Formal Ontology in Information Systems
C. Trojahn et al. (Eds.)

© 2024 The Authors.
This article is published online with Open Access by IOS Press and distributed under the terms of the Creative Commons Attribution Non-Commercial License 4.0 (CC BY-NC 4.0).
doi:10.3233/FAIA241298

Ontology as Structure, Domain and Definition

Brandon BENNETT [a,1] and Giulia SINDONI [a,2]
University of Leeds
ORCiD ID: Brandon Bennett https://orcid.org/0000-0001-5020-6478, Giulia Sindoni https://orcid.org/0000-0003-4003-2317

Abstract. The paper presents a spatio-temporal ontology guided by a particular methodology, in which the semantics is constructed within a *spatio-temporal interpretation structure* that is built up in three stages. The first, stage stipulates a standard classical model of time and space. This structure forms the *grounding* for the interpretation. The next stage is the specification of domains of entities, which are either elements of the grounding structure (time points and regions) or constructions from these elements (mappings from time to space associated with individuals existing within the spatio-temporal structure). The final stage is the definition of conceptual vocabulary in terms of the grounding structure and the specified domains. This definitional stage can be further subdivided into three types of specification: direct grounding of primitives onto the underlying structure, indirect grounding by defining additional vocabulary in terms of grounded primitives, partial grounding by specifying semantics types and axioms to constrain the meaning of vocabulary that is not explicitly defined.

The main goal of the paper is to advocate a methodology rather than a specific ontology. We suggest that building up in this way, results in robust ontologies, whose assumptions can be clearly seen, since they are encapsulated within the grounding stage and domain specifications. Although the definitional stage may incorporate a diverse and expressive vocabulary, its terms are essentially just labels for properties and relations that were already implicit within the grounding structure.

Keywords. Foundational Ontology, Ontology Construction Methodology, Definitions, Spatio-Temporal Semantics

1. Introduction

The paper you are now reading is the result of a long process of reformulation and reorientation. Our original aim was to develop an ontology of physical and temporal entities that could describe the behaviour of moving objects and clearly explain the distinctions and interrelations between categories such as ‘state’, ‘event’ and ‘process’, and would compare and contrast the categorisation with other established ontologies. However, as the work developed in that direction, it seemed that the original goal might have been poorly suited to our methodology. What we found is that, although we could specify a large variety of different kinds of temporal entity, it was unclear how one should com-

[1]Corresponding Author: Brandon Bennett, School of Computing, University of Leeds, LS2 9JT.
[2]Now at Fondazione Bruno Kessler, Trento.

pare these to the categories of other ontologies or whether such comparison would be informative.

Consequently, our main aim is now to present our methodology. Although it does not result in neat categorisations of different kinds of entity, it does provide a rigorously grounded and highly expressive representation language which will believe to be a powerful tool for knowledge representation and ontological analysis.

1.1. Can Upper Ontologies Play a Foundational Role?

For certain types of application, an information system can make use of an ontology to get a large benefit with little pain. These would typically be applications where we want to organise and identify objects that have a wide variety of specific properties and relationships but all objects lie within a small number of general categories (e.g. people, gifts and birthday dates), and the relevant relationships between objects of different categories are limited and well-understood. Although the relationship between a person and their birthday is ontologically complex, a simple but useful information system may just take this as a primitive relational fact which may trigger certain actions (e.g. sending alerts to a person's friends). It could also do some clever reasoning about attributes of possible gifts to make a helpful recommendation.

Yet we only need to consider a slightly broader range of information to find that the complex ontology of birthdays could easily cause a headache for an information system designer. Maybe we want to store actual dates of birth as well as calendar birthdays, and dates of birthday parties that could be different from the actual birthday (and we need to order the right number of candles, and we need to stop reminding people to send toiletries to dead people). Perhaps, our range of gifts includes perfumes and books. We realise that a person may be content to receive exactly the same perfume on their next birthday as they received on their last; but they would be less happy to receive exactly the same book. Is this just because perfumes are more relentlessly advertised, or is their some significant category distinction between perfumes and books?

By distinguishing fundamental types of entities, upper ontologies can prevent confusions that may arise from ambiguities of natural language terminology (such as different senses of 'birthday' or 'book') insinuating themselves within information systems. Established general ontologies such as SUMO [24], DOLCE [21], BFO [2], UFO [17] aim to ensure clear differentiation between entity types, and hence mitigate such issues. In this capacity, despite some differences in their basic categorisation of entities, they can be very effective.

However, to support flexible interpretation of information involving disparate categories, one also needs to articulate the nature of connections between categories — for example the relationship between the event of a birth to the calendar date of a birthday. In this regard, we may be a little less sure about the success of established ontologies. Ontologies employ words such as 'instantiation', 'constitution', 'realisation' and 'manifestation' to express such relationships between high-level categories. According to DOLCE and BFO 'endurants' 'participate' in 'perdurants' (although their interpretations of 'participate' may be a little different). In the terminology of GFO, a 'continuant' 'exhibits' a 'presential' at a point in time. In DOLCE the quality of redness may 'inhere' in a rose. Providing such terminology certainly helps one recognise and refer to significant relationships, but does little to support information processing that requires computational reasoning based on the meaning of these relationships. For that, we would need to give them a more explicit semantics.

Of course, we can give axioms. But, when dealing with such general categories, adequate axioms seem to be very difficult to formulate. Existing ontologies do little more that specify the categories of the entities related and generic relation properties (e.g. functional, 1:1 etc.). What we really want is concrete specifications of how, say, events arise from spatio-temporal configurations or the specific conditions under which a physical object might manifest an intellectual artifact. We believe that such specifications are only feasible if one starts with a solid foundation and employs precise machinery for constructing complex concepts and entities from simpler ones.

1.2. Foundational Theories in the Wake of Logical Positivism

It is notable that these relatively modern formal ontology frameworks are of a very different character from the foundational theories that had been developed during the mid-part of the 20th century. Carnap's *Der logische Aufbau der Welt* [11] proposed that a general theory of reality could be built up from simple basic facts by means of formal constructions and logical axioms. Originally, this programme had been oriented towards an empiricist view of reality, and had tried to construct theories based on 'sense data' (as advocated by Russell [28]). But it turned out to be very difficult to build up any substantial theory directly from perceptual information; so the approach shifted towards considering scientific theories as a starting point. These theories had been developed by empirical observation and were much better suited to precise specification and formal development.

Intensive work on this project was conducted by a small but highly renowned community of logicians with interests in both philosophy and physics. Its high-point was perhaps the international symposium on *The Axiomatic Method with Special Reference to Geometry and Physics* held at UC Berkeley over the New Year of 1958 [19]. All papers in that collection were influenced by the axiomatisation of classical particle mechanics of [22], in which a set theoretic structure representing particles and their fundamental properties is constrained by equations of motion. Subsequently, Montague [23] showed that classical mechanics can in fact be formulated on the basis of a far simpler underlying mathematical structure than had been employed in [22]. He assumed only the sets $\mathbb{R}$ and $\mathbb{N}$ of real and natural numbers and operations of addition and multiplication. The logical status and interpretation of such set-theoretic formulations of physical theories was investigated in detail in [29].

Another distinctive aspect of the work from this period is the prominence given to questions regarding what are the 'primitive' terms of a theory and to the notion of *definability*. Tarksi [31] (referenced many times in [19]) had shown that, any concepts that can be defined within a theory that includes a complete axiomatisation of geometry can actually be defined directly in terms of geometric primitives. Hence, development of foundational theories of this period was guided by the idea that building a theory from a very small number of primitive concepts was both desirable and feasible.

1.3. Our Approach

Drawing inspiration from the foundational theories of scientific positivism mentioned in the last section, our aim in this paper is to advocate and illustrate an approach to ontology development based on building up from well-understood set-theoretic structures, by means of definitions. We shall see that a huge variety of physical properties and temporal conditions can be defined. The proposed framework makes use of ideas in previous work of one of the authors [4,5,9,6] but gives more explanation of the design methodology and presents this in a modular way. The key modules of our framework: structure, domain and definition, will be explained in Sec 2.

One of the few recent ontology developments based on mathematical models of space and time Bittner's framework, based on classical mechanics [10]. Similarly to the current paper, it develops an object language that is grounded on spatio-temporal structures. Possibly that could be used in a foundational way. However, that work was presented more as a way to augment or amend the content of existing ontologies rather than to provide foundation upon which to build a new form of general ontology.

1.4. Aversion to Idealistic Models and Low Level Primitives

A widely held objection to our type of ontology construction is that it is based on mathematical structures that are too idealised to correspond either to reality or to any human conceptual framework, and therefore are not appropriate foundation for a ontology applicable to describing either the real world or its conceptualisation.

The idea that computational knowledge representation and inference should be formulated along the lines of *commonsense* reasoning has a long tradition in AI. An early crystalisation of this view was presented in Hayes' *Naive Physics Manifesto* [18]. Associated wit this general programme is the idea that entities such as points, lines (and perhaps numbers) that are seen as 'mathematical' rather than 'commonsense' in nature are to be avoided. This position was adopted in the construction of several formal representation languages for spatial reasoning [27,3], and more recently has been used to critique the presence of 'instants' in ontologies of time [16].

Our view is that idealisation is reasonable and perhaps essential to foundational ontology. This is because in order to capture the logic of the *conceptualisations* of reality that underlie natural ways of thinking and talking about reality, we must identify formal structures and axiomatic principles, which abstract away from accidental superficial oddities of natural languages, in order to provide a systematic and consistent basis for representation and reasoning. And, once we do that, we implicitly determine certain abstract structures that satisfy these axioms.

We believe that proponents of 'naive' conceptualisations often have a double standard in their view of what is ideal as opposed to real. If we want to reject idealisation in a consistent way, there are strong reasons to reject the characterisation of objects and spatial relationships that is intrinsic to most foundational ontologies. Objects do not have clear boundaries. Whether two objects are in contact breaks down when we examine objects at a microscopic level. To go even further, there are strong arguments for the view that all the categories and objects that seem to exist within our conceptualisation of the world are idealisations of what is actually present in reality. Regarding categories, the inherent vagueness in terms such as 'cup' or 'mountain' indicate that considering portions of the world as instances of such categories is a patent idealisation, since there are not strict criteria for applicability of ordinary language terminology.

Another concern one might have about our approach is that we assume only the most basic types of entity, a few simple primitives yet are attempting to define categories, properties and relationships spanning the range of human conceptualisation. It may seem unfeasible that complex entities such as cups, cats and countries, and events such as birthday parties and wars could be defined just from spatio-temporal structures and object trajectories. This may be so. In fact we believe that some structure accounting for *possibility* (and perhaps also *desirability*) is required. This could be a possible worlds structure, rather than just a single spatio-temporal structure. Nevertheless, we believe that very rich conceptual vocabularies can be defined from just a few quite low level primitives.

2. Structure, Domain and Definition

We consider an *Interpretation Structure* to be a structure $\langle \Gamma, \mathscr{D}, \Delta \rangle$ consisting of a grounding structure Γ, a domain specification $\mathscr{D}$ and a symbol definition specification Δ.

2.1. Structure

Following the approach used in works such as [22] and [23] and examined in detail in [29], the starting point for our ontology is a set-theoretic model of time and space. We specify the semantics of our spatio-temporal language in terms of a structure of the form $\langle \langle T, \preceq \rangle, \langle R, G, V \rangle \rangle$.

$\langle T, \preceq \rangle$ is the time structure. For the ontology developed in this paper to fit our intuitive understanding of its vocabulary, we require that T is infinite and $\preceq$ is a dense total order. We also presume that $\langle T, \preceq \rangle$ is unbounded and continuous, although we believe that our definitions would still make sense in relation to a bounded and/or countable structure of time points.

$\langle R, G, V \rangle$ is the spatial structure, consisting of a set R of regions, a set G of geometrical relations and functions over R, and V, a total order with a minimal element (0), which is the range of metrical functions over R. Specifically, we assume that R is the set of all regular open subsets of $\mathbb{R}^3$ and $G = \{ \subseteq, \cong, d, v \}$, with the value set being $\mathbb{R}$. Here, $\subseteq$ is the subset relation over R, $\cong$ is the congruence relation. The function $d : (R \times R) \rightarrow \mathbb{R}$, gives the distance between regions — i.e. $d(r_1, r_2) = GLB(\{ed(p_1, p_2) \mid p_1 \in r_1, \; p_2 \in r_2\})$, where $ed(p_1, p_2)$ is the Euclidean distance between points. And $v : R \rightarrow \mathbb{R}$ gives the volume of regions (in units commensurate with d).

In terms of the methodology we are highlighting, the particular choice of structure is not so important. What is important is that the structure is declared and its properties are definite. But there are some reasons why we chose this structure. As we shall see later, density of the time series is important for the way that we define dynamic attributes of objects, such at the condition of an object moving or growing. The need for time to be continuous is less clear. However, if one takes space to be continuous then a coherent theory of motion may also require time to be continuous, so that the trajectory of a moving object could correspond to a mapping from time points to spatial locations.

With regard to the structure of space, one may argue that continuity is essential for coherence. If a line is drawn from a point within a sphere to a point outside a sphere, we would expect that the line must intersect the sphere at some point. It would seem rather odd if this point were somehow missing. Yet for a unit sphere located at the origin of an $\mathbb{R}^3$ coordinate system, and a line passing through the origin and oriented at 45° to an axis, their intersection will be at a irrational coordinate ($\sqrt{2}/2$) along that axis. Related issues persist even if we allow only extended 3D regions, since certain configurations of 3D balls are only possible if their points of contact have irrational coordinates in $\mathbb{R}^3$. Against this kind of argument for continuity, one could contend that such issues only arise for ideal objects such as spheres, which do not exist in reality, whereas the actual, imperfect objects of reality can happily exist in a coarse-grained, discontinuous space.

Another objection to our proposed model of time and space is that does not accord with the geometries underlying the relativistic theories used in modern physics, which are certainly more accurate and more general in scope than the, now superseded, Newtonian theory. Thus the separated structures space and time, should be replaced by the integrated Minkowski spacetime $\mathbb{R}^4_1$, or perhaps $\langle \mathbb{R}^4_1, F \rangle$, with F being a distinguished reference frame or set of frames [30], or perhaps even $\langle \mathbb{R}^4_1, \gamma, F \rangle$, where γ is a Riemannian metric on $\mathbb{R}^4_1$ describing gravitational curvature of the space.

2.2. *Domain*

The second item of our proposed interpretation structure is the domain specification. Frege and Russell introduced the operations of *quantification* into logical languages in order to specify properties of predicates in relation to a domain of discourse. In the standard form of first-order logic the domain of discourse is a single non-empty set D of elements. Neither the set nor its elements have any presumed special structure. But they become structured by formulae that specify axioms or facts in relation to the domain by means of predicates expressing properties and relationships.

In one sense of ‘ontology’, the ontology of a logical theory may be identified with its domain of discourse. This is the meaning of Quine’s famous pronouncement: “To be is to be the value of a bound variable” [26]. But in discussing the foundational ontologies developed in recent decades, we would usually consider the ontology to consist primarily of certain key disjoint sub-domains of the domain of discourse corresponding to fundamental kinds of entity. In contrast to this, in his paper *The study of ontology* [14], Fine gave a general analysis of the structure of ontologies in terms of complex entities being constructed from simple ‘given’ elements. Our methodology adopts this approach, but rather that assuming general use of constructive rules, we only specify certain domains of constructed entities. In fact, in the current presentation we only employ one constructed domain: the set $D \subseteq R^T$, is the domain of individual objects. The set R^T is the set of all functions from T to R. Thus each individual is a mapping from time points to spatial regions, giving the individual’s *extension* at each time.

2.3. *Definition*

Tarski [31] gave formal proofs of a method of determining *definability* that had been proposed by Padoa [25]. The method rests on the presumption that if we have a theory expressed in using the predicate terms $\{\tau, \pi_1, ..., \pi_n\}$, then τ is definable by means of the vocabulary $\{\pi_1, ..., \pi_n\}$ if and only if, all models that have the same valuation of all atomic propositions expressed with terms $\{\pi_1, ..., \pi_n\}$ must also have the same valuation for all atoms involving τ. Or more informally, we can say that fixing the situation with respect to descriptions using $\{\pi_1, ..., \pi_n\}$ will also fix all descriptions using τ.

The potential repercussions of this definability criterion in relation to the design of ontologies, was previously explored in [5,6]. In relation to the methodology that we are presenting here, a key observation is that any property or relation, whose applicability in a given situation can be judged from knowledge of the physical characteristics of that situation (i.e. positions and constitution of physical objects), can be defined in terms of any set of primitives that would be sufficient to describe the physical situation. This gives strong support for our belief that a very rich conceptual vocabulary can be defined from a simple spatio-temporal structure. However, as mentioned above, we believe that some notion of *possibility* would need to be added to get a comprehensive framework. (We will not here consider the issue of whether mental concepts can be reduced to physical concepts and possibilities.)

3. A Formal Spatio-Temporal Language and its Semantics

We now give a concise specification of a formal language, whose semantics is directly fixed to the most standard classical model of time and space. Such structures all include the same fixed 'grounding' structure consisting, a set-theoretic model of time and space. But anchored to that are non-fixed structures that specify: the domain of individuals, and the interpretation function for the object language vocabulary.

3.1. Spatio-Temporal Interpretation Structures

A *spatio-temporal interpretation structure* takes the form:

$$\mathscr{I} = \langle \Gamma, \mathscr{D}, \Delta \rangle \ = \ \langle \ \langle \langle T, \preceq \rangle, \langle R, G, V \rangle \rangle, \ \langle T, R, D \rangle, \ \langle \mathscr{V}, \delta, \Theta \rangle \ \rangle \ ,$$

where:

- T is a set of time points with a *dense*, *total* order $\preceq$.
- R is a set of spatial regions and $G = \{ \ \subseteq, \cong, d, v \ \}$ specifies geometrical relations subset and congruence over R, distance $d : (R \times R) \to V$ and volume $R \to V$.
  As explained in Sec. 2.1, we assume R to be the regular open subsets of $\mathbb{R}^3$, and the value range V to be $\mathbb{R}$.
- $\langle T, R, D \rangle$ gives the domains of quantification, with T and R being just the sets of time points and regions in the grounding structure Γ. The set $D \subseteq R^T$ is the domain of individuals. This is a constructed domain, as explained in Sec. 2.2 above.
- $\mathscr{V} = \langle \mathscr{N}, \mathscr{T}, \mathscr{F}_1, \mathscr{F}_2, \mathscr{G}, \mathsf{ext} \rangle$ is a vocabulary comprising:
  - $*$ $\mathscr{T} = \{\ldots, \ t_i, \ldots\}$ — time point symbols,
  - $*$ $\mathscr{R} = \{\varnothing, \ldots, \ r_i, \ldots\}$ — spatial region symbols (including null region, $\varnothing$),
  - $*$ $\mathscr{N} = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \ldots\}$ — names (denoting individuals),
  - $*$ $\mathscr{F}_1 = \{\ldots, \ f_i, \ldots\}$ — fluent predicates (with one time point argument),
  - $*$ $\mathscr{F}_2 = \{\ldots, \ g_i, \ldots\}$ — bi-fluent predicates (with two timepoint arguments),
  - $*$ $\mathscr{G} = \{ \ \subseteq, \cong, \mathsf{dist}, \ \mathsf{vol}, \ \ldots\}$ — primitive geometrical relations and operators,
  - $*$ ext — the extension function,
- δ is a denotation function mapping vocabulary symbols to semantic objects.

- Θ is an auxiliary theory that is employed to define and constrain further vocabulary in terms of the grounded primitives in $\mathscr{V}$.

We specify the denotation function δ as the union of functions giving denotations for each of the non-logical symbol types. Thus, $\delta(\alpha)$, will be equal to $\delta_\tau(\alpha)$, where τ is the symbol type of α. These functions are defined by:

- $\delta_{\mathscr{T}} : \mathscr{T} \to T$
- $\delta_{\mathscr{R}} : \mathscr{R} \to R$ with $\delta_{\mathscr{R}}(\varnothing) = \emptyset$,
- $\delta_{\mathscr{N}} : \mathscr{N} \to D$, where $D \subseteq R^T$,
- $\delta_{\mathscr{F}_1} : \mathscr{F}_1 \to 2^{(D^n \times T)}$, n is the arity of the fluent predicate,
- $\delta_{\mathscr{F}_2} : \mathscr{F}_2 \to 2^{(D^n \times T^2)}$, n is the arity of the bi-fluent predicate,
- $\delta_{\mathsf{ext}}(\mathsf{ext}(\mathsf{a},t)) = \delta_{\mathscr{N}}(\mathsf{a})(\delta_{\mathscr{T}}(t))$, abbreviated as $\delta(\mathsf{a},t)$.

$\delta_{\mathscr{F}_1}$ gives the truth sets of fluent predicate symbols, which for an n-ary fluent will be a subset $D^n \times T$ such that $\langle x_1,\ldots,x_n,t\rangle \in \delta_{\mathscr{F}_1}(f)$ iff the fluent $f(x_1,\ldots,x_n)$ is true at time t. This could also be expressed as a mapping $D^n \to 2^T$, so a fluent predicate is associated with a mapping from tuples of individuals sets of time points. Similarly, $\delta_{\mathscr{F}_2}$ provides a mapping from tuples of individuals to sets of *pairs* of time points (these will typically correspond to intervals).

The specific vocabulary in $\mathscr{G}$ will be directly interpreted by functions in G as specified by the semantics of atomic propositions given below.

The interpretation of ext is defined so that $\delta_{\mathsf{ext}}(\mathsf{ext}(\mathsf{a},t))$ denotes the extension of individual a at time t. Since $\mathsf{a} \in \mathscr{N}$, we have $\delta(\mathsf{a}) \in R^T$; so, $\delta(\mathsf{a})$ is a function from T to R, which we can apply to the time $\delta(t)$, to get some region $r \in R$. To reduce symbol clutter, we abbreviate $\delta_{\mathsf{ext}}(\mathsf{ext}(\mathsf{a},t))$ to $\delta(\mathsf{a},t)$.

3.2. Syntax and Satisfaction

We present syntax and semantics in tandem, by means of clauses that both specify syntactic forms and also stipulate the conditions under which they are true according to any given interpretation structure. We write $\mathscr{I} \Vdash \varphi$ to mean that interpretation $\mathscr{I}$ satisfies the formula φ — in other words, φ is true according to $\mathscr{I}$.

Satisfaction conditions relating to 'grounded' atomic formula, can be specified directly in terms of the spatio-temporal structure. For basic relations comparing terms we can define:

- $\mathscr{I} \Vdash t_1 = t_2, \quad \Vdash t_1 \leq t_2$ iff, respectively, $\delta(t_1) = \delta(t_2),\ \delta(t_1) \preceq \delta(t_2)$,
- $\mathscr{I} \Vdash \rho_1 = \rho_2, \Vdash \rho_1 \subseteq \rho_2$ iff, respectively, $\delta(\rho_1) = \delta(\rho_2),\ \delta(\rho_1) \subseteq \delta(\rho_2)$,
- $\mathscr{I} \Vdash \rho_1 \cong \rho_2$, iff $\delta(\rho_1) \cong \delta(\rho_2)$,
- $\mathscr{I} \Vdash \mathsf{dist}(\rho_1,\rho_2) \leq \mathsf{dist}(\rho_3,\rho_4)$ iff $d(\delta(\rho_1),\delta(\rho_2)) \leq d(\delta(\rho_3),\delta(\rho_4))$,
- $\mathscr{I} \Vdash \mathsf{vol}(\rho_1) \leq \mathsf{vol}(\rho_2)$ iff $v(\delta(\rho_1)) \leq v(\delta(\rho_2))$.

Here, ρ denotes a spatial region and is either a spatial symbol, $\rho \in \mathscr{R}$, or an extension term, $\rho = \mathsf{ext}(\mathsf{a},t)$, with $\mathsf{a} \in \mathscr{N}$ and $t \in \mathscr{T}$.

Our choice of which concepts to ground directly upon the spatio-temporal structure is not intended to be prescriptive. Rather, it can be customised to suit one's purposes. In any case, there is considerable flexibility, since, once we have defined sufficiently expressive basic concepts, we can easily extend the vocabulary by definitions in the object language. We note that, as proved in [8] the relations of spatial parthood ($\subseteq$), and congruence ($\cong$) are sufficient primitives to axiomatise elementary Euclidean geometry.

We now specify in a general way, the interpretation of atomic propositions formed by combining arbitrary (i.e. not directly grounded) fluents or bi-fluents with time symbols. To turn fluents into propositions, that are true or false in an interpretation structure, we append '$@t$', where $t \in \mathscr{T}$. The semantics for these expressions is:

- $\mathscr{I} \Vdash f(a_1,\ldots,a_n)@t$ iff $\langle \delta(a_1),\ldots,\delta(a_n),\ \delta(t)\rangle \in \delta(f)$,
- $\mathscr{I} \Vdash g(a_1,\ldots,a_n)@[t_1,t_2]$ iff $\langle \delta(a_1),\ldots,\delta(a_n),\ \delta(t_1),\delta(t_2)\rangle \in \delta(g)$.

In the above clauses, we see that the satisfaction conditions will depend on the particular interpretation functions $\delta_{\mathscr{F}_1}$ and $\delta_{\mathscr{F}_2}$, which are completely unconstrained by the general specification an interpretation structure. However, as we shall see shortly, these interpretations will typically be constrained by axioms or definitions given in the theory Θ, that ground particular fluents and bi-fluents in terms of the spatio-temporal structure and the extensions of individuals. Hence, only certain interpretation structures will satisfy conditions appropriate to particular intended meanings of fluents and bi-fluents.

Finally, the semantics for complex formula formed with truth functional operators and quantifiers are specified in the usual way:

- $\mathscr{I} \Vdash \neg\varphi$ iff $\mathscr{I} \nVdash \varphi$,
- $\mathscr{I} \Vdash (\varphi \wedge \psi)$ iff $\mathscr{I} \Vdash \varphi$ and $\mathscr{I} \Vdash \psi$
- $\mathscr{I} \Vdash \forall\upsilon[\varphi]$ iff $\Vdash \mathscr{I}' \Vdash \varphi$, for all $\mathscr{I}'$ such that $\mathscr{I}' \approx_\upsilon \mathscr{I}$.

In the quantification clause, υ is a symbol in $\mathscr{N} \cup \mathscr{T}$. Variables are just name or time symbols that are bound to a quantifier. $\mathscr{I}' \approx_\upsilon \mathscr{I}$ means that $\mathscr{I}'$ is exactly like $\mathscr{I}$ except possibly in the value of υ — i.e. $\delta'(\upsilon)$ may differ from $\delta(\upsilon)$. We can define additional connectives, '$\vee$', ' $\rightarrow$ ', ' $\leftrightarrow$ ', and the existential quantifier '$\exists$' in the usual way.

3.2.1. Temporal Qualification of Propositions

So far, our semantics specifies which fluents are true at each time point. However it does not tell us how to interpret complex propositions as being true or false at a given time point. The non-fluent atomic propositions express relations between time points or between spatial regions. Thus they do not change from time to time. So for any spatial or temporal terms τ_1 and τ_2 and any comparison relation '*', qualification by the '@' operator is redundant:

- $\Vdash (\tau_1 * \tau_2)@t$ iff $\Vdash (\tau_1 * \tau_2)$.

Also, since truth functions do not depend on time, we can give the following simple specification for temporal qualification of truth-functional compound formulae:

- $\Vdash (\neg\varphi)@t$ iff $\nVdash \varphi@t$,
- $\Vdash (\varphi \wedge \psi)@t$ iff $\Vdash \varphi@t$ and $\Vdash \psi@t$.

In the case of quantified formula considered with respect to a time point, the interpretation of quantification with respect to time and region variables is straightforward. However, if the quantification is with respect to an object name variable, in relation to a particular time point t, we have two possible ways to specify the domain of quantification. We can either quantify over the whole domain of objects including those that have no physical manifestation at time t; or, we can restrict quantification to only those objects that are manifest (i.e. whose extension is not $\varnothing$) at the time t:

- $\Vdash \forall \upsilon[\Phi(\upsilon)]@t$ iff $\Vdash \forall \upsilon[\,\Phi(\upsilon)@t\,]$
- $\Vdash \forall^{*}\mathsf{x}[\Phi(\mathsf{x})]@t$ iff $\Vdash \forall \mathsf{x}[\,\neg(\mathsf{ext}(\mathsf{x},t) = \varnothing) \rightarrow \Phi(\mathsf{x})@t\,]$ (where $\mathsf{x} \in \mathscr{N}$)

Here, υ can be any symbol in $\mathscr{T} \cup \mathscr{R} \cup \mathscr{N}$, but the second interpretation only makes sense for object symbols in $\mathscr{N}$. If we interpret quantification over objects according to the first option, we can consider this to be a 4-Dimensional or eternalist view of objects. By taking the second option we are considering that only those objects present at a time can be considered to exist from the point of view of that time.

3.3. Axioms and the Auxiliary Theory Θ

According to our view of ontology, the core of our ontology framework is now complete. The fundamental commitments of the ontology are encapsulated within the grounding structure and the domain specification. In relation to those, we defined temporal and spatial primitives and a language for relating entities both within and between the three domains (time, space and individuals). The rest of the ontology will be constructed by an additional theory Θ. This can be divided into two parts. Purely definitional axioms will use the primitive vocabulary to ground the meanings of additional terms. Further axioms can be used to specify meaning postulates that will constrain the interpretation of further terminology for which it is difficult, perhaps impossible, to give a rigorous grounding.

4. Objects in Space

In this section we aim to give just a small sample of the potentially huge variety of terminology that can be defined in terms of the structure presented above, and also of the many ontologically significant distinctions that can between types of entity.

- $\mathsf{C}(\rho_1,\rho_2) \equiv_{def} \forall r_1 \forall r_2[\mathsf{dist}(\rho_1,\rho_2) \leq \mathsf{dist}(r_1,r_2)]$
- $\mathsf{O}(\rho_1,\rho_2) \equiv_{def} \exists r[\neg(r = \varnothing) \wedge r \subseteq \rho_1 \wedge r \subseteq \rho_2]$
- $\mathsf{Compl}(\rho_1,\rho_2) \equiv_{def} \forall r[\neg \mathsf{O}(r,\rho_1) \rightarrow r \subseteq \rho_2]$

4.1. Fluents: changeable relations between individuals

We now specify some properties and relations among individuals. Certain properties of individuals vary at different time points, in other words they are fluents.

- $E(\mathsf{a})@t \equiv_{def} \neg(\mathsf{ext}(\mathsf{a},t) = \varnothing)$
- $C(\mathsf{a},\mathsf{b})@t \equiv_{def} \mathsf{C}(\mathsf{ext}(\mathsf{a},t),\mathsf{ext}(\mathsf{a},t))$
- $O(\mathsf{a},\mathsf{b})@t \equiv_{def} \mathsf{O}(\mathsf{ext}(\mathsf{a},t),\mathsf{ext}(\mathsf{a},t))$

From these definitions, we see that these fluents are fully defined purely in terms of objects, times and spatial properties of object extensions. So they are essentially just convenient ways of characterising the synchronous configurations of multiple trajectories.

4.2. Permanent Properties of Individuals

- $\mathsf{P}^*(\mathsf{a},\mathsf{b}) \quad \equiv_{def} \forall t[\mathsf{ext}(\mathsf{a},t) \subseteq \mathsf{ext}(\mathsf{b},t)]$
- $\mathsf{DR}^*(\mathsf{a},\mathsf{b}) \quad \equiv_{def} \forall t[\neg O(\mathsf{a},\mathsf{b})@t]$
- $\mathsf{Rigid}^*(\mathsf{a}) \quad \equiv_{def} \forall t_1 t_2[\mathsf{ext}(\mathsf{a},t_1) \cong \mathsf{ext}(\mathsf{a},t_2)]$
- $\mathsf{Enduring}^*(\mathsf{a}) \quad \equiv_{def} \forall t_1 t_2 t_3[\ ((t_1 < t_2 < t_3) \wedge E(\mathsf{a},t_1) \wedge E(\mathsf{a},t_3)) \rightarrow E(\mathsf{a},t_2)\ ]$

4.3. Constraints on Object Types

Upper ontologies often define categories according to high level properties such as persistence through time, identity conditions, and the types of change entities of that category they can undergo. As we just saw, our framework makes it easy to specify many of these kinds of property as they affect individual objects and pairs of objects.

- $\mathsf{Rigid}_{\mathsf{x}}[\Phi(\mathsf{x})] \quad \equiv_{def} \quad \forall \mathsf{x}[\Phi(\mathsf{x}) \rightarrow \mathsf{Rigid}^*(\mathsf{x})]$
- $\mathsf{Enduring}_{\mathsf{x}}[\Phi(\mathsf{x})] \quad \equiv_{def} \quad \forall \mathsf{x}[\Phi(\mathsf{x}) \rightarrow \mathsf{Enduring}^*(\mathsf{x})]$

We can also define more complex quantifier-like expressions. For example:

- $\mathsf{Sep}_{\mathsf{x}}[(\Phi(\mathsf{x})] \quad \equiv_{def} \quad \forall \mathsf{a} \forall \mathsf{b}[\ \neg(\mathsf{a}=\mathsf{b}) \wedge \Phi(\mathsf{a}) \wedge \Phi(\mathsf{b})) \rightarrow \mathsf{DR}^*(\mathsf{a},\mathsf{b})\ ]$

So we could write $\mathsf{Sep}_{\mathsf{x}}[\mathsf{Sock}(x)]$ to assert that no two distinct socks may overlap.

5. Temporal Conditions and Entities

The definitions of the spatial relation fluents C and O illustrate the form of definition, by which we can specify the meaning of fluents (properties and relationships true at a time point) and bi-fluents (properties and relationships true in relation to two time points. The general form of such definitions is:

- $\alpha(a_1,\ldots,a_n)@t \quad \equiv_{def} \Phi(a_1,\ldots,a_n,t)$
- $\beta(a_1,\ldots,a_n)@[t_1,t_2] \quad \equiv_{def} \Psi(a_1,\ldots,a_n,t_1,t_2)$

5.1. Dynamic Homeomeric Fluents

In natural languages we often refer to *processes* — that is, ongoing changes of certain kinds. Processess may have some structure (such as the process of setting a table) but an important type of process is those that have been called *homeomeric*, meaning that all temporal parts of the process are, in some relevant sense, equivalent. For a significant class of homeomeric processes, the conditions under which it occurs can be defined by specifying a constraint that must satisfied over some arbitrarily small interval that surrounds the time point. For example, the times at which the process of 'moving' is ongoing (in a very general sense of 'moving') might be captured by the following definition:

$$\mathsf{Moving}(x)@t \;\equiv_{def}\; \exists t_1 t_2[\,(t_1 < t < t_2) \wedge \forall t_1' t_2'[(t_1 < t_1' < t_2' < t_2) \rightarrow \neg(\mathsf{ext}(x,t_1') = \mathsf{ext}(x,t_2'))]\,]$$

This definition says that an individual x satisfies the fluent Moving at time t just in case there is some time interval $[t_1,t_2]$ that strictly contains t, and is such that for any two distinct times within $[t_1,t_2]$ the x extension of x is distinct. Since the definition refers only to change in extension it includes processes such as growing, which might not usually be described as 'moving'. For example, a perfect cylinder would not count as moving if it rotated about its axis. There are also cases where an object hits a wall or oscillates in such a way that it exactly retraces its trajectory immediately after a certain time point. In such cases the object would not satisfy our definition of Moving at the point of reversal.

A possible definition of the process of growth in terms of a dynamic fluent Growing is as follows:

$$\mathsf{Growing}(x)@t \;\equiv_{def}\; \exists t_1 t_2[\,(t_1 < t < t_2) \wedge \forall t_1' t_2'[\,(t_1 < t_1' < t_2' < t_2) \rightarrow (\mathsf{vol}(\mathsf{ext}(x,t_1')) < \mathsf{vol}(\mathsf{ext}(x,t_2')))\,]\,]$$

Open ended processes tend to be associated with changes in some observable value: movement is a change in position, growth is an increase in size, shrinkage is a decrease in size. But this may not always be the case. For instance we may not be able to define 'burning' as a change in some measure of burntness.

Our analysis and representation of dynamic fluents sheds light on the *Dividing Instant Problem* [20]. This is the presumed problem that, when there is some change from the state where a condition holds to one where it does not hold, there is a dilemma concerning whether or not the condition holds at the dividing instant between holding and not holding. What we find (as also noted in [15]) is that some fluents (e.g. Moving) can only hold over open intervals of time, whereas others must hold over closed intervals.

5.2. Event Tokens

Natural language sentences often assert properties of particular occurrences of some event type — for example: 'Sue carefully brushed her teeth for 5 minutes in the bathroom'. These occurrences, or instances, of an event type are called event *tokens*. To reference an event token within a formula we assume, following Davidson [12] that the logical structure of sentences referring to event tokens. Thus, the form of a sentence with an event token would be:

$$(\exists \varepsilon : \beta(a_1,\ldots,a_n))[\,\Phi(\varepsilon)\,]$$

But what kind of entity would the token variable ε be associated with? We do not have any suitable type of entity in our given domain specification $\langle T,R,D\rangle$; however we may be able to construct a suitable domain.

The assignment functions $\delta_{\mathscr{F}_1}$ and $\delta_{\mathscr{F}_2}$ for fluents and bi-fluents can be recast into the forms: $\delta_{\mathscr{F}_1} : (\mathscr{F}_1 \times D^n) \rightarrow 2^T$ and $\delta_{\mathscr{F}_2} : (\mathscr{F}_2 \times D^n) \rightarrow 2^{(T\times T)}$. Hence, we can specify $\delta(f(a_1,\ldots,a_n))$ as denoting the set of time points when the fluent holds and $\delta(g(a_1,\ldots,a_n))$ denoting the set of intervals at which a bi-fluent occurs. We can now construct an event token as a pair consisting of an event type (set of intervals) and a particular interval of occurrence of an event of that type. Thus the domain of event tokens is a constructed domain, which is a subset of $2^{(D^n \times T^2)} \times T^2$. Allowing event token variables to range over this constructed domain, we could rewrite the previous formula as:

$$(\exists \varepsilon t_1 t_2)[\ \varepsilon = \langle(\beta(a_1,\ldots,a_n)),[t_1,t_2]\rangle \wedge \beta(a_1,\ldots,a_n)@[t_1,t_2] \wedge \Phi(\varepsilon)\ ]\ .$$

where, $\delta(\varepsilon) \in (2^{(D^n \times T^2)} \times T^2)$, and the tuple $\delta\langle\tau_1,\tau_2\rangle$ is evaluated as $\langle\delta(\tau_1),\delta\tau_2\rangle$.

So, in the case of the tooth-brushing example, we would have:

$$(\exists \varepsilon t_1 t_2)[\ \varepsilon = \langle \textbf{brush\_teeth}(\mathsf{sue}),[t_1,t_2]\rangle\rangle\ \wedge \textbf{brush\_teeth}(\mathsf{sue})@[t_1,t_2]$$
$$\wedge\ \mathsf{dur}([t_1,t_2]) = 5m \wedge \mathsf{Loc}(\mathsf{sue},\mathsf{bathroom}) \wedge \mathsf{Careful}(\varepsilon)\ ]$$

6. Conclusion

We have hope to have explained and motivated a methodology for ontology design that is significantly different from some well known ontologies, but draws on ideas from the earlier tradition of scientific positivism. Whereas those theories dealt only with quite low level physical properties and relationships, we have indicated ways in which a framework built in this way can be extended by precise definitions to encompass higher-level properties and relations, potentially generating a rich conceptual vocabulary.

Obviously, we are still a long way from constructing a practical ontology incorporating, cups, cats and camping holidays. To do that we would need to build up through many stages both in terms of making abstractions and articulating relevant details. This would require an extensive mid-level ontology, which would be both grounded in the lower spatio-temporal primitives and also suitable for describing more every-day items and circumstances. As we have said, this would almost certainly require the introduction of a notion of possibility. We envisage that this might be modelled in terms of branching possible histories, as was proposed in [9]. We would also need to account for terminology that is not fully grounded because of its ambiguity or vagueness. For that one might employ a supervaluation approach [13], or standpoint semantics as advocated in [7,1].

References

[1] L Gómez Álvarez and Sebastian Rudolph. Standpoint logic: Multi-perspective knowledge representation. In *Proceedings of the 12th International Conference on Formal Ontology in Information Systems*, volume 344, pages 3–17, 2022.

[2] Robert Arp, Barry Smith, and Andrew D Spear. *Building ontologies with basic formal ontology*. Mit Press, 2015.

[3] N. Asher and L. Vieu. Toward a geometry of common sense: A semantics and a complete axiomatization of mereotopology. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI-95), Montreal*, 1995.

[4] Brandon Bennett. Space, time, matter and things. In C. Welty and B. Smith, editors, *Proceedings of the 2nd international conference on Formal Ontology in Information Systems (FOIS'01)*, pages 105–116, Ogunquit, 2001. ACM. http://www.comp.leeds.ac.uk/qsr/pub/bennett-fois01.pdf.

[5] Brandon Bennett. The role of definitions in construction and analysis of formal ontologies. In *Sixth Symposium on Logical Formalizations of Commonsense Reasoning*, AAAI Spring Symposium, Palo Alto, 2003.

[6] Brandon Bennett. Relative definability in formal ontologies. In A. Varzi and L. Vieu, editors, *Proceedings of the 3rd International Conference on Formal Ontology in Information Systems (FOIS-04)*, pages 107–118. IOS Press, 2004.

[7] Brandon Bennett. Standpoint semantics: a framework for formalising the variable meaning of vague terms. In Petr Cintula, Christian Fermüller, Lluis Godo, and Petr Hájek, editors, *Understanding Vagueness — Logical, Philosophical and Linguistic Perspectives*, Studies in Logic, pages 261–278. College Publications, 2011.

[8] Brandon Bennett, Anthony G. Cohn, Paolo Torrini, and Shyamanta M. Hazarika. A foundation for region-based qualitative geometry. In W. Horn, editor, *Proceedings of ECAI-2000*, pages 204–208, 2000.

[9] Brandon Bennett and A.P. Galton. A unifying semantics for time and events. *Artificial Intelligence*, 153:13–48, 2004.

[10] Thomas Bittner. Formal ontology of space, time, and physical entities in classical mechanics. *Applied Ontology*, 13(2):135–179, 2018.

[11] Rudolf Carnap. *Der logische Aufbau der Welt*. Weltkreis, Berlin, 1928. Translated by R. George as *THe Logical Structure of the World*, University of California Press, 1969.

[12] Donald Davidson. The logical form of action sentences. In N. Rescher, editor, *The Logic of Decision and Action*. University of Pitsburgh Press, 1967. Reprinted in *Essays on Actions and Events*, OUP, 1980.

[13] Kit Fine. Vagueness, truth and logic. *Synthèse*, 30:263–300, 1975.

[14] Kit Fine. The study of ontology. *Noûs*, 25(3):263–294, 1991.

[15] A P Galton. *Qualitative Spatial Change*. Oxford University Press, 2000.

[16] Antony Galton. The treatment of time in upper ontologies. In Stefano Borgo, Pascal Hitzler, and Oliver Kutz, editors, *Formal Ontology in Information Systems - Proceedings of the 10th International Conference, FOIS 2018, Cape Town, South Africa, 19-21 September 2018*, volume 306 of *Frontiers in Artificial Intelligence and Applications*, pages 33–46. IOS Press, 2018.

[17] Giancarlo Guizzardi, Gerd Wagner, João Paulo Andrade Almeida, and Renata SS Guizzardi. Towards ontological foundations for conceptual modeling: The unified foundational ontology (ufo) story. *Applied ontology*, 10(3-4):259–271, 2015.

[18] P J Hayes. The second naive physics manifesto. In J R Hobbs and B Moore, editors, *Formal Theories of the Commonsense World*, pages 1–36. Ablex, 1985.

[19] L. Henkin, P. Suppes, and A. Tarski. *The Axiomatic Method — with special reference to geometry and physics*. North-Holland, Amsterdam, 1959.

[20] Jixin Ma and Brian Knight. Representing the dividing instant. *The Computer Journal*, 46(2):213–222, 2003.

[21] Claudio Masolo, Stefano Borgo, Aldo Gangemi, Nicola Guarino, and Alessandro Oltramari. WonderWeb Deliverable D18: DOLCE Ontology Library. Technical report, Laboratory For Applied Ontology - ISTC-CNR, 2003. http://wonderweb.semanticweb.org/deliverables/documents/D18.pdf.

[22] J. C. C. McKinsey, A. C. Sugar, and P. C. Suppes. Axiomatic foundations of classical partical mechanics. *Journal of Rational Mechanics and Analysis*, 2:253–272, 1953.

[23] R. Montague. Deterministic theories. In *Decisions, values and groups 2*. Pergamon Press, 1962. Reprinted in *Formal Philosophy: selected papers of Richard Montague*, ed. R. H. Thomason, Yale University Press, 1974.

[24] Ian Niles and Adam Pease. Towards a standard upper ontology. In *Proceedings of the international conference on Formal Ontology in Information Systems-Volume 2001*, pages 2–9, 2001.

[25] Alessandro Padoa. Un nouveau système irréductible de postulats pour l'algèbre. In *Proceedings of the Deuxième Congrès International des Mathématiciens*, pages 249–256, Paris, 1902.

[26] Willard Van Orman Quine. On what there is. *Review of Metaphysics*, 2:21–38, 1948.

[27] David A. Randell, Zhan Cui, and Anthony G. Cohn. A spatial logic based on regions and connection. In *Proc. 3rd Int. Conf. on Knowledge Representation and Reasoning*, pages 165–176, San Mateo, 1992. Morgan Kaufmann.

[28] B. A. W. Russell. *The Problems of Philosophy*. Williams & Norgate, London, 1912. Reprinted 2001, OUP.

[29] J. D. Sneed. *The Logical Structure of Mathematical Physics*. Reidel, 1971.

[30] Patrick Suppes. Axioms for relativistic kinematics with or without parity. In *The Axiomatic Method (with special reference to geometry and physics)* [19], pages 291–307.

[31] A. Tarski. Some methodological investigations on the definability of concepts. In *Logic, Semantics, Metamathematics*. Oxford Clarendon Press, 1956. trans. J.H. Woodger.