use of negative semantics can be motivated in terms of a metasemantic thesis, Truth Priority, according to which singular terms refer in virtue of their featuring in true atomic sentences embedded in extensional contexts. Third, the existence of abstracts cannot be made defensible merely on the basis of Truth Priority without reliance on abstraction principles. From the abstractionist perspective, any warrant to regard abstract expressions as referring depends on the resources supplied by the associated abstraction principle. And fourth, Truth Priority, as such, does not entail any claim concerning what it is for a singular term to have a uniquely determined reference. In order to determine

[21] See Hale (1987: ch. 7) and Linnebo (2018: ch. 2) on the role of criteria of identity in this context. Hale (1987: 223–4) argues that a combination of HP with a further principle about counting (i.e. the number of Fs is n if and only if there are n Fs) can be used to avoid the indeterminacy of reference. A recent attempt in terms of real definitions has been made by Rosen and Yablo (2020). See Assadian (2019) for a critical examination of Hale's arguments for the determinacy of reference.

reference, abstractionists must enrich their metasemantics with some further principles.[22]

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