Geomechanics for Energy and the Environment 39 (2024) 100572

Contents lists available at ScienceDirect

Geomechanics for Energy and the Environment

journal homepage: www.elsevier.com/locate/gete

Analytical solution for transient radial interaction between energy piles and soils: Thermo-elastic cavity expansion analysis

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ARTICLE INFO

Keywords:
Energy pile
Cavity expansion
Transient thermal stress
Radial stress
Pile-soil interaction

ABSTRACT

Bearing capacity of energy piles may be affected by the Radial Interaction between Energy Piles and Soils (RIEPS) such as energy pile expansion and transient radial heat conduction. This paper proposes a cavity-expansion-based solution to investigate the thermo-elastic RIEPS. Transient temperature distributions are shown by assuming heat conduction in the radial direction and constant temperature at the pile-soil interface. With the temperature distributions, a thermo-elastic solution is obtained to capture the changes in stresses and displacements around energy piles. It is found that the solution under the combined thermal-mechanical loading pattern is the linear superposition of those under the thermal loading and mechanical loading patterns. Hence, the stresses, strains and displacements in soils are determined by the competitive relationships between thermal and mechanical loading patterns. The expression for radial stress change at the pile-soil interface is discussed by the cavity expansion analysis and comparison with field data. For typical soil and pile parameters, the expression could be quite general considering transient temperature distributions and soil/pile moduli. This paper can benefit to the capacity design of energy piles by taking the RIEPS into account.

1. Introduction

Energy piles that serve as pile foundations and heat exchangers for superstructures can provide an efficient and sustainable way to make use of geothermal energy. Temperatures of energy piles and surrounding soils will change when the systerms are operating in different seasons. This thermal effect may be important for the bearing capacity design of energy piles[3]. For example, a large number of studies have shown the changes in shaft friction and displacements of energy piles in the heating/cooling process[17,19,2,21–25,29,30,33,39,41,48,5].

The radial interaction between energy piles and soils (RIEPS) may be one of the reasons for the shaft friction change of energy piles[1,28,29,3,32,43]. This interaction mechanism has been investigated by various methods such as model tests[16,19,29,44,9], in-situ tests[10,11,19,26,43], numerical simulation methods[13,28,31,37], and analytical methods[14,46,47]. Some of these studies indicated that the radial contact stress change at the soil-pile interface, $\Delta\sigma_n$, plays a limited role in shaft resistance of energy pile, while some found the thermal-induced $\Delta\sigma_n$ should not be ignored. For instance, Olgun et al.[31] demonstrated that $\Delta\sigma_n$ induced by the radial expansion of the concrete pile is negligible. On the other hand, a full-scale field test conducted by Xiong et al.[43] showed that higher temperatures can result in an increase of $\Delta\sigma_n$ by more than 50kPa at a sand-pile interface (36.5 m in depth). The discrepancy in the significance of RIEPS may be dependent on different temperatures, water content, and thermo-mechanical properties of soils and energy piles (e.g., stiffness, strength and thermal expansion coefficients). $\Delta\sigma_n$ may also be affected by temperature circles due mainly to cumulative irreversible deformation of soil[10,19,28], but it remains to be validated whether the complex soil temperature distributions under temperature circles have a considerable impact on $\Delta\sigma_n$.

To better describe the radial interaction between energy piles and soils, it is necessary to develop an explicit analytical solution, considering two physics-based factors:

(a) the radial thermal expansion of energy piles, which will squeeze the surrounding soils;

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https://doi.org/10.1016/j.gete.2024.100572
Received 16 January 2024; Received in revised form 18 April 2024; Accepted 18 June 2024
Available online 2 July 2024

Notation List

A_a	integral constant
a	radius of energy pile
b	large radius representing infinite value
c_{ht}	thermal diffusivity of soils
c	specific heat of soils
E, E_{pile}	Elastic moduli of soils and energy piles
J_0, J_1	first Bessel functions of zero and first orders
r	radial position of a soild particle
t	time
T_0	initial ambient temperature
T_a	contant temperature at the pile-soil interface
ΔT	temperature change
ΔT_a	temperature change at the pile-soil interface
$\Delta\sigma_n$	radial stress change at the pile-soil interface
$\Delta\sigma_r$	radial stress change
$\Delta\sigma_\theta$	circumferential stress change
$\Delta\sigma_z$	vertical stress change
U_0, U_1	composite Bessel functions of zero and first orders
u	radial displacement
Y_0, Y_1	second Bessel functions of zero and first orders
α, α_{pile}	thermal expansion coefficients of soils and energy piles
β_n	root of composite Bessel function
ε_r	radial strain
ε_θ	circumferential strain
ε_z	vertical strain
μ, μ_{pile}	Poisson's ratios of soils and energy piles
ρ	soil density
σ_0	initial ambient stress in soil

(b) soil temperature changes, which will generate thermal strains in soils.

There have been a few analytical solutions for modelling the RIEPS[14,46,47]. These solutions are developed primarily based on the cavity expansion theory that investigates stress and displacement changes around an expanding/contracting cavity. Zhou et al.[46] presented a cavity expansion solution in thermoplastic soils with the ACMEG-T model proposed by Laloui and François[20]. Gaaloul et al.[14] showed a thermo-mechanical solution for the limit pressure of cavity expansion in Mohr-Coulomb soils. The temperature distribution in soils was ideally assumed to be uniform and time-independent in their solutions. In reality, soil temperatures vary with the radial distance because the heating/cooling effect of energy piles gradually decreases along the radial direction. On the other hand, some thermo-mechanical solutions were proposed in the fields of solid and structure mechanics to study the transient thermo-mechanical behaviour of hollow cylinders[18,36,38,40,42,8]. These solutions may not be directly applied to the analysis of RIEPS because non-stress or constant-stress boundary conditions were assumed at the inner cylinder wall (i.e., pile-soil interface). Overall, a thermo-mechanical cavity expansion solution is still not available for the transient analysis of RIEPS, which makes the analysis rely mainly on numerical techniques[27,31].

This paper proposes an analytical solution for the calculation of thermo-elastic stresses and displacements during cavity expansion under transient temperature fields. The solution is a modification of Kandil et al.[18] by giving an analytical form and considering a radial displacement boundary at the pile-soil interface. It is therefore more suitable for RIEPS analysis in geotechnical engineering considering the combined effects of transient soil temperatures and energy pile expansion. Finally, the radial contact stress at the pile-soil interface is discussed focusing on the influences of soil temperature distributions and soil moduli.

2. Problem definition and assumptions

The radial interaction between an energy pile and the surrounding soil during heating/cooling is modelled by the expansion/contraction of a cylindrical cavity[13,31]. As shown in Fig. 1, a single energy pile with a diameter of $2a$ is embedded in a homogeneous and isotropic soil of infinite radial extent. Initially, the soil is subjected to a hydrostatic stress σ_0 and the ambient temperature of the pile/soil is denoted as T_0. When t=0 (t denotes time), the energy pile is heated/cooled and soil temperature is assumed to be T_a at the pile-soil interface. Later, the energy pile will expand/contract, the soil temperature will increase/decrease, and stresses and displacements in the soil will change during the

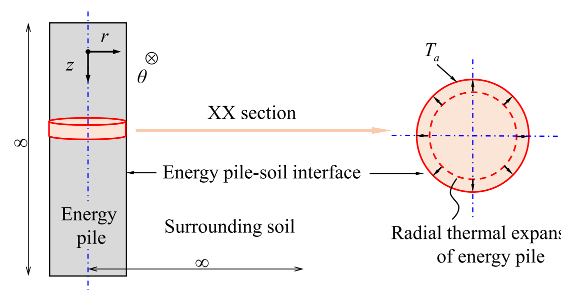

Fig. 1. Schematic of radial energy pile-soil interaction.

heating/cooling process[12,26,31]. Prior to showing the detailed derivation, necessary assumptions are introduced as follows and their reasonability and/or limitations are also demonstrated.

The length-to-diameter ratios of piles are usually large enough and the radial energy pile-soil interaction can be studied by concerning a certain horizontal plane under plane strain and axisymmetric conditions (i.e., XX section in Fig. 1)[13,31,34,7]. The cylindrical coordinate system (r, θ, z) is adopted for convenience with the origin at the pile centre.

The thermal expansion coefficients of soils and energy piles are usually in the order of $10^{-6} \sim 10^{-5}/°C$[31,4], and thermal strains of soils will be in the order of $10^{-4} \sim 10^{-5}$ for $|T_a - T_0| \leq 30°C$. Hence, the thermal-induced stresses and strains may be assumed to obey small strain definitions and Hooke's law (i.e., linear elastic stress-strain relationship) to simplify the solution derivation.

Assumptions in terms of soil temperatures can be summarised into three main aspects. At first, the change of soil temperature is mainly resulted from radial heat conduction, rather than heat radiation and convection. By comparing the finite and infinite line/cylinder heat sources, Philippe et al.[35] validated that pile-soil heat conduction is mainly in the radial direction for a short period (i.e., several days for thermo-mechanical behaviour of energy piles). Secondly, the temperature at the pile-soil interface, T_a, is assumed to be uniform in the vertical direction, as field test results showed a slight variation of this temperature with depth, less than 0.5°C/m[10]. Finally, to derive an analytical form of transient temperature distributions, constant T_a is applied to soils at $r = a$ when t=0. In reality T_a may slightly increase/decrease with time during the heating/cooling process until a steady temperature is reached, but this kind of thermal boundary condition would make it difficult to find analytical-form solutions for transient temperature distributions, if not impossible. As reviewed in Bourne-Webb et al.[4], the boundary of constant T_a is preferred in most publications to simplify the analyses of energy pile-soil interaction.

All the input parameters for energy piles and soils are assumed to be independent of temperature, stress state, and time[13,31]. Changes in stresses and displacement are caused by radial thermal expansion of energy piles and soil temperature changes. The heat generated/consumed by soil deformation is excluded for small deformation of piles/soils during the quasi-static heating/cooling process. Moreover, pore water pressure, water flow, and phase changes are beyond the scope of this paper.

3. Solution for transient thermal stresses and displacement

This section shows the analytical solution for transient thermo-mechanical stresses and displacements in the radial pile-soil interaction system. Transient temperature distributions are calculated and applied to soils to derive transient thermal stresses and displacements in soils.

3.1. Transient temperature distribution

As the vertical heat flow is neglected, the partial differential equation (PDE) for soil temperature in the radial direction can be obtained by Fourier's law[6], as

$$\frac{\partial(\Delta T)}{\partial t} = c_{\mathrm{ht}}\left[\frac{\partial^2(\Delta T)}{\partial r^2} + \frac{1}{r}\frac{\partial(\Delta T)}{\partial r}\right] \tag{1}$$

where ΔT denotes the temperature change related to T_0; r means the current radial position of a soil particle; $c_{\mathrm{ht}} = k/c\rho$ is soil thermal diffusivity, in which k, c, and ρ are the thermal conductivity, specific heat, and density of the soil, respectively.

The initial and boundary conditions for temperature defined in Section 2 can be expressed as

$$\Delta T = 0\ , for\ r > a\ and\ t = 0 \tag{2}$$

$$\Delta T = 0, for\ r = \infty\ and\ t \geq 0 \tag{3}$$

$$\Delta T = \Delta T_a\ , for\ r = a\ and\ t \geq 0 \tag{4}$$

where $\Delta T_a = T_a - T_0$.

With the given PDE, initial conditions and boundary conditions, transient temperature distributions in the surrounding soil can be derived following Carslaw and Jaeger[6]:

$$\Delta T = \Delta T_a \frac{\ln(b/r)}{\ln(b/a)} + \Delta T_a \pi \sum_{n=1}^{\infty} \frac{J_0(\beta_n a) J_0(\beta_n b) U_0(\beta_n r)}{J_0^2(\beta_n a) - J_0^2(\beta_n b)} \exp\left(-c_{\mathrm{ht}}\beta_n^2 t\right) \tag{5}$$

$$U_0(\beta_n r) = J_0(\beta_n r) Y_0(\beta_n b) - J_0(\beta_n b) Y_0(\beta_n r) \tag{6}$$

where J_0 and Y_0 are the first and second kinds of Bessel functions of zero order, respectively; $U_0(\beta_n r)$ is the composite Bessel function defined by Eq. (6) and β_n is the n-th root of $U_0(\beta_n a) = 0$ (counted from 0 to infinite); b is a sufficiently large radius to replace the infinity boundary condition of (3) (e.g., $50a$ in this study).

3.2. Transient stress and displacement analyses

Since the initial and current stresses are in equilibrium states, stress increments should also be in equilibrium in the radial direction, as

$$\frac{\partial(\Delta\sigma_r)}{\partial r} + \frac{\Delta\sigma_r - \Delta\sigma_\theta}{r} = 0 \tag{7}$$

where $\Delta\sigma_r$ and $\Delta\sigma_\theta$ are the changes in radial and circumferential stresses, respectively, during the heating/cooling process.

The total of soil strains may be induced by the changes in both stresses and soil temperatures. Taking compression as positive for stresses and strains, the stress-strain relationship can be expressed as

$$\varepsilon_r = -\frac{\partial u}{\partial r} = \frac{1}{E}[\Delta\sigma_r - \mu(\Delta\sigma_\theta + \Delta\sigma_z)] - \alpha\Delta T \tag{8}$$

$$\varepsilon_\theta = -\frac{u}{r} = \frac{1}{E}[\Delta\sigma_\theta - \mu(\Delta\sigma_r + \Delta\sigma_z)] - \alpha\Delta T \tag{9}$$

$$\varepsilon_z = 0 = \frac{1}{E}[\Delta\sigma_z - \mu(\Delta\sigma_r + \Delta\sigma_\theta)] - \alpha\Delta T \tag{10}$$

where ε_r, ε_θ and ε_z are the total radial, circumferential, and vertical strains, respectively; u denotes the radial displacement of a soil particle; E, μ and $\alpha$are the elastic modulus, Poisson's ratio and thermal expansion coefficient of the soil, respectively.

The compatibility equation regarding strains can be derived by combining Eq. (8) and (9):

$$\frac{\partial\varepsilon_\theta}{\partial r} + \frac{\varepsilon_\theta - \varepsilon_r}{r} = 0 \tag{11}$$

Substituting Eq. (8) and (9) into Eq. (11), the compatibility equation can be transformed into:

$$\frac{\partial}{\partial r}(\Delta\sigma_\theta - \mu\Delta\sigma_r - \mu\Delta\sigma_z - E\alpha\Delta T) + (1+\mu)\frac{\Delta\sigma_\theta - \Delta\sigma_r}{r} = 0 \tag{12}$$

Substituting Eqs. (7) and (10) into Eq. (12) gives

$$\frac{\partial}{\partial r}[(1-\mu)(\Delta\sigma_r + \Delta\sigma_\theta) - E\alpha\Delta T] = 0 \tag{13}$$

Integrating Eq. (13) along the radial direction, one can get that

$$(1-\mu)(\Delta\sigma_r + \Delta\sigma_\theta) = E\alpha\Delta T \tag{14}$$

The stress components in soils can then be solved by combining Eqs. (7), (10), and (14), as

$$\Delta\sigma_r = \frac{A_a}{r^2} + \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int \Delta T \cdot r\mathrm{d}r \tag{15}$$

$$\Delta\sigma_\theta = \frac{E\alpha\Delta T}{1-\mu} - \frac{A_a}{r^2} - \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int \Delta T \cdot r\mathrm{d}r \tag{16}$$

$$\Delta\sigma_z = \frac{E\alpha\Delta T}{1-\mu} \tag{17}$$

where A_a is an integral constant that can be determined by the boundary condition at $r = a$. $\int \Delta T \cdot r\mathrm{d}r$ can be integrated after combination with Eq. (5), as

$$\int \Delta T \cdot r\mathrm{d}r = \frac{\Delta T_a r^2}{2\ln(b/a)}\left(\frac{1}{2} + \ln\frac{b}{r}\right) + \Delta T_a \pi \sum_{n=1}^{\infty} \frac{J_0(\beta_n a) J_0(\beta_n b)}{J_0^2(\beta_n a) - J_0^2(\beta_n b)} \exp\left(-c_{\mathrm{ht}}\beta_n^2 t\right) \frac{rU_1(\beta_n r)}{\beta_n} \tag{18}$$

in which

$$U_1(\beta_n r) = \frac{\beta_n}{r}\int U_0(\beta_n r) \cdot r\mathrm{d}r = J_1(\beta_n r) Y_0(\beta_n b) - J_0(\beta_n b) Y_1(\beta_n r) \tag{19}$$

where J_1 and Y_1 are the first and second kinds of Bessel functions of first order, respectively.

When substituting Eqs. (15), (16), and (17) into Eq. (9), the radial displacement of a given soil particle can be obtained as

$$u = -r\varepsilon_\theta = \frac{1+\mu}{E}\frac{A_a}{r} + \frac{1+\mu}{1-\mu}\frac{\alpha}{r}\int \Delta T \cdot r\mathrm{d}r \tag{20}$$

The boundary condition for radial displacement at the pile-soil interface (derivation is detailed in Appendix):

$$\frac{u(a)}{a} = \alpha_{\text{pile}}\Delta T_a - \frac{1-\mu_{\text{pile}}-2\mu_{\text{pile}}^2}{E_{\text{pile}}}\Delta\sigma_{\text{n}} \tag{21}$$

where E_{pile}, μ_{pile} and α_{pile} are the thermal expansion coefficient, Poisson's ratio and elastic modulus of energy piles, respectively. The first term on the right hand represents the thermal-induced radial strain at the pile-soil interface while the second term accounts for the mechanical radial strain generated by soil restriction.

By combining Eqs. (20) and (21), A_a can be obtained as

$$A_a = \frac{E\alpha_{\text{pile}}\Delta T_a a^2}{1+\mu} - \frac{E\left(1-\mu_{\text{pile}}-2\mu_{\text{pile}}^2\right)}{E_{\text{pile}}(1+\mu)}\Delta\sigma_{\text{n}}a^2 - \frac{E\alpha}{1-\mu}\int \Delta T\cdot r\text{d}r \tag{22}$$

Finally, transient thermo-elastic stresses and displacements in the soil can be simplified by substituting Eq. (22) into Eqs. (15), (16), (17), and (20):

$$\Delta\sigma_r = \frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} - \frac{E\left(1-\mu_{\text{pile}}-2\mu_{\text{pile}}^2\right)}{E_{\text{pile}}(1+\mu)}\Delta\sigma_{\text{n}}\frac{a^2}{r^2} + \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{23}$$

$$\Delta\sigma_\theta = \frac{E\alpha\Delta T}{1-\mu} - \frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} + \frac{E\left(1-\mu_{\text{pile}}-2\mu_{\text{pile}}^2\right)}{E_{\text{pile}}(1+\mu)}\Delta\sigma_{\text{n}}\frac{a^2}{r^2} - \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{24}$$

$$\Delta\sigma_z = \frac{E\alpha\Delta T}{1-\mu} \tag{25}$$

$$u = \alpha_{\text{pile}}\Delta T_a\frac{a^2}{r} - \frac{1-\mu_{\text{pile}}-2\mu_{\text{pile}}^2}{E_{\text{pile}}}\Delta\sigma_{\text{n}}\frac{a^2}{r} + \frac{1+\mu}{1-\mu}\frac{\alpha}{r}\int_a^r \Delta T\cdot r\text{d}r \tag{26}$$

3.3. Special cases

Three special cases of the present solution are discussed in this subsection, including cavity expansion under the purely mechanical loading (ML) pattern, purely thermal loading (TL) pattern, and combined thermal-mechanical loading (TML) pattern without soil restriction on radial expansion of energy piles.

When removing the transient temperature field and applying a radial displacement $\alpha_{\text{pile}}\Delta T_a a$ at $r=a$, the present solution can reduce to the conventional elastic solution for cavity expansion in isothermal soils[45], namely

$$\Delta\sigma_r = \frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} \tag{27}$$

$$\Delta\sigma_\theta = -\frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} \tag{28}$$

$$\Delta\sigma_z = 0 \tag{29}$$

$$u = \alpha_{\text{pile}}\Delta T_a\frac{a^2}{r} \tag{30}$$

If the radial contact stress at the soil-pile interface is ignored, (i.e., $\Delta\sigma_{\text{n}}=0$), the present solution recovers to the solution of Kandil et al.[18] as

$$\Delta\sigma_r = \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{31}$$

$$\Delta\sigma_\theta = \frac{E\alpha\Delta T}{1-\mu} - \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{32}$$

$$\Delta\sigma_z = \frac{E\alpha\Delta T}{1-\mu} \tag{33}$$

$$u = \frac{1+\mu}{1-\mu}\frac{\alpha}{r}\int_a^r \Delta T\cdot r\text{d}r \tag{34}$$

Note that in this paper $\int \Delta T\cdot r\text{d}r$ is integrated analytically with the temperature distributions of Eq. (5), instead of using numerical integration techniques in Kandil et al.[18].

Since E_{pile} is much larger than E, radial thermal expansion of the energy pile can hardly be prevented by the surrounding soil. Therefore, the particle displacement at the pile-soil interface, defined by Eq. (21), can be simplified as

$$u(a)/a = \alpha_{\text{pile}}\Delta T_a \tag{35}$$

Following the similar procedures in Section 3.2, stresses and displacements in the soil are simplified to be

$$\Delta\sigma_r = \frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} + \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{36}$$

$$\Delta\sigma_\theta = \frac{E\alpha\Delta T}{1-\mu} - \frac{E\alpha_{\text{pile}}\Delta T_a}{1+\mu}\frac{a^2}{r^2} - \frac{E\alpha}{1-\mu}\frac{1}{r^2}\int_a^r \Delta T\cdot r\text{d}r \tag{37}$$

$$\Delta\sigma_z = \frac{E\alpha\Delta T}{1-\mu} \tag{38}$$

$$u = \alpha_{\text{pile}}\Delta T_a\frac{a^2}{r} + \frac{1+\mu}{1-\mu}\frac{\alpha}{r}\int_a^r \Delta T\cdot r\text{d}r \tag{39}$$

Interestingly, the cavity expansion solution under the TML (Eq. (36)~(39)) is the sum of the solutions under purely ML (Eq. (27)~(30)) and purely TL (Eq. (31)~(34)). This is because, for free radial expansion of the energy pile, the TML solution coincides with the superposition of the ML and TL solutions in the hypothesis of linear elasticity.

4. Results and discussion

This section shows validation and parametric studies of the proposed solution. Unless stated otherwise, the following input parameters are adopted: $E = 5\times10^4$kPa, $E_{\text{pile}} = 3\times10^7$kPa, $\alpha = 5\times10^{-5}/^\circ$C, $\alpha_{\text{pile}} = 1\times10^{-5}/^\circ$C, μ=0.3 and μ_{pile}=0.15[31]; a=1 m, b=50 m, and $\Delta T_a = 20^\circ$C.

4.1. Solution validation

The proposed solution is validated by comparison with the finite element method via Comsol Multiphysics (CM) 6.0 software. The numerical model established in CM is a two-dimensional axisymmetric model, as shown in Fig. 2. Initially, the numerical model is free of stresses, strains, and temperature change. Then at the left boundary (i.e., $r=a$), there is a radial displacement $\alpha_{\text{pile}}\Delta T_a a$ representing thermal expansion of the energy pile and a constant temperature ΔT_a for heating/cooling at the pile-soil interface. Vertical heat flow and displacements are restricted at the top and bottom boundaries to simulate plane strain conditions in terms of the vertical direction, and the right boundary (i.e., $r=b$) is free of stress and temperature. The soil model and input parameters are the same as those used in the proposed solution, and the mesh number is set as 400. Thermo-mechanical coupling is undertaken by the thermal expansion model built in CM. Fig. 3 shows the distributions of temperatures, stresses and displacements calculated by the present solution and FEM when $c_{\text{ht}}t$=1m^2. The results predicted by the present solution match well with those calculated by FEM, thereby validating the solution accuracy.

4.2. Cavity expansion under the ML, TL, and TML patterns

Fig. 4 shows the stress and displacement distributions for cavity expansion under three loading patterns shown in Section 3.3. The

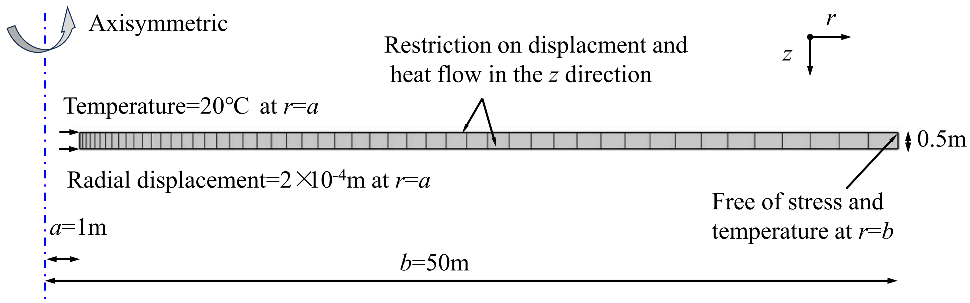

Fig. 2. Numerical simulation model in CM.

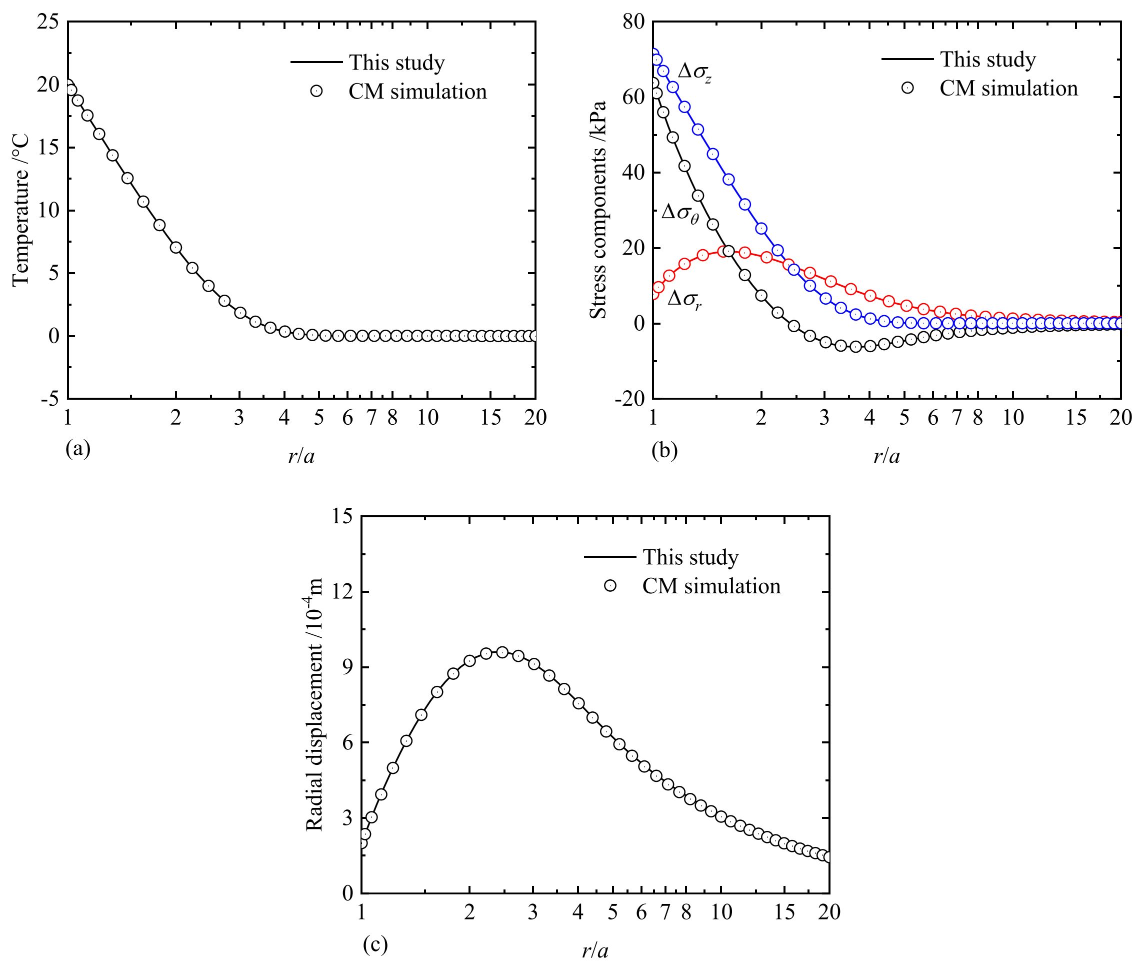

Fig. 3. Results calculated by the present solution and FEM: (a) temperature distribution; (b) stress distribution; (c) radial displacement distribution.

stresses and displacements under the TML pattern are determined by the competitive relationship between those under ML and TL patterns. ML plays an important role in the stresses and displacements in the (approximate) range of $a \le r \le 4a$, out of which ML effect becomes insignificant. With the present input parameters, TL dominates the increase/decrease of stresses and displacements under the TML pattern. For instance, the radial stresses under the TL and TML patterns show a similar trend that $\Delta\sigma_r$ first increases and then decreases with radial positions. When comparing Eq. (27)~(29) under ML pattern and Eq. (31)~(33) under the TL pattern, the stress components are proportional to soil moduli, indicating that soil moduli do not affect the competition between ML and TL patterns. However, the stresses and displacements under ML patterns heavily rely on the thermal expansion coefficients of energy piles (α_{pile}), while the stresses and displacements under TL patterns depend on thermal expansion coefficients of soils (α). In other words, the competitive relationship between TL and ML patterns is primarily determined by the relative values of α_{pile} and α.

4.3. Transient temperature, stress, strain and displacement distributions

Fig. 5 shows the transient distributions of temperatures, strains, stresses and displacements in soils, which are predicted by the present solution. In Fig. 5(a) the soil temperature remains 20°C at $r = a$, and gradually decreases with radius and increases with time. Similar trends can also be observed for vertical stress distributions as shown in Fig. 5 (d). This is because the vertical stress $\Delta\sigma_z$ is proportional to soil temperature if pile and soil parameters are given (see Eq. (17)).

On the contrary, the radial stress $\Delta\sigma_r$ in Fig. 5(b) firstly increases and

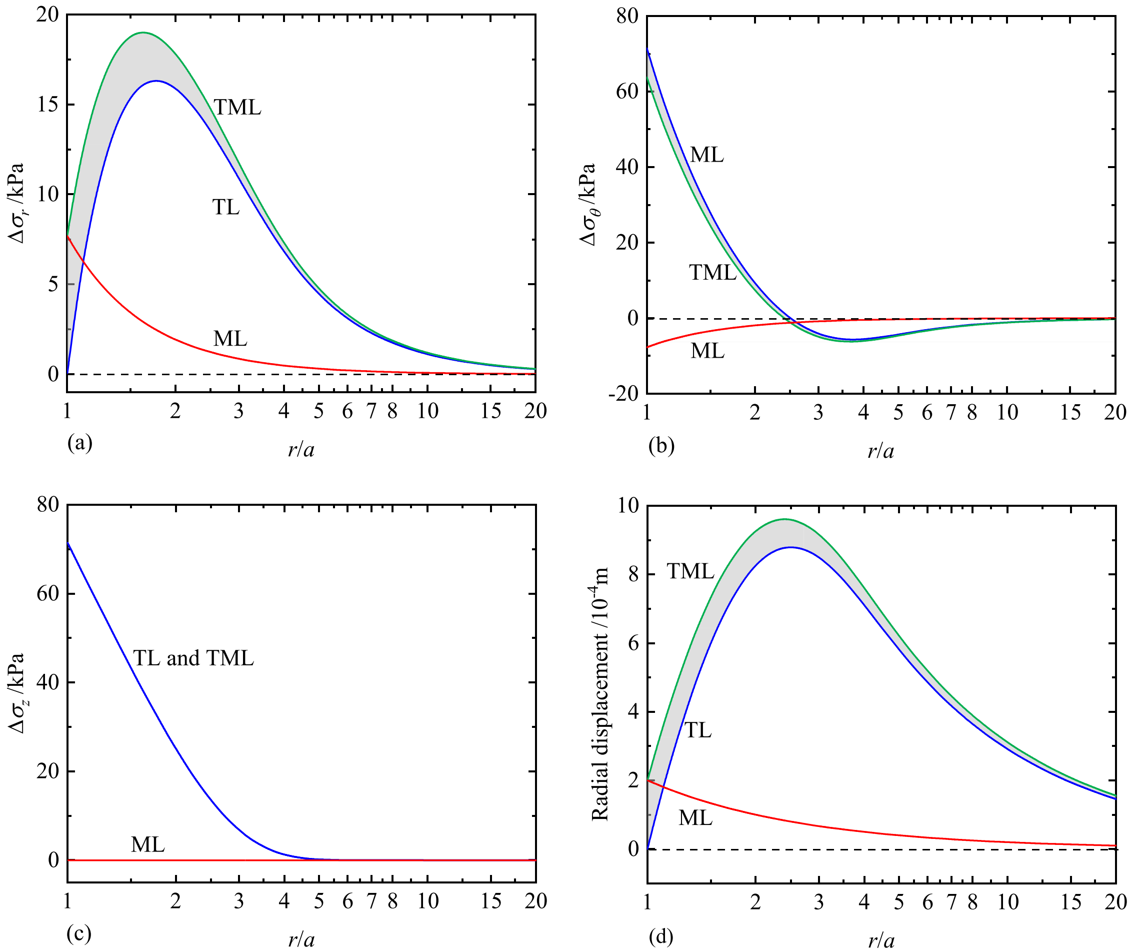

Fig. 4. Stress and displacement distributions under ML, TL and TML patterns: (a) radial stress; (b) circumferential stress; (c) vertical stress; (d) radial displacement.

then decreases with the increase of the radius, which is consistent with the observations in Olgun et al.[31] by FEM. The maximum value of $\Delta\sigma_r$ increases with time and appears at a larger radial position for a longer time. As a result, the thermal expansion and heat conduction of energy piles may impose an increasingly significant impact on adjacent geo-structures (e.g., pile groups) if transient temperature fields are considered. It is also worth noting that the maximum value of $\Delta\sigma_r$ could not increase infinitely with time, because temperature distributions will eventually be in a steady state[15,35].

Fig. 5(c) shows that $\Delta\sigma_\theta$ firstly decreases and then increases with radius, and the minimum value of $\Delta\sigma_\theta$ is negative. This exactly reflects the competitive relationship between the impacts of energy pile expansion (ML) and transient temperature distributions (TL). It is also for this reason that radial displacements and radial strains first increase and then decrease with the increasing radius, as shown in Fig. 5(e) and (f).

5. Radial contact stress at the energy pile-soil interface

If energy piles can expand freely without the radial restriction from surrounding soils, the radial stress change at the pile-soil interface ($r = a$) can be derived from Eq. (36) as

$$\Delta\sigma_{\mathrm{n}} = \frac{E\alpha_{\mathrm{pile}}\Delta T_a}{1+\mu} = \frac{E\varepsilon_\theta|_{r=a}}{1+\mu} \tag{40}$$

While Eq. (40) has been used to estimate $\Delta\sigma_{\mathrm{n}}$[10,26], it is revisited here based on the present solution to discuss the applicability of Eq. (40).

Firstly, Eq. (40) is quite general regarding various temperature fields, although specific temperature distributions are adopted in this paper. This conclusion can be proven as follows. By setting $r = a$ into Eq. (27), $\Delta\sigma_{\mathrm{n}}$ under the ML pattern is the same as Eq. (40). Similarly, $\Delta\sigma_{\mathrm{n}}$ is zero under the TL pattern after substituting $r = a$ into Eq. (31). Based on the superposition principle, $\Delta\sigma_{\mathrm{n}}$ is determined by the radial displacement at $r = a$, but is not affected by different temperature distributions. Therefore, Eq. (40) could be applied to estimate $\Delta\sigma_{\mathrm{n}}$ with arbitrary temperature distributions, as long as the radial displacement (or strain) can be obtained (note that the radial displacement may be time-dependent for complex temperature distributions, such as cyclic heating and cooling).

Secondly, energy pile expansion may be prevented by surrounding soils in real field conditions, and $\Delta\sigma_{\mathrm{n}}$ will be expressed as Eq. (41) after substituting $r = a$ into Eq. (23).

$$\Delta\sigma_{\mathrm{n}} = \frac{E\alpha_{\mathrm{pile}}\Delta T_a}{1+\mu+\left(1-\mu_{\mathrm{pile}}-2\mu_{\mathrm{pile}}^2\right)\left(E/E_{\mathrm{pile}}\right)} \tag{41}$$

Fig. 6 presents the variation of $\Delta\sigma_{\mathrm{n}}$ with soil moduli, taking $E_{\mathrm{pile}} = 3\times10^7$kPa, $\mu_{\mathrm{pile}} = 0.15$, and $\Delta T_a = 10$°C. The figure shows that Eq. (41) predicts similar $\Delta\sigma_{\mathrm{n}}$ as that in Olgun et al.[31], and the slight difference lies in Poisson's ratio effect (plane strain assumption in this study and plane stress assumption in Olgun et al.[31]). Eq. (41) can be simplified as Eq. (40) for typical soil moduli within the range of $10^3\sim10^5$kPa[31], revealing that energy piles can expand with negligible restrictions from the surrounding soils. In this case, the mechanism of $\Delta\sigma_{\mathrm{n}}$ increasing with E can therefore be explained by Eq. (40) that $\Delta\sigma_{\mathrm{n}}$ is proportional to E, but the increase of $\Delta\sigma_{\mathrm{n}}$ is hardly relevant to soil restriction on radial expansion of energy piles. Moreover, Eq. (41) indicates that soil restriction is

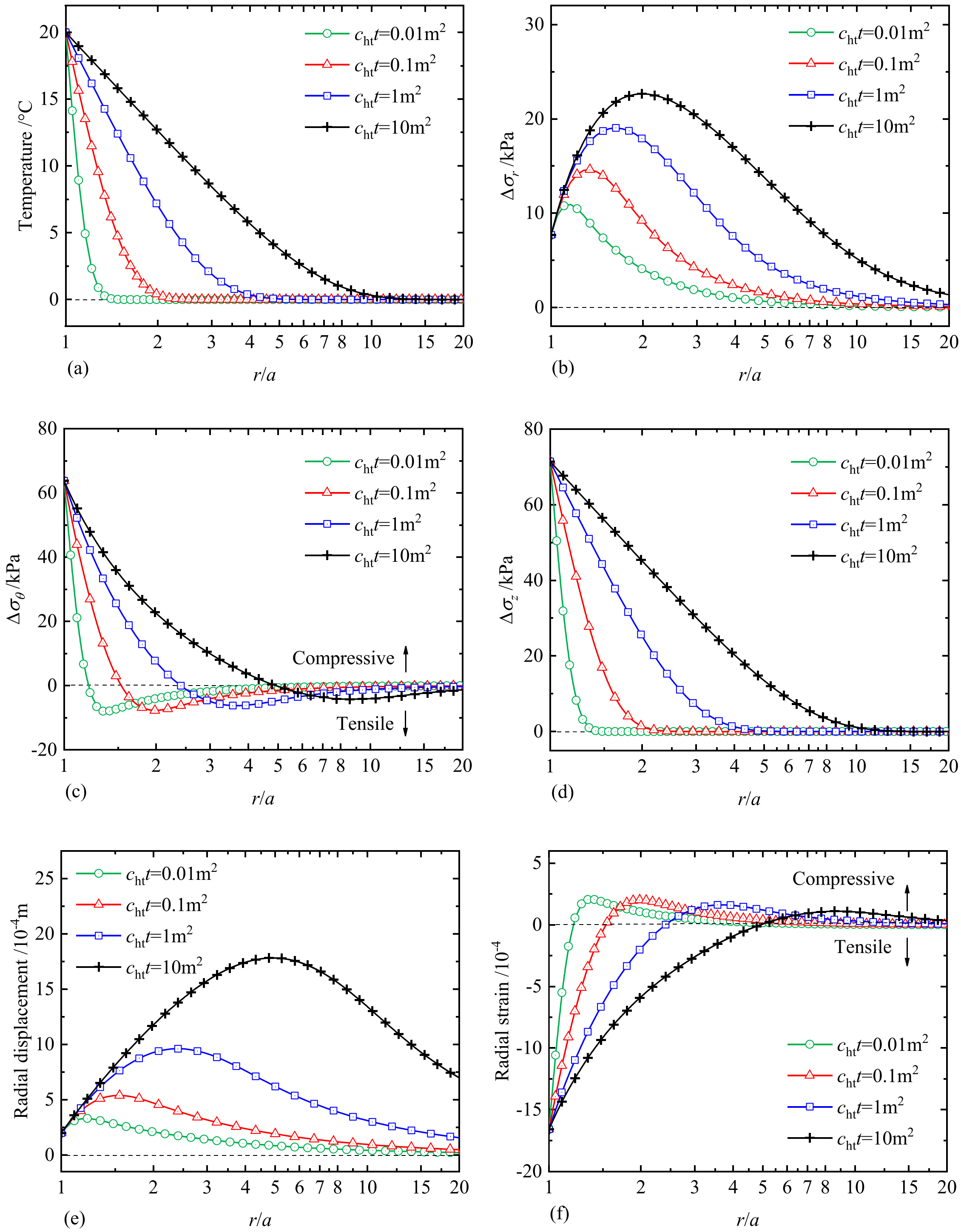

Fig. 5. Field distributions with time: (a) temperature; (b) radial stress; (c) circumferential stress; (d) vertical stress; (e) radial displacement; (f) radial strain.

mainly dependent on the soil-to-pile modulus ratio, which may become important when energy piles are installed into hard rocks with high elastic moduli.

In summary, Eq. (40) may be able to estimate $\Delta\sigma_n$ with the consideration of various temperature distributions and pile/soil moduli. Fig. 7 shows the comparison of $\Delta\sigma_n$ predicted by Eq. (40) and measured by field tests that were conducted in red cliff sand with E=60 MPa and μ=0.3 at the Melbourne test site[10,12,27] and hard clay with E=6.9 MPa and μ=0.35 at Nanjing Liuhe test site[26]. Good agreement can be seen between measured and predicted $\Delta\sigma_n$, thereby validating the accuracy of Eq. (40) for quantifying the RIEPS. In the practical capacity design of energy piles, $\Delta\sigma_n$ can be neglected for soft soils with low temperature change, while it may become noticeable for stiff soils with high temperature change. Finally, care should be taken because Eq. (40) is derived by assuming that soil deformation is purely elastic. Soil plasticity may occur as the stress states may be disturbed by energy pile installation and be influenced by vertical expansion of energy piles.

6. Conclusions

This paper proposes a thermo-elastic cavity expansion solution for radial interaction between energy piles and soils (RIEPS), considering

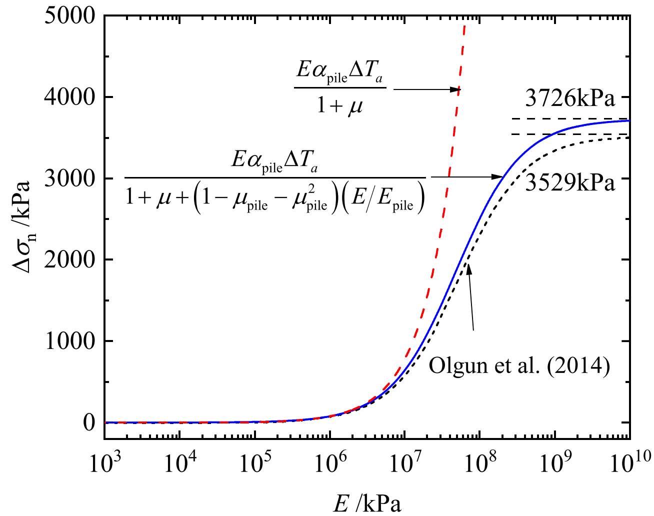

Fig. 6. Variation of radial stress at the pile-soil interface with soil moduli.

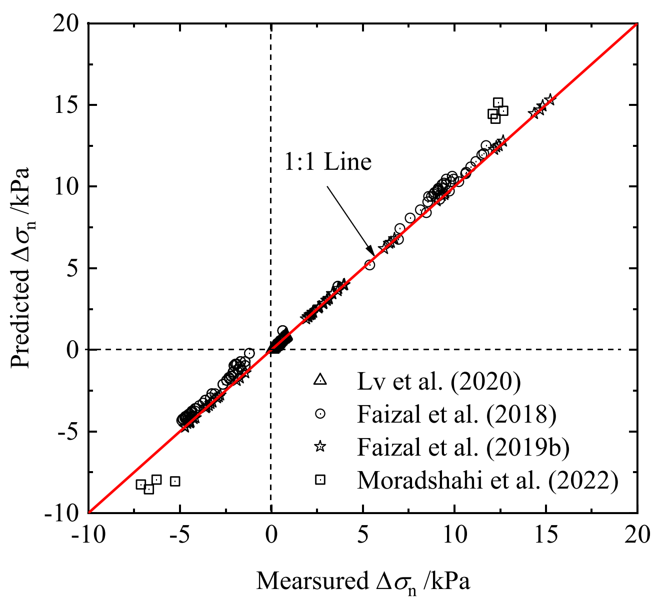

Fig. 7. Comparison of predicted and mearsured radial stress changes.

transient temperature distributions and radial displacement boundary at the pile-soil interface. The solution is adopted to investigate transient stresses and displacements in soils and analyse the radial contact stress change at the pile-soil interface. It is found that the cavity expansion solution under the combined thermo-mechanical loading (TML) pattern is the linear superposition of the solutions under purely mechanical loading (ML) and thermal loading (TL) patterns. The thermal-mechanical stresses and displacements in soils depend on the competitive relationship between ML and TL patterns, more specifically, the thermal expansion coefficients of soils and energy piles. For this reason the stresses and displacements in soils evolve with time and radial positions in various ways. The change in radial contact stress at the pile-soil interface is revisited with a particular focus on the influences of transient temperature distributions and soil/pile moduli. After showing parametric studies and comparison with field test data, Eq. (40) may be utilised to estimate the change of radial contact stress at the energy pile-soil interface. The proposed solution can be considered as a useful benchmark for future research and contributes to the mathematical validation of Eq. (40) under transient thermal fields. However, the application of the solution to real foundation could be limited since the radial stress change can be neglected after considering typical variation range of soil stiffness, which is in accordance with past studies on this topic.

CRediT authorship contribution statement

He Yang: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Pin-Qiang Mo:** Writing – review & editing, Conceptualization. **Pei-Zhi Zhuang:** Writing – review & editing, Supervision, Conceptualization. **Xiaohui Chen:** Writing – review & editing, Supervision, Conceptualization. **Hai-Sui Yu:** Writing – review & editing, Supervision, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgement

The first author thanks the financial support from the China Scholarship Council for his study at the University of Leeds. The second author would like to acknowledge the financial support from the National Natural Science Foundation of China (52108374), the "Taishan" Scholar Program of Shandong Province, China (tsqn201909016), the Shandong Provincial Natural Science Foundation (ZR202102250562). Program of Shandong Province, China (tsqn201909016), the Shandong Provincial Natural Science Foundation (ZR202102250562). The third author wants to thanks the financial support provided by National Natural Science Foundation of China (52178374), Basic and Applied Basic Research Foundation of Guangdong Province (2024A1515011552)

Appendix A. Derivation of Eq. (21)

The radial deformation of energy piles at the pile-soil interface (i.e., $r = a$) may be split into the thermal expansion part $\alpha_{pile}\Delta T_a a$ and the soil restriction part with a confining pressure of $\Delta\sigma_n$ at $r = a$. The latter induced by soil restriction can be derived as follows.

The stress equilibrium equation remains the same as Eq. (7), and the elastic stress-strain relationship for energy piles can be simplified as

$$\varepsilon_r = -\frac{\partial u}{\partial r} = \frac{1}{E_{pile}}\left[\Delta\sigma_r - \mu_{pile}(\Delta\sigma_\theta + \Delta\sigma_z)\right] \tag{42}$$

$$\varepsilon_\theta = -\frac{u}{r} = \frac{1}{E_{pile}}\left[\Delta\sigma_\theta - \mu_{pile}(\Delta\sigma_r + \Delta\sigma_z)\right] \tag{43}$$

$$\varepsilon_z = 0 = \frac{1}{E_{\text{pile}}}\left[\Delta\sigma_z - \mu_{\text{pile}}(\Delta\sigma_r + \Delta\sigma_\theta)\right] \tag{44}$$

As the boundary conditions that $\Delta\sigma_r = \Delta\sigma_{\text{n}}$ at $r = a$ and $\Delta\sigma_r \neq \infty$ at $r = 0$ should be satisfied, the stress components will be

$$\Delta\sigma_r = \Delta\sigma_\theta = \Delta\sigma_{\text{n}} \tag{45}$$

$$\Delta\sigma_z = 2\mu_{\text{pile}}\Delta\sigma_{\text{n}} \tag{46}$$

The radial displacement at $r = a$, which are resulted from soil restriction, can be derived by substituting Eqs. (45) and (46) into Eq. (43):

$$u(a) = -\frac{\left(1 - 2\mu_{\text{pile}}\right)\left(1 + \mu_{\text{pile}}\right)}{E_{\text{pile}}}\Delta\sigma_{\text{n}}a \tag{47}$$

Finally, the radial displacement of piles at $r = a$ can be obtained by adding $\alpha_{\text{pile}}\Delta Ta$ and Eq. (47), thereby giving the expression of Eq. (21):

$$\frac{u(a)}{a} = \alpha_{\text{pile}}\Delta T_a - \frac{1 - \mu_{\text{pile}} - 2\mu_{\text{pile}}^2}{E_{\text{pile}}}\Delta\sigma_{\text{n}} \tag{48}$$

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