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# A Bayesian network development methodology for fault analysis; case study of the automotive aftertreatment system

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## ABSTRACT

This paper proposes a structured methodology for generating a Bayesian network (BN) structure for an engineered system and investigates the impact of integrating engineering analysis with a data-driven methodology for fault analysis. The approach differs from the state of the art by using different initial information to build the BN structure. This method identifies the cause-and-effect relationships in a system by Causal Loop Diagram (CLD) and based on that, builds the Bayesian Network structure for the system. One of the challenges in identifying the root cause for a fault is to determine the way in which the related variable causes the fault. To deal with this challenge, the proposed methodology exploits Dynamic Fault Tree Analysis (DFTA), CLD and the correlation between variables. To demonstrate and evaluate the effectiveness of the presented method, it is implemented on the data-driven methodology applied to the automotive Selective Catalytic Reduction (SCR) system and the obtained results have been compared and discussed. The proposed methodology offers a comprehensive approach to build a BN structure for an engineered system, which can enhance the system's reliability analysis.

## 1. Introduction

As engineered systems, such as road vehicles, continue to evolve, ensuring their safety and reliability becomes an escalating challenge. These systems, featuring numerous interconnected components and subsystems, exhibit high nonlinear and stochastic dynamics, while operating in diverse loads and noisy environments, leaving them susceptible to various failure modes. Given these challenges, the development and implementation of effective diagnostics and prognostics methods for these systems become imperative [1–3].

Over the years, various health-related programs have been introduced and developed, with the most common and well-known being different types of maintenance programs (e.g., corrective maintenance, preventive maintenance, condition-based maintenance (CBM)), as well as prognostics and health management (PHM). Diagnostics and prognostics play vital roles in all of these health-related programs. Diagnostic analysis is responsible for detecting, isolating, and identifying faults or failures when they occur, while prognostics aim to predict the future faults, failures, or states of a system based on its current conditions [4,5].

In addressing the necessity of the research, it is crucial to highlight the ongoing advancements in diagnostics and prognostics, which have undoubtedly yielded commendable results. However, within these achievements, there exist compelling opportunities for further improvement of diagnostic and prognostic capabilities, particularly within the realm of complex engineered systems. The imperative

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for such endeavours stems from the challenges inherent in meeting the escalating demands and stringent requirements for the reliability and safety of these systems. Notably, current methodologies may fall short in comprehensively encompassing all the complex characteristics of these systems, prompting the need for a more detailed and comprehensive approach. A comprehensive discussion on complex systems and how different methods can address one or more of their characteristics in diagnostics and prognostics analysis can be found in [1].

Sensor reliability and the accuracy of sensor measurements have consistently been integral to data-driven methods [6]. However, the measurement precision of sensors may not always meet acceptable standards, and they could occasionally yield biased outputs randomly. In the context of autonomous cars, where safety is of paramount concern, the measurement precision of sensors becomes a critical area of focus, prompting numerous research endeavors to address this challenge [7]. If the measurement accuracy of sensors is a matter of concern in such a safety-critical system, the situation will be worse in non-safety-critical systems such as road vehicles. To compare predicted values and sensor measurements, model-based methods are widely employed. Building a precise mathematical model that accurately describes the real physical process is arduous, and the process of simplification and making assumptions may lead to a decrease in the model's accuracy [8,9].

In general, data for engineered systems are obtained from sensor measurements or model-based calculations. Sensor data possess uncertainty due to the limited accuracy of sensors, and the use of more precise yet expensive sensors in mass production is not cost-effective [10,11]. Data-driven methods for diagnostics and prognostics of engineered systems may not yield highly reliable results, given their reliance on the available data. In the literature, various methods, such as different forms of Kalman filters [12,13] and Bayesian approach [14,15], are employed to address the challenges posed by uncertain data in physics-based diagnostics and prognostics approaches. Nonetheless, the application of physics-based methods for complex engineered systems remains limited, with data-driven or hybrid methods being widely adopted.

Valuable information and knowledge concerning the system's function can be derived from engineering analysis. Consequently, it holds a significant role in supporting data-driven approaches for the reliability analysis of engineered systems. In the literature, a few experience-based or knowledge-based approaches have been employed for diagnostics and prognostics purposes [1]. These methods rely on engineering experience and historical failure data to automate the representation of problem-solving in a manner akin to human reasoning [16]. However, there is no well-established methodology for utilising engineering analysis and functional information to bolster data-driven methods.

In the literature, the focus of diagnostics and prognostics research has primarily been on well-known systems such as the Tennessee Eastman process (e.g., [17]) and rolling bearings (e.g., [18]), while studies on other complex systems like automotive systems are scarce. The limited application of proposed diagnostics and prognostics methodologies to complex engineered systems with real-world data poses a challenge in validating the applicability and robustness of these approaches. Addressing the complexities of diagnostics and prognostics for complex systems is considerably more demanding than dealing with well-known systems with thoroughly analysed data [1].

A Bayesian network is a graphical model based on probability theory, employed to visually and conceptually elucidate the relationships between faults and symptoms. In recent decades, the utilisation of Bayesian Networks (BN) and Dynamic Bayesian Networks (DBN) has gained considerable popularity in dependability analyses, encompassing reliability, availability, and maintainability, owing to their modeling capabilities (e.g., ability to represent complex inter-relationships) and advantages over traditional approaches [19,20]. While not universally applicable, Bayesian network modeling offers flexibility and adaptability across diverse domains.

Bayesian networks have demonstrated their efficacy as tools in diagnostics and prognostics across a broad spectrum of applications, including the aeronautics industry [21], rotating machines [22], industrial process [23], and power systems [24]. Their effectiveness in these domains has been further enhanced by integrating them with other methods. For instance, the combination of Hidden Markov Models (HMM) with BN/DBN has found use in various studies for fault analysis of systems like the Tennessee Eastman process (e.g., [17,25]) and rotating machinery (e.g., [18,26]).

It is essential to highlight that the application of Bayesian networks for fault analysis is predominantly limited to extensively researched and well-established systems, exemplified by widely studied processes like the Tennessee Eastman process, where an established BN structure is readily available. Rarely do research studies for fault analysis encounter instances where the BN has been systematically constructed for the specific system under investigation, with a comprehensive explanation of its development process. This clear limitation underscores the pressing need for this research.

The overall research was motivated by experiences with automotive propulsion systems, where the presence of high variability in usage duty cycles gives rise to significant dynamic transient behaviour. Effectively managing this behaviour is crucial to ensure optimal performance and adherence to regulatory requirements. The primary aim of this study is to construct a BN structure through engineering analysis for the benefit of diagnostics and prognostics for complex engineered systems. Additionally, it entails a comparative analysis of the impact of the BN structure on the results obtained from a data-driven methodology.

The remainder of this paper is organised as follows: In Section 2, a comprehensive background is provided on building BN structures in the existing literature, alongside the discussion of research objectives and contributions. In Section 3, the proposed methodology is introduced and discussed within the context of the Selective Catalytic Reduction (SCR) system, utilising causal loop diagrams (CLD) and Dynamic Fault Tree Analysis (DFTA) to develop and validate the BN structure. In Section 4, the root cause identification results of the data-driven fault analysing methodology are presented and supported by DFTA and data correlation. Finally, the last section contains the conclusions drawn from the study.

## 2. Background and objectives

In this research paper, the review of related work is presented in two distinct sections. The first section provides a succinct overview of diagnostics and prognostics in complex systems, drawing from the summary of pertinent literature featured in [2]. The second section, conversely, delves into the Bayesian Network structure development. In this context, the latter part of this section highlights the distinct contributions of this study, focusing on what makes the methodology proposed unique and advantageous. As a result, this research aims to bring a noteworthy and fresh perspective to the academic field.

### 2.1. Review of related work on data-drive diagnostics and prognostics

In the realm of fault analysis in engineered systems, Diagnostics and Prognostics are crucial and essential for ensuring the reliability and optimal performance of complex systems. The growing complexity of these systems has led to an increasing reliance on data-driven methods, highlighting their rising significance, particularly in the domain of Diagnostics and Prognostics. The effective application of digital twin technologies for fault diagnosis in engineering systems has been demonstrated by recent studies [27,28], showcasing the growing importance of integrating virtual and real data in diagnostic methodologies.

Among these data-driven techniques, Hidden Markov modelling and Bayesian Networks have emerged as important key tools, offering a robust framework for modelling and addressing faults in engineered systems.

The utilisation of BN or dynamic BN (DBN) in the analysis of system dependability, including aspects like reliability, availability, and maintainability, has significantly surged in recent decades due to its modeling capabilities and advantages compared to classical methods [29]. While it's essential to recognize that Bayesian network modelling doesn't offer a universal solution, it does exhibit considerable adaptability and relevance across a broad spectrum of applications, such as aeronautics industry [21], offshore plant [30], automotive industry [31] and nuclear technology [32].

The integration of BN and other methods have been utilized by many researchers for the benefits of Diagnostics and Prognostics [25,33–36]. One of these integrations is with Hidden Markov modelling (HMM) which has been used recently for Diagnostics and Prognostics in engineered systems in some studies. For example, in [33], HMM and BN are integrated to introduces a novel method for fault detection and diagnosis for Tennessee Eastman (TE) chemical process. In a similar work, [25], HMM and BN are combined to detect and predict the identified faults in the TE process by comparing the mean of log-likelihood as the main methodology. More recently Yassenjiang et al. [34] have use the same integration for fault diagnosis and prediction of complex continuous industrial processes.

### 2.2. Background of developing a BN structure

In general, reliability analysis, as well as the use of BNs, is driven by the architecture or structure of a system. Although this principle holds true, complex engineered systems present a reality where tracing the system's behaviour back to its components is not straightforward. While the flow basis (energy, material, and information) remains one of the primary approaches for analysis, deriving an actual behaviour model from the structure becomes significantly more complicated and requires a comprehensive understanding and/or mapping.

Deriving a reliability model in the conventional way, based on a Reliability Block Diagram (RBD) or system structure, is not immediately feasible or indeed practical for two key reasons. Firstly, the complexity of interactions (i.e., dynamic linkages between components), some of which may not be fully defined. And secondly, the probability of component failure needs to be defined as conditional probability in relation to the covariates, rendering it largely unknown a priori.

The important aspect lies in learning the dependency graph of a Bayesian network. Building a Bayesian network relies on two primary sources of information: knowledge from domain experts, sometimes referred to as prior knowledge, and statistical data [37]. In the literature, Bayesian network structures are constructed either from data (e.g. [38–40]), the knowledge of experts (e.g. [20,41,42]), or a combination of both data and expert knowledge (e.g. [43]). Essentially, expert knowledge is not flawless, and the absence of effective communication among experts can lead to errors in the BN structure. Similarly, the uncertainty of data and the lack of availability of appropriate and sufficient data may result in an incorrect BN structure [44].

Various methods to extract BN structure typically begin by utilising initial information about the system, which may involve data containing valuable insights [39] or behavioural/functional information such as Fault Tree Analysis (FTA) [45] or engineering system's descriptions [46]. For complex systems, understanding the dynamic behaviour and interdependencies poses a challenge. Additionally, the data for engineered systems, which normally comes from sensor measurements, is characterised by relatively high uncertainty. With these challenges highlighted, the question that arises is how to construct a BN structure for a complex engineered system.

One way to construct a BN structure is to consider each component of a system as a parent node and connect all of them into a child node representing the system's functional state, which has been utilised in some research works (e.g. [2,17]). However, the main problem with this approach is that it mostly ignores the probabilistic dependencies or interrelationships between components. Furthermore, it requires introducing a single parameter representing each component, which can be challenging as, in reality, a combination of parameters may be needed to represent the functional status of a component.

BN extraction from FTA/DFTA is an approach that has been employed by some researchers for different systems such as fire alarm system [47], train network control management system [48], multiprocessor system [45], and aircraft fuel distribution system [49]. FTA, as a top-down diagnostics approach, can identify the basic events (component-level failures) that contribute to the occurrence of

the top event (system-level failure). It has commonly been used for diagnostics and reliability assessment due to its advantages, including the ability to highlight critical components associated with system failure and visually demonstrate and diagnose the root cause of a top event in a logical manner that leads to failure. However, the applicability and effectiveness of implementing FTA for extracting BN structures in complex engineered systems need further investigation. This is because FTA generally defines only two states (normal or faulty) in the events and uses two logic gates (AND and OR gates) to link events in a hierarchy. Moreover, FTA itself cannot represent a system’s dynamic behaviour and complex functional interrelationships. Additionally, even DFTA may not capture all functional relationships in a complex system since the gates presented in DFTA are also limited and can represent only a few more functional relations than FTA. Another issue lies in generating the FTA for a complex system; extracting BN structures from FTA also presents its own challenges (e.g., extracting conditional probabilities and dealing with cyclic graphs).

BN structure extraction from RBDs [50] is also carried out as an alternative reliability modelling technique; however, their application is limited, and they are generally found to be less efficient compared to FTA/DFTA.

2.3. Research objectives and contributions

In the context of extending the work presented in [2], this study sets out to accomplish an objective: the development of an optimised data-driven methodology (which exploits Bayesian Network) for fault analysis with the overarching aim of achieving more accurate results. While a few hybrid knowledge-based methods, (e.g., [51]), have been previously employed for diagnostic and prognostic purposes, there’s a clear need for a well-established methodology that can systematically integrates engineering analysis and functional information to improve data-driven diagnosis and prognosis in complex engineered systems. This research seeks to bridge this gap by introducing a methodology to develop an optimised Bayesian network structure for an engineered system.

Striving for an accurate data-driven model, the integration of comprehensive functional insights into the engineered system is essential. Specifically, the construction of a BN structure, endowed with the ability to effectively handle causality using probabilities in real-world applications demands a comprehensive exploration and understanding of intricate interdependencies.

This study centres around a core research question: “How can engineering analysis be leveraged to develop an optimised Bayesian network structure for an engineered system, with a particular focus on the critical parameters inherent to the system?” The subsequent sections of this paper will thoroughly explain how this research question is addressed, with a methodological focus on the integration of engineering analysis and cause-and-effect information to establish a BN for an engineered system, emphasizing the identified critical parameters. Notably, rather than segregating the proposed methodology from a case study, the paper presents it within the context of a case study – the Selective Catalytic Reduction (SCR) system.

The distinctive advantages of the proposed methodology in BN structure construction, compared to current state-of-the-art

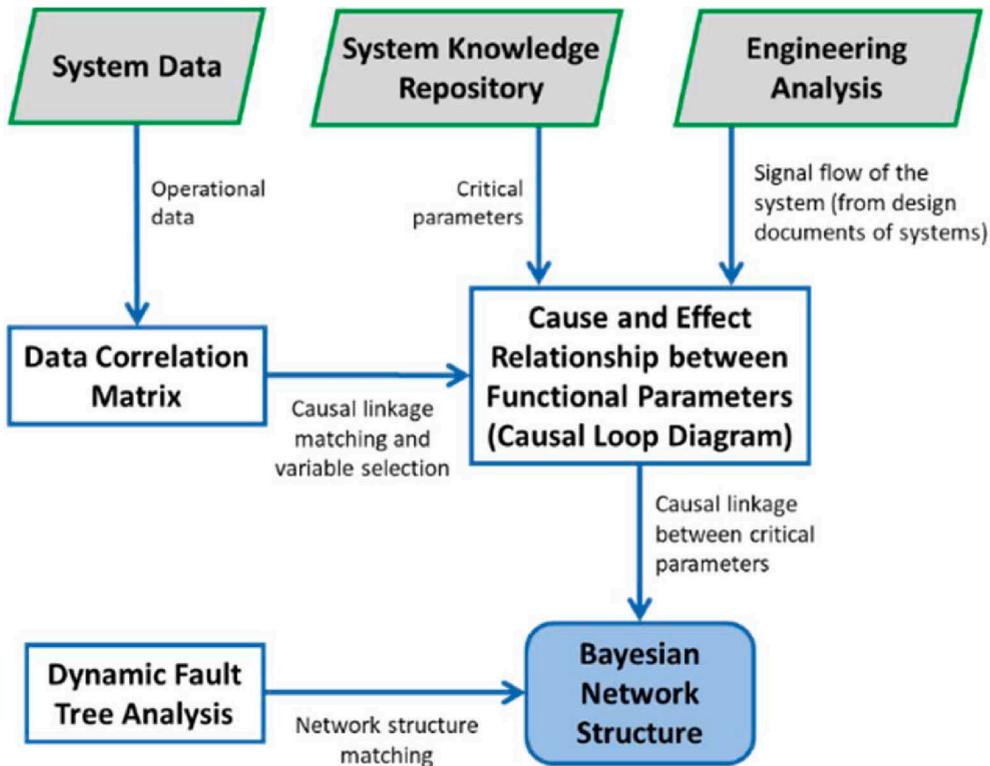


Fig. 1. Process flow diagram of generating BN structure.

methods, stem from its unique utilisation of diverse initial information to construct the BN framework. These advantages become particularly pronounced when applied to systems like SCR, which lack distinct subsystems or individual components with discrete functions. Unlike typical BN structures found in the literature, the SCR system, with its integrated control system regulating key parameters in chemical reactions within the SCR catalyst, presents a unique characteristic. The proposed methodology strategically shifts its focus from individual components to control variables and parameters, offering clear advantages for precise modelling and analysis. Its primary strength lies in providing customized BN structures for systems like SCR, where distinct subsystems or components are absent and no pre-established BN structure exists, distinguishing it from approaches applied to systems like the Tennessee Eastman process, which features well-defined subsystems and an established BN structure.

In summary, this research goes beyond the current paradigm by providing a comprehensive framework that utilises engineering analysis and a distinct BN structure to offer enhanced fault analysis capabilities within complex engineered systems, exemplified by the SCR system. The method's significance lies in its potential to bring about more accurate and reliable diagnostics and prognostics, thus offering practical benefits in terms of system reliability, reduced downtime, and cost savings in complex engineering systems.

### 3. Proposed methodology to develop a BN structure

The proposed methodology presents a structured approach for generating a BN structure tailored to an engineered system. This methodology – an extension of research work presented in [2] – is illustrated in the context of the SCR system as a case study. As highlighted in the previous section, the distinction in constructing the BN for this system compared to systems studied in the literature lies in the reliance on available information, specifically operational data and the signal flow of the system (i.e., the control model of the system). Unlike many engineered systems comprising multiple subsystems or components with distinct functions, the SCR system incorporates a control system that regulates certain parameters within the chemical reactions occurring in the SCR catalyst. Consequently, instead of individual components, it involves control variables/parameters, which serve as key elements to build the BN structure.

Based on the research requirements and the available information for the system, the illustrated process in Fig. 1 is proposed to generate the BN structure. This methodology incorporates inputs such as the system's operational data, critical parameters identified by domain experts, and the signal flow of the system obtained from a confidential technical design document provided by a private organization. In considering the practical application of this methodology and its applicability for different engineered systems, it is essential to specify the scenarios under which it is most effective. This approach thrives in situations where comprehensive operational data, a well-defined control model, and critical parameters of the system are available for the system under study.

The identification of cause-and-effect relationships between functional parameters of the system is achieved through the construction of a Causal Loop Diagram (CLD), forming the foundation for developing the BN structure. The CLD is generated based on the system's control variables and signal flow, and data correlation is employed to validate the causal linkages and simplify the diagram to isolate critical parameters. Subsequently, the BN is derived from the refined CLD. To ensure the congruence of the BN structure, DFTA is employed. The following subsections discuss different steps of the process.

#### 3.1. Cause and effect relationship between functional parameters

In the case of a simple system, the relationships between its components are straightforward, as addressed by classic approaches based on RBDs that comprehensively capture and explain system behaviour. However, for a complex system, these relationships can become challenging to manage. Acknowledging this fact, a fundamental requirement is to grasp the causal linkages between parameters within the system to proceed with its reliability analysis.

Various tools exist for modelling and illustrating cause-and-effect relationships in a system, such as Causal Loop Diagrams (CLDs), fishbone diagrams, design structure matrices, and Failure Mode and Effects Analysis (FMEA). In this work, the focus is on identifying the interrelations between critical parameters of a system and constructing a BN structure. Consequently, CLD was selected as the preferred method for mapping the system's structure and identifying causal linkages between functional variables of the system.

##### 3.1.1. Causal loop diagram for visualising causal linkages

A CLD is a causal diagram that aids to capture the system structure, and feedback processes and visualising the way different variables in a system are interrelated [52]. The CLD is composed of a set of edges and nodes, where nodes symbolise the variables, and edges represent the connections or relationships between two variables. By collecting and illustrating all causal relations in a single view, the CLD provides a clear description of the system's causal dynamics. In the literature, it is commonly used in a qualitative manner to understand and visualise the relationships between the variables of a system [53–55].

Given that the aim is to qualitatively specify causal linkages between the functional parameters of a system, CLD is employed over other cause-effect tools. This preference stems from CLD's ability to visualise and indicate the direction of relationships between functional parameters, which closely aligns with the structure of the BN. In contrast, other introduced tools lack such visualisation capabilities or the ability to indicate the direction of causal linkages. For instance, FMEA does not provide qualitative visualisation or illustration, and the fishbone diagram is primarily used to identify potential causes of a problem, presenting a one-directional graph. Although the design structure matrix captures correlations between variables, CLD offers a more comprehensive view of the causal relationships between the system's parameters and also bears the closest resemblance to the architecture of the BN structure.

The CLD development process involves three critical steps (i.e., problem articulation, identification of endogenous variables, and mapping of system structures), each contributing to the creation of a robust structural model [56]. Firstly, the process begins with

problem articulation, a conceptualisation phase often performed during the design process. Here, the system is already designed based on an articulated problem. Subsequently, the identification of endogenous variables takes place, focusing on the variables involved in the energy, material, and information flow of the system. The causal linkages between those variables can be identified from the design signal flow of the system. The Last step entails mapping the system structures. By considering the variables identified in the signal flow of the system, their causal linkages can be established. This mapping of relevant variables forms the initial CLD for the system. Importantly, these causal linkages represent the interdependencies between variables and contribute to the overall structural integrity of the model. To enhance the robustness of the CLD, data can be leveraged to validate the established causal linkages between variables. Notably, variables demonstrating a direct relationship in the CLD should exhibit a relatively strong data correlation.

To construct a BN structure, the CLD requires simplification, which can be accomplished by reducing the number of variables to the identified critical parameters, facilitated by data correlation. The developed CLD, with its inclusion of control variables, may encompass some variables that are not functional parameters for the system. In other words, certain variables may solely serve as connectors in the control system and can be eliminated to streamline the CLD. Additionally, variables referring to a similar entity exhibiting strong data correlation can be amalgamated into a single variable/parameter in the CLD (referred to as variable selection). This process yields a diagram that contains causal linkages between the variables of interest (i.e., the critical parameters) for the system, represented in the form of a causal loop diagram.

3.1.2. Causal loop diagram for the SCR system

Prior to discussing the CLD for the SCR system, it is important to briefly discuss the Automotive Aftertreatment system and the role of SCR within this system. The automotive exhaust gas Aftertreatment system is an advanced assembly of interdependent subsystems designed to meet increasingly stringent emission standards in the automotive industry. It comprises three main catalysts with distinct functions—Diesel Oxidation Catalyst (DOC), Diesel Particulate Filter (DPF), and Selective Catalytic Reduction [57]. The system’s complexity is illustrated in Fig. 2, emphasizing essential variables, parameters, and the catalytic impact of DOC, DPF, and SCR on gas composition. This study specifically focuses on the SCR subsystem, employing a proposed framework.

The control signal flow of the SCR system, obtained from a confidential technical design document, forms the foundation for mapping the relevant variables available in the ECU data [2]. By establishing causal linkages between these variables, valuable insights were gained into the interdependencies within the SCR system’s control mechanisms. Table 1 presents the list of these variables and their corresponding names. Throughout this paper, these variables will be referred to by their index numbers (i.e.,  $V_i$ ).

Fig. 3 illustrates the causal relationships between the variables in the form of a causal loop diagram, which is based on the signal flow of the SCR system. To generate this CLD, the relationship between each variable in the control signal flow of the SCR system is identified and mapped in the diagram. For instance,  $V_2$  and  $V_7$  are connected through the feedback controller responsible for regulating the amount of injected urea. Consequently, these variables exhibit a direct relationship in the CLD, signifying that  $V_2$  directly affects  $V_7$ .

It is worth noting that the variables listed in Table 1 are not solely collected from the SCR system, as some of them are associated with the engine or other subsystems in the Aftertreatment system (i.e., DOC, and DPF). The CLD includes identification boxes, designating the related system for each variable to distinguish variables within the system hierarchy.

3.1.3. Causal loop diagram matching and simplifying based on operational data

To validate the established CLD, the causal linkages between variables should closely align with the correlation observed in almost all related data sets. To verify this, a specific ECU data set for the SCR system is utilised as an example. Due to confidentiality agreements, the exact source details of the data are withheld. Nevertheless, Fig. 4, generated by MATLAB, presents the related Pearson pairwise correlation in the form of a heat map for the variables introduced in Table 1 for the CLD. Pearson correlation serves as a measure of linear correlation between two data sets, calculated as the ratio between the covariance of two variables and the product of their standard deviations. Consequently, it provides a normalised measure of covariance, and the resulting correlation value always falls within the range of  $-1$  to  $1$ .

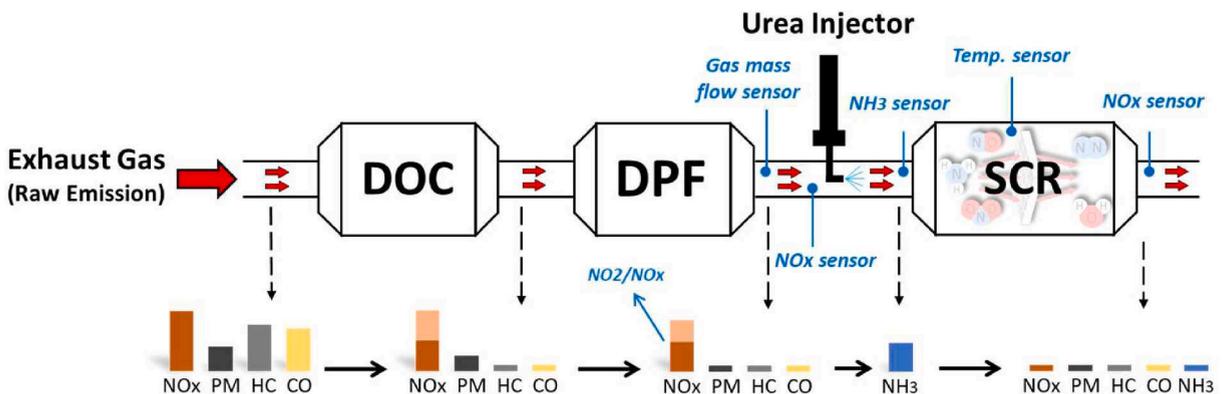
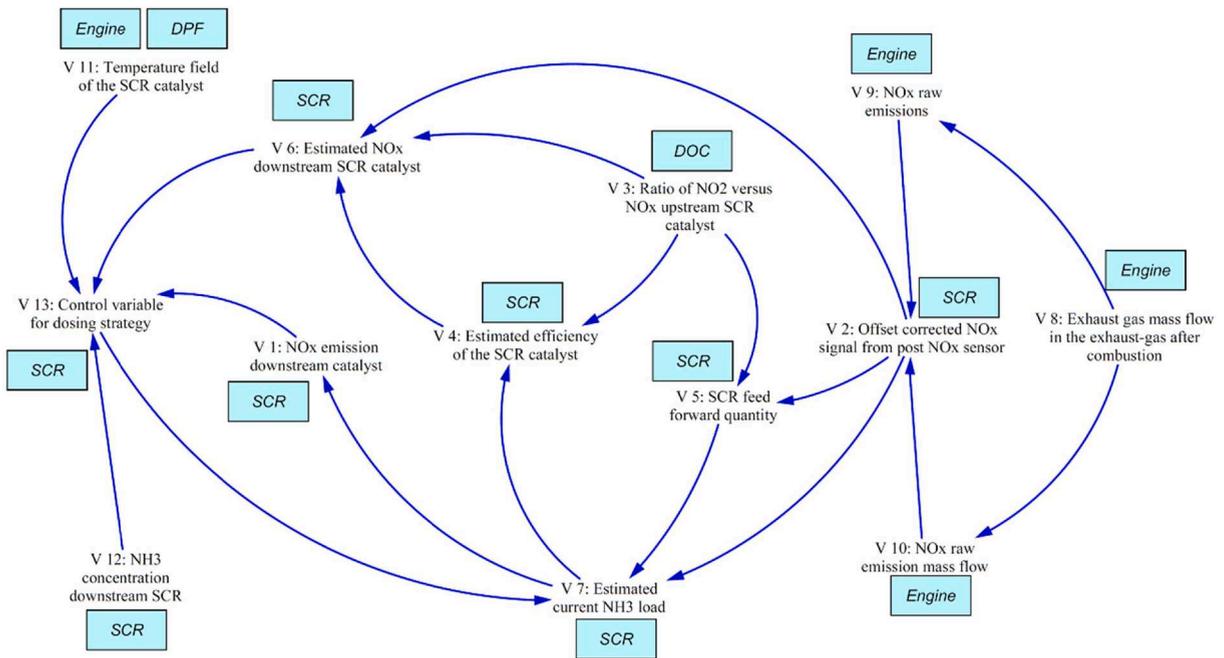


Fig. 2. A schematic illustration of the automotive Aftertreatment system, focused on SCR subsystem (Modified from [2]).

**Table 1**

List of the variables in the control signal flow of the SCR system that are available in ECU data.

Variable	Description	Unit
V_1	NOx emission downstream of secondary catalyst (outlet NOx)	ppm
V_2	Offset corrected NOx signal from post NOx sensor (inlet NOx)	ppm
V_3	Ratio of NO2 versus NOx upstream SCR catalyst	–
V_4	Estimated efficiency of the SCR catalyst	–
V_5	SCR feed forward quantity	mg/s
V_6	Estimated NOx downstream SCR catalyst	ppm
V_7	Estimated current NH3 load	g
V_8	Exhaust gas mass flow in the exhaust-gas after combustion	Kg/h
V_9	NOx raw emissions	ppm
V_10	NOx raw emission mass flow	mg/s
V_11	Temperature of the SCR catalyst	°C
V_12	NH3 concentration downstream SCR	ppm
V_13	Control variable for dosing strategy	–



**Fig. 3.** Causal Loop Diagram for parameters in the SCR system, along with identifying variables within the system hierarchy.

It is important to note that the relationships between variables in the CLD are not necessarily linear. As a result, the corresponding data correlation might not be linear as well. Thus, it is expected that variables with a direct relation in the CLD, pertaining to a unique entity, will exhibit a very strong data correlation. On the other hand, for other variables associated with different entities, the data correlation might not be as strong since the heat map measures the linear correlation between variables.

In the CLD, V\_8 is observed to have a direct influence on V\_9 and V\_10. Notably, this variable demonstrates a strong correlation with both V\_9 and V\_10 in the data set as well. Another significant example is the direct impact of V\_4 on V\_6, and these two variables also exhibit a robust correlation in the data. Additionally, there is a direct relationship between V\_9 and V\_10 with V\_2 in the CLD, where all three variables pertain to inlet NOx level. As anticipated, this correspondence results in very strong linear correlations (nearly 1) between them in the data. This alignment between the CLD and data correlation serves to validate the accuracy and reliability of the generated CLD.

To obtain a BN structure comprising the identified critical parameters, the current CLD requires simplification. Considering the collinearity among variables referring to a similar entity, only the important variables are selected from the highly correlated ones as critical parameters. This variable selection process eliminates linearly correlated variables from the BN. For instance, V\_9 and V\_10 directly influence V\_2 and exhibit high correlation with this variable. Therefore, these three variables are deemed as one of the critical parameters for the SCR functionality, denoted as “inlet NOx” or “Upstream NOx level” in Table 1. This decision stems from the fact that although V\_2, V\_9, and V\_10 may differ in terms of their values, they all pertain to the same entity, which is the inlet NOx level. The strong correlations between these variables in Fig. 4 further support this variable selection.

Similarly, employing a similar rationale, V\_1 and V\_6 are regarded as one critical parameter (“outlet NOx”/“Downstream NOx

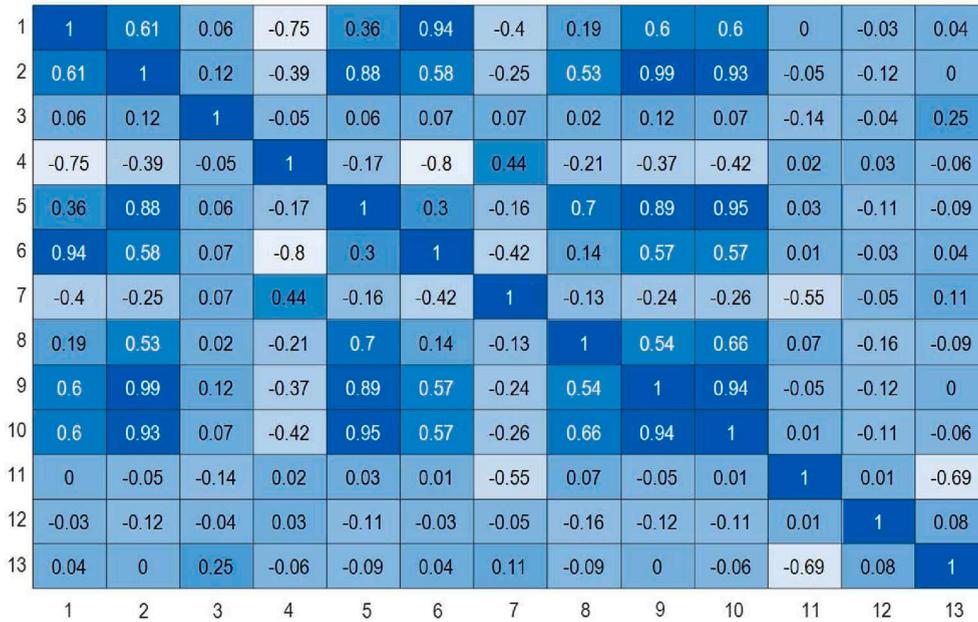


Fig. 4. Pairwise correlation heat map for the data set of the SCR system.

level”), where one variable represents the sensor measurement value, and the other one indicates the model-based simulation value for a unique entity. The correlation value between these two variables (0.94) also substantiates this aspect.

The degradation factor of the SCR catalyst is acknowledged as one of the critical parameters and serves as a coefficient factor in the control strategy of the SCR system. Due to its influence on the catalyst’s efficiency, V\_4 (“Estimated efficiency of the SCR catalyst”) is selected as a representative for this critical parameter in this study. Although V\_7 (“Estimated current NH3 load”) and V\_12 (“NH3 concentration downstream SCR”) are both related to the NH3 level, their values are not correlated since V\_7 pertains to the amount of NH3 in the catalyst, while V\_12 indicates the slipped ammonia. The weak correlation (nearly zero) between these variables in Fig. 4 further validates this distinction.

In the CLD, three additional critical parameters that represent their respective entities have been identified: V\_3 (“Ratio of NO2 versus NOx upstream SCR catalyst”), V\_8 (“Exhaust gas mass flow in the exhaust-gas after combustion”), and V\_11 (“Temperature of the SCR catalyst”). On the other hand, two remaining variables in the CLD, namely V\_5 (“SCR feed forward quantity”) and V\_13 (“Control variable for dosing strategy”), do not directly relate to any critical parameter and have connecting roles within the control algorithm of the SCR system.

Table 2 provides a summary of the variable selection process used to simplify the CLD. The table presents the relationship between the 13 variables introduced in Table 1 and the seven critical parameters identified for the SCR system’s functionality in the study presented in [2]. Additionally, it identifies whether these variables serve as functional variables or have connector roles within the system. Please note that the variable numbers have been slightly modified in this study compared to the research presented in [2].

**Table 2**  
The list of variables in the CLD and their relationship with the identified critical parameters.

Control variable	Role	Related critical parameter
V_1	Functional variable	Downstream (outlet) NOx level
V_6	Functional variable	
V_3	Functional variable	NO2 / NOx upstream SCR catalyst
V_4	Functional variable	Degradation factor of SCR catalyst
V_5	Connector variable	–
V_8	Functional variable	Exhaust gas mass flow
V_2	Functional variable	Upstream (inlet) NOx level
V_9	Functional variable	
V_10	Functional variable	
V_11	Functional variable	Temperature of the SCR catalyst
V_7	Functional variable	Quantity of supplied NH3 (NH3 level)
V_12	Functional variable	
V_13	Connector variable	–

3.2. Bayesian network structure underpinned by causal Loop diagram

The aim of this study is to construct a Bayesian network (BN) for a system based on its critical parameters. To achieve this, the causal linkages between the critical parameters are represented in the form of a CLD. By employing the simplified CLD, which comprises the critical parameters for the system, as the BN structure, the causal relationships between the variables in the system can

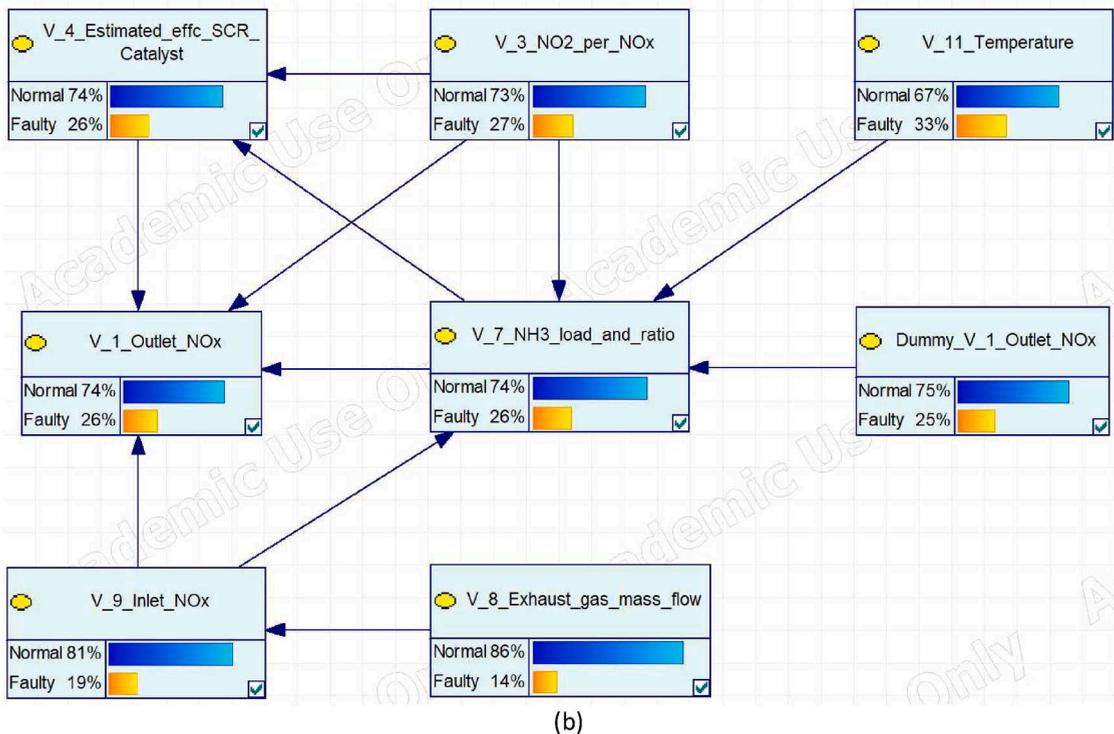
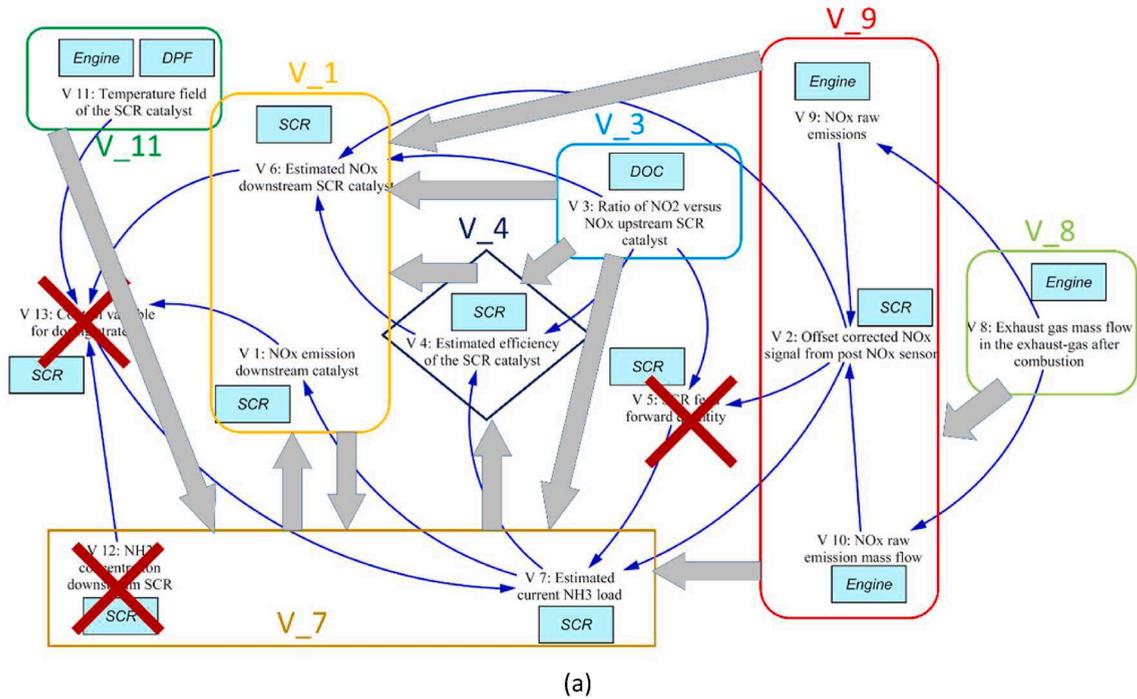


Fig. 5. (a) Simplified CLD for the SCR system, (b) The Bayesian network for SCR system based on engineering analysis.

be visualised and explained.

Learning a Bayesian network involves two subtasks: structural learning and parametric learning. Structural learning entails identifying the topology of the Bayesian network, while parametric learning involves estimating the conditional probabilities. Among these tasks, learning the structure of a BN is considered the most challenging [56], and it is discussed in this section.

Based on the variable selection process, only seven variables representing the identified critical parameters were chosen from the available set of 13 variables. The selection was based on retaining only one variable among those referring to a unique parameter and discarding variables with connecting roles, i.e., variables that do not represent specific functional parameters. Fig. 5(a) provides an illustration and summary of the simplification process of the CLD, depicting how the selected variables were connected. Consequently, Fig. 5(b) presents the constructed BN structure for the SCR system. Each node in the BN structure corresponds to the respective variable number in the CLD. To maintain the acyclic nature of the BN, a dummy node representing “Downstream (outlet) NOx level” was introduced into the BN structure.

In certain Aftertreatment system configurations, the function of Ammonia Slip Catalyst (ASC), responsible for removing excess NH3 before releasing gases into the atmosphere, is delivered by the SCR system through an optimised NH3 dosing control mechanism. V\_12 (NH3 concentration downstream SCR) is measured to regulate the injection of urea and reduce its amount when it exceeds the functional requirements of the SCR system. Although excessive outlet NOx levels can indicate a failure of the SCR system, V\_12 does not directly influence the primary function of the system, which involves the oxidation of NOx into nitrogen and water. Its measurement aims to prevent the release of excess ammonia into the atmosphere. Due to the limited availability of data for V\_12, it was omitted in the BN structure for this study. Although this variable could have been considered, its omission was necessary in light of the data constraints.

The BN structure presented here is designed to be generic and applicable for the analysis of any system fault related to the SCR system, encompassing seven key variables as primary nodes. GeNIe software [58] was used to construct the BN structure and perform belief updating.

### 3.3. Bayesian network structure matching with DFTA

The proposed methodology utilises DFTA to validate the structure of the generated BN. Accordingly, this section initiates the construction of a DFTA for the SCR system and subsequently compares it with the BN to ensure their alignment.

#### 3.3.1. Dynamic fault tree analysis for the SCR system

For fault analysis in complex engineered systems, it is essential to comprehend the causal linkages among (i) defect or fault initiation, (ii) pathways for fault propagation, and (iii) symptoms and their impact on the system. While these relationships are straightforward in simple systems, they can become intractable in complex systems, making classic methods less effective.

Fault Tree Analysis (FTA) is a top-down deductive technique employed to categorise the instrumental relationships that lead to a

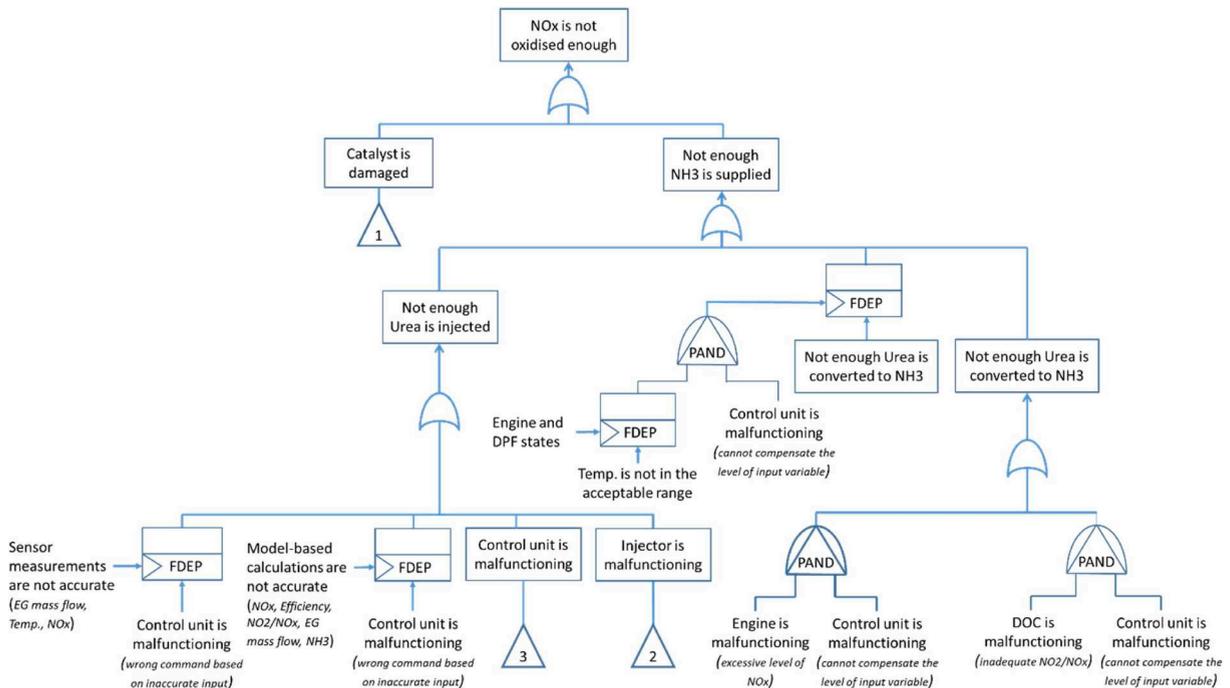


Fig. 6. Dynamic fault tree for SCR system.

specific top event, such as a failure mode. One major advantage of FTA is its ability to provide insight into the operation and potential failure of a system, facilitating the investigation of strategies for eliminating and minimising product failures. By exploring the various ways a top event can occur, FTA generates a list of faults. Furthermore, FTA aids in identifying critical elements within a system that are associated with system failure, providing crucial information about fault initiation, fault propagation, and their effects on the system [59]. To extend the capabilities of static fault trees to model time-dependent failures, DFTA is employed as a powerful tool for reliability modeling of complex systems.

Fig. 6 illustrates the developed DFTA for SCR system. The process of constructing the DFTA involved identifying the hazard, understanding the system's function, and subsequently creating the dynamic fault tree. Through the DFTA, important elements of the system related to its failure are highlighted, and a list of root causes for the top event is provided.

The undesirable event (top event) in the SCR Dynamic Fault Tree is "NOx is not oxidised enough," which can be attributed to several different combined lower-level events. This selection is based on the primary function of the SCR system, which aims to oxidise NOx to comply with emission regulations. Notably, the physical event of catalyst damage is included in the DFTA but has not been further analysed in this work, as it falls beyond the scope of this study's focus. Another crucial event is "Not enough supplied amount of NH3," which may arise from two potential causes: "Not enough Urea is injected" or "Not enough urea is converted to NH3."

The combination of several events results in "Not enough Urea is injected," primarily related to control unit or injector malfunctions. The analysis of physical failures such as "Injector is malfunctioning" (e.g., restricted flow) is not pursued further. Other events contributing to "Not enough Urea is injected" are associated with control unit malfunctions, possibly stemming from incorrect input or software issues. However, in this work, further analysis of control unit malfunctions not caused by incorrect input has been omitted, as software uncertainty is relatively lower compared to sensor measurements and model-based calculations.

The events triggering the control unit malfunction depend on critical parameters related to the SCR system functions (sensor measurements and model-based calculations as triggers in the FDEP gates of the DFTA). Control unit malfunction here refers to the provision of incorrect commands based on inaccurate inputs. These inputs, which are the functional variables (identified as critical parameters), originate from sensor measurements or model-based calculations (or both). Considering uncertainties in these values (measurements or model uncertainties), they can serve as major root causes for faults in the SCR system.

The SCR control unit is the event leading to "Not enough urea is converted to NH3," influenced by other subsystem malfunctions in the Aftertreatment system. The control unit's primary role is to provide the appropriate urea amount based on input variables. However, if certain input variables, such as "Excessive level of NOx" or "Inadequate NO2/NOx," reach extreme levels that cannot be compensated by the control unit, "Not enough urea is converted to NH3" may occur. These extreme levels are typically caused by malfunctioning in the Engine and DOC subsystems. In such cases, the Priority AND gate adequately models these situations. Additionally, "Excessive level of NOx" can directly contribute to "NOx is not oxidised enough" since, beyond a certain NOx level, even an excessive amount of NH3 cannot oxidise the desired NOx amount.

The level of converted urea to ammonia is highly dependent on the SCR catalyst temperature. Therefore, the event "Not enough urea is converted to NH3" can be triggered by temperature variations. Engine and DPF states may lead to temperature ranges unsuitable for efficient urea-to-ammonia conversion. While the control unit can compensate for slight temperature deviations by adjusting urea injection, in many cases, achieving the desired level of ammonia becomes challenging.

The DFTA confirms the significance of the previously identified critical parameters for SCR system functionality [2]. The basic events in the DFTA are primarily related to these seven critical parameters, highlighting their substantial impact on the system's functionality and potential contribution to the top event.

### 3.3.2. Validation of the BN structure by DFTA

In this research, the incorporation of engineering analysis is proposed to complement the data-driven fault analysis methodology. DFTA is specifically employed for two primary objectives: first, to assess whether the structure of the generated BN aligns with the qualitative aspects DFTA (as discussed in this section), and second, to validate the results of root cause identification obtained from the data-driven methodology – which will be addressed later in the paper. The rationale behind the adoption of DFTA lies in its ability to highlight critical system elements related to failure and provide a list of root causes for the top event in the system. These aspects are vital for validating the data-driven outcomes.

In DFTA, the root causes for the top event are derived from basic events, encompassing temperature, inlet NOx, outlet NOx, NO2/NOx, exhaust gas mass flow, NH3 level, and efficiency of SCR catalyst all of which are either measured or calculated. These variables correspond to the ones presented in CLD and BN, contributing to distinct events in different manners, ultimately leading to the top event. The commonality of these variables in DFTA, CLD, and BN (as nodes) demonstrates the alignment between these modelling tools.

Regarding structure matching, the bottom events (root causes) in DFTA exhibit connections among critical parameters, utilising various gates. For instance, temperature, inlet NOx, outlet NOx, and NO2/NOx correlate with NH3 load and ratio, akin to their appearance in the BN structure. The control unit malfunctioning is also linked to these critical parameters as it bases decisions on the level of these parameters. However, V\_5 and V\_13 (considered connector variables in CLD and excluded from BN) are absent in DFTA, signifying no functional significance. This confirmation of variable correspondence and connections validates the generated BN structure.

Given the mutual validation of DFTA and BN, the proposed methodology for constructing the BN structure in this study not only meets the requirements of established approaches in the literature (i.e., extracting BN structure from FTA) but also accounts for additional causal relationships among functional variables, which might not be captured by FTA/DFTA. This distinction arises as FTA/DFTA predominantly focus on system elements rather than its data (pertaining to functional variables).

### 3.4. Conditional probability tables for the BN

At the core of a Bayesian network lies the conditional probability table (CPT), which describes the probability distribution of a variable given the values of its parent variables. In other words, a CPT shows how the probability of a particular outcome changes depending on the values of other related variables. These tables allow to capture and analyse the dependencies between different variables and make predictions about the behaviour of the system being modeled [37]. In addressing the generation of CPTs, the primary focus of this work is on the construction of the BN structure rather than the detailed training of CPTs. The statistical algorithm introduced in [2] was employed to extract CPTs from historical data, with a detailed discussion available in [2].

## 4. Root cause identification for SCR system: Results and discussion

In this section, the root cause identification results of the methodology introduced in [2] are presented, utilising two distinct BN structures for the same fault within the SCR system.

The method introduced in [20] is employed for root cause identification, wherein the parent node with the highest percentage change following BN updating is considered the root cause of the fault. This approach utilises soft evidence for BN updating, resulting in more precise and accurate root cause diagnosis for dynamic process analyses.

The BN solution necessitates only the graph structure and CPTs [60], both of which are readily available. By providing evidence, the BN can be updated. Rather than relying on hard evidence, which involves observing a random variable in a specific state, virtual evidence employs likelihood information to update the BN. However, obtaining hard evidence for a variable in a particular state is not always feasible, and claimed observations may be unreliable, leading to uncertainty in the evidence. Virtual evidence offers a means of directly incorporating uncertain observations, in the form of probability values across potential states of the observation, into the normally unobservable variable.

To establish the likelihood evidence, all variables in the data are assessed up to a certain time before the fault detection, with the objective of determining the probabilities of each variable being in the danger zone [2]. These probabilities are subsequently utilised as virtual evidence to update the BN. The root cause of the fault is then specified by identifying the parent node with the highest percentage following the BN update.

To examine the significance of incorporating engineering analysis in constructing the BN structure, the root cause identification results obtained from two different BN structures have been compared. The first BN structure, shown in Fig. 5(b), is established based on engineering analysis. On the other hand, the second BN structure, depicted in Fig. 7 and employed in [2], is constructed using the method presented in [17]. The reason for selecting this method is that, among the referenced research works ([17,25,33]), only this reference introduces a methodology for generating the BN structure, while others solely used the well-established BN structure for the TE process.

To identify the root cause of the detected fault depicted in Fig. 10 of [2], the BN structures were updated by incorporating the same likelihood evidence (virtual evidence) into both structures. Table 3 presents a comparison of the prior and posterior probabilities of the faulty state after updating the BNs with the same virtual evidence. It is evident that, in both BN structures, the variable associated with SCR temperature (V\_11) exhibits the most significant percentage change in the posterior probabilities, notwithstanding the variations in some of the initial and posterior probabilities. Consequently, this variable is identified as the root cause of the introduced fault

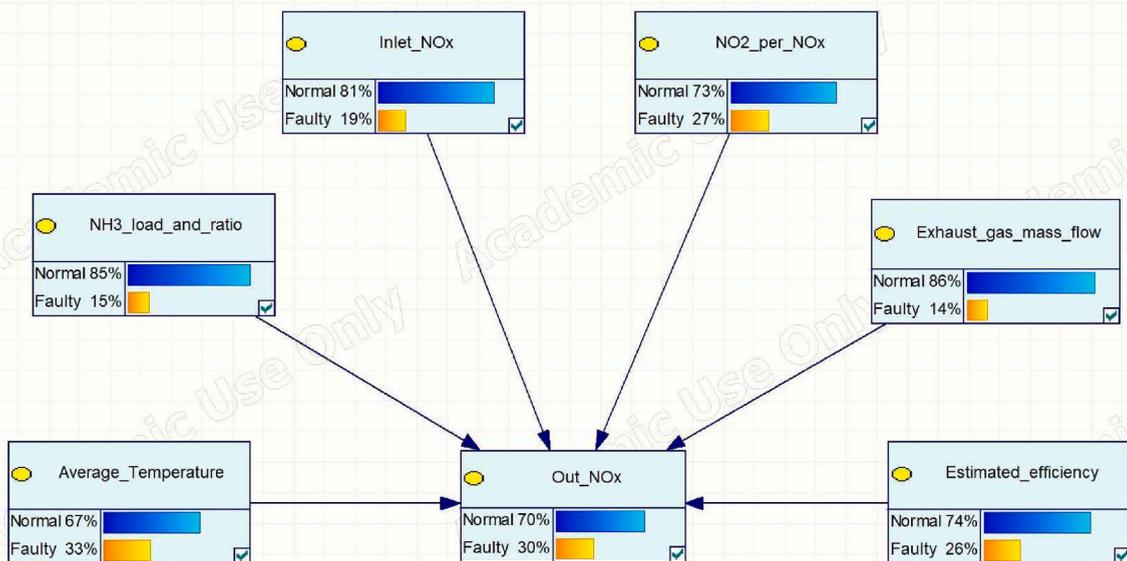


Fig. 7. BN structure for SCR system, built based on a method introduced in the literature [2].

**Table 3**

Comparing the probabilities for two different BN structures, related to the first detected fault.

Node	The BN structure based on engineering analysis		The BN structure based on a method in the literature [2]	
	Initial probability	Updated probability	Initial probability	Updated probability
Temperature of the SCR catalyst	<b>0.33</b>	<b>0.83</b>	<b>0.33</b>	<b>0.92</b>
Estimated current NH3 load	0.26	0.43	0.15	0.25
Inlet NOx level	0.19	0.28	0.19	0.43
NO2/NOx	0.27	0.24	0.27	0.42
Exhaust gas mass flow	0.14	0.10	0.14	0.22
Estimated efficiency	0.26	0.40	0.26	0.43
Outlet NOx level	0.25	0.51	0.30	0.41

illustrated in Fig. 10 of [2]. The analysed data set utilised to detect and predict this fault pertains to instances where the temperature exceeds acceptable levels, thereby impacting the SCR system's performance. As a result, the root cause is accurately identified.

It is worth noting that the difference between the initial probability values for 'Estimated current NH3 load' and 'Outlet NOx level' in Table 3 is due to the deliberate selection of variables representing the same entity but measured in different ways. The variable names, although slightly different, essentially capture variations in measurement methods, such as sensor readings and model-based simulations. This intentional choice is highlighted to demonstrate that variables with strong correlations, referring to the same entity, can be used interchangeably, as evidenced by fairly similar changes in updated probabilities.

The root cause identification result obtained from the BN must be verified for alignment with the DFTA. Additionally, it is crucial to ascertain the mechanism through which the root cause variable contributes to the fault, as the root cause identification output of the data-driven methodology does not provide such information. To address this, DFTA and the correlation between variables are employed to identify the actual root cause of a fault from among the potential causes.

The SCR catalyst temperature appears in two distinct contexts within the DFTA: one involves triggering the "Not enough urea is converted to NH3" due to an undesirable temperature level, while the other relates to a wrong sensor measurement. To specify which one is them is the root cause for this fault, the CLD and correlation between the variables can be leveraged. Temperature (V\_11), being a critical variable for the system's functionality, demonstrates a direct correlation with V\_13 in the CLD. In Fig. 4, the correlation value of  $-0.69$  verifies this relationship. Had the sensor measurement been incorrect, the correlation between these variables might have been weak or even positive. Therefore, the observed correlation value implies that the measurement is accurate. Consequently, the actual temperature level (deemed unacceptable) emerges as the true cause of this fault, as depicted in Fig. 8.

As anticipated, the modification in the BN structure yielded posterior probabilities that were close but not identical. Consequently, while the identified root cause remained the same for the analysed fault, altering the BN structure could lead to different outcomes in root cause identification. Therefore, as a future work, it is recommended to explore root causes for other potential faults and subsequently compare the obtained results.

## 5. Conclusions

This study has investigated the integration of engineering system analysis with a data-driven methodology to enhance fault analysis. The development of a novel Bayesian network structure, incorporating insights from engineering analysis, particularly CLD and DFTA, has been presented. The systematic methodology employed system engineering principles, utilising CLD to construct the BN structure based on identified critical parameters and the system's signal flow. Validation of the CLD and BN structure was performed through operational data correlation and DFTA, respectively, within the context of the SCR system.

The demonstrated efficacy of the proposed methodology in enhancing fault analysis for complex engineered systems is noteworthy. Root cause identification results obtained from the new BN structure were compared with an existing BN structure built based on a method from the literature, emphasising the added value of engaging engineering analysis in the BN development process. The incorporation of DFTA and data correlation further bolsters the reliability of root cause identification outcomes through the data-driven methodology.

This research contributes to a novel approach for building BN structures in engineered systems, elevating reliability analysis, and enabling more accurate fault diagnosis and prognosis. By leveraging engineering insights and data-driven techniques, the proposed methodology offers a comprehensive framework to support the robust analysis and understanding of complex systems in various engineering domains.

While the demonstrated efficacy of the proposed methodology in enhancing fault analysis is substantial, it is crucial to acknowledge certain limitations. This methodology necessitates a substantial volume of data for the specific system under study, along with the system's control model and an understanding of critical parameters. These requirements pose practical challenges that should be considered when applying the methodology to a new system. In terms of future research directions, leveraging digital twin capabilities could potentially mitigate the data requirements and enhance the applicability of the methodology to a wider range of systems. This avenue offers a valuable prospect for future investigations, aligning with the evolving landscape of digital technologies in engineering analysis and fault diagnosis.



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