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The role of AI in engineering: Towards rapid inverse blast analysis

A. **Jay Karlsen**, *University of Sheffield, United Kingdom*
B. **Sam E. Rigby**, *University of Sheffield, United Kingdom*

ABSTRACT

As artificial intelligence grows increasingly commonplace in areas of analysis that were once the exclusive domain of the engineer, it is critical to establish the scope for cooperation and competition between the two. This paper therefore evaluates their individual performance in addressing a problem of extreme practical significance: increasing the computational efficiency of inverse explosion analysis. At present, methods for determining the equivalent yield and position of an explosive lack the speed necessary to meaningfully inform life-saving, post-blast response; a consequence of their use of inefficient exhaustive 'brute force' algorithms. Therefore, this paper details and evaluates two alternative searching routines, one representative of AI and the other physics-informed. Aiming to retain the equivalent charge yield and position estimation accuracy of traditional methods, but with reduced computation time, the first scheme is a genetic algorithm that uses the principles of machine learning optimisation to more efficiently explore the domain. The second, trilateration, continues to employ an exhaustive search, but on a smaller domain that is strategically constrained using prior understanding of the governing physics. The schemes' predictive accuracy and computation time are then assessed through the inverse analysis of blast wave arrival time data from six small-scale, free-field experiments. The alternative algorithms exhibit an insignificant reduction in accuracy compared to exhaustive search; the maximum increase in estimation error is 6.6 mm for position and equates to 0.21% of the true charge mass for the yield. There is, however, considerable increases in computational efficiency, with the genetic algorithm and trilateration requiring 1,600- and 109,000-times fewer computational iterations to complete, respectively. Overall, the implementation of a more intelligent searching routine within iterative inversion is proven to generate considerable reductions in computation without a significant loss in accuracy, thereby supporting the rapid analyses necessary to meaningfully inform the coordination of life-saving, post-blast response. Additionally, while artificial intelligence is seeing increasing integration within engineering, this work demonstrates the immense value that continues to be brought by a deep, physical understanding.

1. BACKGROUND

Explosive incidents, be they accidental or intentional, induce high-magnitude pressures and impulses which have the potential to result in significant structural damage and loss of life. The consequences of an explosion strongly depend on its yield, and distance from the source. As such, in order for effective and immediate life-saving action to be taken, the size and location of an explosion need to be accurately and rapidly estimated.

This necessitates some form of inverse analysis (effect → cause), typically where the forward problem (cause → effect) is iteratively solved until the difference between model and observation has been minimised. This is particularly challenging in the field of blast analysis due to the strongly nonlinear nature of shock waves. Further, more sophisticated methods for solving the forward problem (e.g. finite element analysis or computational fluid dynamics) – especially given the often excessive number of iterations required to solve the inverse problem – render current methods prohibitively time-consuming and lacking the versatility, robustness and accuracy needed to reliably estimate both charge yield and position.

The focus of this paper is therefore to investigate two types of schemes for expediting the inversion. The first, 'trilateration', developed in [1], exploits an understanding of blast wave physics to strategically constrain the exhaustive search, improving its efficiency. The second scheme is a genetic algorithm, an example of Machine Learning (ML) optimisation [2], that biases its search of the domain, focussing on the regions most likely to contain the optimal solution. The latter is considered a proxy for artificial intelligence (AI) herein, given its use of ML optimisation principles. With AI seeing increased integration into engineering, it is imperative to understand its role within design, particularly the circumstances in which its application is more effective and efficient than other available techniques.

Both methods are used to perform inverse analysis of blast wave arrival time data from six small-scale, free-field experiments. Their accuracy in predicting equivalent charge yield and position, alongside the computational efficiency of the respective analyses, is then compared to the datum set by an exhaustive search. This work aims to establish the benefits and drawbacks of solutions employing artificial intelligence and those founded upon prior understanding of the governing physics. Whilst the work herein is focussed on the context of blast analysis, the overarching philosophical discussion applies more widely within engineering and data science.

2. EXPERIMENTAL CASE STUDY

Farrimond et al. [3] conducted a series of free-field experiments to measure blast parameters from 250 g hemispherical PE10 charges. This paper uses data from six of those tests. The arena schematic is depicted in Figure 1, in plan view, detailing the charge positions in each trial and the four fixed gauge locations.

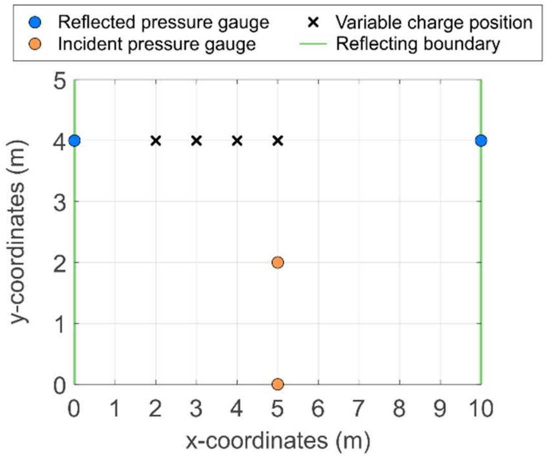

Figure 1: Schematic of the free-field experimental set-up, from Farrimond et al. [3]. All dimensions in metres.

It is the relative blast wave time of arrival (TOA) that is used in the subsequent inverse analyses, given its reported reliability for such purposes [4,5]. Note, the original publication contained only the reflected gauge data and the incident values were subsequently published elsewhere [1].

The values of equivalent charge yield and position are to be found via the subsequent inversion. Mass is assumed to vary between 1-1000 g_{PE10} and the explosive can be located anywhere in the 5×10 m experimental arena.

3. ITERATIVE INVERSION

3.1 PROCEDURE OVERVIEW

The general procedure for iterative inversion is summarised in Figure 2. It highlights the three principal processes within iterative inversion: forwards-direction prediction (FDP), iteration evaluation, and an iterative algorithm. While the nature of the scheme overall is largely invariant between applications, the specific details of these processes are dictated by the context to which iterative inversion is applied. Consequently, the three governing components of the solver are detailed herein with respect to determining a charge's equivalent yield and the *x*- and *y*-coordinates of its position from measured blast wave time of arrival.

3.2 FORWARDS-DIRECTION PREDICTION

Many practical applications of post-blast iterative inversion may demand the use of computational fluid dynamics (CFD) for forwards-direction effect prediction, e.g., due to the significant influence of blast-obstacle interaction effects on wave propagation [6]. However, the free-field nature of the experimental case study instead facilitates employing well-validated empirical laws for the modelling of blast effects with significantly less computational expense.

Equation 1 is used as the FDP for modelling blast wave TOA, t_a (ms). It was developed by Rigby et al. [7] as a convenient approximation for the semi-empirical free-field predictions of Kingery and Bulmash [8] that are well-validated over a large range of scaled distances [9-11]. Through scaled distance, $Z = {}^{R}/_{W^{1/3}}$, the inputs to Equation 1 are the assumed values of equivalent charge yield, W (kg_{TNT}), and equivalent charge position (R (m): the stand-off distance from the charge centre to the point of interest). For compatibility with the experimental case study, a TNT-equivalency for PE10 of 1.22 is used [3].

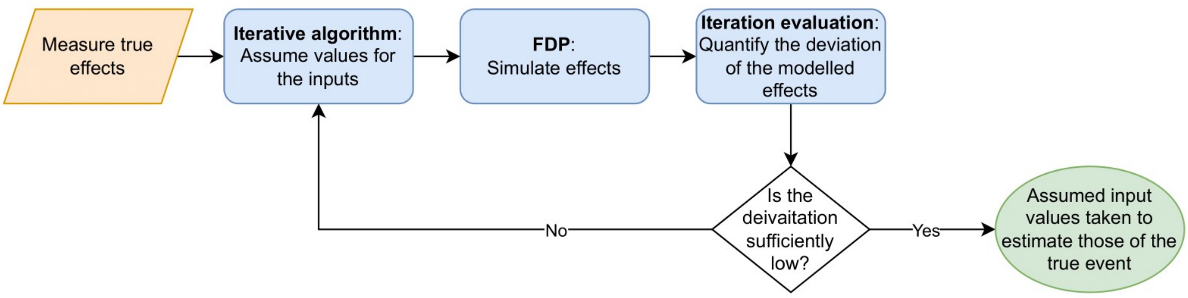

Figure 2: Flowchart describing the iterative inversion algorithm.

$$log\left(\frac{t_a}{W^{\frac{1}{3}}}\right) = 0.717\cdot(log\,Z)^5 - 0.0567\cdot(log\,Z)^4 - 0.3192\cdot(log\,Z)^3 + 0.1495\cdot(log\,Z)^2 + 1.8165\cdot log\,Z - 0.3215 \quad \textbf{(1)}$$

3.3 ITERATION EVALUATION

For some assumed initial conditions, the deviation of the arrival times modelled by the FDP from those observed during the real event, t_{exp}, at n number of measurement locations, is found for each individual data point, j. This deviation is quantified as the mean squared error, MSE, using Equation 2.

$$MSE = \frac{1}{n}\sum_{j=1}^{n}\left(t_{exp,j} - t_{a,j}\right)^2 \quad \textbf{(2)}$$

An MSE of zero implies that the assumed values of charge yield and position *exactly* match those of the true event; while this perfect case is desired, it is practically impossible due to, e.g., variability/noise in the observation data, imprecision in the assumed initial conditions, inaccurate assumptions within the FDP, etc. Therefore, the values of assumed initial conditions that generate the lowest MSE are considered to be most closely representative of the real-life event.

3.4 ITERATIVE ALGORITHM

In order for the initial conditions of an explosive event to be inversely estimated with confidence, modelling and evaluation of a single potential solution is insufficient. Instead, the domain of potential solutions must be rigorously explored by trialling several paired values of assumed charge yield and position. Previous related works have adopted an exhaustive search for this [12,13], however this entails significant computational inefficiency that renders the approach impractical in applications where rapid estimation is required. An alternative searching algorithm is therefore needed, and this paper assesses two potential options on the basis of their accuracy and computational efficiency with respect to traditional iterative inversion. These two methods are described in the following section.

4. ALTERNATIVE ALGORITHMS

4.1 ARTIFICIAL INTELLIGENCE

4.1 (a) Machine learning

The comprehensive search of a high-complexity, multi-dimensional optimisation space is a challenge encountered frequently across engineering disciplines [14]. In many such cases, the time-intensity of exhaustive search demands its replacement on the basis of practicality and expense, and machine learning is a common solution, e.g., [15-17]. It is therefore considered for integration into post-blast iterative inversion.

Machine learning is an application of AI that enables a scheme to "learn" from the data it encounters, detecting patterns in the performance of past iterations to inform subsequent ones [18]. Effectively the search is biased to focus on the regions most likely to contain the optimum, whereas an exhaustive search gives equal consideration to the entire domain which then increases the total number of iterations required. The learning mechanism is dictated by the optimisation approach adopted, and this selection is broadly governed by the availability of the gradient of a scheme's loss function with respect to the variables of interest [19]. When the gradient is not directly calculable and it is infeasible to estimate, as is the case with more generalised blast effect FDPs (like CFD), then zeroth-order 'random search' routines are favoured. A random search is also robust to non-convexity [20], a critical benefit considering the intrinsic and extrinsic variability inherent in blast events [21,22]; gradient-based solutions present a susceptibility to the associated saddle points and local minima, potentially preventing the identification of the optimum [23,24].

The specific metaheuristic ML optimisation selected for evaluation as an alternative to exhaustive search is the genetic algorithm as it is a solution favoured by other simulation-based optimisation schemes [25].

4.1 (b) Genetic algorithm for post-blast inversion

Inspired by the mechanics of Darwinian evolution [26], the genetic algorithm (GA) is an adaptive heuristic search that drives optimisation through the competition of iterations via natural selection [27]. The general scheme of a simple GA is summarised in Figure 3 wherein the relative performance of several unique trialled solutions is evaluated and the highest-

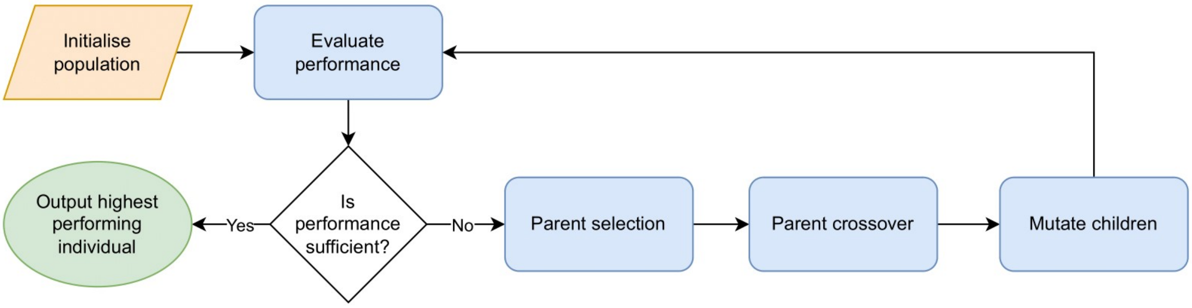

Figure 3: Flowchart describing the simple genetic algorithm [32].

performing individuals are randomly selected to inform the subsequent generation of 'children'. Through this process, the least erroneous variable values are largely retained whilst those that perform poorly are removed; the variety introduced through sharing traits from multiple 'parents' facilitates the strategic exploration of the domain with bias towards zones expected to improve performance. Additionally, population stagnation is mitigated through the random 'mutation' of inherited variable values [28]. The procedure is terminated when some end condition is satisfied, e.g., the achievement of sufficient convergence, performance stagnation and/or the exceedance of an allocated budget [29].

A continuous genetic algorithm [30] is adopted for incorporation into a post-blast iterative inversion scheme that analyses the experimental case study introduced in Section 2. The specific procedures of the scheme are detailed herein with respect to the general structure depicted in Figure 3. The hyperparameter values impacting analytical performance were optimised using a coarse grid search with manual tuning [31].

The population is initialised as a scattering of guesses generated randomly using a uniform distribution. This is because strategically seeding the population with better-than-random individuals produces no meaningful increase in computation efficiency [20]. The hypothetical effects of these iterations are simulated using Equation 1 as the FDP, with individual performance then quantified using the MSE.

The end condition specified in this work is a limit on the total number of permitted iterations. This is adopted to facilitate an effective comparison between the GA and an exhaustive search because the evaluative parameters being considered are both dependent on generation number. For a simple GA, the estimation accuracy incurs diminishing returns with increasing iteration, whilst the computation time grows linearly [20]. Section 5.2(c) therefore undertakes a sensitivity analysis to optimise this limit.

Should the end condition not be satisfied, then a subsequent generation of iterations is created through the reproduction of parents. To cultivate improvements in accuracy, it is advantageous to propagate the positive traits of only the highest-performing individuals. However, this cannot be done exclusively else the population's diversity will stagnate, reducing the exploration of the optimisation domain and limiting estimation accuracy [33]. Consequently, stochastic selection via a 'roulette-wheel' [34] is adopted. Inspired by simulated annealing [35], an individual's probability of selection

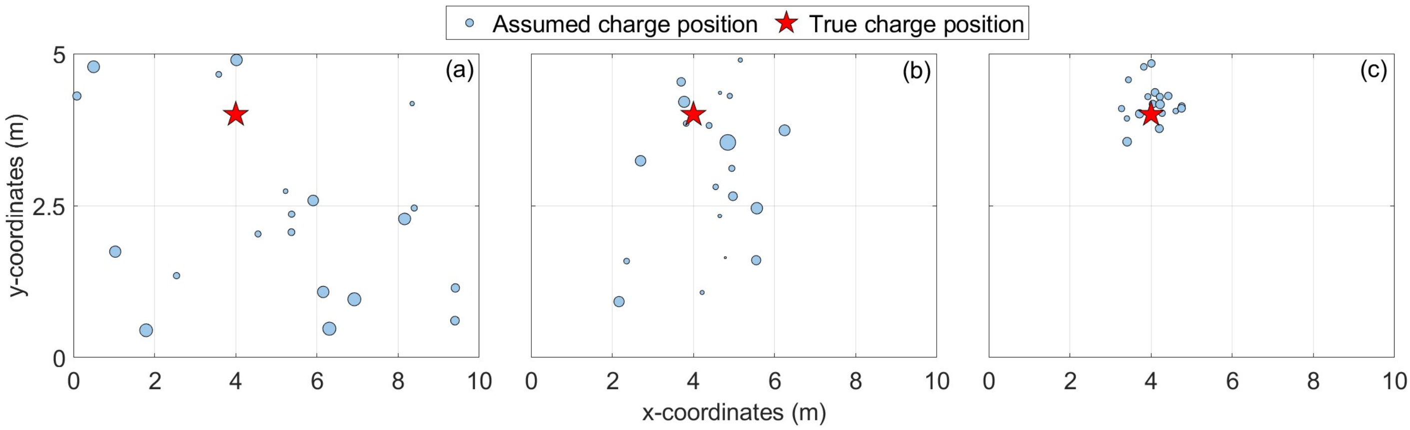

Figure 4: A demonstration of the genetic algorithm's generational convergence, where marker size is proportional to assumed charge yield. (a) 1st generation. (b) 2nd generation. (c) 10th generation.

increases exponentially with fitness to promote the propagation of proven performance whilst supporting diversification.

To maintain a steady population size, two breeding parents must produce two children. The children inherit parental traits via 'genetic crossover' which is the stochastic mixing of the parents' variable values. Arithmetic crossover, the weighted averaging of parental variable values by some randomly generated proportion, [36] is adopted in this work. Finally, a generated child may be subjected to mutation in order to further diversify the population, supporting the avoidance of early convergence within local minima [28]. For each independent variable of each generated child, there is a probability that its value is adjusted by a random increment.

The successful generation of a new population of children enables their performance evaluation, with the cycle then repeating per Figure 3 until the end condition is satisfied. Across the generations, convergence towards a common solution manifests – as illustrated in Figure 4. Ultimately, the highest-performing, fittest individual with the minimum MSE is output as the scheme's best estimate for the true event's initial conditions.

4.2 PHYSICS-INFORMED

In response to the need for a rapid post-blast inversion tool, Karlsen et al. [1] developed a novel implementation of 'trilateration' that is founded upon an engineering understanding of blast wave behaviour. This technique is to be trialled as a potential alternative to an exhaustive search within post-blast iterative inversion.

Trilateration is a well-established source localisation method, seeing applications in seismology [37] and the Global Positioning System [38]. It estimates the geospatial coordinates of a point of interest by measuring its distance from several probes whose locations are known. Karlsen et al. [1] adapted this for post-blast applications by leveraging the physical behaviours of blast waves, namely the scalability of blast effects fundamentally inherent in the free-field [21]. Critically, by framing the problem in terms of Hopkinson-Cranz scaled distance [39,40], the dependency between an explosion's unknown initial conditions can be exploited to reduce the search domain exclusively to charge yield. Therefore, despite trilateration employing brute force incrementation, it is considerably more efficient than a traditional exhaustive search which must also trial all possible spatial coordinates.

The complete methodology of post-blast inversion with trilateration is detailed and discussed in [1] wherein it is successfully applied in the inverse analysis of the 2020 Beirut explosion.

5. RESULTS AND DISCUSSION

5.1 EVALUATION STRATEGY

In order to establish a datum for performance to which the proposed alternative searching algorithms can be evaluated, exhaustive search is first used to inversely analyse the experimental data from Section 2. Both the accuracy of the inverse estimate and its computational efficiency are the parameters by which algorithmic performance is measured. The metric for mass estimation accuracy is W_{err}, the relative error between the estimated charge yield and the true value (250 g_{PE10}), averaged across the six experiments. Similarly, the quality of the position estimate is measured as the scaled distance between the true charge centre and the estimated one, termed "radial error", Z_{err}.

It is appreciated that the FDP adopted herein is not reflective of the sophistication likely to be demanded in practical post-blast inversion. Whereas the analytical run-time of Equation 1 is comparable in magnitude to the algorithmic procedures, a practical FDP, e.g., CFD, is likely to render the algorithmic time insignificant, with the efficiency therefore being directly proportional to the number of FDP simulations required. Therefore, the metric for computational efficiency used throughout this paper is the total number of FDP simulations required to complete an inverse analysis of the six experimental case studies, termed N_{sim}.

5.2 PERFORMANCE ASSESSMENT

5.2 (a) Exhaustive search

The grid-based nature of exhaustive search demands the user to specify the fidelity of the routine. A searching increment of 1 g_{PE10} in equivalent charge mass is specified, alongside a 1 cm grid for each of the charge position's Cartesian coordinate axes. The mean performance of the defined exhaustive search in the post-blast iterative inversion of the six Section 2 experimental case studies is detailed in Table 1.

Note, the estimation accuracy achieved by exhaustive search is indicative of the upper-bound on the accuracy of any iterative inversion scheme that uses Equation 1 as the FDP (for the user-defined precision presented). This is because exhaustive search trials all possible solutions and thus is guaranteed to identify the global minimum in MSE that lies on the search grid.

5.2 (b) Trilateration

Trilateration continues to employ an exhaustive search, but on a strategically constrained domain having removed the need to increment through assumed charge position. Consequently, only the charge yield increment must be input and this is again selected to be 1 g_{PE10}. The mean performance metrics for trilateration in its analysis of the experimental case study are summarised in Table 1.

5.2 (c) Genetic algorithm

Unlike trilateration and exhaustive search, the genetic algorithm is not constrained by a grid search. Instead, however, GA's stochastic nature means that any two repeat analyses of the same problem can generate different values of the estimated equivalent charge yield and position. With a lack of confidence in the outputs of a single routine, there is a need to perform several repeats to increase reliability, taking the average of the estimate outputs as being representative of the true scenario. To ensure statistical significance through the law of large numbers [41], 30 repeat analyses are adopted.

As discussed in Section 4.1(b), both the estimation accuracy and computation time increase as the number of permitted GA generations increases. With both of these parameters being the basis for the evaluation of the algorithm's performance, it is critical that they be optimised. Figure 5 displays the sensitivity to the number of permitted generations of both output accuracy and the number of required FDP simulations.

It is shown that accuracy is efficiently maximised after 500 generations, with subsequent computation effectively becoming redundant. Consequently, the performance of the algorithm following 500 generations is selected as representative of the wider scheme ahead of comparison to exhaustive search and trilateration; its performance is summarised in Table 1.

Table 1: Mean analytical accuracy and computational expense of each of the iterative algorithms in the inversion of the case study.

Algorithm	W_{err} [%]	Z_{err} $[m/kg_{TNT}{}^{1/3}]$	N_{sim}
Exhaustive search	4.15	0.166	1.2×10^{10}
Trilateration	4.15	0.175	1.1×10^{5}
Genetic algorithm	4.36	0.169	7.2×10^{6}

5.3 PERFORMANCE ASSESSMENT

5.3 (a) Comparison to exhaustive search

The performance of the alternative algorithms' with respect to the both evaluative criteria, inverse estimation accuracy and computational expense, are assessed relative to exhaustive search in Figure 6.

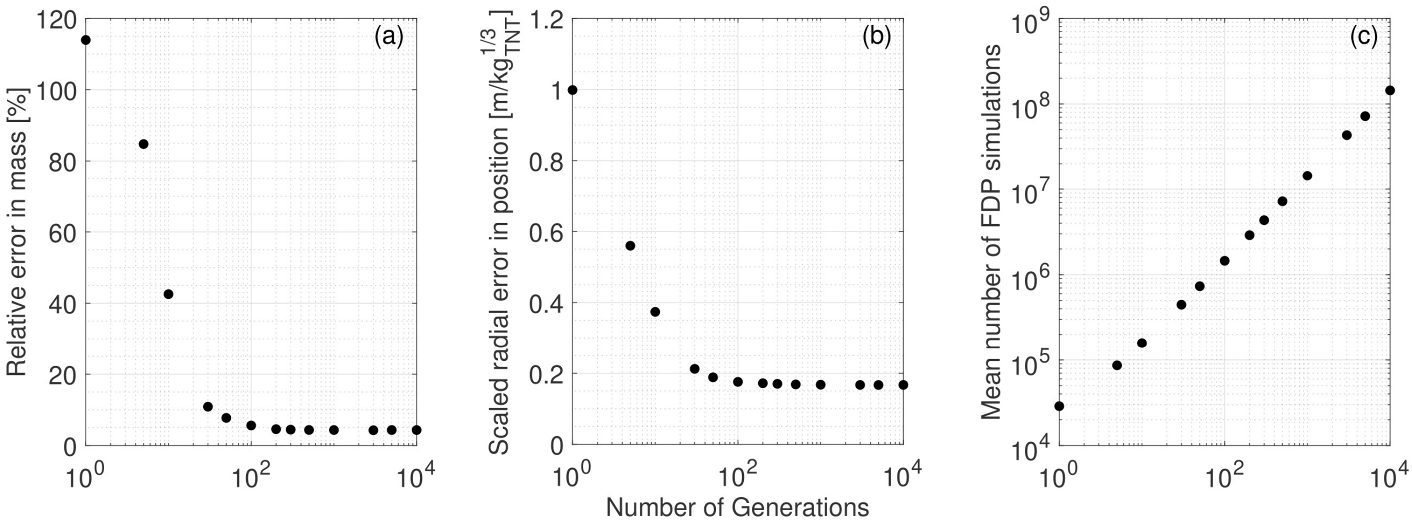

Figure 5: Sensitivity of the genetic algorithm's analytical performance to the number of permitted generations. (a) Equivalent yield estimation accuracy. (b) Charge centre estimation accuracy. (c) Computational expense.

Figure 6(a) demonstrates trilateration to perform equally as well as exhaustive search for the estimation of equivalent charge yield through iterative inversion. GA incurs a slight reduction in accuracy, generating an output that is 5% more erroneous but this deviation is of little practical significance, however, as it equates to just 0.21% of the true charge yield (~0.5 g_{PE10}).

Both GA and trilateration incur additional error in the estimation of equivalent charge position over exhaustive search (1.8% and 5.5%, respectively), per Figure 6(b). The deviation of the former is attributed to its aforementioned asymptotic convergence, whilst trilateration's error is a consequence of removing charge position as a degree of freedom within the search. Effectively, exploration of the MSE domain with respect to charge yield alone identifies the saddle along which the global minimum is located, but it cannot be pinpointed exactly due to an inability to locally explore the position variables. The reduction in position estimation accuracy is, however, relatively insignificant as the error increases by a maximum of just 6.2 mm, which is less than 1/10th of the charge's diameter (assuming the nominal density of PE10 to be 1.55 g/cm^3 as reported [3]).

Therefore, the equivalent charge yield and position estimation accuracy of both trilateration and the genetic algorithm is deemed to be nominally identical to exhaustive search as any discrepancy in their estimation is minor and considered practically inconsequential.

Performance against the second criterion, computational expense, is evaluated in Figure 6(c) which directly compares the average number of FDP simulations required by each algorithm to conduct their inversion. Both alternative algorithms are evidenced to be more computationally efficient than an exhaustive search, with them each demanding significantly fewer FDP simulations to achieve a successful inversion. The GA completed its inverse analysis of the experimental case studies using 0.06% of the FDP models required by an exhaustive search (1,600-times faster). Trilateration generated an even greater reduction in computation having completed its inversion after using just 0.0009% of the FDP simulations required by an exhaustive search (109,000-times faster).

Therefore, both alternative algorithms are deemed to be demonstrably superior to exhaustive search for post-blast applications of iterative inversion and can be readily incorporated to obtain a decrease in computation time with no meaningful reduction in estimation accuracy.

5.3 (b) Competing the alternative schemes

The intention of this work is to support the development of a rapid post-blast analytical tool for informing the coordination of emergency response efforts. Comparing the relative performance of trilateration and the genetic algorithm in this regard, it may then be concluded that the former is the more effective routine, having completed its inversion in two orders of magnitudes less time than the latter. However, this is only true for the narrow case study analysed in this paper. In practice, trilateration cannot be realistically adopted due to its free-field assumption considerably restricting applicability.

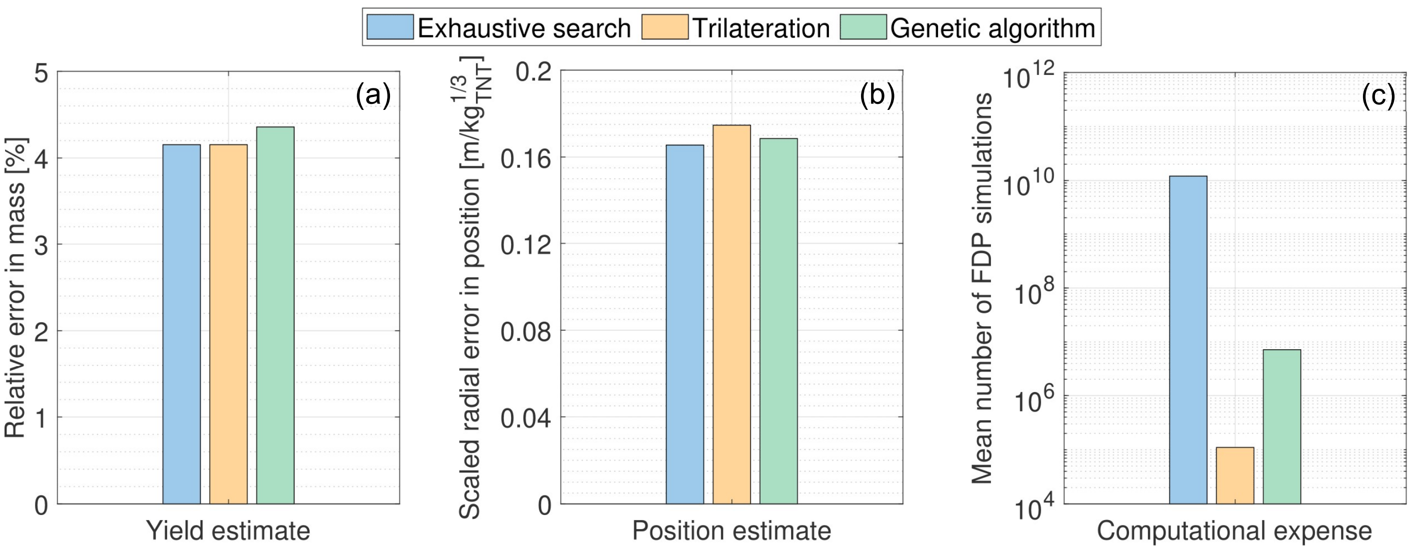

Figure 6: Sensitivity of GA analytical performance to the number of permitted generations.
(a) Equivalent yield estimation accuracy. (b) Charge centre estimation accuracy. (c) Computational expense.

Moreover, this contrast offers insight into the qualities of the general analytical approaches being investigated. Physics-informed methods (by extension of trilateration) offer superior efficiency, whilst the GA and machine learning approaches, with no understanding of a problem's physicality (and thus no restriction on application), support enhanced versatility. Thus far, throughout this paper, the two have been treated as being in competition but these traits are in fact complementary and thus the schemes would mutually benefit from acting compositely. This concept is presently driving the advancement of physically-valid analytical tools in many contexts, including the prediction of explosive effects, as discussed by Pannell et al. [42]. Therefore, it is concluded that a hybrid physics-informed machine learning optimisation algorithm offers the greatest potential for achieving practically-applicable, rapid and robust post-blast iterative inversion.

One conceptualised composite algorithm for post-blast iterative inversion is inspired by the findings of Section 5.3(a). It may first employ a physics-informed search to rapidly approximate the optimal solution before a metaheuristic scheme then relaxes the constraints to maximise analytical accuracy. The relative performance enhancements of such an approach are indicatively presented in Figure 7.

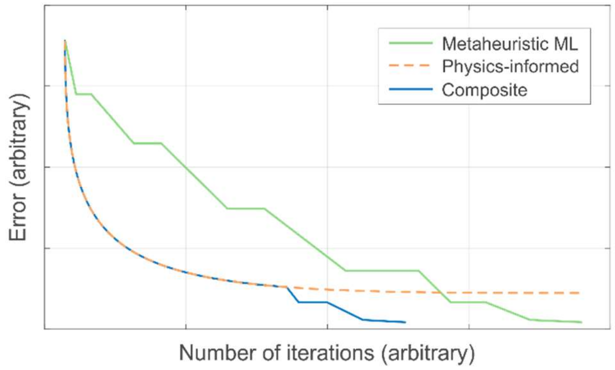

Figure 7: Hypothetical performance of a hybrid physics-informed machine learning optimisation algorithm for post-blast iterative inversion.

More generally, the above can be more generally considered a commentary on the integration of artificial intelligence within engineering practice. Where presently, there exists an anxiety that AI may invoke the obsolescence of the engineer [43], it is instead suggested that AI becomes another tool at the engineer's disposal, which in no way diminishes the significant value uniquely offered by physical understanding and professional judgement. The two are thus free to act compositely for the benefit of the design process.

This principle is elegantly summarised by Lagaros et al. [44]: *"the expert needs the optimizer, but the optimizer also needs the expert"*.

6. CONCLUSIONS AND OUTLOOK

Post-blast iterative inversion could support the efficient coordination of life-saving emergency response efforts in the event of an explosion. However, the scheme presently relies on an inefficient exhaustive searching routine, incurring prohibitively protracted computation times that render it impractical and ineffectual in these applications. This paper therefore investigated alternative searching algorithms in an effort to improve the computational efficiency of post-blast iterative inversion, without any significant loss in accuracy.

Two schemes were proposed and evaluated against exhaustive search with respect to the accuracy of their inverse estimation of equivalent charge yield and position, and their associated computation time. The first alternative routine, a bespoke genetic algorithm, employed machine learning optimisation, as a proxy for artificial intelligence, to more efficiently explore the domain of potential solutions; it attained an accuracy nominally identical to an exhaustive search, but in just 0.06% of the time. Trilateration, a post-blast physics-informed inversion approach developed by Karlsen et al. [1], was the second searching algorithm evaluated. Its computational efficiency was also considerably greater than exhaustive search, completing its inversion 109,000-times faster, with no significant reduction in estimation accuracy. For the limited case study considered, both trilateration and the genetic algorithm are therefore practically-superior alternatives to exhaustive search within post-blast applications of iterative inversion.

Finally, this study identified that machine learning should not be treated as being in competition with solutions founded upon physical understanding. The benefits of each are mutually complementary, and a hybrid physics-informed machine learning optimisation scheme may extend the post-blast iterative inversion performance improvements achieved in this work to scenarios beyond the free-field. Furthermore, this may attest to the integration of AI within the engineering industry more broadly. Particularly that a perception of AI being in competition with the engineer may unhelpfully disguise their independent benefits, thereby inhibiting a collaboration that is ultimately more effective than the sum of its parts.

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