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Published online: 01 May 2024.

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GWmodelS: a standalone software to train geographically weighted models

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ABSTRACT

With the recent increase in studies on spatial heterogeneity, geographically weighted (GW) models have become an essential set of local techniques, attracting a wide range of users from different domains. In this study, we demonstrate a newly developed standalone GW software, **GWmodelS** using a community-level house price data set for Wuhan, China. In detail, a number of fundamental GW models are illustrated, including GW descriptive statistics, basic and multiscale GW regression, and GW principle component analysis. Additionally, functionality in spatial data management and batch mapping are presented as essential supplementary activities for GW modeling. The software provides significant advantages in terms of a user-friendly graphical user interface, operational efficiency, and accessibility, which facilitate its usage for users from a wide range of domains.

1. Introduction

The principle of spatial heterogeneity (Goodchild 2004) suggests that geographical variables and their relationships frequently present heterogeneous or non-stationary patterns. To explore this fundamental phenomena, place-based methods and local statistical techniques that assume data relationships to be spatially variant have been extensively developed (Fotheringham and Brunsdon 1999; Goodchild and Li 2021). These include the expansion method (Casetti 1972), multilevel modeling (Duncan and Jones 2000; Jones 1991) and random coefficient modeling (Rao 1965; Swamy, Conway, and LeBlanc 1988; Swamy, Roger, and Michael 1988). Extending such approaches further, geographically weighted regression (GWR) was proposed to incorporate spatial weights into local regression model calibrations and to highlight spatial heterogeneities in data relationships (Brunsdon, Fotheringham, and Charlton 1996; Fotheringham, Charlton, and Brunsdon 1998). This reflects the principle of spatial dependence proposed by Tobler (1970) and a hypothetical "bump of influence" in which observations closer to the location being considered are assigned larger weights in the estimation of local regression coefficients (Fotheringham, Brunsdon, and Charlton 2002).

Utilising this distance-decaying schema, a set of place-based techniques, termed geographically weighted (GW) models have been developed to calculate local parameters and measures, in addition to GWR. These techniques include GW descriptive statistics (Brunsdon, Fotheringham, and Charlton 2002b; Harris, Charlton, and Fotheringham 2010), GW principal components analysis (Fotheringham, Brunsdon, and Charlton 2002; Harris, Brunsdon, and Charlton 2011), GW discriminant analysis (Brunsdon, Fotheringham, and Charlton 2007; Foley and Demšar 2012), GW visualization techniques (Dykes and Brunsdon 2007) and GW artificial neural network (Du et al. 2020; Hagenauer and Helbich 2021). These GW models form a continually evolving technical framework for identifying spatially non-stationary features or patterns in a wide range of disciplines or fields including geography (Cao et al. 2019; Nawrotzki and Bakhtsiyarava 2017; Tu et al. 2020), the social sciences (Jin, Xu, and Huang 2019; Li et al. 2009; Yu and Wu 2004), biology (Bai et al. 2022; Liu et al. 2019a; Samec et al. 2020), public health (Xu et al. 2021, 2022; Yin et al. 2018) and environment science (Liu et al. 2019b; Wang et al. 2019; Xiao et al. 2022; Zhan, Yang, and Liu 2024).

The increasing popularity of GW models, particularly GWR, has resulted in the development of several software packages, standalone programs and toolkits. These tools include **GWR3.X** (Charlton, Fotheringham, and Brunsdon 2007), **GWR4.0** (Nakaya et al. 2009), GWR tools in the modeling spatial relationships toolset of **ArcGIS Pro** (ESRI Corp 2023) and **MGWR** (Li et al. 2019) which provide standalone and user-friendly software for undertaking GWR. A range of R and Python packages have been developed to incorporate different GW models, including **spgwr** (Bivand and Yu 2020), **mgwrsar** (Geniaux and Martinetti 2018), **GWLelast** (Yoneoka, Saito, and Nakaoka 2016), **spMoran** (Murakami 2017), **gwer** (Araujo, Cysneiros, and Cysneiros 2020), **lctools** (Kalogirou 2020), **gwrr** (Wheeler 2013) and **GWmodel** (Gollini et al. 2015; Lu et al. 2014). Among these packages, **GWmodel** provides a comprehensive set of GW models (Comber et al. 2023), including GW summary statistics, GW principal components analysis, GW discriminant analysis and a variety of alternative GW regression forms and tools. The **GWmodel** package has been used in a number of workshops such as at Spatial Accuracy 2018 (https://www.spatialaccuracy2018.org/work shops) and 7th Channel Network Conference (https://biometricsociety.org.uk/events/cnc2019/courses). In the 10 years or so since its first release on CRAN (https://CRAN.R-project.org/package=GWmodel), the number of downloads of the **GWmodel** package is more than 150, 000 according to the results returned by the R package **cranlogs** (Csárdi 2019).

Despite its popularity, fundamental R programming skills are required to use the GW functionalities in **GWmodel**. This is not always an easy task for researchers, particularly for beginners or scholars in different application fields. Recently, **GWmodel** was implemented as **GWmodelS** using C++ and the cross-platform software **Qt** (Lu et al. 2022). **GWmodelS** is a new, free, user-friendly and high-performance computational framework to continually integrate different and new GW models (Lu et al. 2022). Its continuous evolution provides a stable platform for the future development of GW models together with **GWmodel** and **gwverse**, a new template for GW R packages (Comber et al. 2022). **GWmodelS** is a standalone software and incorporates spatial data management and mapping tools as well as the GW model functions. It has a user-friendly graphical user interface (GUI) making it easier to construct GW models, but no requirement for any programming skills. In the study by Lu et al. (2023), we have briefly summarized the background and development design of **GWmodelS**, but with a very simple example of applying standard GWR. For further details, in this article we will demonstrate the use of **GWmodelS** through a series of typical GW analyzes that undertake GW descriptive statistics (GW averages and correlations), GW regression (basic and multiscale GWR) and GW principal components analysis. It explains the GUI and parameter settings in detail, and provides a comprehensive illustration and tutorial for undertaking GW models with **GWmodelS**. This paper is structured as follows: In Section 2, different GW models and the training data set (community-level house price data for Wuhan (WHHP)) are introduced. Section 3 details the operations of **GWmodelS** in conducting GW analyzes, and Section 4 summarizes this study. Similar to the other well-known software for spatial analytics, **GeoDa** (Anselin, Syabri, and Kho 2010), All the analyzes presented in this study can be reproduced and used in courses or workshops with relative ease.

2. Models, software and data

2.1. GW models and GWmodelS

GW models form a branch of spatial statistics, including GWR and its extensions, GW descriptive statistics, GW principal components analysis that address the need for local place-based, local models that reflect hypothetical "bump of influence" described above. They form a generic and rapidly evolving technical framework that meet different tiers of spatial data analysis, i.e. descriptive analysis, exploratory analysis, diagnostic analysis, predictive analysis and prescriptive analysis (Calzon 2022). To facilitate the usage of such GW models, a stand-alone software tool has been developed, **GWmodelS**. It was constructed with the C++ programming language based on the geospatial data abstraction library (GDAL) (GDAL/OGR contributors 2022), **QGIS** development libraries (QGIS.org 2022a) and **Armadillo** C++ library (Sanderson and Curtin 2016). In this regard, **GWmodelS** largely inherits some core functionalities from **QGIS**, including spatial data overview and cartography tools, which greatly facilitates the operations for mutual users from this popular open GIS software.

The main interface of **GWmodelS** is shown in Figure 1. The menu bars at the top contain the project data/layer management and a number of fundamental GW tools: GW summary statistics (GWSS, Brunsdon, Fotheringham, and Charlton 2002b), GWPCA (Harris, Brunsdon, and Charlton 2011) and basic GWR (Brunsdon, Fotheringham, and Charlton 1996). It also
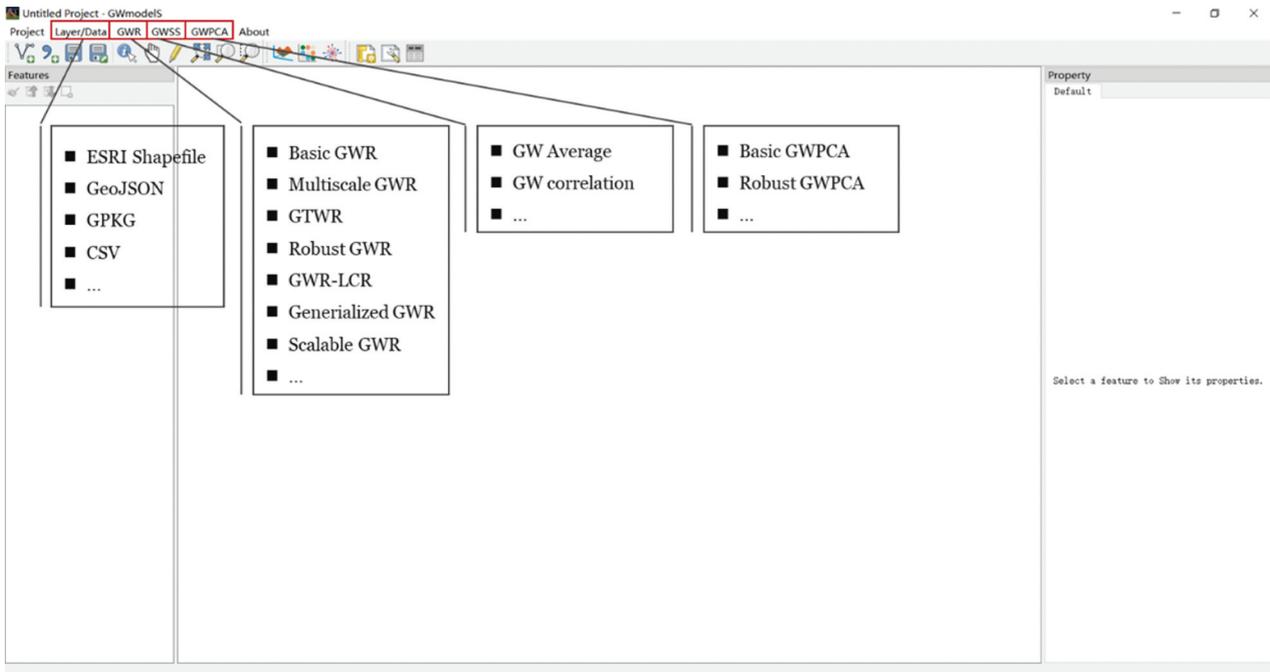
Figure 1. The main interface of GWmodelS.

includes a range of extensions applicable to large-scale data sets (Murakami et al. 2020), multiscale GWR (Lu et al. 2017, 2018, 2019), geographically and temporally weighted regression (GTWR) (Huang, Wu, and Barry 2010), robust GWR (Harris, Fotheringham, and Juggins 2010), locally compensated GWR (GWR-LCR), generalized GW regression (Nakaya et al. 2005). The plan is that this software will continue to be developed and augmented to include additional GW tools, e.g. GW interaction model (Kordi and Fotheringham 2016), GW discriminant analysis (Brunsdon, Fotheringham, and Charlton 2007) and GW artificial intelligence techniques (Hagenauer and Helbich 2021). The key idea behind **GWmodelS** is to provide a comprehensive, usable and free platform of local techniques under the GW modeling framework for researchers from a wide range of fields. Software installation and the use of tools for GWSS, GWR, and GWPCA are presented in this study with the WHHP data set.

2.2. House price data of Wuhan

As a training tool to facilitate usage of **GWmodelS**, the WHHP data set is provided. It is located in the software installation directory. WHHP describes community-level house prices in Wuhan, the capital of Hubei province in the central region of China (Lu et al. 2022). As shown in Figure 2, the boundary of the WHHP data set is central built-up area of Wuhan, where most of the houses for sale are located. The WHHP data set contains 9 variables collected in 2015: *Annual_AQI*, *Pop_Den*, *Green_Rate*, *GDP_per_Land*,

Rev_per_Land, FAI_per_Land, TertI_Rate, Den_POI. They are fully described in Table 1.

3. Usage details of GWmodelS

This section details the application of **GWmodelS** to calculate GW summary statistics, GW PCA and GWR with the WHHP data. Some data management and visualization tools are also introduced.

3.1. Spatial data management

In **GWmodelS**, several formats of spatial datasets can be directly imported, including ESRI Shapefile, GeoJSON, OGC GeoPackage (GPKG) and CSV. ESRI Shapefile is the most popular format and widely compatible with most of the GIS software or platforms with a majority of geospatial vector data provided in this format. With the increasing amount of spatial data collected from the internet, GeoJSON, a text-based format extend from the JSON format is a commonly used alternative. However, these two formats are inefficient for large-scale data sets, and so the GPKG format is included as it is more efficient in terms of storage overhead (spatial data sets are stored in SQLite databases.). Finally, CSV format is incorporated for data interchanges across a range of software and platforms.

The WHHP data is provided in two formats: GeoPackage (*. Gpkg) and ESRI Shapefile (*. Shp). As shown in Figure 3, WHHP data in the GPKG format is loaded via the menu "Layer/ Data → GPKG" and shown in the map zone
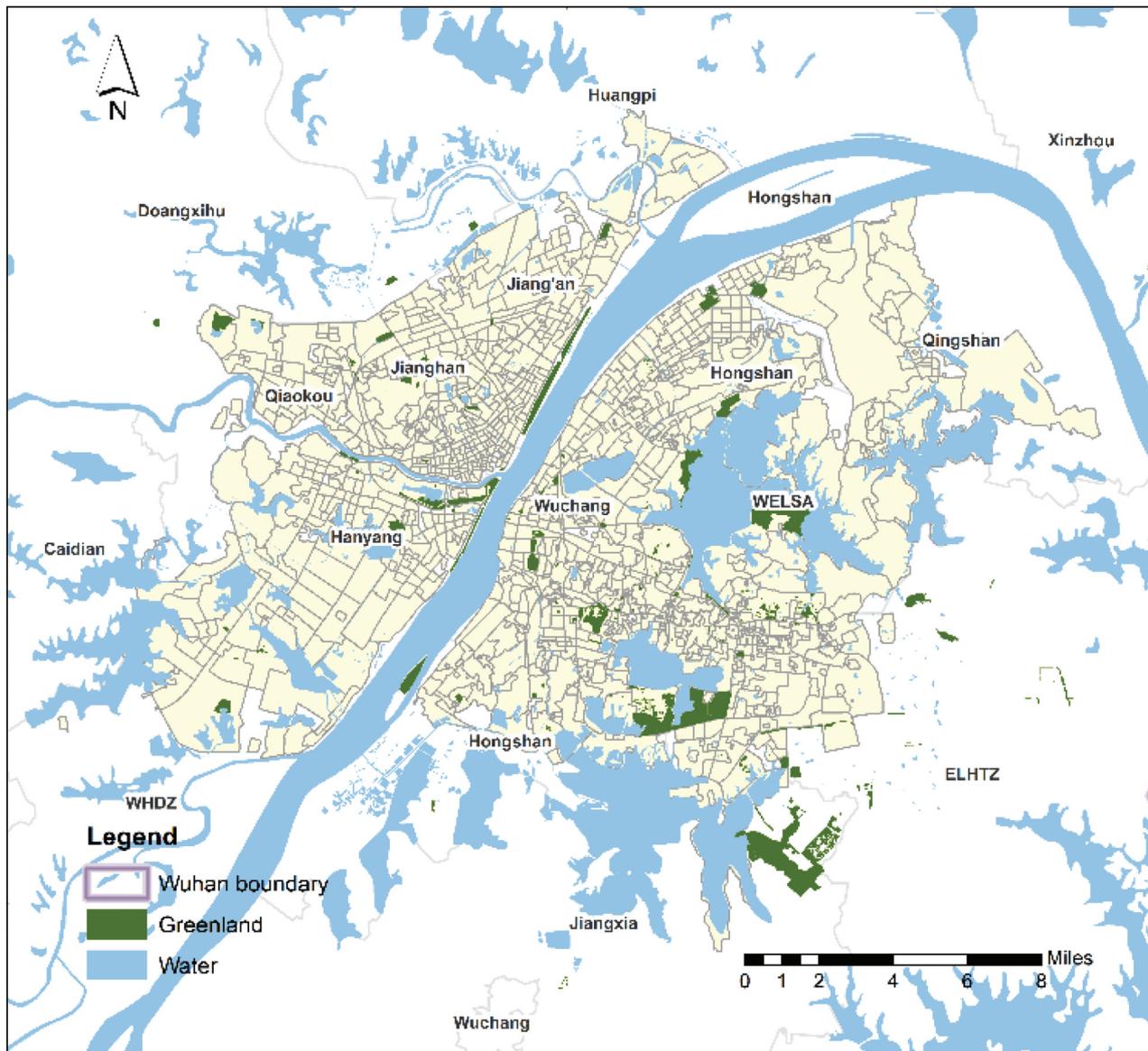
Figure 2. Boundary of the WHHP data set.

Table 1. Variable names and descriptions in the WHHP data.

Variable	Description
Avg_HP	Average house price at the community-level (Yuan/m^2)
Annual_AQI	Annual air quality index (AQI)
Pop_Den	Population density (10^5/km^2)
Green_Rate	Percentage of green space (%)
GDP_per_Land	GDP per land area (10^4 Yuan/km^2)
Rev_per_Land	Revenue per land area (10^4 Yuan/km^2)
FAI_per_Land	Fixed assets investment per land area (10^4 Yuan/km^2)
Tertl_Rate	Percentage of tertiary industry (%)
Den_POI	Density of places of interest (POI) (10^{-2}/km^2)

(shown in Figure 4). Alternatively, the ESRI Shapefile data set can be imported by clicking the menu item "Layer/Data → ESRI Shapefile".

To examine the imported data set and its attributes, the user right clicks on the layer's name (i.e. WHHP here) in the "Features" panel, and selects "Property" on the popup menu to display summary information about the layer in the property panel, including feature count, type of geometry, coordinates of bounding box, type of each field, and the spatial reference, as shown in Figure 5.
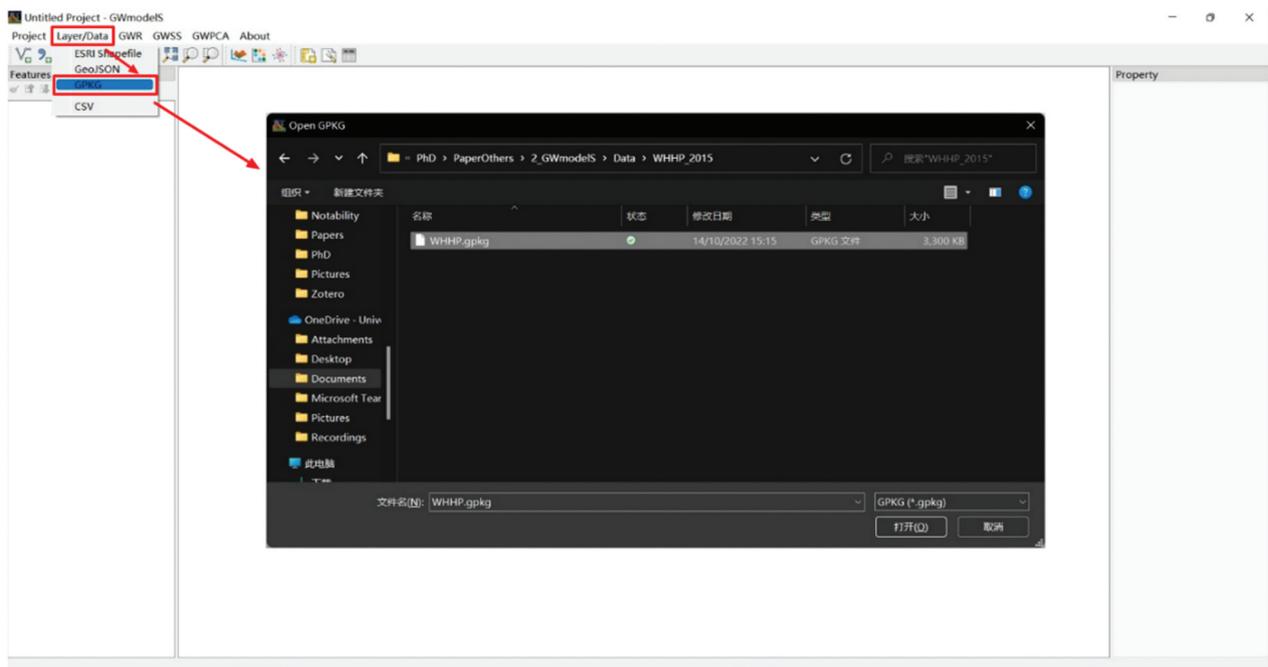
To examine the attributes, right click a layer's name and select "Attribute Table". Similar to **QGIS**, all the fields are presented in the pop-up window (as shown in Figure 6). With data manipulation tools inherited from **QGIS**, edition, pan, selection and calculation tools are also available in the top toolbar (see details in QGIS.org 2022b).

To visually explore the imported data, click the "Symbology" button in the right popup menu, and set the style for visualizing the imported layer. Symbol, color and geometric types can be set accordingly. As shown in Figure 7(a)–(d), a choropleth map of the variable, *Avg_HP* can be visualized by setting a "Graduated" blue ramp according with naturally separated values. This functionality is also essential to produce thematic maps of the results of GW models.

Table 2. Options in the GW descriptive statistics configuration windows[1].

Option	Explanation
Layer name	Data layer to be used for calculating GW averages and correlations
→ Candidate Variables	Candidate variables available in the data layer and to be selected for GW modelling by clicking the button ">>".
Weighting Scheme	Group of parameters to be set for the weighting scheme.
→ Bandwidth Type	Two types of bandwidth: 'adaptive' and 'fixed' (Lu et al. 2014), - 'fixed' is default.
→ Bandwidth Size	The Bandwidth size for calculating spatial weights, referring to a constant distance threshold for fixed bandwidth and number of nearest neighbours for the adaptive type.
→ → Auto-selection	By default, the auto-selection option is ticked, and the bandwidth will be optimized via the golden section search procedure utilising the cross-validation (CV) approach (Loader 1999); otherwise, the user-defined value will be used.
→ → User-defined	If auto-selection is unticked, a user-defined bandwidth will be used for GW model calibration, and note scaling options are also provided here, i.e. ×1, ×10, ×100, and ×1000.
→ Kernel Function	Kernel function to calibrate GW models, including Gaussian, exponential, bi-squared, tri-cube, and box-car.
Distance Metric	Parameters for calculating distance metrics
→ According to CRS	If ticked, spatial distance will be calculated according to the coordinate reference system (CRS) of the source data, i.e. Euclidean distance for a projected CRS while great circle distance for the a geographic (longitude and latitude-based) CRS.
→ Minkowski distance	If ticked, the Minkowski distance function will be used to calculate distance, and it could be expressed as follows (Lu et al. 2016): $d_{ij}(p, \theta) = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}(|\sin(\theta + \alpha)|^p + |\cos(\theta + \alpha)^p|)^{\frac{1}{p}}$, where (u_i, v_i) and (u_j, v_j) are spatial coordinates, $\alpha = \tan^{-1}\left(\frac{u_1 - u_2}{v_1 - v_2}\right)$, p is a positive constant and θ is the rotation angle of the CRS.
→ → theta	The rotation angle θ.
→ → p	The value of p for the Minkowski distance function.
→ Self-defined (*.dmat)	If this option is ticked, a self-defined distance matrix provided in a binary file (*.dmat) will be used for weight calculation. Note that this distance matrix could be calculated via GWmodelS or customized with an n ×m matrix where n is the number of observations and m is the number of calibration locations.
→ → Select a distance matrix file	File path of the self-defined distance matrix.
Computing configuration	Parameters for computational configuration
→ None	No computational configuration is specified and the GW modules will be carried out in a stand-alone mode
→ Multithreading	If ticked, the GW module will be carried out with open multi-processing (Open-MP).
→ → Threads	Number of the parallel threads
Others	Further options of the GW average module
→ Quantile	If this option is ticked, the GW quantiles will be calculated.

Figure 3. Load the spatial data of the GPKG format into GWmodelS.

Fundamental functionality for spatial data manipulation, exploration and mapping are included in **GWmodelS** as it imports GDAL and QGIS development libraries. Other operations can be explored by users, with the potential to include additional functionalities available, such as other data formats supported by the GDAL library.

3.2. GW models calibration

This section describes four common GW models and demonstrates their application with **GWmodelS**, including GW descriptive statistics (i.e. GW averages and correlations – see technical details in Brunsdon, Fotheringham, and Charlton 2002a), regression (basic and multiscale – see technical details in
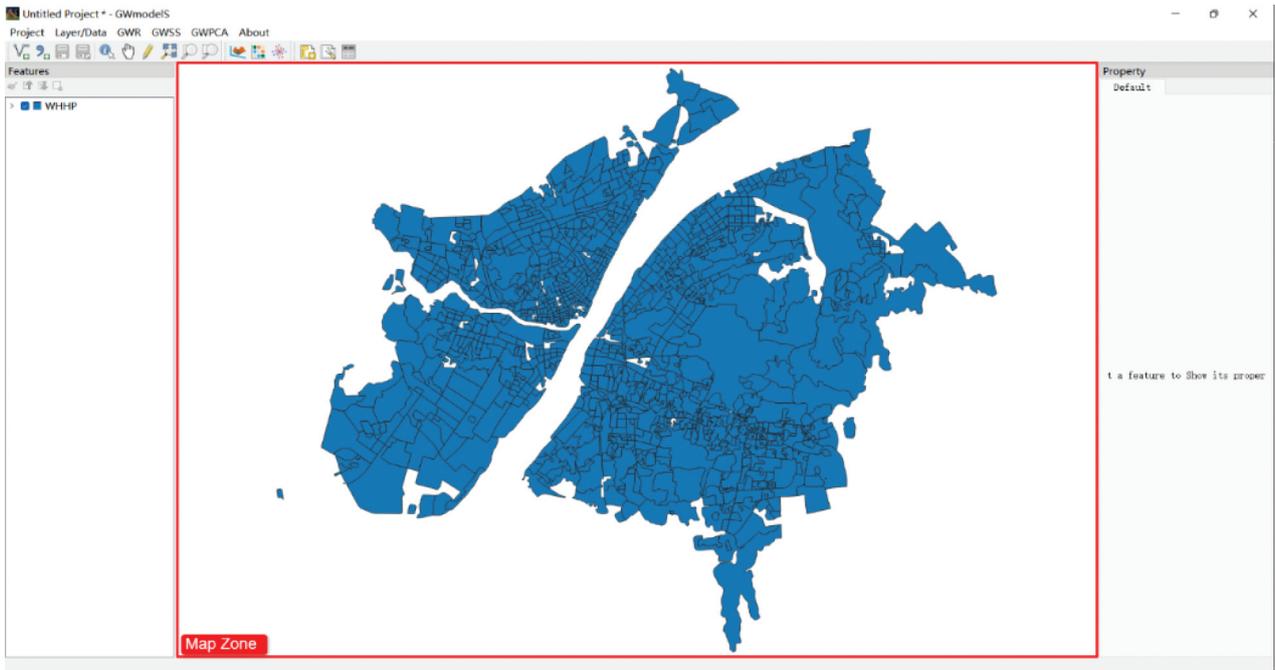
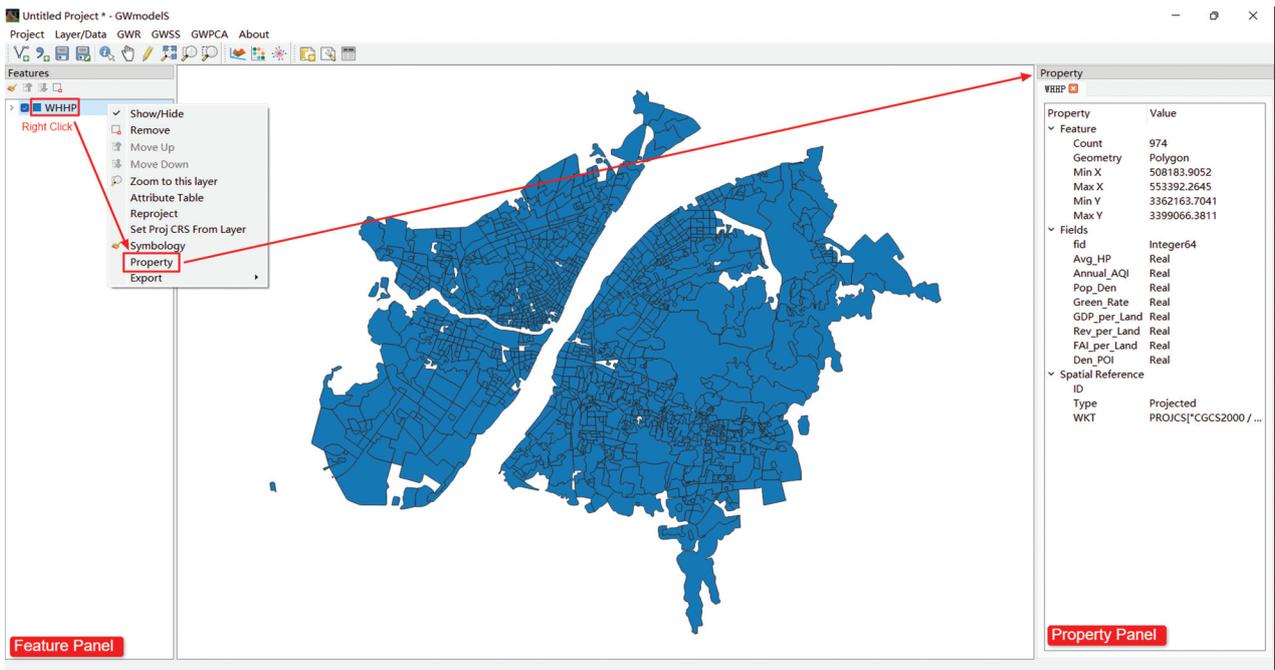**Figure 4.** Examine spatial data (WHHP) in the map zone.

Figure 5. Overview and information of the imported spatial data.

Fotheringham, Yang, and Kang 2017; Lu et al. 2017, 2019) and PCA techniques (Harris, Brunsdon, and Charlton 2011). This section could be used as introductory materials in GW model training with **GWmodelS**. Note that a wide range of GW models are available, including a number of GW regression extensions applicable to large-scale data set (Murakami et al. 2021), spatiotemporal data (Huang, Wu, and Barry 2010), outliers (Harris,

Fotheringham, and Juggins 2010) and local collinearity (Wang, Mei, and Yan 2008). These are not described in detail here, but their usage can be inferred with similar interfaces in **GWmodelS** (Lu et al. 2023).

3.2.1. GW descriptive statistics
GW descriptive statistics are always vital pre-cursors to preliminarily explore a spatial data set and provide
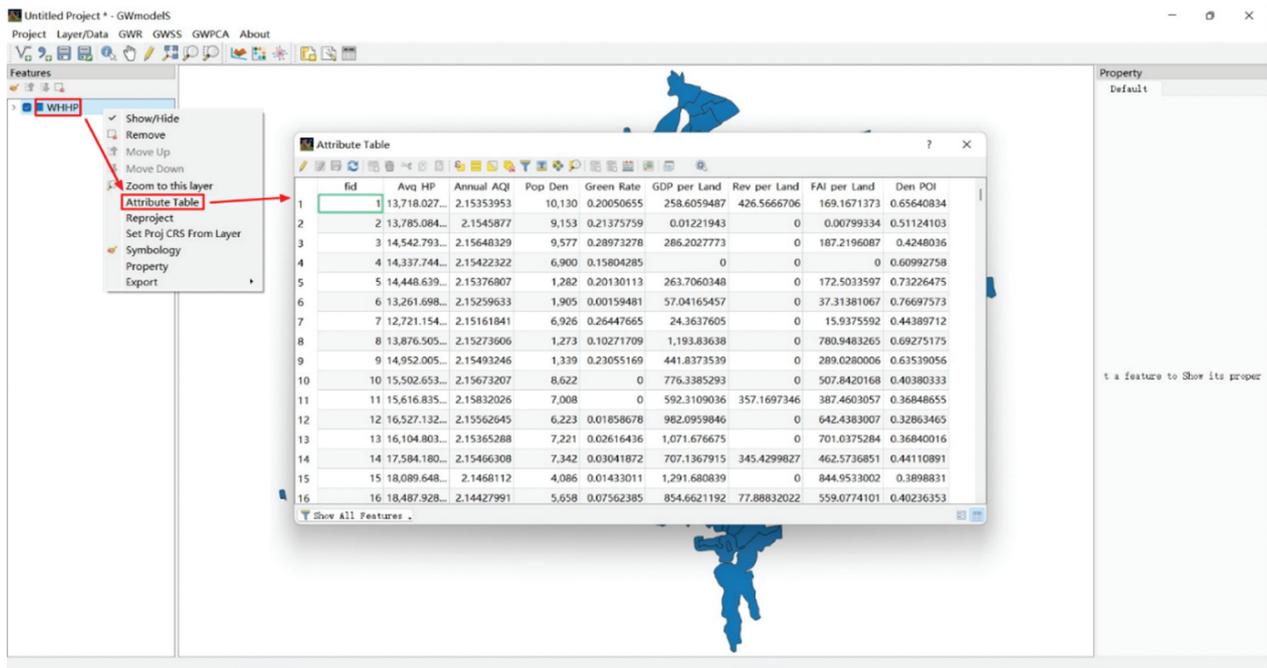
Figure 6. View attribute table of the imported layer.

a localized perspective. **GWmodelS** incorporates GW averages and correlations that locally summarize univariate and binary variables. To calculate the GW averages, click the menu item "GWSS → GW Averages". As shown in Figure 8, the required parameters can be configured in the pop-up window, and interpreted as in Table. With all the parameters configured properly, GW averages (LM) plus standard deviations (LSD), variances (LVar), skewness (LSkew), and CV (LCV) can be calculated and summaries returned in the property panel, as shown in Figure 9.

Figure 10 and Table 2 present the configuration window and setting interpretations for calculating bivariate GW correlations. Note that there are two groups of variables, namely X and Y, referring to two sets of variables, $\{X_1, X_2, \cdots, X_m\}$ and $\{Y_1, Y_2, \cdots, Y_n\}$. With them, the bivariate GW correlations between each pair of X_i and Y_j (where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$) are calculated. By default, all the GW correlations are calculated via a uniformly defined kernel function (e.g. Gaussian), while bandwidths are optimized specifically for each pair of variables. On clicking the text of pairwise variables in the left textbox, the configuration for each bivariate GW correlation can be customized individually, i.e. different kernel function and bandwidth settings can be used for each single calculation. In this way, it is possible to undertake more elaborate calculations than in **GWmodel**. The GW correlation coefficients are returned as properties in a new layer and summarized in the property panel, as shown in Figure 11. Observe that both Pearson and Spearman (robust) correlation coefficients can be calculated, as in **GWmodel** (Lu et al. 2014).

3.2.2. Basic GWR

To calibrate a basic GWR model, click the menu item "GWR → Basic GWR", and set the parameters in the specific configuration window, as shown in Figure 12. Their meanings and options are described in Table 3, but note the parameters in Table 1 and that the remaining configuration windows will not be repeated from here. With all the parameters set properly, the status of this dialog in the bottom left corner will be changed into "Valid", and the basic GWR model is ready for calibration by clicking the "OK" button.

Once the GWR model calibration is achieved, a property window with model summarizing information appears on the right side, as shown in Figure 13.

In the top subpanel, the results from a global regression (OLS model) are summarized if the OLS option is ticked, including coefficient-wise estimates, standard error and t-values, following by its diagnostic information, i.e. AIC, AIC_c, residual sum of squares (RSS), R^2 and adjusted R^2 values.

In the "Results of Geographically Weighted Regression" panel, the model calibration information, including kernel function, bandwidth size, regression locations and distance metric are reported. Each GWR coefficient estimate is summarized in a table with minimum, quantiles, and maximum. For model performance comparison to the OLS model, the diagnostic information is also included.

Model specification functionality is incorporated through a stepwise procedure as detailed in Lu et al. (2014). To view this procedure, a circle view and line chart are drawn here. The former plot presents the procedure of candidate variables
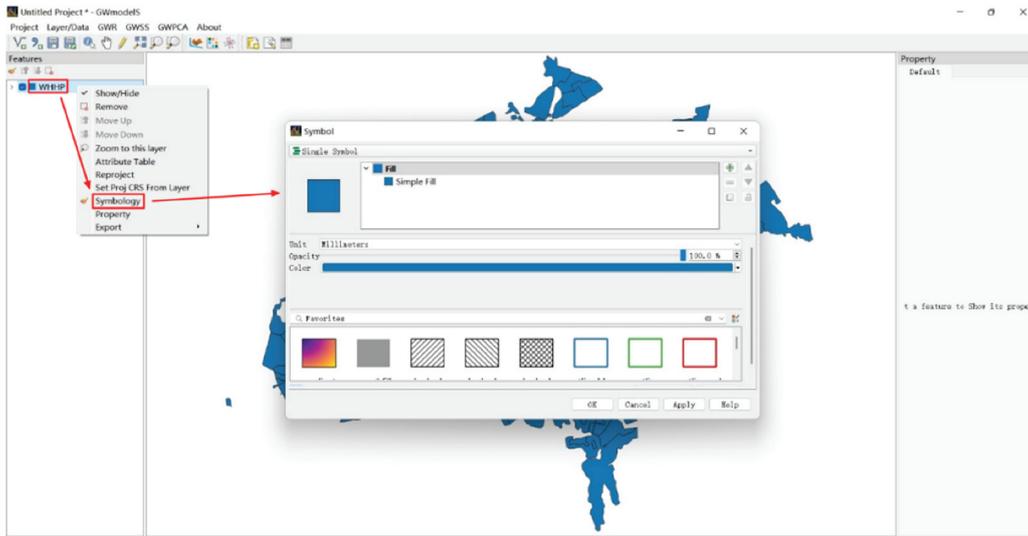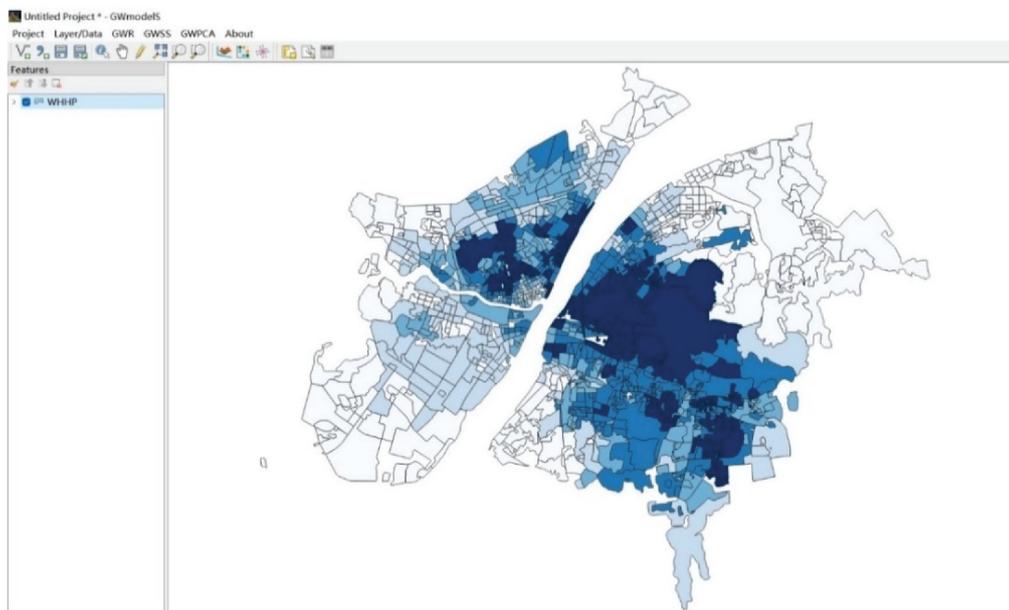
(a) Click the "Symbology" button
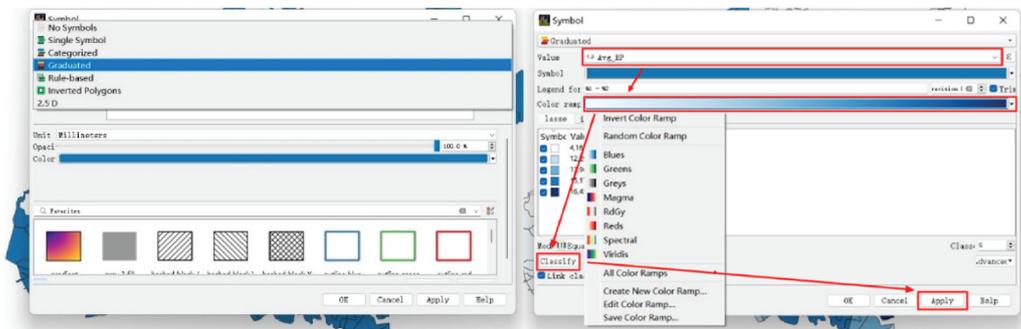
(b) Choose symbol type

(c) Modify symbol settings.

(d) Choropleth map visualization

Figure 7. Visually explore the spatial data imported.

included into the model in a "forward" direction, where the dependent variable located in the center of the chart and the independent variables are represented as nodes differentiated by shapes and colors. The latter line chart presents the corresponding AICc values from each candidate regression model.

The GWR bandwidth size is optimized via an AICc or CV approach. To view this procedure, a dot-line chart is provided in the sub-section "Bandwidth
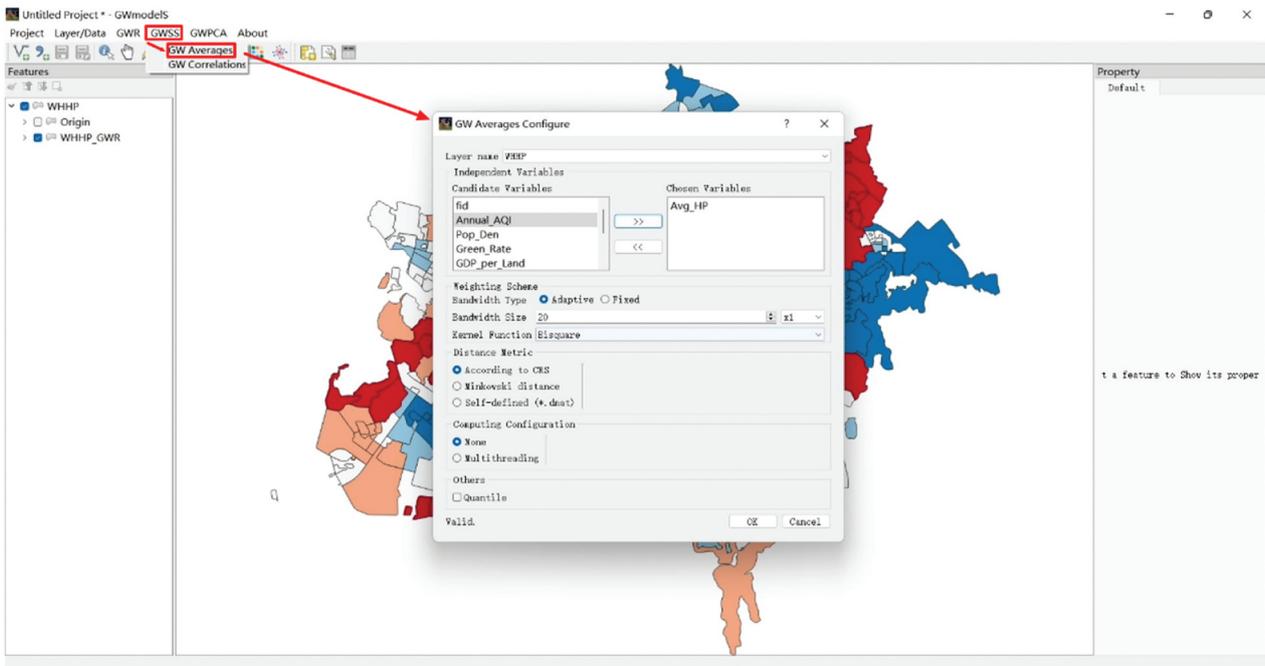
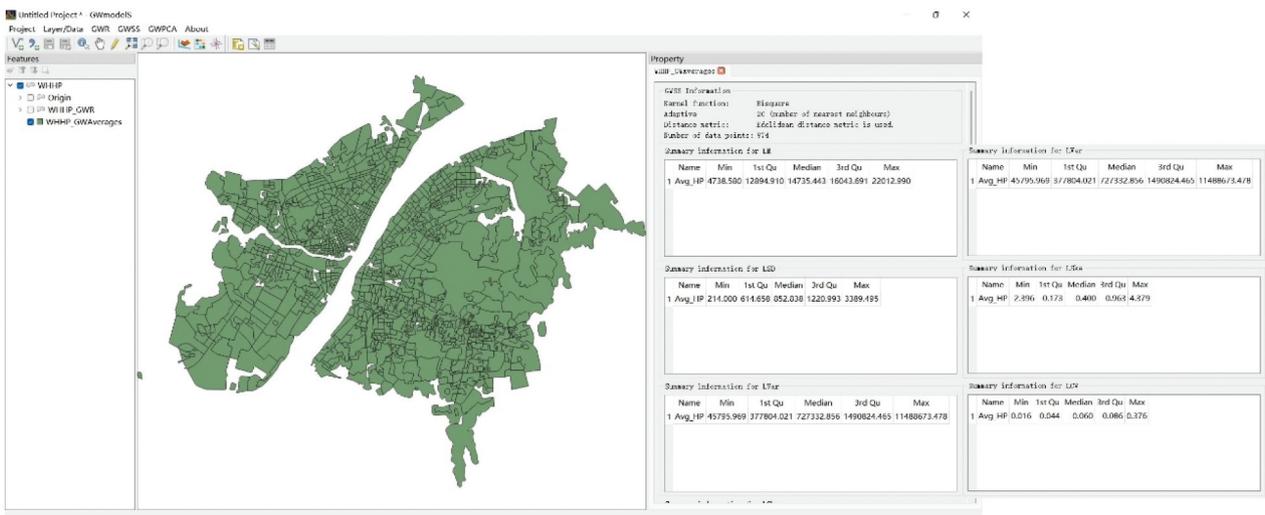**Figure 8.** Configuration window of GW descriptive statistics.

Figure 9. Result panel of GW descriptive statistics.

Selection View", in which the horizontal axis refers to the bandwidth sizes and vertical axis are the corresponding AICc or CV values. All these tree charts can be scaled freely, and zoomed into or out of to examine details.

For further analysis, the localized coefficient estimates and diagnostic information (e.g. residuals, local R^2 and t-test values) are written into a new layer with a suffix, "_GWR". Similarly, this can be explored using the fundamental data management and visualization tools, as shown in Figure 14.

3.2.3. Multiscale GWR
Multiscale GWR provides information about the different scales of relationships in the data, and

should always be explored in GWR analyzes (Comber et al. 2023). On clicking the menu item "GWR → Multiscale GWR", the configuration window for calibrating a multiscale GWR model is activated, as shown in Figure 15. Its configuration is largely similar to that of a basic GWR model, like the "Layer Name", "Dependent Variable", and "Independent Variable". The key difference between calibrating a multiscale GWR and basic GWR model is the iteratively back-fitting algorithm adopted for the former one (Fotheringham, Yang, and Kang 2017; Leong and Yue 2017; Lu et al. 2017), and thus the "Iteration Configuration" parameters are incorporated. In **GWmodelS**, the algorithm is from GWR with parameter-specific distance metrics (Lu et al.
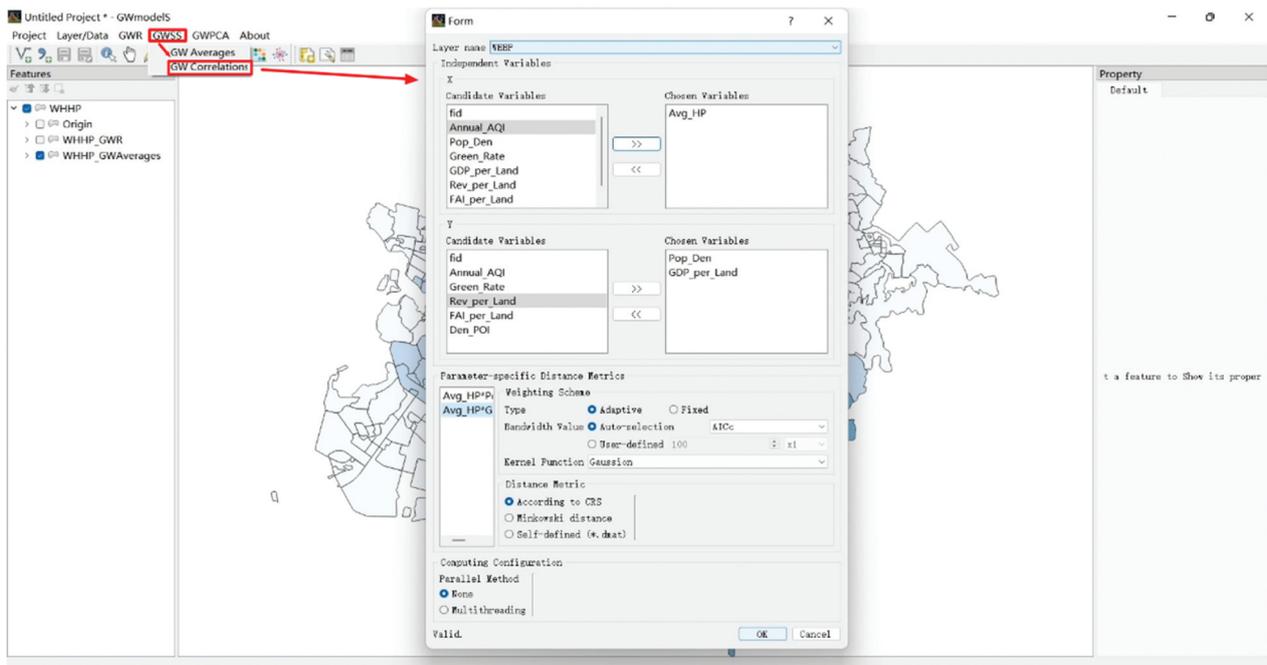
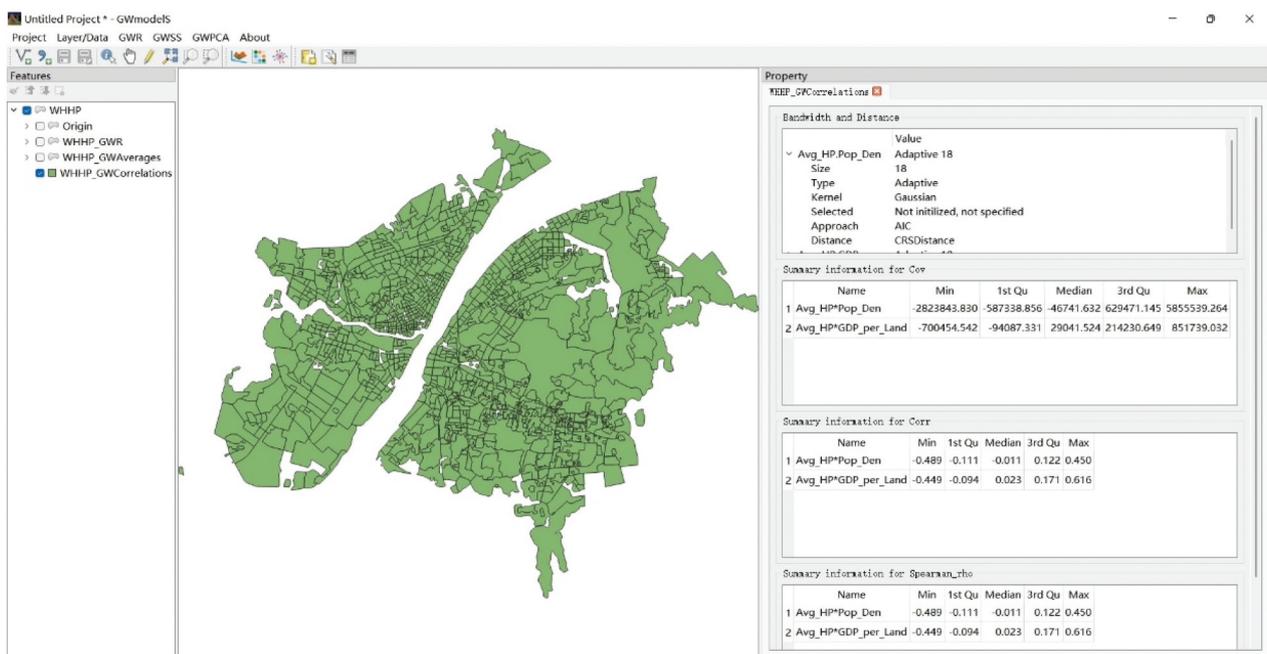**Figure 10.** Configuration window of GW correlations.

Figure 11. Result panel of GW correlations.

2017), and thus a subsection namely "Parameter-specific Distance Metrics" is available for setting the weighting scheme (including both kernel function and bandwidth size) and distance metric for each varying coefficient estimate. This algorithm is different from the other two multiscale GWR techniques in terms of bandwidth optimization strategy and convergence criteria (see details in Lu et al. 2018, 2019). Thus, there could be some discrepancy between the results from this tool and the MGWR

tool (Oshan et al. 2019), and that in **ArcGIS Pro**, where gradient-based optimization is adopted (Zhou et al. 2023).

With all the options configured properly, the status of this dialog in the bottom left corner appears as "Valid". On clicking the "OK" button, the specific multiscale GWR model is calibrated and progress indicated through a progress bar. As presented in Figure 16, the results of the multiscale GWR model are returned in two parts:
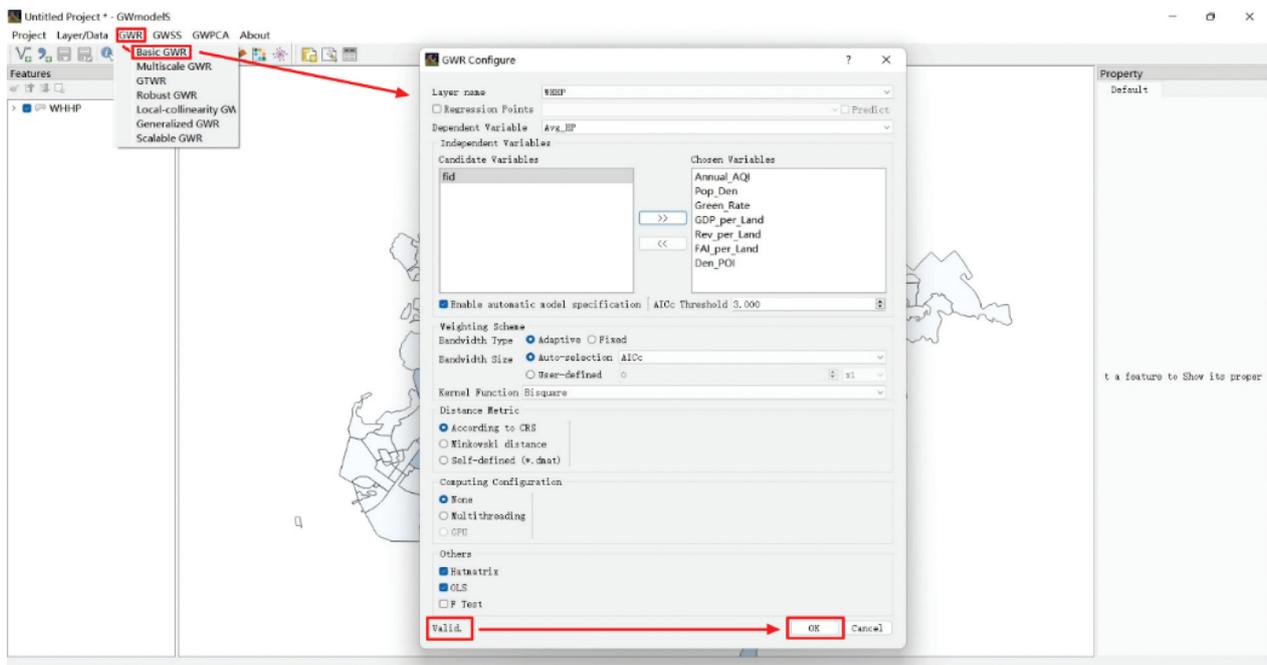
Figure 12. Configuration window for calibrating a basic GWR model.

Table 3. Options for the basic GWR configuration.

Option	Explanation
Layer name	Data layer of observations for calibrating the GWR model
Regression Points	If ticked, an individual layer should be provided as regression locations.
→ Predict	If ticked, predictions will be made at each regression location with observations of the independent variables provided.
Dependent Variable	Dependent variable selected for the GWR model.
Independent Variable	Independent variables selected for the GWR model.
→ Candidate Variables	Candidate variables available in the imported layer to be chosen as the independent variables of the GWR model by clicking the button ">>".
→ Chosen Variables	Variables chosen as independent variables.
→ Enable automatic model specification	If ticked, a step-wise GWR model specification procedure will be conducted, see details in Lu et al. (2014).
→ AICc Threshold	Threshold value set for the step-wise model specification, and in the forward procedure the rest variables will be excluded from the model when the reduction of AICc is less than it.
Others	Further options for GWR model calibration
→ Hatmatrix	If ticked, the hat matrix will be calculated and diagnostic information of the GWR model will be returned.
→ OLS	If ticked, result from the ordinary linear square (OLS) approach will be returned.
→ *F* Test	If ticked, F tests of spatial heterogeneity will be conducted according to Leung, Mei, and Zhang (2000).

(1) the results of a global regression, model configuration parameters and diagnostic information are summarized in the property dialog on the right side, and

(2) the localized coefficient estimates and diagnostic information are written into a new layer with a suffix "_MGWR".

As parameters are specifically configured for each coefficient, tree view controls are used which can be expanded to check details of settings, e.g. kernel function, bandwidth size and distance metric. The specific options for configuring multiscale GWR are shown in Table 4.

3.2.4. *Gw pca*

To activate the GWPCA tool, click the menu item "GWPCA → basic GWPCA" and the configuration window will pop up, as shown in Figure 17. Most of the options are the same as those for the previous GW tools, and so are not repeated here. Specific options for GWPCA are indicated in Table 5. In the subsection "PCA Settings", the key parameters for calculating GWPCA are collected, including the number of retained components, the version of the GWPCA (basic or robust) and scores. Moreover, a glyph plot tool is provided to visualize local loadings of all the variables, and a new graphic device is activated if this option is ticked.

As shown in Figure 18, the results of the GWPCA tool are outlined in the property panel, including calculation settings and summarized percentages of variance (PV) for each retained component. The localized PV and cumulative PV are written in a new layer with a suffix
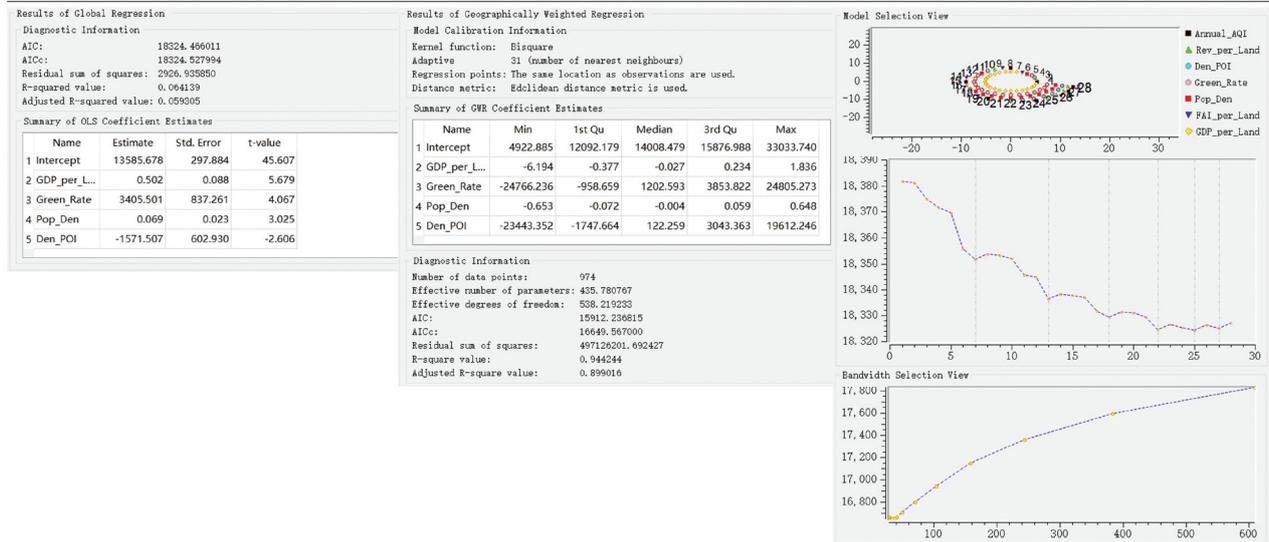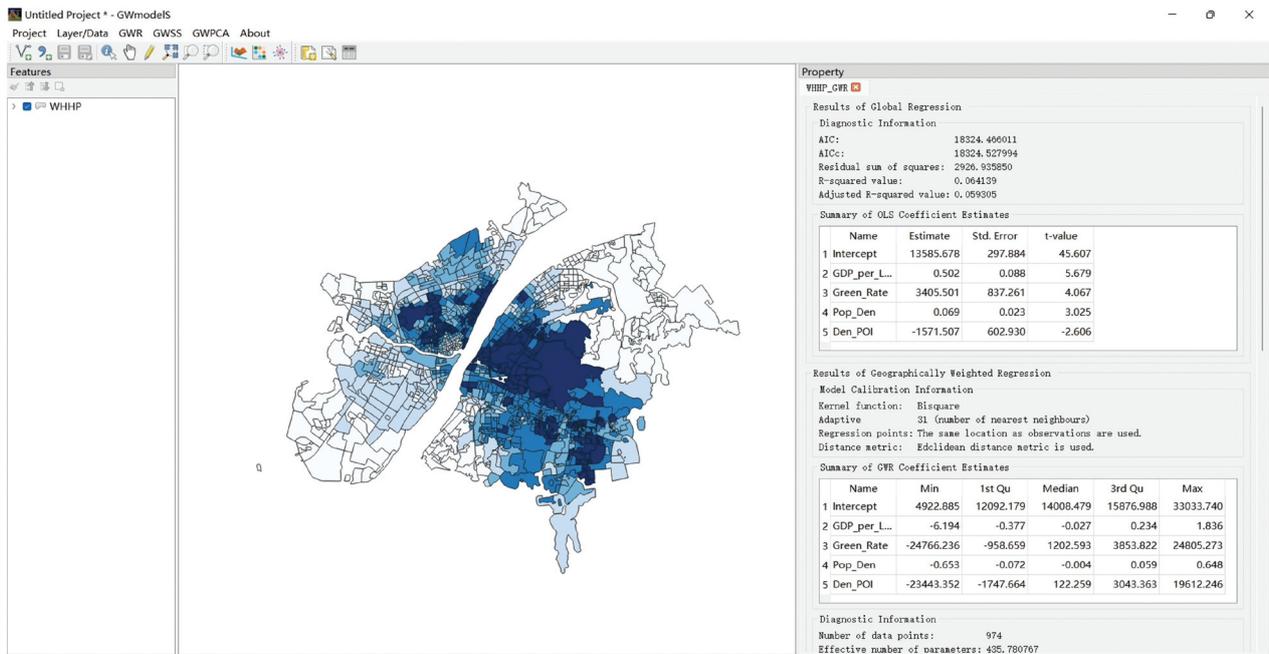
Figure 13. Property panel of the results of the basic GWR model, with panels highlighted at the bottom of the figure.

Figure 14. Mapping GWR coefficient estimates.

Figure 15. Configuration window of multiscale GWR.

Figure 16. Results from a multiscale GWR model.

Table 4. Specific options for multiscale GWR configuration.

Option	Explanation
Iteration Configuration	Options for the back-fitting iteration algorithm.
→ Criterion	Criterion of determining convergence, including changing value of RSS (CVR) and a differential version (dCVR) (see definitions in Lu et al. 2017)
→ Max Iterations	Maximum number of iterations.
→ Converged Threshold	Convergence threshold value to terminate the back-fitting iterations.
→ Bandwidth Type	Type of bandwidth, i.e. fixed or adaptive.
→ Bandwidth Optimize Retry	The bandwidth sizes for each coefficient generally converge fast, and could stop being re-optimized in the following iterations. The number of maximum retries in bandwidth optimization is defined here.
→ Minimum Adaptive Bandwidth	To avoid risk of over-fitting, the lower bound of an adaptive bandwidth is defined for optimization.
Parameter-specific Distance Metrics	Options for setting parameter specific weighting schemes and distance metrics. Switch the target variable by clicking the variable label in the left list.
→ Other	Further options for coefficient estimates
→ → Predictor centralization	Variable will be centralized if ticked (see details in Lu et al. 2019)
Computing Configuration	Refer to the "Computing Configuration" part in Table, but note GPU is not supported for this tool.
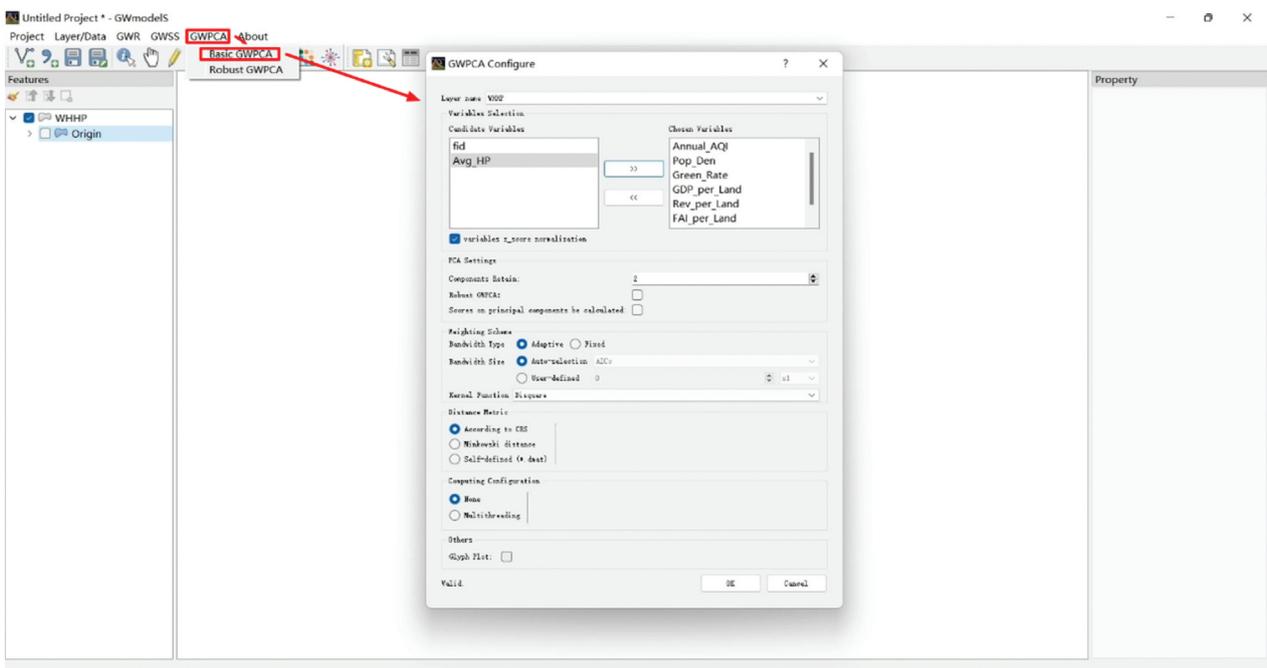
Figure 17. Configuration window of the GWPCA tool.

Table 5. Specific options for GWPCA configuration.

Options	Explanation
Variables z_score normalization	If ticked, all the variables will be normalized via z-scores approach, defined by $\tilde{x} = \frac{x - \bar{x}}{SD(x)}$, where $\tilde{x}$ is the normalized value, x is the original value, $\bar{x}$ is the mean value, $SD(x)$ is the standard deviation of x.
PCA Settings	Options to control the PCA algorithm.
→ Components Retain	Number of retained components and must be less than the number of variables
→ Robust GWPCA	If ticked, robust GWPCA will be applied (Lu et al. 2014).
→ Scores on principal components be calculated	If ticked, the scores of the supplied data on the principal components will be calculated.
Others	
→ Glyph Plot	If ticked, a glyph plot will be drawn to visualize local loadings of all the variables.

"_GWPCA". Note that the localized loadings for each component are not retuned as an explicit output due to its complex dimensionality, but a glyph plot provides a good way to summarize its properties and is enabled as an option in the configuration window. With the local loadings, a new string-type column is added to the output layer to indicate the localized winning variable for
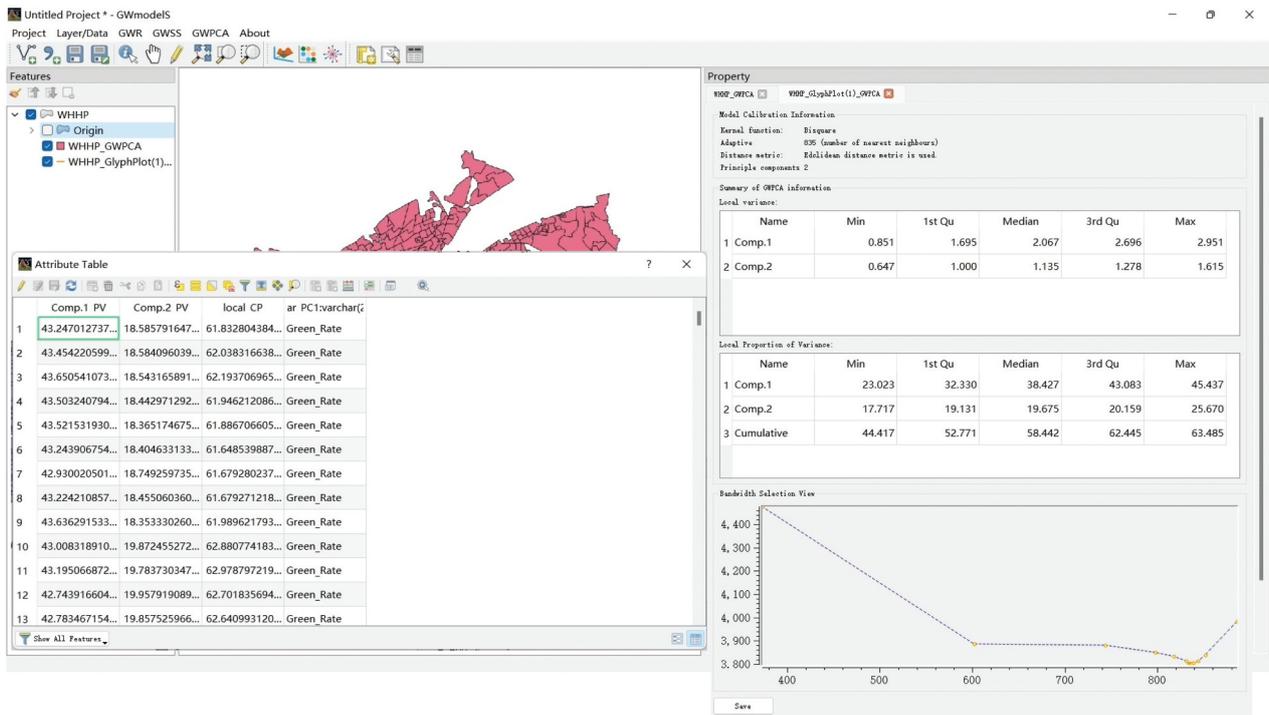
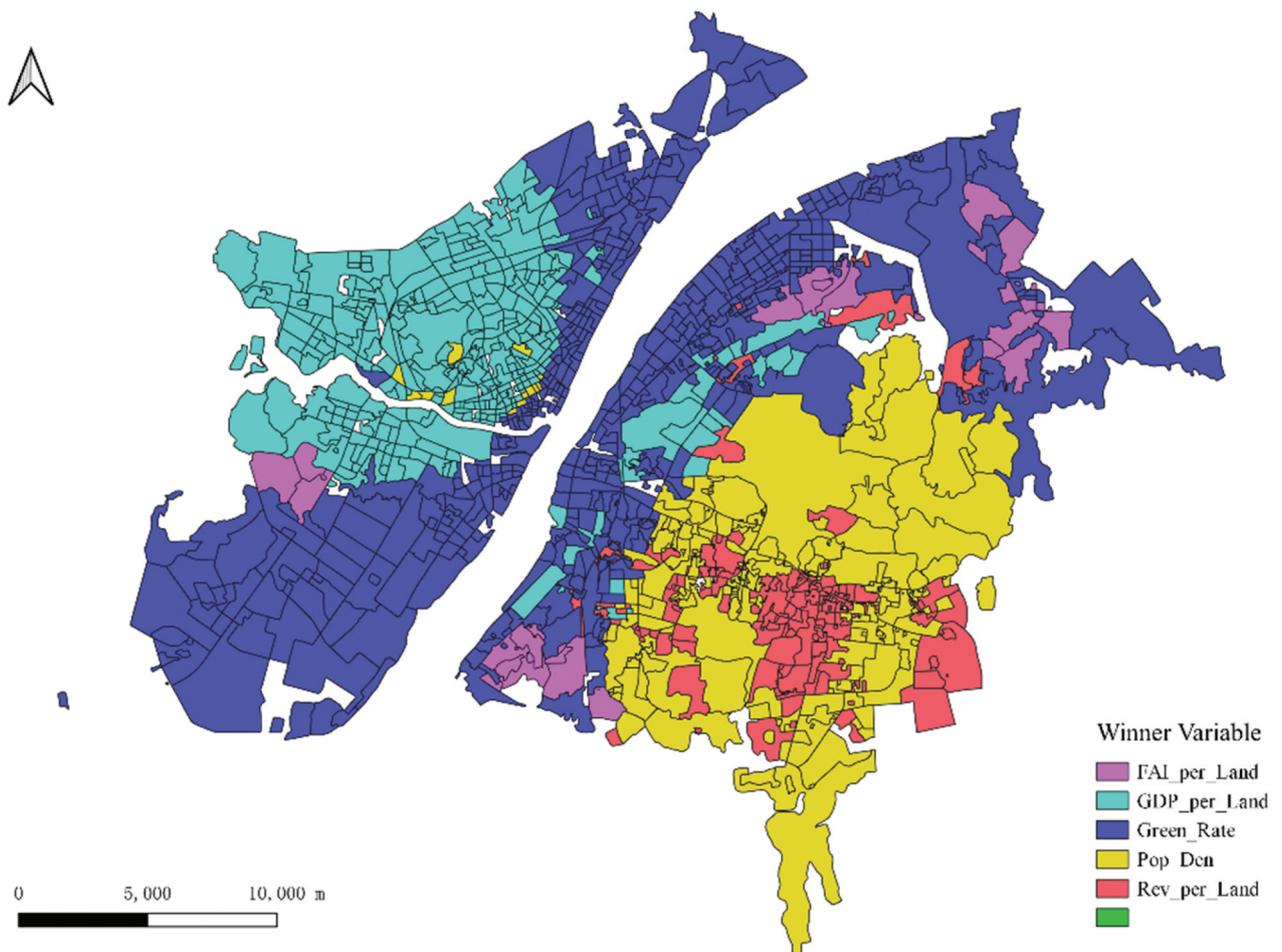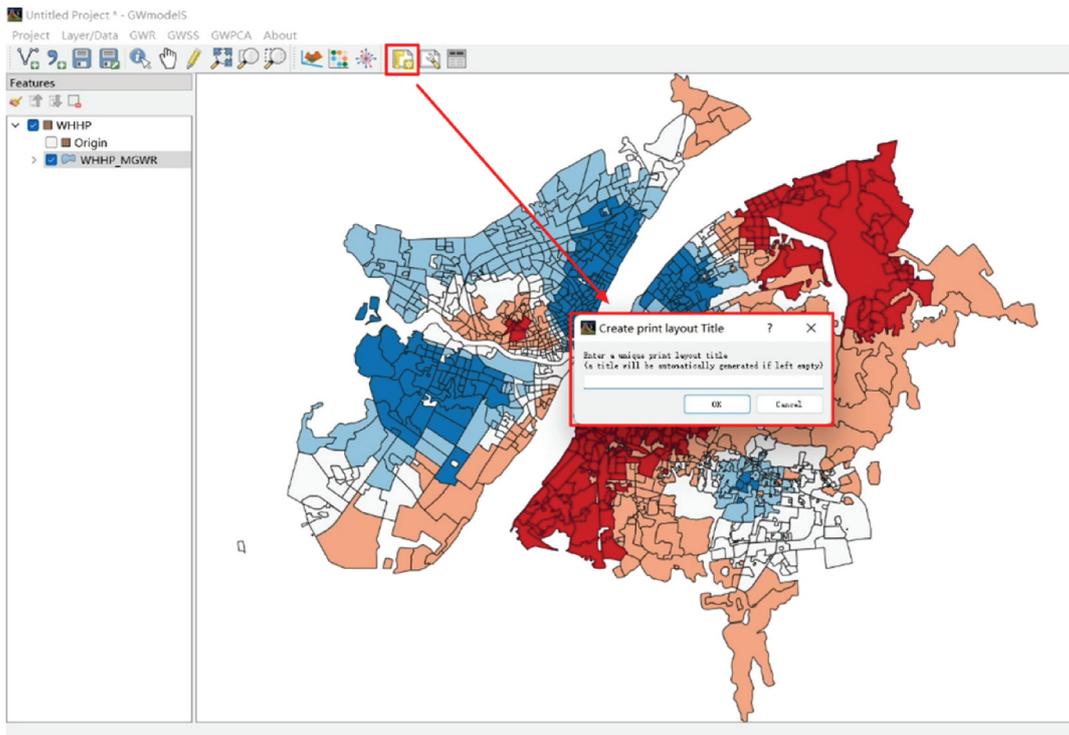**Figure 18.** Results from the GWPCA tool.

Figure 19. Mapping wining variables from GWPCA results.

(a) Give the layout a title
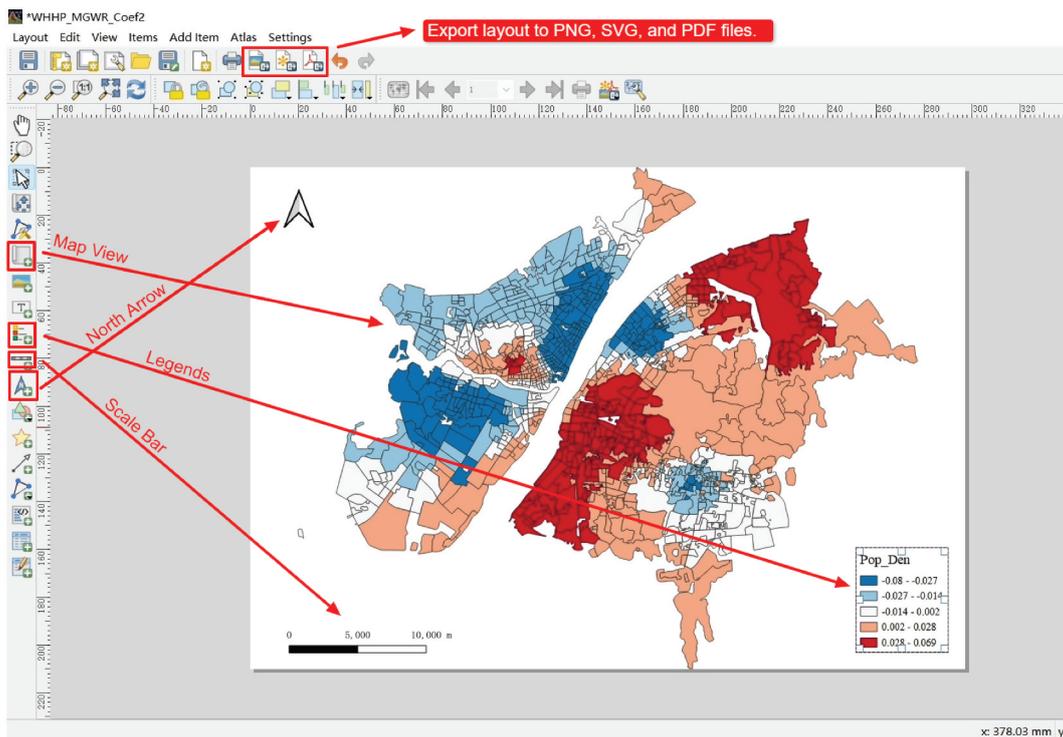
(b) Set the cartographical parameters

Figure 20. Set a layout for mapping.

the first principal component. This identifies the most important variables across the study area and can be used to produce a map, as shown in Figure 19. In this case, observe the clear geographical variation in the influence of each variable on the first component where in this case, the *Greem_Rate* is clearly dominant along the Yangtze River, whilst *GDP_per_land* and *Pop_Den* dominate the Hankou (east) and Wuchang (west) regions, respectively.
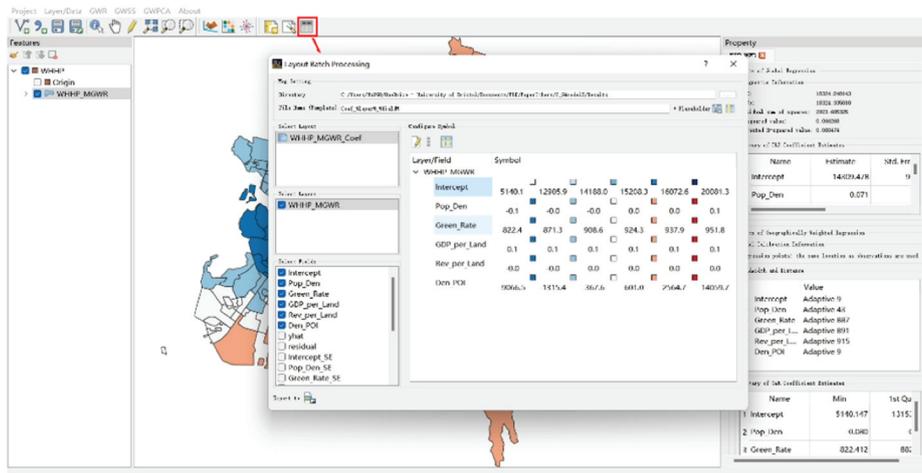
(a) Symbol settings for batch mapping
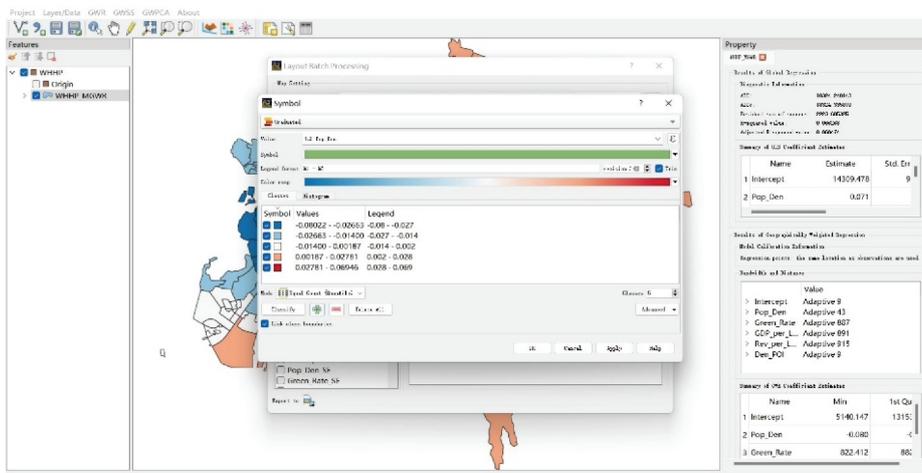
(b) Customize the symbol setting for a specific variable
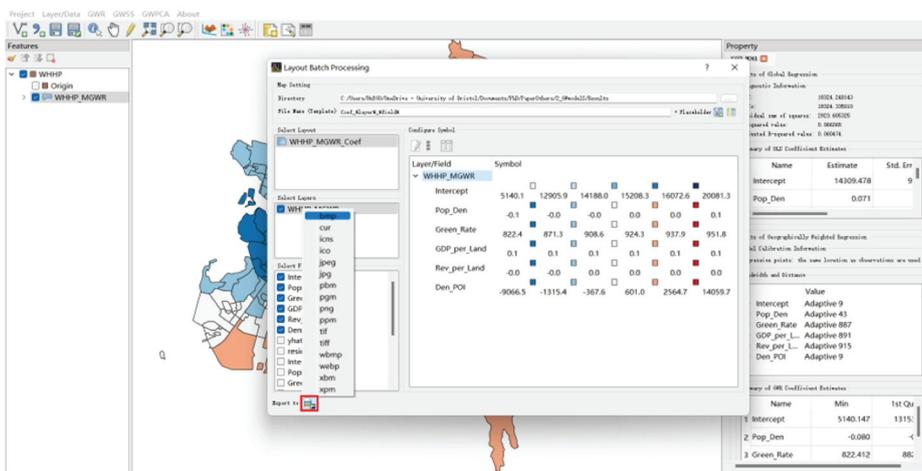
(c) Set the image format for out maps

Figure 21. Parameter settings and output configurations for batch mapping.

3.3. Batch mapping

The key feature of GW models is to generate mappable outputs for presenting the localized results from the GW models (e.g. Comber et al. 2023; Gollini et al. 2015; Lu et al. 2014). This is exemplified in Figures 20(a)–(b) by clicking the layout button in the tool bar to initialize the mapping tool. This functionality is inherited from **QGIS**, and for a more detailed description the user could refer to its manual (QGIS community 2022). Once the layout is prepared,

Table 6. Options for configurating batch mapping.

| Option	Explanation
Directory	Path of the directory for outputs
File Name (Template)	File name of the output file, and could be a template if placeholders "%field%" or "%layer%" is used.
+Placeholder	Two buttons for quickly add placeholders, the left one refers to "%layer%" whilst the right one refers to "%field%".
Select Layout	Select a template layout.
Select Layer	Select target layers by ticking the check box in front of each layer name
Select Fields	When a layer in "Select Layer" list view is checked, its fields will be displayed in this list view and enabled for further selection. On ticking the check box in front of each field name, they will be accordingly selected for batch mapping.
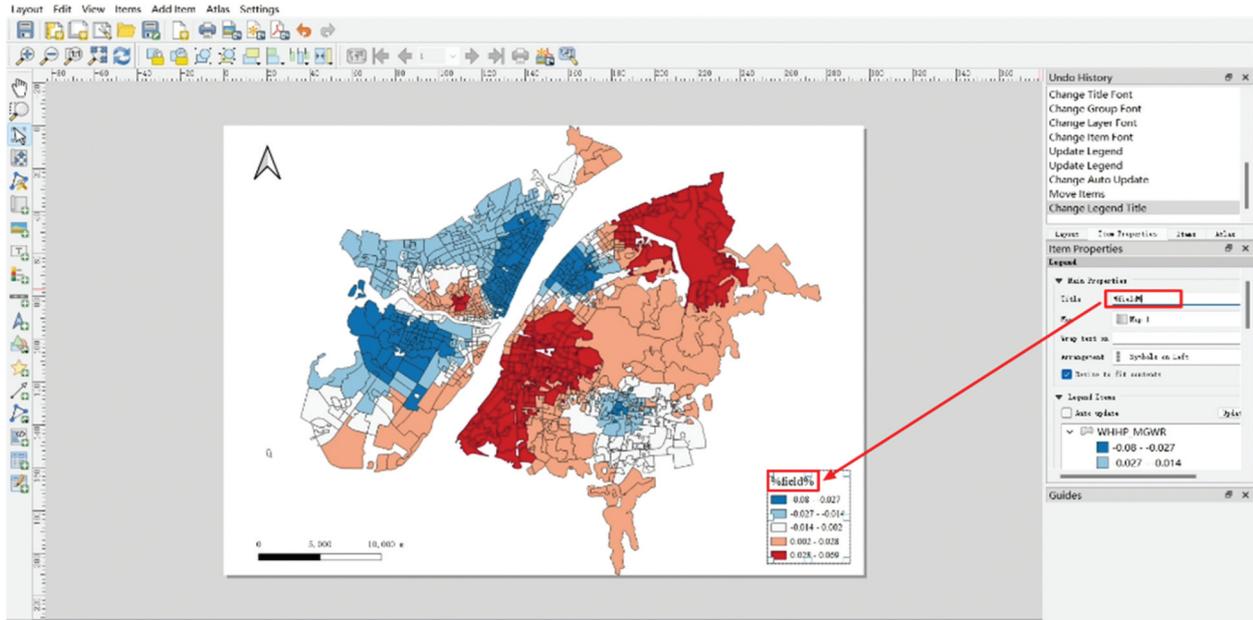
Figure 22. Legend setting for all the variables with placeholders.

(a) *Den_POI* (b) *GDP_per_Land* (c) *Green_Rate*

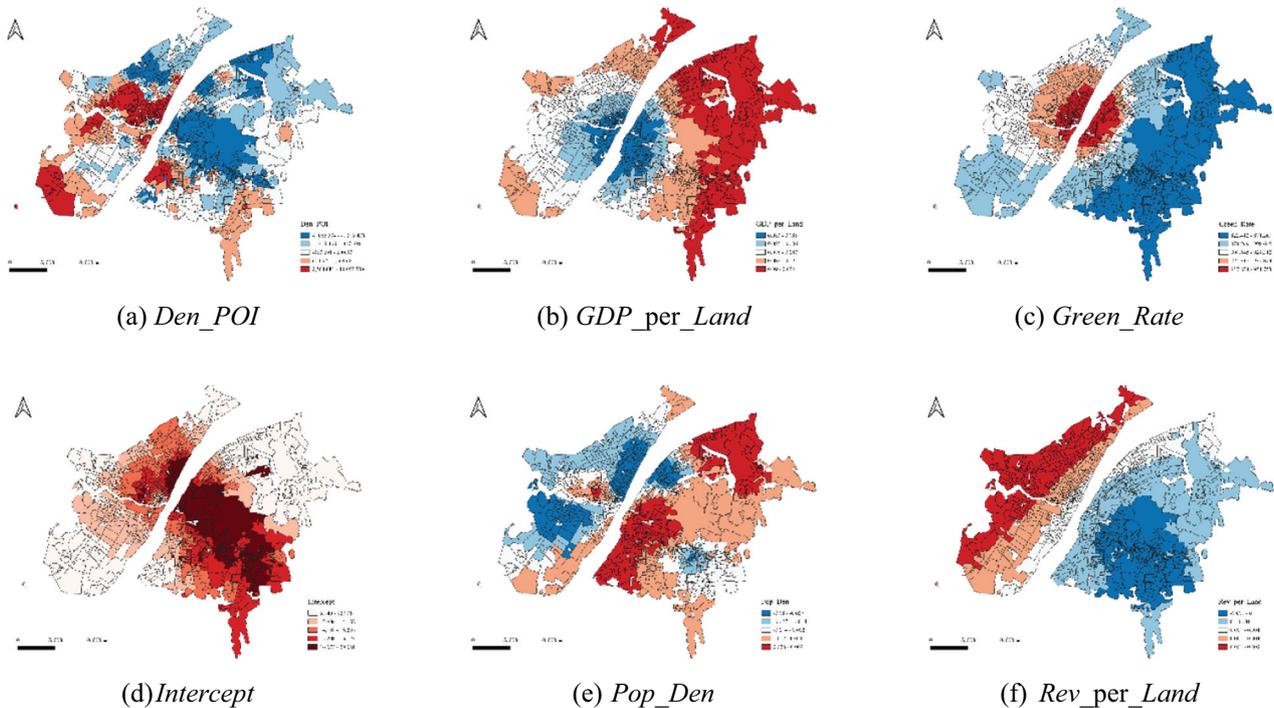(d)*Intercept* (e) *Pop_Den* (f) *Rev_per_Land*

Figure 23. Output maps from the batch mapping tool.

a single thematic map can be exported to an image file in several formats, including PNG, SVG, or PDF. This cartographic tool makes it much easier to generate mapped outputs from **GWmodelS** analyzes than its twin R package, **GWmodel**.

However, many maps are usually required to describe any given case study, particularly with a relatively large number of variables are involved in a GW model. Repetitive cartographic jobs can be frustrating, and difficult for nonprofessional GIS users. In **GWmodelS**, a batch mapping tool is provided, allowing the user to set all the cartographical parameters once, and to create all the maps in high quality using a uniform template. This is done by clicking the batch mapping button in the toolbar and activating the "Layout Batch Processing" window to set the corresponding parameters, as shown in Figures 21(a)–(c). In Table 6, the options for configurating batch mapping are described. Notably, two placeholders are defined as wildcards, i.e. "%field%" and "%layer%". The former one refers to the name of a field, whilst the latter one refers to the name of a layer. As shown in Figure 21(a), the selected variables are listed in the "Configure Symbol" area with a unified style by default. If the user selects a specific item and clicks the button in the middle, a dialog will pop-up to customize the symbol setting and value cuts for the specific variable. As illustrated in Figure 21(b), the symbology type is set as "Graduated" and the blue-red color ramp selected for mapping the coefficient *Pop_Den*. Most popular image formats are supported for output maps, as shown in Figure 21(c). To use the layout template for batch mapping, the placeholder "%layer%" should be inserted for legend setting, as shown in Figure 22. With the batch mapping tool, large numbers of thematic maps can be produced and saved as high-quality images as shown in Figures 23(a)–(f). The maps of coefficient estimates can be easily obtained through the configurations described above. In this way, the batch tool greatly facilitates the descriptions of GW models in academic studies or data science reports, particularly for interdisciplinary users.

4. Concluding remarks

With the recent rise of research on spatial heterogeneity (Goodchild and Li 2021), GW models have emerged as an essential subset of local techniques, attracting a wide range of users. This article demonstrates a newly developed software tool, **GWmodelS**, which calibrates a number of fundamental GW models and statistics, including GW descriptive statistics, basic and multiscale GWR and GWPCA. It provides significant advantages in terms of user-friendly graphical user interface (GUI), operational efficiency, and accessibility[2]

(Lu et al. 2023). Additionally, tools for spatial data management and batch mapping significantly facilitate its usage for users from diverse domains, such as biology (Zheng et al. 2023), environmental science (Mayfield et al. 2018) and social science (Marek et al. 2020). As a twin software tool, when paired with the R package **GWmodel**, the functionalities of **GWmodelS** are almost equivalent, both having a number of GWR extensions available, including robust GWR (Harris, Fotheringham, and Juggins 2010), locally compensated GWR (Brunsdon, Charlton, and Harris 2012), generalized GWR (Nakaya et al. 2005) and scalable GWR (Murakami et al. 2021). Their usages are not introduced here but can be easily deduced from the examples provided in this study.

Notes

1. The symbol "→" means sub-option affiliation, and note that the public parameters will be not repeatedly interpreted in the following sections.
2. Free download via https://gwmodel-lab.github.io/GWmodelS/

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability statement

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