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Jet Noise in the 'Zone of Silence'

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We analyze the far field acoustic power spectrum of a jet flow convecting a region of turbulent activity by formulating an acoustic analogy. The analogy is defined using Goldstein's linearized Euler equation system (J. of Aero-Acoustics 2002, vol. 1, pp. 1) with the propagation based on a parallel mean flow, and the source statistics on a Reynolds averaged Navier Stokes (RANS) calculation.

Our analysis confirms the jet noise spectrum can be thought of as being composed of two terms. The first is significant at high frequencies, while the second term is important at low frequencies and is especially dominant at small observation angles to the jet flow—in the so-called 'zone of silence'. This additional term appears to account for the experimentally observed low frequency noise in the 'zone of silence' (Lush, JFM 1971, vol. 46, pp. 477) and has previously been discussed by Goldstein (JFM 1975, vol. 70, pp. 595). We show that the low frequency term does not occur if the fluctuating Reynolds stress source term in the momentum equation is assumed to be isotropic at all times, but that it is significant if that source is only supposed to be statistically isotropic—if the cross power spectral density of the fluctuating Reynolds stress source term is taken to be isotropic. The low frequency term depends on the gradient of the mean flow velocity. In this paper, we assess the relative magnitudes of the terms in noise spectrum, and show the behavior of the additional low frequency term.

Our work contributes to showing that a unified treatment of the jet noise problem is possible using an acoustic analogy to predict both the high and low frequency noise within reasonable accuracy and a modicum of empirical tuning.

I. Introduction

The existence of different mechanisms that generate noise from a jet flow has long been argued.[1,2] One of these mechanisms was thought to dominate the jet noise spectrum at high frequency. Another type, was found to be most significant at very low frequencies. This point of view began to prevail after some rather surprising experiments with jet flows. Lush[3] and Ahuja[4] found, when jet velocity was high, the peak noise occurred at a lower frequency, and this peak grew the closer to the jet axis the observations were made—in the so-called 'zone of silence'. The low frequency behavior in the 'zone of silence' was interesting—not least because it was somewhat difficult to explain; but also the noise spectrum, here, seemed to be dependent on the mean velocity profile.

Now, most of the theoretical attempts to explain the observed noise in the 'zone of silence' have used the acoustic analogy.[5] The standard procedure is to re-arrange the Navier-Stokes equations so that the linear fluctuations (from any base flow) appear on the left hand side, and the non-linear fluctuations are put to the right hand side. The linear fluctuations represent the acoustic propagation and the non-linear terms are the sources of the noise. For example, in the Lilley equation[6] the mean flow is based on a parallel shear layer in the streamwise direction. The real breakthrough to explain the noise in the 'zone of silence' came after analyzing the Lilley equation. Goldstein[7] (and later, Balsa[8]) proved at very low frequencies the noise spectrum, using Lilley's equation, would be proportional to the square of the mean velocity gradient. The *startling result* Goldstein[7] found was this mean flow dependent term in the spectrum did account for the noise the experimenters were observing in the 'zone of silence' of the jet flow.

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Hitherto, the standard approach in jet noise prediction is to model the source terms that appear on the right hand side of the acoustic analogy equation, and solve the wave propagation problem, using a parallel mean flow for example.[9–11] Reynolds Averaged Navier Stokes (RANS)-based models have been successful in predicting the noise that dominates at high frequency—outside of the 'zone of silence'; for example, take the model by Tam and Aurialt.[9] But what has remained somewhat elusive is a method that predicts *both* the high frequency and low frequency noise.

Our aim in this paper is to show, for a cold jet flow, the noise spectrum *can* indeed be understood by two terms. The first, is significant at high frequency, while the second term is important at low frequencies and dominant in the 'zone of silence'. The low frequency term is proportional to the local mean velocity gradient and was discussed by Goldstein[7] and Balsa.[8] We show the low frequency term can exist if the cross power spectral density of the fluctuating Reynolds stress source term is assumed to be isotropic. However, this low frequency term will not occur if the Reynolds stress source term is, itself, taken to be isotropic for all values of time. The acoustic analogy we use, is based on Goldstein's[12] linearized Euler equation system; and, we solve the wave propagation problem for a parallel shear layer. The source statistics and mean flow are defined by a RANS calculation of the Stromberg jet: the Reynolds number is 3600 and the Mach number is 0.9.[13] We focus on noise predictions at an observation angle of 30^o to the jet flow, and the jet noise directivity. For both of these cases, we compare our predictions to the available DNS data.[14,15]

II. Acoustic Analogy

A. Governing Equations

Imagine a region of turbulence convected by a jet flow. The turbulent field is confined to a region near the jet, so that at large distances from the flow, all of the turbulent motion has ceased. Within the jet region, momentum is transferred in a random fashion, the jet flow interacts with this transfer causing energy changes in the field. The physical processes of *momentum transfer* and *energy change* are governed by the Navier Stokes equations, and generate acoustic waves that propagate to the far field.

The outcome of this section is Eq. (25)—a formula for the power spectral density of the far field pressure due to the turbulent activity in the jet flow. We represent the turbulent field by a 'source term', which is simply a stationary random function described by field variables $(\mathbf{y}, \tau)$. The sources represent the dynamics of the turbulent field and generate noise in the far field, at the observer position $(\mathbf{x}, t)$. A picture of the jet flow is shown in Fig. (1).

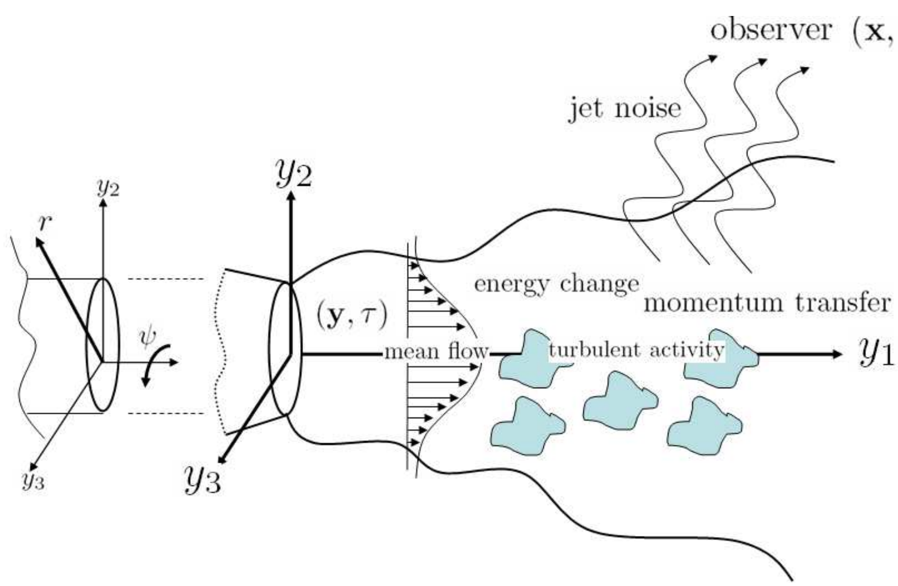


Figure 1. Coordinate system for a jet flow

We define the analogy using the linearized Euler equations formulated by Goldstein.[12] In his system, the Euler equations are linearized about a base flow with density $\bar{\rho}$, pressure $\bar{p}$, and velocity $\tilde{v}_j$. The bar and single prime represent the time average and its perturbation, and the tilde and double prime represent the Favre average and its perturbation. The averaging operations are defined in the usual way:

$$\overline{(\bullet)}(\mathbf{y}) \equiv \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (\bullet)(\mathbf{y},\tau)d\tau \quad \text{and} \quad \bar{\rho}\widetilde{(\bullet)} \equiv \overline{\rho(\bullet)}. \tag{1}$$

The momentum variable is defined with zero time average, $u_i = \rho v_i''$; and the Favre averaged stagnation enthalpy, and its perturbation, take the special definitions:

$$\tilde{h}_o = \tilde{h} + \frac{1}{2}\tilde{v}^2 \quad \text{and} \quad h_o'' = h'' + \tilde{v}_i v_i'' + \frac{1}{2}v''^2. \tag{2}$$

The system is given by the set of equations:

$$\frac{\partial \rho'}{\partial \tau} + \frac{\partial}{\partial y_j}(\rho'\tilde{v}_j + u_j) = 0 \tag{3}$$

$$\frac{\partial u_i}{\partial \tau} + \frac{\partial}{\partial y_j}(\tilde{v}_j u_i) + \frac{\partial p'}{\partial y_i} + u_j\frac{\partial \tilde{v}_i}{\partial y_j} - \left(\frac{\rho'}{\bar{\rho}}\right)\frac{\partial \tilde{\tau}_{ij}}{\partial y_j} = \frac{\partial T'_{ij}}{\partial y_j} \quad i = 1,..3. \tag{4}$$

$$\left(\frac{1}{\gamma-1}\right)\frac{\partial p'}{\partial \tau} + \left(\frac{1}{\gamma-1}\right)\frac{\partial}{\partial y_j}(p'\tilde{v}_j) + \frac{\partial}{\partial y_j}(u_j\tilde{h}) + p'\frac{\partial \tilde{v}_j}{\partial y_j} - \left(\frac{u_i}{\bar{\rho}}\right)\frac{\partial \tilde{\tau}_{ij}}{\partial y_j} = Q. \tag{5}$$

(Throughout this paper, summation applies across repeated indices.)
(γ is the ratio of the specific heat capacities of air, $\gamma = 1.4$.)

The nice feature of Goldstein's equations are the source terms, which are in a simple form, viz.:

$$\underbrace{T'_{ij} = -(\rho v_i'' v_j'' - \bar{\rho}\widetilde{v_i'' v_j''})}_{\text{Noise due to momentum transfer}} \tag{6}$$

$$Q = \underbrace{-\tilde{v}_j\frac{\partial T'_{ij}}{\partial y_i} + \frac{1}{2}\delta_{ij}\left[\frac{DT'_{ij}}{D\tau} + \frac{\partial \tilde{v}_k}{\partial y_k}T'_{ij}\right]}_{\text{Noise due to energy change}} - \underbrace{\frac{\partial}{\partial y_j}\left(\rho v_j'' h_o'' - \bar{\rho}\widetilde{v_j'' h_o''}\right)}_{\text{Noise due to enthalpic heating}} \quad . \tag{7}$$

In this paper, however, we concentrate on the cold jet flow where the noise from enthalpic heating is negligible compared to momentum transfer. Moreover, for the same reason, the perturbations in density can be neglected, i.e. $\rho(\mathbf{y},\tau) \approx \bar{\rho}(\mathbf{y})$. Then, the source terms reduce to:

$$T'_{ij} = -\bar{\rho}(v_i'' v_j'' - \widetilde{v_i'' v_j''}) \tag{8}$$

$$Q = -\tilde{v}_j\frac{\partial T'_{ij}}{\partial y_i} + \frac{1}{2}\delta_{ij}\left[\frac{DT'_{ij}}{D\tau} + \frac{\partial \tilde{v}_k}{\partial y_k}T'_{ij}\right]. \tag{9}$$

$D/D\tau$ is the usual convective derivative, given by:

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + \tilde{v}_j\frac{\partial}{\partial y_j}. \tag{10}$$

The tensor, $\tilde{\tau}_{ij}$, defined by $\tilde{\tau}_{ij} = \delta_{ij}\bar{p} + \bar{\rho}\widetilde{v_i'' v_j''}$, appears in the propagation operator of the linearized equations. It drives the jet evolution in the flow by interacting with the mean pressure. For a parallel mean flow the vector $\partial\tilde{\tau}_{ij}/\partial y_i$ is identically zero.

B. Representation Theorem

The wave propagation problem is solved in the frequency domain using the adjoint Green function. This method has been used many times in the past, for example the work by Dowling et al,[16] and by Tam and Aurialt.[17] The adjoint Green function method is advantageous because it allows one to specify the turbulent field for a particular observer point. The method amounts to solving a set of homogeneous equations (when the right hand side is zero) in the jet region. Any unknown constants are easily found, because this solution in the jet must reduce to the solution of the wave equation in the far field, where the mean flow is all zero. In this paper we use the Fourier transform pair (with angular frequency ω):

$$\hat{f}(\omega) = \int\limits_{-\infty}^{\infty} f(\tau) e^{-i\omega\tau}\, d\tau \quad \text{and} \quad f(\tau) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega\tau}\, d\omega. \tag{11}$$

It is algebraically straightforward to formulate a representation theorem for the far field pressure. For example, by taking the momentum-like adjoint Green function and performing the inner product of this, and the momentum equation (Eq. 4), gives the *adjoint momentum equation.* One must integrate each term by parts and apply the divergence theorem; we have to suppose, then, that any surface terms at infinity, far away from the jet, are zero. A similar operation gives the *adjoint mass equation* and *adjoint energy equation.* Hence, the representation theorem for the far field pressure is:

$$\hat{p}(\mathbf{x},\omega) = -\int\limits_{V_\infty(\mathbf{y})} \left(\hat{G}_i(\mathbf{y},\omega \mid \mathbf{x}) \frac{\partial \hat{T}_{ij}}{\partial y_j}(\mathbf{y},\omega) + \hat{G}_4(\mathbf{y},\omega \mid \mathbf{x}) \hat{Q}(\mathbf{y},\omega) \right) d^3\mathbf{y}. \tag{12}$$

$\hat{T}_{ij}(\mathbf{y},\omega)$ is the Fourier transform of $T'_{ij}(\mathbf{y},\tau)$, and $\hat{\mathbf{G}}(\mathbf{y},\omega \mid \mathbf{x})$, the Fourier transform of the adjoint Green's function, satisfies the *adjoint equations*:

$$i\omega \hat{G}_o + \tilde{v}_j \frac{\partial \hat{G}_o}{\partial y_j} + \left(\frac{\hat{G}_i}{\bar{\rho}} \right) \frac{\partial \tilde{\tau}_{ij}}{\partial y_j} = 0 \tag{13}$$

$$i\omega \hat{G}_j + \frac{\partial \hat{G}_o}{\partial y_j} + \tilde{v}_i \frac{\partial \hat{G}_j}{\partial y_i} - \hat{G}_i \frac{\partial \tilde{v}_i}{\partial y_j} + \tilde{h} \frac{\partial \hat{G}_4}{\partial y_j} + \left(\frac{\hat{G}_4}{\bar{\rho}} \right) \frac{\partial \tilde{\tau}_{ij}}{\partial y_i} = 0 \quad j = 1,..3. \tag{14}$$

$$\left(\frac{i\omega}{\gamma - 1} \right) \hat{G}_4 + \left(\frac{\tilde{v}_j}{\gamma - 1} \right) \frac{\partial \hat{G}_4}{\partial y_j} - \hat{G}_4 \frac{\partial \tilde{v}_j}{\partial y_j} + \frac{\partial \hat{G}_j}{\partial y_j} = \delta(\mathbf{y} - \mathbf{x}). \tag{15}$$

$\hat{G}_o$ is the adjoint density-like variable and $\hat{G}_1$-$\hat{G}_3$ are the adjoint momentum-like variables. $\hat{G}_4$, the pressure-like quantity, is the variable in the adjoint energy equation.

C. An equivalent representation theorem: far field pressure budget

An important aspect of the present analysis is that the source term in the representation theorem is $\hat{T}_{ij}$ only. The term Q, is, a function of $\hat{T}_{ij}$ as well—see Eq. (9). If we now insist that $\hat{T}_{ij}$ is *continuous* throughout the field space $(\mathbf{y},\tau)$ we can integrate Eq. (12) by parts. It seems sensible to do this. Computing spatial derivatives of a function that we can, at best, model, would be numerically challenging. Especially given that one would be relying upon a CFD solution that is only ever known on a discrete set of points. The Green function, on the other hand, can be determined, and differentiated, with accuracy. Substituting Eq. (9) into Eq. (12) and integrating each term by parts, so that we isolate $\hat{T}_{ij}$, gives the equivalent sound field representation:

$$\hat{p}(\mathbf{x},\omega) = \int\limits_{V_\infty(\mathbf{y})} \hat{I}_{ij}(\mathbf{y},\omega \mid \mathbf{x}) \hat{T}_{ij}(\mathbf{y},\omega)\, d^3\mathbf{y}. \tag{16}$$

$\hat{I}_{ij}$ is a propagation tensor defined by:

$$\hat{I}_{ij}(\mathbf{y},\omega \mid \mathbf{x}) = \frac{\partial \hat{G}_j}{\partial y_i}(\mathbf{y},\omega \mid \mathbf{x}) \quad - \quad \left[\frac{\partial \tilde{v}_j}{\partial y_i}(\mathbf{y})\hat{G}_4(\mathbf{y},\omega \mid \mathbf{x}) + \tilde{v}_j(\mathbf{y})\frac{\partial \hat{G}_4}{\partial y_i}(\mathbf{y},\omega \mid \mathbf{x})\right] \\ + \quad \frac{\delta_{ij}}{2}\left[i\omega\left(1+\frac{\tilde{v}_k}{i\omega}\frac{\partial}{\partial y_k}\right)\hat{G}_4(\mathbf{y},\omega \mid \mathbf{x}) - \frac{\partial \tilde{v}_k}{\partial y_k}(\mathbf{y})\hat{G}_4(\mathbf{y},\omega \mid \mathbf{x})\right]. \tag{17}$$

The equivalent representation states, mathematically, what we said in words at the beginning of this section. For a cold jet flow, momentum transfer and energy change in the field of turbulence (through $\hat{T}_{ij}$) generates acoustic waves (through Green function terms) that propagate to the far field. The far field "pressure budget" is shown in Fig. (2).

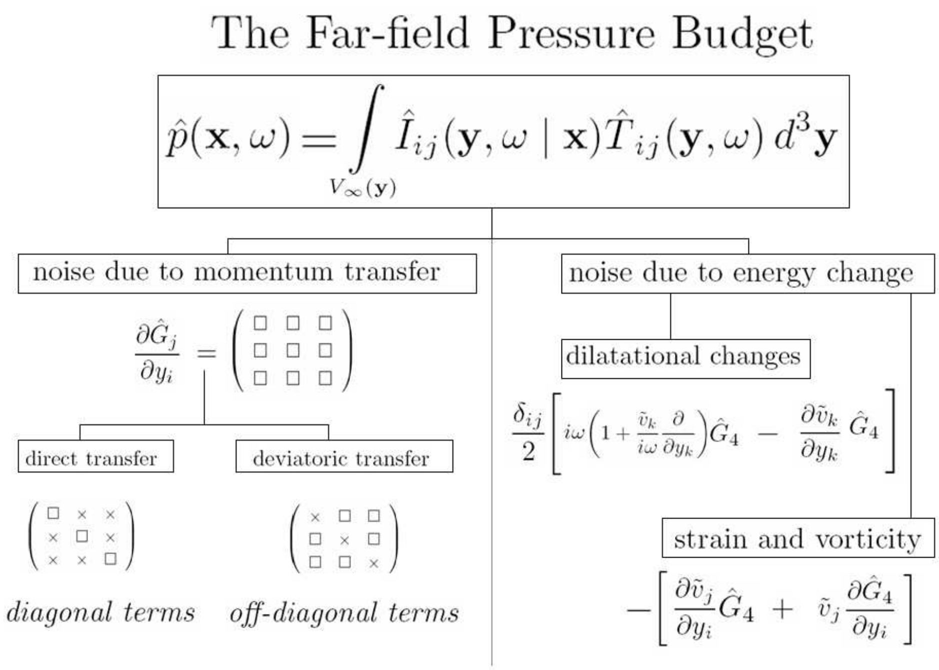


Figure 2. The equivalent representation theorem, Eqs. (16) & (17)

D. Power spectral density formula

The power spectral density of the far field pressure is:

$$\hat{P}(\mathbf{x},\omega) = \int\limits_{V_\infty(\mathbf{y})}\int\limits_{V_\infty(\eta)} \hat{I}_{ijkl}(\mathbf{y},\eta,\omega \mid \mathbf{x})\hat{R}_{ij,kl}(\mathbf{y},\eta,\omega)\, d^3\eta\, d^3\mathbf{y}. \tag{18}$$

η is the vector separation between the correlation positions $\mathbf{y}$ and $\mathbf{y}+\eta$; in Cartesian coordinates $\eta = (\eta_1,\eta_2,\eta_3)$. The integrand is expressed in terms of the 4th rank tensors: $\hat{I}_{ijkl}(\mathbf{y},\eta,\omega \mid \mathbf{x})$—for the wave propagation, and $\hat{R}_{ijkl}(\mathbf{y},\eta,\omega)$ for the turbulent field. They are easily defined from standard theory,[18] i.e.,

$$\hat{I}_{ijkl}(\mathbf{y},\eta,\omega \mid \mathbf{x}) = \hat{I}_{ij}(\mathbf{y},-\omega \mid \mathbf{x})\hat{I}_{kl}(\mathbf{y}+\eta,\omega \mid \mathbf{x}), \tag{19}$$

where the second rank tensors $\hat{I}_{ij}$ are given by Eq. (17). $\hat{R}_{ijkl}(\mathbf{y},\eta,\omega)$, the 4th rank cross power spectral density of the stationary random function $\hat{T}_{ij}$ is:

$$\begin{aligned}\hat{R}_{ij,kl}(\mathbf{y},\eta,\omega) \quad &= \int_{-\infty}^{+\infty} R_{ij,kl}(\mathbf{y},\eta,\tau_0)e^{-i\omega\tau_0}\,d\tau_0 \\ &= \int_{-\infty}^{+\infty} \overline{T'_{ij}(\mathbf{y},\tau)T'_{kl}(\mathbf{y}+\eta,\tau+\tau_0)}e^{-i\omega\tau_0}\,d\tau_0.\end{aligned} \tag{20}$$

We simplify the wave propagation tensor $\hat{I}_{ijkl}$ by assuming the variation of second rank tensor $\hat{I}_{kl}$ is small over η_2 and η_3 compared to the correlation length. The dependence on η_1 can be approximated, for example, by following Tam and Aurialt,[9] i.e.,

$$\hat{I}_{kl}(\mathbf{y}+\eta,\omega \mid \mathbf{x}) \approx \hat{I}_{kl}(\mathbf{y},\omega \mid \mathbf{x})e^{ik_o\eta_1 cos\theta} \tag{21}$$

where the observer is in the far field at an angle θ to the jet flow; and $k_o = \omega/c_\infty$ is the wave number in the far field. The power spectral density then simplifies to,

$$\hat{P}(\mathbf{x},\omega) = \int\limits_{V_\infty(\mathbf{y})} \hat{I}_{ij}(\mathbf{y},-\omega \mid \mathbf{x})\hat{I}_{kl}(\mathbf{y},\omega \mid \mathbf{x}) \int\limits_{V_\infty(\eta)} \hat{R}_{ij,kl}(\mathbf{y},\eta,\omega)e^{ik_o\eta_1 cos\theta}\,d^3\eta\,d^3\mathbf{y} \tag{22}$$

We will find it useful to label $\hat{I}_{ijkl}(\mathbf{y},\omega \mid \mathbf{x})$ as

$$\hat{I}_{ijkl}(\mathbf{y},\omega \mid \mathbf{x}) = \hat{I}_{ij}(\mathbf{y},-\omega \mid \mathbf{x})\hat{I}_{kl}(\mathbf{y},\omega \mid \mathbf{x}); \tag{23}$$

and the integral of the turbulent field tensor over η, $\hat{R}^{\mathrm{total}}_{ij,kl}(\mathbf{y},\omega)$, as

$$\hat{R}^{\mathrm{total}}_{ij,kl}(\mathbf{y},\omega) = \int\limits_{V_\infty(\eta)} \hat{R}_{ij,kl}(\mathbf{y},\eta,\omega)e^{ik_o\eta_1 cos\theta}\,d^3\eta. \tag{24}$$

Then, the final form of power spectral density is rather easier to handle.

$$\hat{P}(\mathbf{x},\omega) = \int\limits_{V_\infty(\mathbf{y})} \hat{I}_{ijkl}(\mathbf{y},\omega \mid \mathbf{x})\hat{R}^{\mathrm{total}}_{ij,kl}(\mathbf{y},\omega)\,d^3\mathbf{y} \tag{25}$$

The rest of this paper is devoted to analyzing Eq. (25) for a parallel shear flow. We adopt a cylindrically based coordinate system for a jet flow that is circular cylindrical, so that the field spaces are $\mathbf{y} = (y_1, r, \psi)$ and $\mathbf{x} = (x_1, R, \Psi)$. The mean flow is directed axially in y_1, and a function of r; and the observer is in the far field at an angle θ to the jet axis. To evaluate the wave propagation tensor, $\hat{I}_{ijkl}$ (using Eqs. 17 & 23), we require a Green function solution for a parallel flow. That problem is relatively straightforward; for example, the method used by Afsar *et al*[19] is quite convenient for a CFD based mean flow, and is used in this paper. For the turbulent field tensor $\hat{R}_{ij,kl}$, on other hand, we must resort to simple modeling.

III. Instantaneous Isotropy: model $T'_{ij}(\mathbf{y},\tau)$

We restrict our attention to the kinematic condition of isotropy that is interpreted in two ways. First, we consider the *instantaneous field* $(\mathbf{y},\tau)$, and model the stationary random function $T'_{ij}(\mathbf{y},\tau)$. Second, we consider the *statistical field*, where we have already averaged over time, and model, therefore, the statistical function $R_{ij,kl}(\mathbf{y},\eta,\tau_0)$. In this section we assess the first of these approaches—model $T'_{ij}(\mathbf{y},\tau)$.

A. Definition

If we suppose the stationary random function $T'_{ij}(\mathbf{y},\tau)$ is isotropic for all values of time τ, then:

$$T'_{ij}(\mathbf{y},\tau) = \delta_{ij} Q_1(\mathbf{y},\tau) \tag{26}$$

Notice that under this definition, $Q_1 = T'_{11} = T'_{22} = T'_{33}$, and $T'_{12} = T'_{13} = T'_{23} = 0$. If we substitute instantaneous isotropy into Eq. (20) for $\hat{R}_{ij,kl}(\mathbf{y},\eta,\omega)$ we get:

$$\begin{aligned} \hat{R}_{ij,kl}(\mathbf{y},\eta,\omega) &= \delta_{ij}\delta_{kl} \int_{-\infty}^{+\infty} \overline{T'_{11}(\mathbf{y},\tau)T'_{11}(\mathbf{y}+\eta,\tau+\tau_0)} e^{-i\omega\tau_0}\, d\tau_0 \\ &= \delta_{ij}\delta_{kl} \int_{-\infty}^{+\infty} R_{11,11}(\mathbf{y},\eta,\tau_0) e^{-i\omega\tau_0}\, d\tau_0. \end{aligned} \tag{27}$$

B. Power spectral density

Substituting Eq. (27) into the power spectral density formula Eq. (25) gives,

$$\hat{P}(\mathbf{x},\omega) = \int_{V_\infty(\mathbf{y})} \hat{I}_{jjkk}(\mathbf{y},\omega \mid \mathbf{x}) \hat{R}^{\text{total}}_{11,11}(\mathbf{y},\omega)\, d^3\mathbf{y} \tag{28}$$

If we retain the momentum transfer term only, in Eq. (17), the power spectrum under instantaneous isotropy is simply,

$$\hat{P}(\mathbf{x},\omega) = \int_{V_\infty(\mathbf{y})} \left| \frac{\partial \hat{G}_j}{\partial y_j}(\mathbf{y},\omega \mid \mathbf{x}) \right|^2 \hat{R}^{\text{total}}_{11,11}(\mathbf{y},\omega)\, d^3\mathbf{y}. \tag{29}$$

A formula like this appeared in Morris and Farrasat[10] and Afsar *et al.*[19]

C. $R_{11,11}(\mathbf{y},\eta,\tau_0)$

In this paper we use a Gaussian model for $R_{11,11}(\mathbf{y},\eta,\tau_0)$, suggested by Tam and Aurialt.[9] The model is scaled on the local values of the turbulent kinetic energy k, the rate of energy dissipation ϵ, and the mean flow.

$$R_{11,11}(\mathbf{y},\eta,\tau_0) = \frac{4}{9}\bar{\rho}^2 A^2 k^2 \exp\left(-\frac{\mid \eta_1 \mid}{U\tau_s} - \frac{\ln 2}{l_s^2}\left[(\eta_1 - U\tau_s)^2 + \eta_2^2 + \eta_3^2 \right] \right) \tag{30}$$

The statistical quantities that represent a characteristic length scale, l_s, and time scale, τ_s, of the function $R_{11,11}(\mathbf{y},\eta,\tau_0)$ are defined by

$$l_s = c_l \frac{k^{\frac{3}{2}}}{\epsilon} \quad \text{and} \quad \tau_s = c_\tau \frac{k}{\epsilon}. \tag{31}$$

The characteristic scales are multiplied by empirical constants c_l and c_τ. The constant, A, multiplies the amplitude of $R_{11,11}(\mathbf{y},\eta,\tau_0)$. All of these constants, (c_l, c_τ, A), are chosen so that the predicted 90^o spectrum is close enough to the experimental data. To use this model in our power spectral density formula, however, we have to first take the Fourier transform, and then integrate in η. Those steps are quite straightforward, and have been spelled out before,[9] we just the final result:

$$\begin{aligned} \hat{R}^{\text{total}}_{11,11}(\mathbf{y},\omega) &= \int_{V_\infty(\eta)} \hat{R}_{11,11}(\mathbf{y},\eta,\omega) e^{ik_o\eta_1 cos\theta}\, d^3\eta \\ &= \frac{4}{9}\bar{\rho}^2 A^2 k^2 2 \left(\frac{\pi}{\ln 2}\right)^{\frac{3}{2}} l_s^3 \tau_s exp\left(-\frac{\omega^2 l_s^2}{4U^2 \ln 2} \right) \frac{1}{1+\omega^2\tau_s^2\left(1-\frac{U}{c_\infty}\cos\theta\right)^2}. \end{aligned} \tag{32}$$

D. Predicted noise spectra

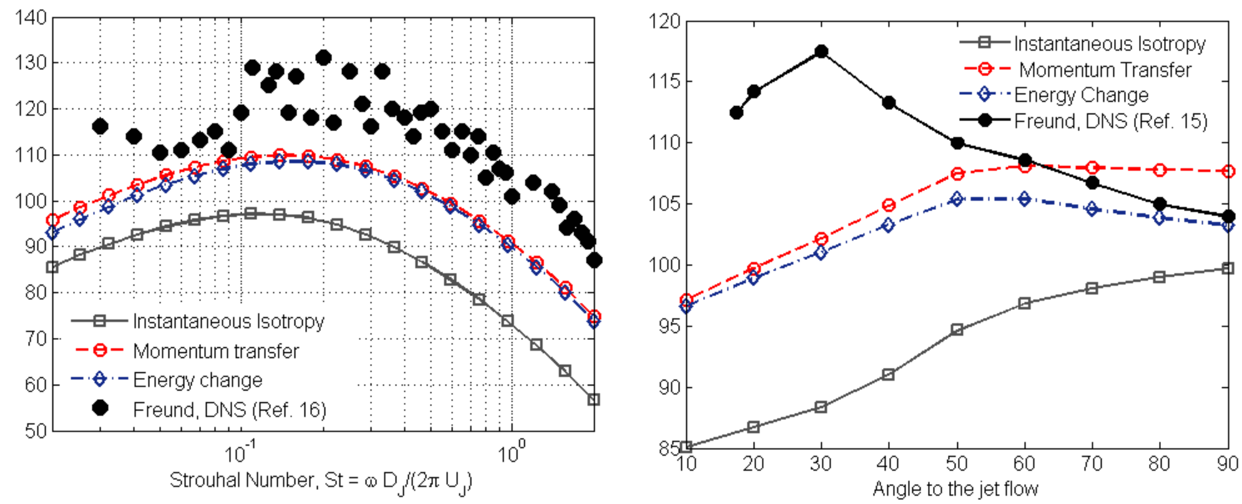
$\hat{R}_{11,11}(\mathbf{y}, \eta, \omega)$ is defined using a $k-\epsilon$ RANS calculation of the Stromberg[13] jet. The Reynolds number is 3600 and the Mach number is 0.9. The Green function is based on the RANS mean flow at 6 jet diameters downstream of the nozzle exit. At this axial location, the kinetic energy from the RANS solution is maximum.

We can now analyze the noise due to momentum transfer and energy change separately. In Figs. (3a) we show the thirty degree prediction, and in Fig. (3b) we show the overall sound pressure level (OSPL).

E. Conclusions

1. Momentum transfer is biggest part of the spectrum at large observation angles, outside of the 'zone of silence'.

2. Instantaneous isotropy with momentum transfer only, gives reasonable predictions compared to the DNS data, at large observation angles outside the 'zone of silence'.



(a) Thirty degree spectrum

(b) Directivity: Overall sound pressure level

Figure 3. Instantaneous isotropy. $SPL = 10\log(4\pi\mathbf{P}(\mathbf{x},\omega)/p_{ref}^2(D_j/U_j))$, **$D_j$ is the nozzle exit diameter; U_j is the nozzle exit velocity and p_{ref}^2 is the reference pressure for Stromberg's jet.[13]** $|\,\mathbf{x}\,| = 30D_j$. **Comparisons are made against the DNS data.[14,15] The coefficients in Eq. (32),** (A, c_l, c_τ), **are** $(0.125, 0.5, 1.0)$.

IV. Statistical Isotropy: model $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$

Now we turn our attention to the statistical field by representing the cross power spectral density of fluctuating Reynolds stress, $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$, directly.

A. Definition

If we suppose the statistical function $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$ is isotropic and symmetric in all its tensor indices, then:

$$R_{ij,kl}(\mathbf{y}, \eta, \tau_0) = (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\, F(\mathbf{y}, \eta, \tau_0) \tag{33}$$

The scalar function, $F(\mathbf{y}, \eta, \tau_0) = \frac{1}{3}R_{11,11}(\mathbf{y}, \eta, \tau_0)$. Under statistical isotropy the Fourier transform of $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$ is:

$$\hat{R}_{ij,kl}(\mathbf{y}, \eta, \omega) = (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\, \frac{1}{3}\int_{-\infty}^{+\infty} R_{11,11}(\mathbf{y}, \eta, \tau_0) e^{-i\omega\tau_0}\, d\tau_0. \tag{34}$$

B. Power spectral density

Substituting Eq. (35) into the power spectral density formula Eq. (25) gives,

$$\hat{P}(\mathbf{x},\omega) = \frac{1}{3} \int\limits_{V_\infty(\mathbf{y})} \left(\hat{I}_{jjkk}(\mathbf{y},\omega \mid \mathbf{x}) + \hat{I}_{jkjk}(\mathbf{y},\omega \mid \mathbf{x}) + \hat{I}_{jkkj}(\mathbf{y},\omega \mid \mathbf{x}) \right) \hat{R}^{\text{total}}_{11,11}(\mathbf{y},\omega)\, d^3\mathbf{y} \tag{35}$$

The quadratic forms, $(\hat{I}_{jjkk}, \hat{I}_{jkjk}, \hat{I}_{jkkj})$, can easily be found using Eqs. (17) and (23). We note here the biggest contribution to the power spectrum will come from the Hermitian quadratic form $\hat{I}_{jkjk}$.

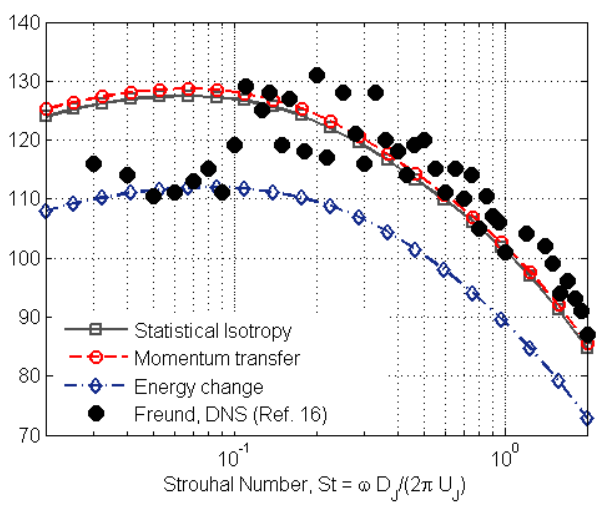
1. *Momentum transfer and Energy change*

In the second rank propagation tensor $\hat{I}_{ij}$, momentum transfer is given by term I. The part of the tensor that corresponds to energy change is the sum of three terms: IIa, IIb and IIc.

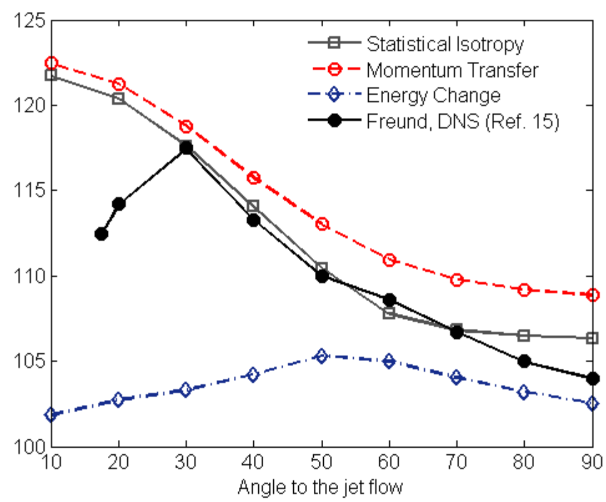
$$\hat{I}_{ij}(\mathbf{y},\omega \mid \mathbf{x}) = \underbrace{\frac{\partial \hat{G}_j}{\partial y_i}(\mathbf{y},\omega \mid \mathbf{x})}_{\text{Momentum transfer: term I}} - \underbrace{\frac{\partial \tilde{v}_j}{\partial y_i}(\mathbf{y})\hat{G}_4(\mathbf{y},\omega \mid \mathbf{x})}_{\text{Energy change: term IIa}} - \underbrace{\tilde{v}_j(\mathbf{y})\frac{\partial \hat{G}_4}{\partial y_i}(\mathbf{y},\omega \mid \mathbf{x})}_{\text{Energy change: term IIb}} + \underbrace{\frac{\delta_{ij}}{2} i\omega \left(1 + \frac{\tilde{v}_k}{i\omega}\frac{\partial}{\partial y_k}\right) \hat{G}_4(\mathbf{y},\omega \mid \mathbf{x})}_{\text{Energy change: term IIc}}. \tag{36}$$

C. Predicted noise spectra

The function $R_{11,11}(\mathbf{y},\eta,\tau_0)$ is given by Eq. (31), and the wave propagation (the three quadratic forms) is based upon a mean flow at 6 D_j downstream of the nozzle exit. In Fig. (4) we assess the contribution momentum transfer, and energy change, play to the total power spectrum. Notice the predictions are very good, and particularly good in the 'zone of silence'.



(a) Thirty degree spectrum: Sound pressure level, SPL (dB)



(b) Directivity: Overall sound pressure level, OSPL (dB)

Figure 4. Statistical isotropy. $SPL = 10\log(4\pi\mathbf{P}(\mathbf{x},\omega)/p^2_{ref}(D_j/U_j))$, **$D_j$ is the nozzle exit diameter; U_j is the nozzle exit velocity and p^2_{ref} is the reference pressure for Stromberg's jet.[13]** $\mid \mathbf{x} \mid = 30D_j$. **Comparisons are made against the DNS data.[14,15] The coefficients in Eq. (32),** (A, c_l, c_τ), **are** $(0.144, 0.5, 1.0)$.

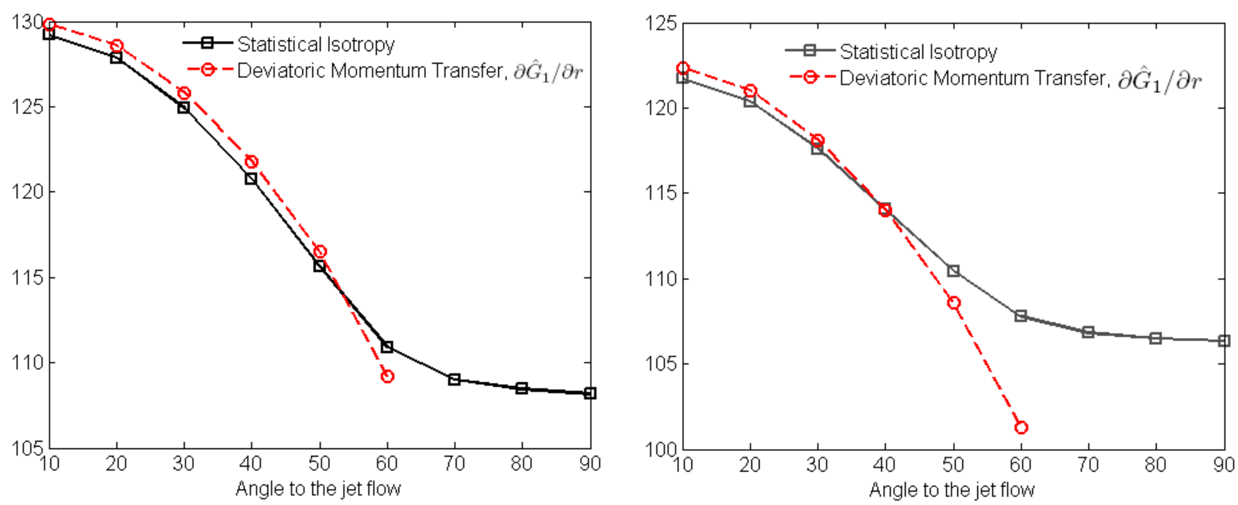
D. Conclusions

The condition of statistical isotropy has revealed some startling findings:

1. For the thirty degree spectrum, the peak noise occurs at a lower frequency (compare Figs. 3a and 4a).
2. The overall sound pressure level is in reasonable agreement with the DNS data, at large observation angles *and* at smaller angles.
3. Momentum transfer is biggest part of the spectrum, especially at small observation angles, in the 'zone of silence'.
4. The noise due to energy change is non-negligible outside of the 'zone of silence', near 90^o.

E. Momentum transfer term, $\partial\hat{G}_1/\partial r$

We have seen that most of the predicted noise in the 'zone of silence' is due to the transfer of momentum. In fact, it is *solely* due to the transfer of momentum by the component $\partial\hat{G}_1/\partial r$. In Fig. (5) we plot the noise due to this component, and compare it to the total noise predicted under the condition of statistical isotropy. The comparison is made for a parallel shear layer Green function based upon a mean flow at 1 D_j (Fig. 5a), and 6 D_j (Fig. 5b) downstream of the nozzle exit . In both cases, whether the Green function is based upon a mean flow that is "plug flow-like", or "Gaussian-like," the noise in the 'zone of silence' (under statistical isotropy) is *completely* due to the component $\partial\hat{G}_1/\partial r$. Indeed, this does show the predicted noise in the 'zone of silence' will depend on the mean flow profile, if a parallel shear layer is used to represent an evolving jet.[20] Notice also that none of the admitted noise sources under an assumption of instantaneous isotropy involves this propagation term, $\partial\hat{G}_1/\partial r$.



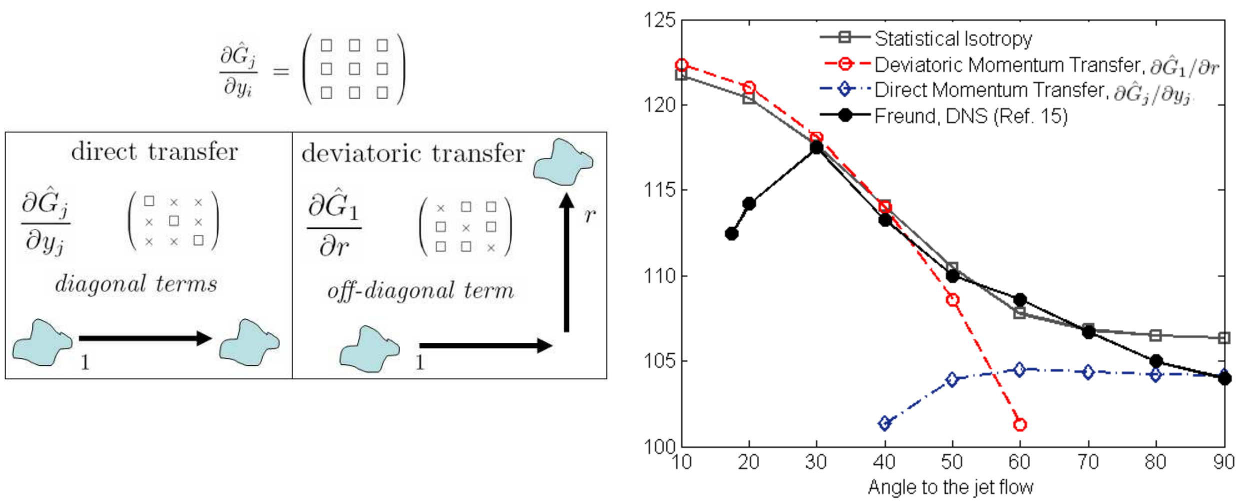
(a) Green function based on: mean flow at 1 D_j

(b) Green function based on: mean flow at 6 D_j

Figure 5. Noise in the 'zone of silence'. Overall sound pressure level (dB), with the same legend as Figure 4.

V. Discussion: two dominant terms in the jet noise spectrum

We have shown it is possible to predict the noise in the 'zone of silence' by supposing, for example, $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$ is isotropic, and symmetric, in all of its indices. The noise in the 'zone of silence' results *entirely* from the transfer of momentum by the off-diagonal term, $\partial \hat{G}_1/\partial r$. Outside of the 'zone of silence' the noise generated by other momentum transfer terms, like the trace, $\partial \hat{G}_j/\partial y_j$, begin to dominate.



(a) Direct and Deviatoric Momentum Transfer

(b) Dominant terms in Statistical Isotropy

Figure 6. Dominant terms in the noise spectrum. Overall sound pressure level (dB), with the same legend as Figure 4.

The behavior of the dominant terms in the noise spectrum, like $\partial \hat{G}_1/\partial r$ and $\partial \hat{G}_j/\partial y_j$, is given by the momentum transfer tensor, Eq. (17). That is,

1. Inside of the 'zone of silence': *Off-diagonal term in the tensor: deviatoric momentum transfer*

$$\begin{aligned} \frac{\partial \hat{G}_1}{\partial r}(\mathbf{y}, \omega \mid \mathbf{x}) \quad & \propto \frac{1}{(1 - M(r)\cos\theta)^2} \frac{\cos\theta}{k_o} \left[\frac{\partial}{\partial r} - \frac{2\cos\theta}{(1 - M(r)\cos\theta)} \frac{dM}{dr}(r) \right] \frac{D_1 \hat{G}_4}{D\tau}(\mathbf{y}, \omega \mid \mathbf{x}) \\ & \sim \frac{2k_o \cos^2\theta}{(1 - M(r)\cos\theta)^3} \frac{dM}{dr}(r) \\ & \text{remains } \; O(\omega) \qquad\qquad \text{(At very low frequency, } k_o \to 0.) \end{aligned} \tag{37}$$

2. Outside of the 'zone of silence': *Diagonal terms in the tensor: direct momentum transfer*

$$\begin{aligned} \frac{\partial \hat{G}_j}{\partial y_j}(\mathbf{y}, \omega \mid \mathbf{x}) \quad & \propto \frac{D_1 \hat{G}_4}{D\tau}(\mathbf{y}, \omega \mid \mathbf{x}) \\ & \text{remains } \; O(\omega^2). \qquad\qquad \text{(At very low frequency, } k_o \to 0.) \end{aligned} \tag{38}$$

If one supposes $R_{ij,kl}(\mathbf{y}, \eta, \tau_0)$ remains isotropic, in the limit of very low frequency the noise spectrum is proportional to the mean flow dependent quantity, $\omega^2 (dU/dr)^2 \cos^2\theta$. This is because, when the frequency is very low, any term in the noise spectrum that is multiplied by $\omega^2 (dU/dr)^2$ (Eq. 37) will dominate over one that is multiplied by ω^4 (Eq. 38) . As Goldstein[7] remarked in his article, "*it is the velocity gradient that acts as a sounding board to increase the efficiency of the source term. Thereby producing a low frequency* *generator with exceptional acoustic efficiency*". In Fig. (6) we show that the condition of statistical isotropy naturally recovers this behavior. Inside the 'zone of silence' the spectrum closely matches the noise due to the deviatoric momentum transfer component, Eq. (37); and outside of the zone the higher frequency terms in the momentum transfer tensor dominate.

VI. Conclusion

The noise spectrum from a cold jet flow can be thought of as being composed of two terms. One of these terms dominates at high frequency, while the other is significant at very low frequency and is multiplied by $\omega^2(dU/dr)^2$ (where dU/dr is the local mean flow gradient). This low frequency term is biggest at small observation angles to the jet flow, in what is termed the 'zone of silence'. It was first shown to exist by Goldstein.[7]

The noise in the 'zone of silence' can be predicted if the cross power spectral density ($R_{ij,kl}$) of the Reynolds stress source term (T'_{ij}) is taken to be isotropic and symmetric in all of the tensor indices. By supposing this, the predicted sound remains in reasonable agreement to the DNS data at large observation angles, *and* at small angles in the 'zone of silence'. The low frequency term, that accounts for the noise in the 'zone of silence', will *not* occur if the Reynolds stress source term (T'_{ij}) itself is taken to be isotropic for all values of time.

Acknowledgments

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