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INFLUENCE OF MEMBER FAILURE TIME ON DYNAMIC LOAD REDISTRIBUTION IN PINNED PLANAR TRUSSES

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ABSTRACT

Optimized structures have, by definition, zero redundancy. That is, if a single member were to suddenly fail (through material degradation, or an extreme event such as fire, earthquake, or blast), the load previously carried by that member would be redistributed to neighbouring members, potentially inducing catastrophic progressive collapse. This is somewhat addressed in the area of "fail-safe" optimization, however dynamic effects within the load redistribution phase are generally not considered, thus leading to potentially unsafe designs. This paper begins to address this through an investigation into the influence of non-zero member removal times on dynamic stress amplification (ratio of peak dynamic stress to static stress) in pinned trusses. A force-replacement method was implemented to model non-instantaneous member removal, where the failed element is substituted by equivalent external forces that decay linearly over a defined duration. Dynamic load redistribution is then analysed using a simple time integration method, which is first verified against existing analytical work. Analysis of a simple example structure found that an increase in member removal time led to a reduction in dynamic amplification, the relation of which was linked with the modal time periods of the damaged structure. Investigating the problem for different structures, a series of parametric studies utilising Monte Carlo sampling encompassed sets of 500 individual truss structures with randomly assigned member cross-sectional areas. Dynamic amplification factors were found to follow an approximate log-normal distribution, with a modal average of 1.58 for instantaneous removal, which was reduced to 1.15 when considering removal times greater than 3ms. This research is a promising first step towards comprehensive consideration of dynamic effects in fail-safe truss optimization, with an ultimate view of designing low-carbon, economical structures that are robust and safe.

1. INTRODUCTION

Reducing material consumption in modern infrastructure is becoming increasingly important in light of the climate crisis. Material optimization in structural design is a promising avenue to address this problem. However, structural safety remains paramount, with all structures requiring an adequate degree of robustness to prevent disproportionate collapse; the event where damage to a small part of a structure results in damage or collapse over a much larger area [1], [2]. Hence, there is a need to develop optimization methodologies that incorporate fail-safe design to ensure structural safety during damaging events.

Current fail-safe optimization work focused on the design of truss and frame structures subject to fast-acting member removal utilises a linear static analysis approach to determine the resulting designs [3], [4], [5], [6]. However, the behaviour of a structure during damage events such as fast member removal is inherently dynamic, with the structure moving to a new equilibrium state through a transient vibration state where overshoot can result in heightened element stresses [7]. Consequentially, standard static analysis of a damaged structure is often non-conservative [7], [8] and using standard amplification factors as compensation can lead to over-conservative designs [9]. Hence, the incorporation of dynamic analysis into fail-safe structural optimization is a necessary step to improve the safety and efficiency of current solutions.

There have been numerous studies looking at performing dynamic finite element analysis (FEA) on truss-type structures subject to fast member removal to assess for disproportionate collapse [10], [11], [12], [13], [14], along with others undertaking practical testing to verify such numerical modelling techniques or to test the effects of various types of truss connections [8], [11], [15], [16]. Much work has been done on critical member identification methods to help avoid a complete dynamic analysis assessment on every damage case [17], [18], [19], [20], [21]. The topic of dynamic amplification factors has also been of interest, with Goto et al. [10], McKay et al. [9] and Khuyen & Iwasaki [12] proposing ways to approximate values and thus avoid the use of the standard dynamical load factors which generally lead to overly conservative designs. Stress wave propagation has also been a topic of interest for some researchers [22], [23], [24], but the importance of its role in fail-safe analysis still lacks a conclusion. In addition, the influence of member removal time on the dynamic response of truss structures has not been greatly investigated, however, a study by Mozos & Aparicio [25] presented interesting findings, which are discussed in detail in section 4.2.

The influence of non-instantaneous member removal time on dynamic action may have a significant influence on a structure's peak dynamic stresses, and thus inform the structural volume of a design. This therefore warrants further investigation for the benefit of developing fail-safe optimization. Furthermore, little work investigating pure pinned truss typologies has been undertaken; the typologies are of concern for many fail-safe optimization designs due to their inherent structural efficiency [26], [27]. Hence, this work looks to investigate the dynamic stress amplification of a simple planar truss structure subject to non-instantaneous member removal damage scenarios. The information on the dynamic amplification of this typology will help inform future fail-safe optimization work which incorporates dynamic behaviour, therefore helping to achieve safe material-efficient structures.

2. METHODOLOGY

2.1 ANALYSIS METHOD & MODELLING DETAILS

An implicit time integration method was developed to solve the dynamic force equilibrium equation, and thus simulate dynamic response. The numerical integration method assumes a linear variation in acceleration between time increments, along with a linear elastic material model, and elastic instability (buckling failure) is neglected. Furthermore, the deflections of the structure are presupposed to be significantly small such that non-linear geometric effects are insubstantial.

The change in the nodal displacements ($\Delta\boldsymbol{u}$) and velocity ($\Delta\dot{\boldsymbol{u}}$) may be determined by equations (1) and (2):

$$\Delta\boldsymbol{u}(t) = \boldsymbol{\beta}^{-1}\boldsymbol{\alpha}(t) \tag{1}$$

$$\Delta\dot{\boldsymbol{u}}(t) = \frac{3}{\Delta t}\Delta\boldsymbol{u}(t) - 3\dot{\boldsymbol{u}}(t_0) - \frac{\Delta t}{2}\ddot{\boldsymbol{u}}(t_0) \tag{2}$$

Where:

$$\boldsymbol{\beta} = \boldsymbol{K} + \frac{3}{\Delta t^2}\boldsymbol{M} + \frac{3}{\Delta t}\boldsymbol{C} \tag{3}$$

$$\boldsymbol{\alpha}(t) = \Delta\boldsymbol{P}(t) + \boldsymbol{M}\left[\frac{6}{\Delta t}\dot{\boldsymbol{u}}(t_0) + 3\ddot{\boldsymbol{u}}(t_0)\right] \tag{4}$$

Δt is the time step value and t_0 is the time duration before the time step has been applied. $\boldsymbol{K}$, $\boldsymbol{C}$, and $\boldsymbol{M}$ are the system's stiffness, damping, and mass matrices respectively. $\boldsymbol{P}(t)$ refers to the vector of externally applied loads. The system's nodal acceleration can be determined from the force equilibrium equation (5).

$$\ddot{\boldsymbol{u}}(t_0) = \boldsymbol{M}^{-1}(\boldsymbol{P}(t_0) - \boldsymbol{C}\dot{\boldsymbol{u}}(t_0) - \boldsymbol{K}\boldsymbol{u}(t_0)) \tag{5}$$

Elements were modelled as 2D bar members with nodal lump massing. All static analyses used a linear stiffness method.

2.2 MODELLING DAMAGE THROUGH INSTANTANEOUS MEMBER REMOVAL

Damage events will be defined by removing a single member, with the damage case naming coinciding with the element's numbering (e.g. damage case 5 refers to removing element 5).

The simplest form of modelling member removal is by assuming the structure experiences the damage event instantly. This may be modelled by first considering the system in its undamaged stressed state, with the displacements of the degrees of freedom (i.e. nodal connection displacements) used as initial displacement conditions for the dynamic analysis. The structure's topology can then be updated with one of the members removed, changing the system's stiffness, damping, and mass matrix. Since the structure is assumed to be in a static equilibrium state before the damaging event, the initial velocities are assumed to be zero. The initial accelerations may be determined by considering equation (5). From here, the dynamic analysis can be initiated, and the peak dynamic stresses can be found (see Fig. 1). A separate static analysis of the damaged structure can be performed to determine the static stresses.

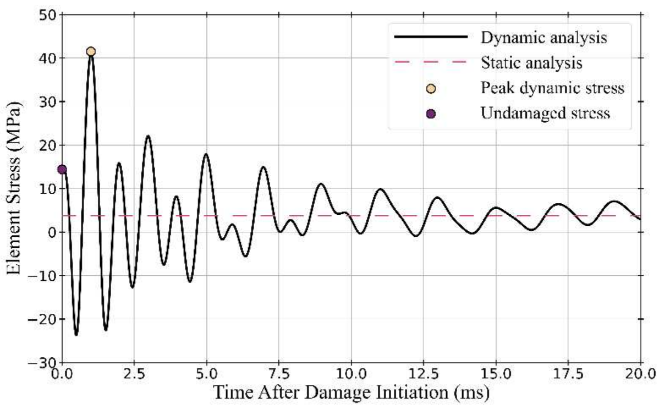


Figure 1: Example truss bar stress-time plot after instantaneous member removal, incorporating structural damping for illustrative purposes. Undamaged stress = 14.6MPa; Peak dynamic stress = 42.7MPa; Static Stress = 3.8MPa (DAF = 11.2)

Dynamic effects for a given damage case will be measured by considering dynamic amplification factors (DAF), which are defined as the ratio of the peak dynamic stress and the static stress of the damaged structure:

$$DAF_{i,j} = \frac{\sigma_{Dyn,i,j}}{\sigma_{Stat,i,j}} \tag{6}$$

Where i refers to the ith member in the structure, and j refers to the jth damage case, defined by a member being removed. Cases when $\sigma_{Stat,i,j} = 0$, thus leading to infinite DAF values, are neglected.

2.3 MODELLING VARIABLE MEMBER REMOVAL

Although simple, assuming instantaneous member removal may not represent realistic damaging events, such as the structure being subject to an impact or some form of blast load. A recent example is the Francis Scott Key bridge collapse, where it appears that the damaged pier still provided some strength for a brief time after impact of the boat. Hence, it is necessary to consider a method of modelling non-zero member removal times. Yan et al. [11] used a cross-sectional area reduction method to model member decay, however, this requires multiple re-evaluations of the system's stiffness, mass, and damping matrix, as well as stiffness matrices inversion computations, thus adding substantial computational cost. Decaying the member's elastic modulus over time is another approach, however, it also shares similar computational demands as the cross-sectional area method. Alternatively, a force replacement method may be implemented [10], [12], [16].

Considering a stressed static truss system, a member may be replaced with an equivalent externally applied load equal to the removed member's internal force. This equivalence can be used to replace an element with an appropriate externally applied load, which can then be decayed to model non-instantaneous member removal. Here, it is assumed the decay follows a linear function, as shown in Fig. 2. The modelling procedure is almost identical to that of the instantaneous member removal, except with the inclusion of updating the externally applied load vector, $\boldsymbol{P}(t)$, which included the member equivalent loads before initialising the dynamic analysis.

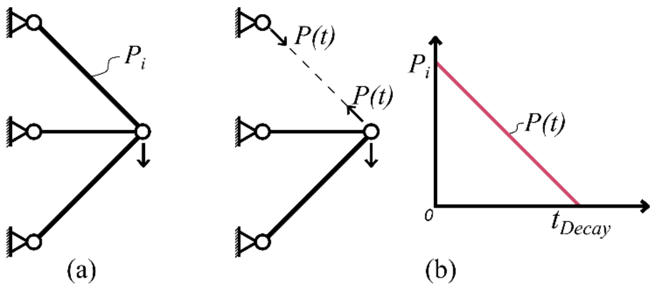
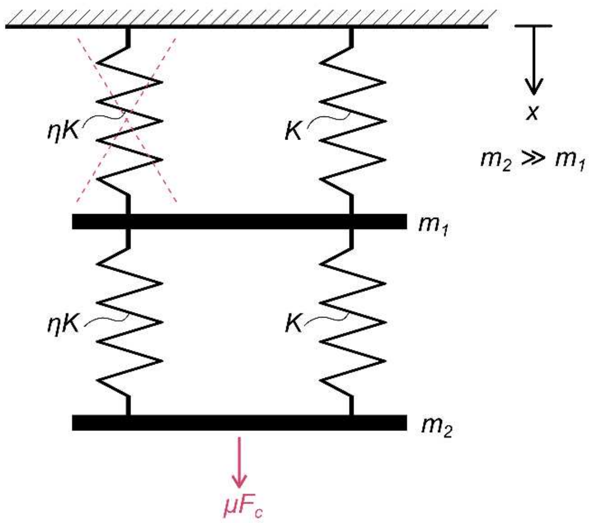


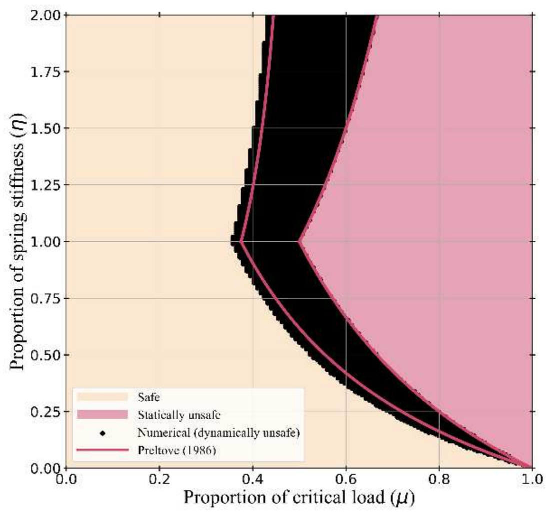
Figure 2: Modelling non-instantaneous member removal by substituting member with equivalent applied load. **(a)** Undamaged stressed structure. **(b)** Damaged structure and force-time graph of equivalent applied load.

3. VERIFICATION OF THE TIME INTEGRATION METHOD

Pretlove [7] investigated the dynamic nature of member removal for a simple 1D parallel spring system (see Fig. 3a). The study analysed different proportions of critical load (μ) and spring stiffness (η) to determine whether, upon removal of the top left spring, the system would not fail ("safe"), fail through dynamic analysis ("dynamically unsafe"), or fail through both static and dynamic analysis ("statically unsafe"). This section looks to verify the



(a)



(b)

Figure 3: Replicating Pretlove [7] fail-safe analysis study using numerical time integration and finite element modelling. **(a)** 1D parallel spring system, adapted from Pretlove [7], showing boundary conditions, properties, and the member to be removed. **(b)** Results of replication.

proposed numerical dynamic analysis method by replicating Pretlove's study.

Values of spring stiffness (K), nodal mass (m_1), and maximum allowable internal force (P_c) are assigned values of unity to facilitate the normalisation of the results. Modal mass m_2 is given an arbitrarily large value $\left(\frac{m_2}{m_1} = 1000\right)$ due to Pretlove's constraint that $m_2 >> m_1$. The system is assumed to have zero damping and the member removal process is instantaneous. Multiple analyses were undertaken, systematically changing the two system parameters (μ and η, as above) and recording the peak dynamic forces (P_{Dyn}) and static forces (P_{Stat}) of the top right spring. Values of μ and η for which the system failed through dynamic analysis but not static analysis ($P_{Dyn} > 1$ and $P_{Stat} < 1$) are then recorded and plotted, as shown in Fig. 3b. The results show a good match with the analytical results derived in the Pretlove study, thus verifying the adopted numerical approach.

4. DYNAMIC AMPLIFICATION WITH VARIABLE MEMBER REMOVAL

4.1 CASE STUDY STRUCTURE

The topology and boundary conditions of the considered structure are illustrated in Fig. 4. All elements are assumed to be made of steel with an elastic modulus of 200GPa and a density of 8000kg/m^3. The sections are given a cross-section area of $4\text{x}10^{-4}$m^2; their cross-section shape is arbitrary due to the system only experiencing axial forces. Furthermore, all connections are assumed to be frictionless pins. Zero damping effects are assumed.

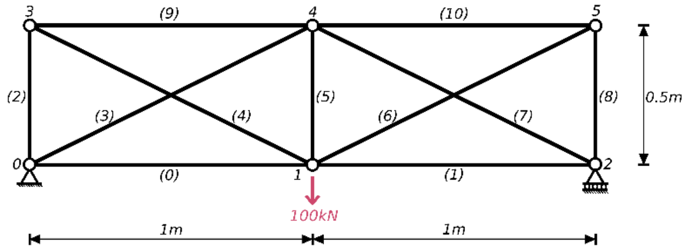


Figure 4: Simply-supported cross-braced planar truss system with a single central vertical point load.

4.2 DAF OVER VARYING REMOVAL TIME

The example structure was subject to 2 damage cases: 2 and 5. The analysis was performed multiple times for each damage case for different member removal times. The results are plotted in Fig. 5.

The results report a rapid decrease in dynamic amplification with an increase in member removal time for both damage cases, with both having an average DAF value of around 1.1 or less after 10ms. However, a somewhat oscillatory behaviour is also observed, showing periodic increases and decreases in DAF. Damage case 8 showed similar behaviour but was not reported for brevity. Upon further investigation, it was found that the 'period' of these oscillations coincided with the damaged structure's modal time periods, with the minimums occurring at intervals of the first and second time periods, as indicated in Fig. 5. Damage case 5, shown in Fig. 5b, has well-defined bumps, characterised by the first and second modal periods being very close together, thus making their minimum points coincide. This modal-related behaviour is consistent with the findings from the analytical study conducted by Mozos & Aparicio [25], who showed that by considering the response of a structure through modal superposition, a relation between maximum modal response and member removal time could be found. An increase in removal time decreases modal response, with minimums occurring at multiples of the modal time period. Consequently, an increase in

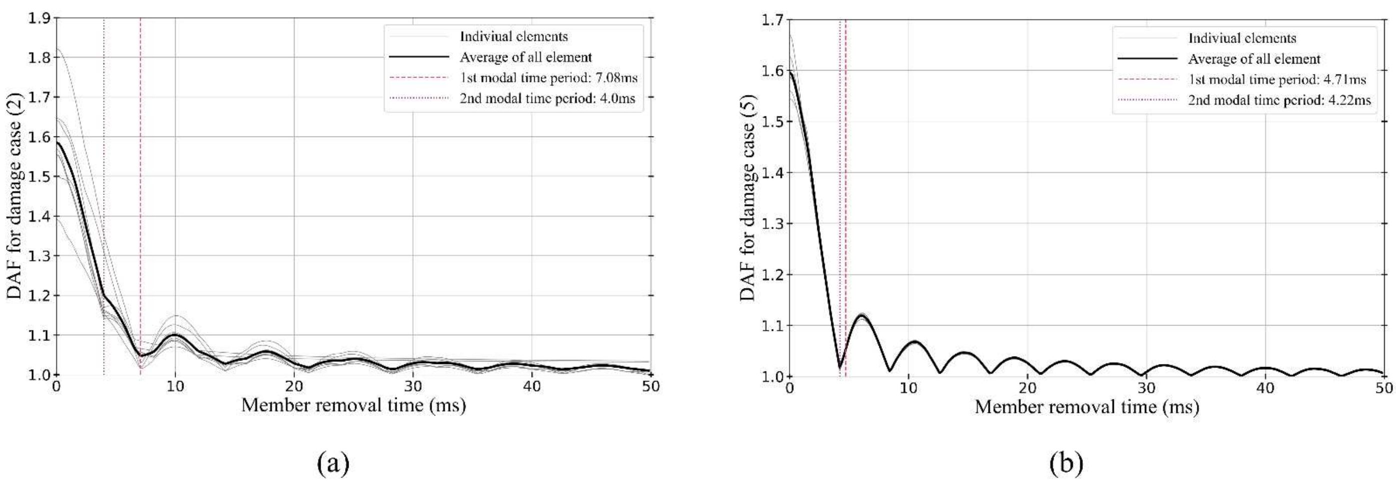


Figure 5: Plots of dynamic amplification over varying member removal times for different damage cases, plotting amplification values of individual members, and a mean average of all members. **(a)** Damage case (2). **(b)** Damage case (5).

removal time prevents a larger number of modes from contributing to the dynamic response of the structure significantly, thus explaining the rapid decrease in DAF values shown in Fig. 5.

Some useful observations can be drawn using the relation between the member removal time and the modal time periods. It can be said that the first minimum DAF will occur when the member removal time is equal to the first modal or second time period of the damaged structure. The initial rapid decay of the DAF values is roughly linear, suggesting that a rule of thumb or predictive approach may be developed as a result, although more substantial analysis is required to achieve this.

4.3 DYNAMIC INFLUENCE OVER ALL DAMAGE CASES

Within this section, dynamic amplification of the entire structure will be investigated, and how the alteration of different member removal times influences the overall dynamic response. For given member removal times of 0.00ms, 1.25ms, and 2.50ms, all damage cases are considered and corresponding DAF values were calculated. The results are illustrated in Fig. 6.

It can be observed from the histograms that not only does the member removal time decrease the mean DAF value, but it also reduces the spread of values, with the 2.50ms removal time having a more consistent grouping of DAF values. Furthermore, an increase in member removal time decreases the overall dynamic effects of the structure.

4.4 DYNAMIC AMPLIFICATION EXPERIENCED BY DIFFERENT STRUCTURES

In this section, Monte Carlo simulations were performed on the topology and boundary conditions from Fig. 4, where the structure is analysed for every damage case multiple times utilising randomly generated cross-sectional areas. DAF values for every bar for every damage case are recorded for each simulation. The cross-sectional areas were allowed to take values between $10^{-5}m^2$ and $10^{-3}m^2$. Six sets of simulations were performed using different removal times, taking values between 0 and 5ms. 500 simulations were performed for each set, collecting over 46,000 DAF values each.

Fig. 7a shows the DAF values calculated between the interval of 1 and 5 considering instantaneous member removal. The distribution appears to be log-normal, showing an increase of frequency around 1 to 2.5 and a long tail that gradually becomes statistically insignificant, with the distribution mode being around 1.58. Fig. 7b shows the DAF values considering 3ms removal times. The distribution can be seen to shift to the left with a taller and steeper peak, reinforcing the findings from the previous section. The peak can be seen around 1 to 2.0, with the mode being around 1.15. The mode values for the six different Monte Carlo simulations can be seen in Fig. 8, showing a quick decrease as the removal time

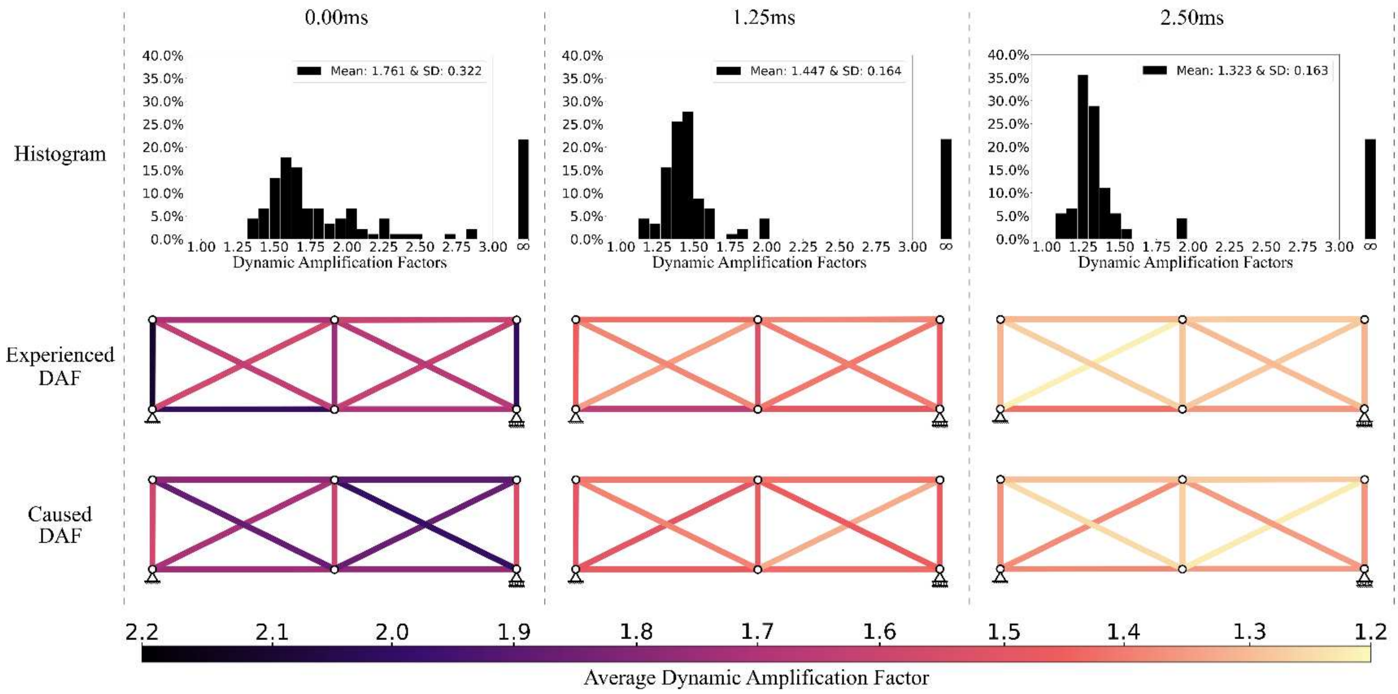


Figure 6: Dynamic amplification impacts on the entire structure for instantaneous member removal (left column), 1.25ms removal time (middle column), and 2.50ms removal time (right column). **Top Row**: Histograms of data gathered from all damage cases. **Middle Row**: Colour plot of average DAF experienced by each member. **Bottom Row**: Colour plot of average DAF caused by each damage case.

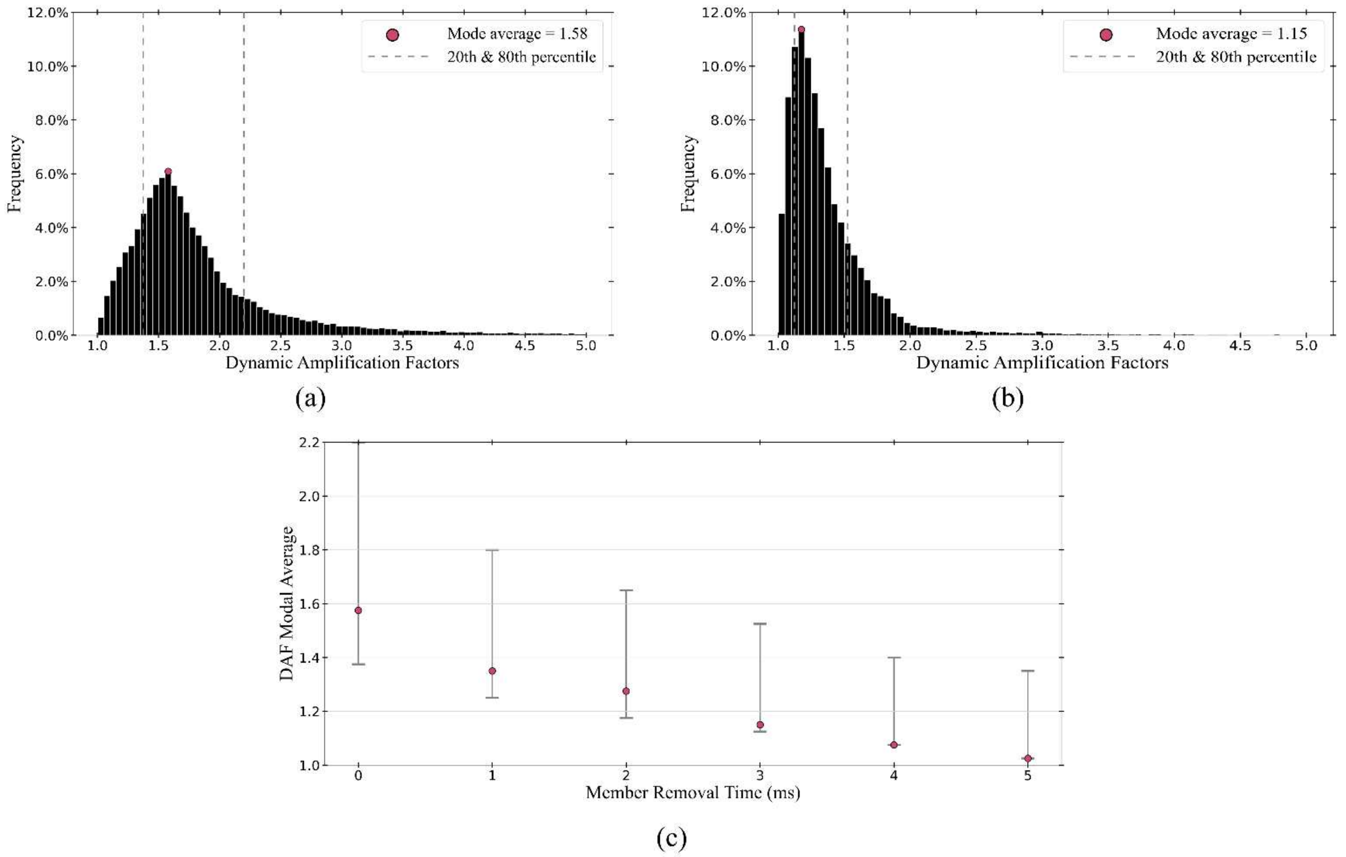


Figure 7: Monte Carlo simulations, determining dynamic amplification factors for randomly selected cross-sectional areas for the example topology and boundary conditions. DAF values are placed in bin sizes of 0.025. **(a)** DAF collected considering instantaneous member removal. **(b)** DAF collected considering 3ms member removal time. **(c)** Plot of modal average DAF values for 0, 1, 2, 3, 4, and 5ms member removal time, with error bars corresponding to 20th and 80th percentiles.

increased along with a decrease in statistical deviation, measured through the 20th and 80th percentiles.

DAF values significantly greater than 5 were also calculated but were found to be in low numbers. It is worth noting that large DAF values do not necessarily indicate extreme stress increases due to the dynamic redistribution, but can be a consequence of the DAF definition. Situations, where the load redistribution causes a reduction in a member's stress in its new static equilibrium state, will naturally produce large DAF values. See Fig. 1 for an example stress-time plot which produces a DAF value of around 11.2 and a damaged stress considerably *lower* than the undamaged stress.

5. CONCLUSIONS

A parametric-based study on the dynamic influence of non-instantaneous member removal damage events on pinned planar trusses has been carried out. The use of a simple time integration method for dynamic analysis was presented and verified through a successful replication of an analytical study by Pretlove [7]. A force replacement-based method for variable member removal time was also presented. The analysis of a simple 2-D structure was considered for this investigation. It was found that for a given damage case, the magnitude of the dynamic amplification is influenced by the modal properties of the damaged structure, supporting the findings from Mozos & Aparicio [25]. An analysis of the structure's overall response for all damage cases and the impact of varying member removal times was then presented. It was found that the introduction of longer member removal times not only reduced dynamic amplification but also reduced the statistical variance of the values. These findings were reinforced by six sets of Monte Carlo simulations, which were undertaken on the example topology, with the first considering instantaneous member removal and the others considering increasing removal time from 1ms to 5ms. The simulations randomly varied the cross sections 500 times and evaluated the structure for all its damage cases. Approximate log-normal distributions were observed, with DAF values around 1.58 representing the distribution's mode. The mode values reduced to around 1.15 when introducing member removal times greater than 3ms.

It can be concluded that assuming instantaneous member removal is conservative, however, due to the necessity to reduce carbon emissions from construction, this may not be suitable. If an engineer can utilise their knowledge about a potential damage event and thus consider non-instantaneous member removal times, a significant reduction in dynamic stress amplification can be found, and thus lead to the design of more material-efficient collapse-resistant structures. The insight gained from this study will help support future developments in fail-safe truss optimization.

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