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PEDESTRIAN LATERAL FOOT PLACEMENT AND LATERAL DYNAMIC INSTABILITY OF BRIDGES

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Abstract. *The most often purported mechanism causing the lateral dynamic instability of the London Millennium Footbridge is the synchronisation of footsteps to the lateral structural motion. However, evidence from full-scale measurements and treadmill tests has challenged this notion. Instead, an active control of foot placement is advocated to be the source of destabilising forces to the structure, occurring even without synchronisation. This is to say that, while walking on a laterally oscillating surface, pedestrians maintain their balance primarily by controlling the position of their feet, rather than adjusting the timing. Similar behaviour was previously observed in experimental tests measuring the response of pedestrians to an impulsive perturbation of gait. The analysis of the collected data suggested a simple linear foot placement control law, whereby the position of the foot at the instant of foot placement immediately following the perturbation depends on the instantaneous lateral velocity of the centre of mass and a constant offset. However, it is has been uncertain whether the same foot placement control law applies while walking on laterally oscillating structures. To test this proposition, an experimental campaign was conducted on a laterally oscillating treadmill with a test subject monitored with an optical motion capture system. The motion of the body centre of mass and the position of the feet were identified and analysed. It was found that a simple linear foot placement control law applies. Further tests were conducted to test the influence of the visual information on pedestrian stepping behaviour using virtual reality delivered via a head mounted display. It was found that the identified foot placement control law is very robust for different walking surface conditions and visual environments.*

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1 INTRODUCTION

Lateral dynamic instability under the loading from a walking crowd has been observed on many bridges around the world [1-8]. Despite intensive research in the last two decades to uncover the nature of this phenomenon, a consensus has not yet been reached. The paradigm still dominating in the field is that, in the presence of perceptible bridge motion, pedestrians adjust their stride frequency to match the vibration frequency of the bridge, hence a direct resonance will occur. This is supposedly amplified by a tendency of pedestrians to synchronise their footsteps to one another. However, some measurements from full-scale structures [5, 6] and laboratory environments [9, 10] have challenged the notion of prevalence of the direct resonance, while the natural tendency of pedestrians to synchronise their footsteps with other pedestrians has been found to be quite weak [11, 12]. Meanwhile, it has been shown that destabilising forces to the bridge can arise from simple mechanics of walking, whereby pedestrians adjust the lateral position of their stepping foot in response to perturbations from the oscillating bridge, even in the absence of synchronisation [13]. This analysis was accomplished by employing an inverted pendulum model (IPM) to idealise the dynamics of pedestrian motion in the frontal plane, i.e. the vertical plane perpendicular to the direction of progression. The IPM consists of a mass supported by a rigid massless leg. The transition between steps is instantaneous and the position of foot placement for the stepping leg, u, is assumed to follow the law proposed by Hof *et al.* [14]:

$$u = x_{\mathrm{CoP}} - x_{\mathrm{CoM}} = \frac{\dot{x}_{\mathrm{CoM}}}{\Omega_p} \pm b_{\min} \tag{1}$$

where x_{CoP} and x_{CoM} is the lateral distance, measured from the same arbitrary reference point on the surface, to the centre of pressure (CoP) and the vertical projection of the centre of mass (CoM), respectively, a dot over the symbol represents differentiation with respect to time, $\Omega_p = \sqrt{g/l_{\mathrm{eq}}}$ is the angular pendulum frequency in which g is the gravitational acceleration and l_{eq} is the equivalent pendulum length, the ± sign accounts for the bipedal nature of human gait (i.e. changing point of support from step to step) and $b_{\min}$ is a constant termed the margin of stability. The IPM based on Eq. (1) has been shown to be capable of explaining the forces generated by a pedestrian on a laterally oscillating bridge [13, 15], with good qualitative agreement with experiments involving unimodal [16, 17] and multi-modal [18] bridge motion. However, little direct experimental evidence has been presented to date to specifically support the adopted foot placement control law [19, 20]. Moreover, it is currently unclear whether $\dot{x}_{\mathrm{CoM}}$ in Eq. (1) should be taken relative to the moving bridge, $\dot{x}_{\mathrm{CoM}}^{\mathrm{rel}}$, or some stationary point outside of the bridge relative to which the bridge is moving, $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ [9].

Considering the gaps in the current state of knowledge, the aim of this study is to verify whether a foot placement control law of the type proposed by Hof *et al.* [14] applies for walking on an oscillating bridge. To this end, tests were conducted on a self-paced treadmill capable of lateral oscillation. Gait kinematics of a test subjects were recorded using an optical motion capture system to obtain an instantaneous position of the body centre of mass (CoM) and feet. Motivated by the form of the Hof *et al.*'s [14] foot placement control law in Eq. (1), a simple linear regression model was fitted to the data relating the step width and lateral velocity of the CoM at the instant of foot placement. To tests the robustness of the pedestrian stepping behaviour, three types of visual environment were provided to the test subject.

2 METHODOLOGY

A 21 year-old healthy female with body mass of 51kg and height 1.64m was recruited for the tests. The test subject wore gym-type clothing and flat-sole shoes. Prior to participating in the tests, the subject familiarised herself with the information letter for participants, completed a physical readiness questionnaire and signed an informed consent form. The tests were approved by the University of Bristol Ethics of Research Committee.

Six tests were conducted in total. The subject walked on the treadmill when stationary (i.e. non-oscillating), denoted NLTM (no lateral treadmill motion), and on the treadmill with it oscillating in the lateral direction with sinusoidal motion of amplitude 10mm and frequency 0.9Hz, denoted LTM (lateral treadmill motion). The test subject was subjected to three visual conditions, including that of the laboratory environment, denoted RL (real life), and two virtual reality environments delivered via a head mounted display, denoted VR1 and VR2. The details of the instrumentation used during the tests and data analysis are given in this section.

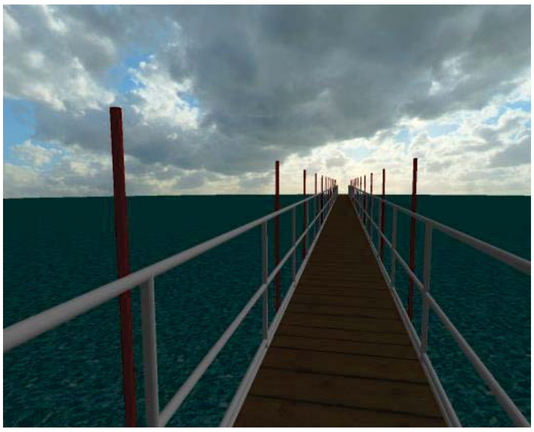
2.1 Treadmill

A custom-built treadmill mounted on a shaking table capable of providing lateral motion was used during the tests. A generous walking area of 2m by 1.5m was provided to facilitate the operation of a treadmill speed feedback control mechanism, enabling automatic adjustment of the treadmill speed to that of the user, and to facilitate adaptive pedestrian stepping behaviour.

2.2 Virtual reality

An immersive and interactive virtual reality (VR) environment was delivered via an nVisor SX111 head mounted display, offering a 76 degrees horizontal and 64 degrees vertical field of view and 1280 by 1024 resolution per eye. The VR environment was developed in NetBeans 7.01 IDE using the Jogl platform providing 3D graphics based on the OpenGL® API for applications written in Java™. It consisted of three main elements, (i) the bridge, (ii) the substratum, i.e. stationary objects rendered below the bridge, providing height reference, and (iii) the superstratum, i.e. objects rendered around the bridge, above the substratum. Two VR scenes were built, differing in terms of the number and type of visual reference cues available to the user. In both scenes, the virtual bridge was translating together with the treadmill belt thus providing optical flow as for normal overground walking, i.e. motion of the objects perceived due to self-motion – here realized by walking on a treadmill. In both scenes, the lateral movement of the virtual bridge was enabled by coupling it with the lateral treadmill motion. The first scene, hereafter referred to as VR1, presented in Figure 1 (a), contained visual information enabling motion parallax, i.e. the perception of relative motion of objects against the rest of the visual field caused by self-motion of the observer. This was achieved by inclusion of stationary objects set in the scene, i.e. the substratum (water beneath the bridge) and red posts positioned alongside the bridge, and also bridge railings moving with the bridge and the deck being rendered in a pattern resembling wood. The second scene, hereafter referred to as VR2, presented in Figure 1 (b), was built by stripping VR1 of most of the information facilitating the perception of self-motion to the point that the only object left in the scene was the deck, rendered in a low contrast pattern, surrounded by dense fog.

(a) (b)



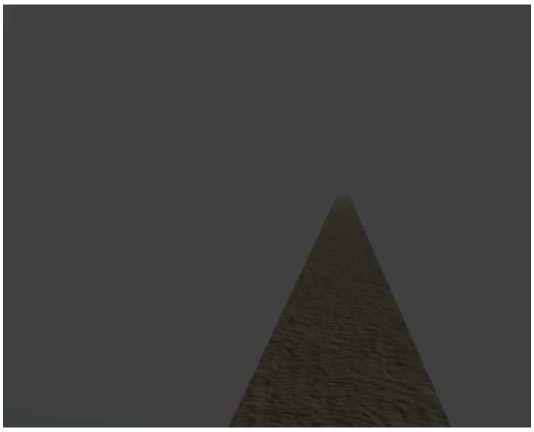


Figure 1: Virtual reality environments projected onto the left-eye screen of head mounted display, (a) VR1, (b) VR2.

2.3 Data capture and analysis

An optical motion capture system consisting of six Qualisys cameras was used to record, at the sampling frequency of 128Hz, the motion of fourteen retroreflective markers attached to the body landmarks of the test subject. This enabled the position of the CoM to be determined using the linked-segments modelling procedure [21]. One marker was attached to the lateral malleolus (i.e. ankle) of each foot to determine the pseudo step width, defined here as the lateral distance between the projections of the position of the CoM and the ankle marker onto the surface at the instant of foot placement. The term *pseudo step width* is used herein, since in the original formulation of the IPM [14] the step width is defined using the centre of pressure (CoP) and the transfer from one leg to the other is instantaneous. However, the CoP was not measured during the tests and the double-support phase of gait is not negligibly short. Therefore, it was assumed herein that (i) the ankle marker, due to anatomical constraints, gives a representative point of reference for the step width in the spatial reference frame, and that (ii) the instant of foot placement gives a representative point of reference for the step width in the temporal reference frame. That instant was determined from the vertical velocity of the ankle markers according to the procedure proposed in [22].

The following function was fitted to the pseudo step width data, compatible with the IPM and lateral foot placement control law in Eq. (1):

$$u^* = x_{\text{ANKLE}} - x_{\text{CoM}} = \rho_1 \dot{x}_{\text{CoM}} \pm \rho_2 \tag{2}$$

where u^* is the pseudo step width, x_{ANKLE} is the lateral position of the ankle marker attached to the stepping foot, $\dot{x}_{\text{CoM}}$ is the lateral velocity of CoM at the instant of heel strike, and ρ_1 and ρ_2 are the coefficients of the fit. Note ρ_1 and ρ_2 in Eq. (2) correspond to the inverse of the IPM angular frequency, Ω_p, and the margin of stability, $b_{\min}$, respectively, from Eq. (1), and have physical units of time and displacement, respectively. The outstanding problem is the definition of the lateral velocity of the CoM, $\dot{x}_{\text{CoM}}$, for LTM tests. Note this ambiguity does not exist for NLTM tests, since $\dot{x}^{\text{rel}}_{\text{CoM}}$ is then equal to $\dot{x}^{\text{abs}}_{\text{CoM}}$. This issue will be taken into consideration in the discussion of the results in Section 3.

The displacement signal of the CoM was differentiated to obtain lateral the CoM velocities $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ and $\dot{x}_{\mathrm{CoM}}^{\mathrm{rel}}$. A fourth-order two-way Butterworth low-pass filter with cut off frequency of 6Hz was applied to all the data.

3 RESULTS AND DISCUSSION

The empirical data relating pseudo step width, u^*, with the absolute lateral velocity of the CoM, $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$, for all the conducted tests are shown in Figure 2.

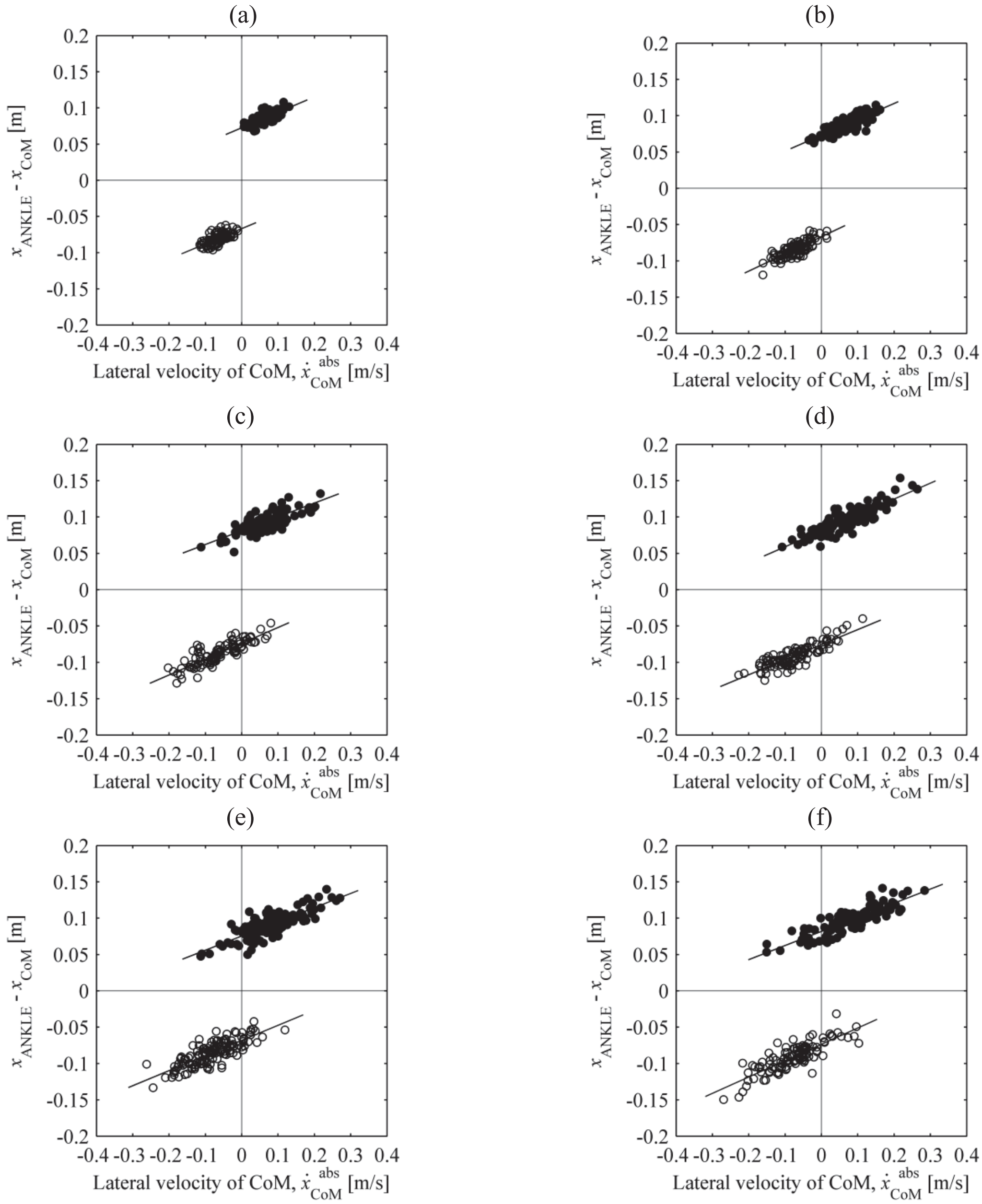


Figure 2: Relationship between the pseudo step width and the absolute lateral velocity of the CoM at the instant of heel strike for (a) RL NLTM, (b) RL LTM, (c) VR1 NLTM, (d) VR1 LTM, (e) VR2 NLTM and (f) VR2 LTM. The data for the right and left leg are denoted by ● and ○ in the top and bottom parts of the plots, respectively.

The least spread of data can be seen in Figure 2 (a) & (b) for walking in the visual environment of the laboratory (RL), without and with lateral treadmill oscillation (NLTM & LTM), respectively. The greatest spread of data can be seen in Figure 2 (e) & (f) for walking in VR2 in NLTM & LTM, respectively. Despite the differences in data variability, the gradients and intercepts (with the vertical axes) of the best linear fits, found separately for the right and left leg data, seem to be compatible.

During all the tests, except for NLTM RL, some steps were taken in which the CoM was travelling towards the stance leg in the frontal plane at the instant of touchdown of the stepping leg. These steps are shown as points in Figure 2 for which $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ is negative for the right leg or positive for the left leg. No excursion of the foot to the contralateral (i.e. abnormal) side of the CoM can be directly observed from the data in Figure 2. This would be evident if u^* was negative for the right leg or positive for the left leg. However, since the ankle markers were not aligned with the CoP, it is possible that this situation occurred for some steps, when the magnitude of u^* (on the vertical axes) was below 0.05m.

The results of fitting a linear function (i.e. simple linear regression) to the experimental data according to Eq. (2), based on the absolute and relative lateral velocity of the CoM, $\dot{x}_{\mathrm{CoM}}$, are given in Table 1. Note ρ_2 takes positive and negative values depending on the stepping leg. Goodness of fit statistics are quantified in terms of the root mean square (RMS) error, denoted as standard error in Table 1, and degrees-of-freedom adjusted R-square, denoted as adjusted R-square in Table 1.

Experimental condition	$\dot{x}_{\mathrm{CoM}}$ [m/s]	Figure 2 reference	Leg	ρ_1 [s]	ρ_2 [m]	Standard error [m]	Adjusted R-square
RL NLTM	abs	(a)	right	0.2071	0.0787	0.0052	0.5381
RL NLTM	abs	(a)	left	0.2003	–0.0748	0.0055	0.4341
RL LTM	abs	(b)	right	0.1985	0.0796	0.0064	0.6654
RL LTM	abs	(b)	left	0.2229	–0.0742	0.0059	0.6768
RL LTM	rel	N/A	right	0.1065	0.0858	0.0096	0.2386
RL LTM	rel	N/A	left	0.0964	–0.0828	0.0091	0.2467
VR1 NLTM	abs	(c)	right	0.1874	0.0824	0.0086	0.6532
VR1 NLTM	abs	(c)	left	0.2217	–0.0746	0.0080	0.7448
VR1 LTM	abs	(d)	right	0.2144	0.0819	0.0090	0.7444
VR1 LTM	abs	(d)	left	0.2044	–0.0762	0.0081	0.7339
VR1 LTM	rel	N/A	right	0.1659	0.0854	0.0139	0.3903
VR1 LTM	rel	N/A	left	0.1252	–0.0824	0.0125	0.3649
VR2 NLTM	abs	(e)	right	0.1895	0.0811	0.0094	0.6525
VR2 NLTM	abs	(e)	left	0.2058	–0.0728	0.0087	0.7044
VR2 LTM	abs	(f)	right	0.1893	0.0814	0.0099	0.7217
VR2 LTM	abs	(f)	left	0.2215	–0.0729	0.0111	0.6931
VR2 LTM	rel	N/A	right	0.1564	0.0771	0.0138	0.4955
VR2 LTM	rel	N/A	left	0.1646	–0.0692	0.0153	0.4355

Table 1: Results of fitting a linear function to experimental data.

A simple linear regression based on the absolute velocity of the CoM, $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$, provides overall better fit to the data than using the relative velocity of the CoM, $\dot{x}_{\mathrm{CoM}}^{\mathrm{rel}}$. The mean adjusted R-square obtained using $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ is 0.71 for LTM. In comparison, the mean adjusted R-square obtained using $\dot{x}_{\mathrm{CoM}}^{\mathrm{rel}}$ for LTM is only 0.36. The values of adjusted R-square obtained using $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ show that a linear foot placement control law can explain 71% of pseudo step width variance for LTM. In the world of biology, and considering the data come from testing a human, this indicates a strong functional relationship between the variables. Therefore, all further discussion is based on the results obtained using $\dot{x}_{\mathrm{CoM}}^{\mathrm{abs}}$ only.

The mean gradients of the fit, ρ_1, for the right and left legs are 0.2s and 0.21s, respectively. The mean intercepts of the fit, ρ_2, for the right and left legs are 0.08m and –0.07m, respectively. The difference in sign reflects the bipedal nature of human gait. The values are consistent throughout all the tests, except RL NLTM, indicating a simple linear foot placement control law is robust against surface oscillation and modifications to the visual field.

The higher variability of data for VR tests visible in Figure 2 is reflected in the higher values of the standard error in Table 1, quantifying the dispersion of the pseudo step width around the fit. This is the case for NLTM and LTM tests. The highest variability in data is observed for VR2. The test subject was then provided the least amount of visual information which, it appears, led to greater variation of the gait.

The values of ρ_1 and ρ_2 differ from the values previously assumed in the use of the IPM [13-15], as defined in Eq. (1). The gradient of the linear foot placement control law is in this case the inverse of the pendulum angular frequency, which for the test subject employed for the test would be 0.34s. This is greater than all of the values from the experiments in Table 1. The intercept is the margin of stability, which for unperturbed walking was assumed to take a value of 0.016m [13-15]. These discrepancies were expected considering the definition of the pseudo step width, stated in Section 2, in particular the difference in the spatial and temporal reference frames used in the formulation of the IPM compared with those used in the experiments.

Overall, it is concluded that the foot placement control law of the type proposed by Hof *et al.* [14], based on lateral velocity of the CoM at heel strike, applies for walking on laterally oscillating bridges. This gives supporting evidence for the assumptions made in the IPM [13,15,19]. Further work will attempt to reconcile the discrepancies between IPM predictions and the experimental data presented in this study.

4 CONCLUSIONS

- A foot placement control law of the form proposed by Hof *et al.* [14] applies for walking on laterally oscillating structures.
- The step width depends on the absolute rather than relative lateral velocity of the centre of mass at the instant of foot placement onto the walking surface.
- The foot placement control law is robust against modifications to the visual environment as well as motion of the walking surface.
- The presented data can help in the calibration of the inverted pendulum pedestrian model for walking on laterally oscillating structures.

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