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A Remark on the Expressivity of Asynchronous TeamLTL and HyperLTL

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Abstract. Linear temporal logic (LTL) is used in system verification to write formal specifications for reactive systems. However, some relevant properties, e.g. non-inference in information flow security, cannot be expressed in LTL. A class of such properties that has recently received ample attention is known as hyperproperties. There are two major streams in the research regarding capturing hyperproperties, namely hyperlogics, which extend LTL with trace quantifiers (HyperLTL), and logics that employ team semantics, extending truth to sets of traces. In this article we explore the relation between asynchronous LTL under set-based team semantics (TeamLTL) and HyperLTL. In particular we consider the extensions of TeamLTL with the Boolean disjunction and a fragment of the extension of TeamLTL with the Boolean negation, where the negation cannot occur in the left-hand side of the Until-operator or within the Global-operator. We show that TeamLTL extended with the Boolean disjunction is equi-expressive with the positive Boolean closure of HyperLTL restricted to one universal quantifier, while the left-downward closed fragment of TeamLTL extended with the Boolean negation is expressively equivalent with the Boolean closure of HyperLTL restricted to one universal quantifier.

Keywords: Hyperproperties · Temporal Logic · Team Semantics · HyperLTL · Verification

1 Introduction

In 1977 Amir Pnueli [16] introduced a core concept in verification of reactive and concurrent systems: model checking of formulae of linear temporal logic (LTL). The idea is to view the accepting executions of the system as a set of infinite sequences, called traces, and check whether this set satisfies specifications expressed in LTL. The properties that can be checked by observing every execution of the system in isolation are called *trace properties*. An oft-cited example of a trace property is *termination*, which states that a system terminates if each of its computations terminates. Classical LTL is fit for the verification of such propositional trace properties, however some properties relevant in, for instance, information flow security are not trace properties. These properties profoundly speak of relations between traces. Clarkson and Schneider [3] coined the term *hyperproperties* to refer to such properties that lie beyond what LTL can express. *Bounded termination* is an easy to grasp example of a hyperproperty: whether every computation of a system terminates within some bound common for all traces, cannot be determined by looking at traces in isolation. In information flow security, dependencies between public observable outputs and secret inputs constitute possible security breaches; checking for hyperproperties becomes invaluable. Two well-known examples of hyperproperties from this field are noninterference [18,15], where a high-level user cannot affect what low-level users see, and observational determinism [21], meaning that if two computations are in the same state according to a low-level observer, then the executions will be indistinguishable. However, hyperproperties are not limited to information flow security; examples from different fields include distributivity and other system properties such as fault tolerance [6].

Given this background, several approaches to formally specifying hyperproperties have been proposed since 2010, with families of logics emerging from these approaches. The two major streams in the research regarding capturing hyperproperties are *hyperlogics* and logics that employ *team semantics*. In the hyperlogics approach, logics that capture trace properties are extended with trace quantification, extending logics such as LTL, computation tree logic (CTL) or quantified propositional temporal logic (QPTL), into HyperLTL [2], HyperCTL* [2], and HyperQPTL [17,4], respectively. An alternative approach is to lift the semantics of the temporal logics from being defined on traces to sets of traces, by using what is known as team semantics. This approach yields logics such as TeamLTL [14,9] and TeamCTL[13,9]. Since its conception, TeamLTL has been considered in two distinct variants: a synchronous semantics, where the team of traces agrees on the time step of occurrence when evaluating temporal operators; and an asynchronous semantics, where the temporal operators are evaluated independently on each trace. An example that illustrates the difference between these two semantics is the aforementioned termination and bound termination pair of properties. If we write F for the future-operator and terminate for a proposition symbol representing the trace terminating, we can write the formula F terminate, which under the synchronous semantics expresses the hyperproperty "bounded termination", while under the asynchronous semantics the same formula defines the trace property "termination". Not only is the above formulation of bounded termination clear and concise, it also illuminates a key difference between hyperlogics and team logics: while each formula of hyperlogic has a fixed number of quantifiers, which restricts the number of traces that can be referred to in a formula, which restricts the number of traces between which dependencies can be characterised by formulae, team logics have the ability to refer to an unbounded number of traces, even an infinite collection.

One of the original motivations behind team semantics [19] was to enable the definition of novel atomic formulae, and this is another important defining feature of team temporal logics as well. Among these atoms the *dependence* *atom* $\mathrm{dep}(\bar{x}, \bar{y})$ and *inclusion atom* $\bar{x} \subseteq \bar{y}$ stand out as the most influential. They respectively state that the variables $\bar{y}$ are functionally dependent on the variables $\bar{x}$, and that the values of the variables $\bar{x}$ also occur among the values of variables $\bar{y}$. As an example of the use of the inclusion atom, let the proposition symbols $o_1, \ldots, o_n$ denote public observable bits and assume that the proposition symbol s is a secret bit. The atomic formula $(o_1, \ldots o_n, s) \subseteq (o_1, \ldots o_n, \neg s)$ expresses a form of non-inference by stating that an observer cannot infer the value of the confidential bit from the outputs.

While the expressivity of HyperLTL and other hyperlogics has been studied extensively, where the many extensions of TeamLTL lie in relation the hyperlogics is still not completely understood. The connections for the logics without extensions were already established in Krebs et al. [14], where they showed that synchronous TeamLTL and HyperLTL are expressively incomparable and that the asynchronous variant collapses to LTL. With regards to the expressivity of synchronous semantics, Virtema et al. [20] showed that the extensions of TeamLTL can be translated to HyperQPTL^{+}, which in turn extends HyperLTL with (non-uniform) quantification of propositions. Relating the logics to the first-order context, Kontinen and Sandström [11] defined Kamp-style translations from extensions of both semantics of TeamLTL to the three-variable fragment of first-order team logic. It is worth noting that recently asynchronous hyperlogics have been considered also in several other articles (see, e.g., [10,1]). An example of the significant rift between asynchronous and synchronous TeamLTL is that the asynchronous semantics is essentially a first-order logic, while the synchronous semantics has second-order aspects. Especially the set-based variant of asynchronous TeamLTL can be translated, using techniques in [11], into first-order logic under team semantics, which is known to be first-order logic [19]. Similarly, HyperLTL is equally expressive as the guarded fragment of first-order logic with the equal level predicate, as was shown by Finkbeiner and Zimmermann [7].

In this article we focus on exploring the connections between fragments of HyperLTL and extensions of TeamLTL. The set-based asynchronous semantics that we consider here was defined in Kontinen et al. [12] in order to further study the complexity of the model checking problem for these logics. Prior to that, the literature on temporal team semantics employed a semantics based on multisets of traces. In the wider team semantics literature, this often carries the name *strict semantics*, in contrast to *lax semantics* which is de facto a set-based semantics. This relaxation of the semantics enabled the definition of normal forms for the logics, which we use in this article to explore the connection with HyperLTL.

Our contribution. We show correspondences in expressivity between the set-based variant of linear temporal logic under asynchronous team semantics and fragments of the Boolean closure of HyperLTL. In particular we show that LTL under team semantics with the Boolean disjunction, TeamLTL($\varovee$), is equi-expressive with the positive Boolean closure of HyperLTL restricted to only one universal quantifier, while the left downward closed fragment of TeamLTL($\sim$) is equi-expressive with the Boolean closure of HyperLTL restricted to one universal quantifier.

2 Preliminaries

We begin by defining the variant of TeamLTL and its extensions, as in [12].

Let AP be a set of *atomic propositions*. The formulae of LTL (over AP) is attained by the grammar:

$$\varphi ::= p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \bigcirc \varphi \mid \mathsf{G}\, \varphi \mid \varphi \,\mathsf{U}\, \varphi,$$

where $p \in \mathrm{AP}$. We follow the convention that all formulae of TeamLTL are given in negation normal form, where $\neg$ is only allowed before atomic propositions, as is customary when dealing with team semantics.

We will consider the extensions of TeamLTL with the Boolean disjunction $\otimes$, denoted TeamLTL($\otimes$), and Boolean negation $\sim$, denoted TeamLTL($\sim$).

A *trace* t over AP is an infinite sequence of sets of proposition symbols from $(2^{\mathrm{AP}})^\omega$. Given a natural number $i \in \mathbb{N}$, we denote by $t[i]$ the $(i+1)$th element of t and by $t[i,\infty]$ the suffix $(t[j])_{j \geq i}$ of t. We call a set of traces a *team*.

We write $\mathcal{P}(\mathbb{N})^+$ to denote $\mathcal{P}(\mathbb{N}) \setminus \{\emptyset\}$. For a team $T \subseteq (2^{\mathrm{AP}})^\omega$ a function $f\colon T \to \mathcal{P}(\mathbb{N})^+$, we set $T[f,\infty] := \{t[s,\infty] \mid t \in T, s \in f(t)\}$. For $T' \subseteq T$, $f\colon T \to \mathcal{P}(\mathbb{N})^+$, and $f'\colon T' \to \mathcal{P}(\mathbb{N})^+$, we define that $f' < f$ if and only if

$$\begin{aligned}\forall t \in T' : &\min(f'(t)) \leq \min(f(t)) \text{ and,}\\ &\text{if } \max(f(t)) \text{ exists, } \max(f'(t)) < \max(f(t)).\end{aligned}$$

Definition 1 (TeamLTL)**.** *Let T be a team, and φ and ψ TeamLTL-formulae. The lax semantics is defined as follows.*

$$\begin{aligned}
T \models l \quad &\Leftrightarrow \quad t \models l \textit{ for all } t \in T, \textit{ where } l \in \{p, \neg p \mid p \in \mathrm{AP}\}\\
&\qquad \textit{is a literal and "}t \models \textit{" refers to } \mathrm{LTL}\textit{-satisfaction}\\
T \models \varphi \wedge \psi \quad &\Leftrightarrow \quad T \models \varphi \textit{ and } T \models \psi\\
T \models \varphi \vee \psi \quad &\Leftrightarrow \quad \exists T_1, T_2 \textit{ s.t. } T_1 \cup T_2 = T \textit{ and } T_1 \models \varphi \textit{ and } T_2 \models \psi\\
T \models \bigcirc \varphi \quad &\Leftrightarrow \quad T[1,\infty] \models \varphi\\
T \models \mathsf{G}\, \varphi \quad &\Leftrightarrow \quad \forall f\colon T \to \mathcal{P}(\mathbb{N})^+ \textit{ it holds that } T[f,\infty] \models \varphi\\
T \models \varphi \,\mathsf{U}\, \psi \quad &\Leftrightarrow \quad \exists f\colon T \to \mathcal{P}(\mathbb{N})^+ \textit{ such that } T[f,\infty] \models \psi \textit{ and}\\
&\qquad \forall f'\colon T' \to \mathcal{P}(\mathbb{N})^+ \textit{s.t. } f' < f, \textit{ it holds that } T'[f',\infty] \models \varphi\\
&\qquad \textit{or } T' = \emptyset, \textit{ where } T' := \{t \in T \mid \max(f(t)) \neq 0\}
\end{aligned}$$

The semantics for the Boolean disjunction and Boolean negation, used in the extensions TeamLTL($\otimes$) *and* TeamLTL($\sim$)*, are given by:*

$$\begin{aligned}
T \models \varphi \otimes \psi \quad &\Leftrightarrow \quad T \models \varphi \textit{ or } T \models \psi\\
T \models \sim \varphi \quad &\Leftrightarrow \quad T \not\models \varphi
\end{aligned}$$

Note that the Boolean disjunction is definable in TeamLTL($\sim$), as the dual of conjunction, i.e. $T \models^l \varphi \otimes \psi$ if and only if $T \models^l \sim (\sim \varphi \wedge \sim \psi)$.

Two important properties of team logics are *flatness* and *downward closure*. A logic has the flatness property if $T \models^l \varphi$ if and only if $\{t\} \models^l \varphi$ for all $t \in T$, holds for all formulae φ of the logic. A logic is downward closed if for all formulae φ of the logic if $T \models^l \varphi$ and $S \subseteq T$ then $S \models^l \varphi$. The following Proposition was proven in [12].

Proposition 2. TeamLTL^l *has both the flatness and the downward closure properties, while* TeamLTL$^l(\varovee)$ *only has the downward closure property.*

We consider the *left-downward closed* fragment of TeamLTL$^l(\sim)$, denoted left-dc–TeamLTL$^l(\sim)$, where every subformula of the form $\mathsf{G}\,\psi$ or $\psi \,\mathsf{U}\, \theta$, the subformula ψ is a TeamLTL($\varovee$)-formula

It was established in [12] that any formula of TeamLTL$^l(\varovee)$ can be equivalently expressed in $\varovee$*-disjunctive normal form*, i.e. in the form

$$\bigovee_{i \in I} \alpha_i,$$

where α_i are LTL-formulae.

Similarly by [12], every formula of left-dc–TeamLTL$^l(\sim)$ can be equivalently stated in *quasi-flat normal form*, which means in the form

$$\bigovee_{i \in I} (\alpha_i \wedge \bigwedge_{j \in J_i} \exists \beta_{i,j}),$$

where α_i and $\beta_{i,j}$ are LTL-formulae, and $\exists \beta_{i,j}$ is an abbreviation for the formula $\sim \beta^d_{i,j}$, where $\beta^d_{i,j}$ is the formula obtained from $\neg\beta$, after $\neg$ has been pushed down to the atomic level.

Next we state the syntax and semantics of HyperLTL, as defined in [2], as well as the Boolean closure concepts we are concerned with.

Definition 3 (Syntax of HyperLTL**).** *Let* AP *be a set of propositional variables and* $\mathcal{V}$ *the set of all trace variables. Formulas of* HyperLTL *are generated by the following grammar:*

$$\begin{aligned} \psi &::= \exists \pi.\psi \mid \forall \pi.\psi \mid \varphi \\ \varphi &::= a_\pi \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc \varphi \mid \varphi \,\mathsf{U}\, \varphi, \end{aligned}$$

where $a \in \mathrm{AP}$ *and* $\pi \in \mathcal{V}$.

We denote the set of all traces by TR and the set of all trace variables by $\mathcal{V}$. For a trace assignment function $\Pi\colon \mathcal{V} \to \mathrm{TR}$, we write $\Pi[i,\infty]$ for the trace assignment defined through $\Pi[i,\infty] = \Pi(\pi)[i,\infty]$, and $\Pi[\pi \mapsto t]$ for the assignment that assigns t to π, but otherwise is identical to Π.

Definition 4 (Semantics of HyperLTL**).** *Let* $a \in \mathrm{AP}$ *be a proposition symbol,* $\pi \in \mathcal{V}$ *be a trace variable,* T *be a set of traces, and let* $\Pi\colon \mathcal{V} \to \mathrm{TR}$ *be a trace assignment.*

$$\begin{aligned}
\Pi &\models_T \exists\pi.\psi && \Leftrightarrow && \textit{there exists } t \in T\colon \Pi[\pi \mapsto t] \models_T \psi \\
\Pi &\models_T \forall\pi.\psi && \Leftrightarrow && \textit{for all } t \in T\colon \Pi[\pi \mapsto t] \models_T \psi \\
\Pi &\models_T a_\pi && \Leftrightarrow && a \in \Pi(\pi)[0] \\
\Pi &\models_T \neg\varphi && \Leftrightarrow && \Pi \not\models_T \varphi \\
\Pi &\models_T \varphi_1 \vee \varphi_2 && \Leftrightarrow && \Pi \models_T \varphi_1 \textit{ or } \Pi \models_T \varphi_2 \\
\Pi &\models_T \bigcirc\varphi && \Leftrightarrow && \Pi[1,\infty] \models_T \varphi \\
\Pi &\models_T \varphi_1 \mathsf{U} \varphi_2 && \Leftrightarrow && \textit{there exists } i \geq 0\colon \Pi[i,\infty] \models_T \varphi_2 \\
& && && \quad \textit{and for all } 0 \leq j < i \textit{ we have } \Pi[j,\infty] \models_T \varphi_1
\end{aligned}$$

Definition 5 (Universal Fragments). *The universal fragment of* HyperLTL*, denoted by* $\forall^*$HyperLTL*, is the fragment of* HyperLTL *with no existential quantification. We write* $\forall$HyperLTL *for the one variable universal fragment of* HyperLTL*, and* QHyperLTL *for the one variable fragment of* HyperLTL*.*

Definition 6 ((Positive) Boolean Closure). *The Boolean closure of a logic* $\mathcal{L}$*, denoted by* $\mathrm{BC}(\mathcal{L})$*, is the extension of* $\mathcal{L}$ *that is closed under* $\wedge$*,* $\vee$ *and* $\neg$*. The positive Boolean closure of a logic* $\mathcal{L}$*, denoted by* $\mathrm{PBC}(\mathcal{L})$*, is the extension of* $\mathcal{L}$ *that is closed under* $\wedge$ *and* $\vee$*.*

The semantics for the Boolean closures are attained by relaxing the definition of conjunction $\wedge$, disjunction $\vee$, and $\neg$ to apply to any formula of the Boolean closure.

Using a suitable algorithm, all $\mathrm{BC}(\mathcal{L})$-formulae can be equivalently expressed in disjunctive normal form, i.e. as a disjunction of conjunctions with possibly a negation in front of each formula of $\mathcal{L}$. Similarly, all $\mathrm{PBC}(\mathcal{L})$-formulae can be equivalently expressed as

$$\bigvee_{i\in I} \bigwedge_{j\in J} \varphi_{i,j}$$

for some formulae $\varphi_{i,j} \in \mathcal{L}$ and index sets I and J. From here on we use I and J to denote arbitrary index sets.

3 Correspondence between TeamLTL and HyperLTL

In this section we will explore the relationship between the logics by proving some correspondence theorems. First, however, we prove some pertinent propositions regarding the Boolean closure of HyperLTL, showing that conjunction, disjunction and negation distribute over the quantifiers in a manner analogous to first-order logic. We go through these propositions in some detail as, although they appear familiar from the first-order setting, HyperLTL is usually considered only in the prenex normal form and thus these basic results are not explicitly addressed in the literature. Moreover, the proofs feature arguments that will be useful in subsequent proofs.

As usual, for logics $\mathcal{L}$ and $\mathcal{L}'$, we write $\mathcal{L} \leq \mathcal{L}'$, if for every $\mathcal{L}$-formula there exists an equivalent $\mathcal{L}'$-formula. We write $\mathcal{L} \equiv \mathcal{L}'$, if both $\mathcal{L} \leq \mathcal{L}'$ and $\mathcal{L}' \leq \mathcal{L}$.

Proposition 7. PBC($\forall^*$HyperLTL) $\equiv$ $\forall^*$HyperLTL

Proof. Let $\bigvee_{i\in I} \bigwedge_{j\in J} \psi_{i,j}$ be an arbitrary formula of PBC($\forall^*$HyperLTL). If all $\psi_{i,j}$ are quantifier free, we are done, as then $\bigvee_{i\in I} \bigwedge_{j\in J} \psi_{i,j}$ is a $\forall^*$HyperLTL-formula. Thus, we may assume that $\psi_{i,j} = \forall\pi_1 \cdots \forall\pi_n \varphi_{i,j}$ for some LTL-formula $\varphi_{i,j}$. Suppose

$$\Pi \models_T \bigvee_{i\in I} \bigwedge_{j\in J} \forall\pi_1 \cdots \forall\pi_n \varphi_{i,j}.$$

Without loss of generality, we may assume a uniform quantifier block in each conjunct, as one can rename variables and take the largest quantifier block as the common one, since redundant quantifiers do not effect evaluation. The previous is therefore equivalent with

$$\Pi \models_T \bigvee_{i\in I} \forall\pi_1 \cdots \forall\pi_n \bigwedge_{j\in J} \varphi_{i,j}.$$

At this point, we wish to push the disjunction past the quantifier block, but the variables would become entangled and different traces could satisfy different disjuncts. We need to distinguish the variables of the disjuncts from each other, so we rename the trace quantifiers. The previous evaluation is therefore equivalent with

$$\Pi \models_T \forall\pi_1^1 \cdots \forall\pi_1^i \cdots \forall\pi_n^1 \cdots \forall\pi_n^i \bigvee_{i\in I} \bigwedge_{j\in J} \varphi_{i,j}(\pi_1^1, \cdots, \pi_n^i).$$

This is a formula of $\forall^*$HyperLTL. $\square$

The following remark, familiar from first-order logics, can be proven with a straight-forward induction over the length of the quantifier block.

Remark 8. For HyperLTL-formula $Q_1\pi_1 \cdots Q_n\pi_n\psi$ it holds that

$$\neg Q_1\pi_1 \cdots Q_n\pi_n\psi \equiv Q_1^-\pi_1 \cdots Q_n^-\pi_n\neg\psi,$$

where for every index i, Q_i are quantifiers $\forall$ or $\exists$, and Q_i^- is $\exists$ if Q_i is $\forall$ and vice versa.

Proposition 9. BC(HyperLTL) $\equiv$ HyperLTL

Proof. Consider a BC(HyperLTL)-formula $\bigvee_{i\in I} \bigwedge_{j\in J} \varphi_{i,j}$ in disjunctive normal form, with either $\varphi_{i,j} \in$ HyperLTL or $\varphi_{i,j} = \neg\psi_{i,j}$ for some formula $\psi_{i,j} \in$ HyperLTL. By Remark 8 $\neg\psi_{i,j} \equiv Q_1^{i,j}\pi_1^{i,j} \cdots Q_n^{i,j}\pi_n^{i,j}\theta_{i,j}$, where $\theta_{i,j} \in$ LTL. Thus we may assume that $\varphi_{i,j}$ only appears positively. By a similar argument to that of the proof of Proposition 7 we get the following:

$$\bigvee_{i\in I} \bigwedge_{j\in J} Q_1^{i,j}\pi_1^{i,j} \cdots Q_n^{i,j}\pi_n^{i,j}\psi_{i,j} \equiv Q_1^{1,1}\pi_1^{1,1} \cdots Q_1^{1,j}\pi_1^{1,j} \cdots Q_n^{i,j}\pi_n^{i,j} \bigvee_{i\in I} \bigwedge_{j\in J} \psi_{i,j}.$$

$\square$

One last remark before we get to the core results of this article, this time relating quantifier-free HyperLTL-formulae with LTL-formulae. The remark can again be proven by induction on the structure of the formula.

Remark 10. Let T be a team, Π be a trace assignment, π be a trace variable, φ be a LTL-formula, and let $\varphi(\pi)$ be the HyperLTL formula identical to φ, except every proposition symbol p is replaced by p_π. Suppose $\Pi(\pi) = t$ for some $t \in T$. Now the following equivalence holds

$$\Pi \models_T \varphi(\pi) \iff t \models \varphi.$$

Using the above propositions we may now proceed with proving our main results: correspondence theorems between team logics and the Boolean closures of hyperlogics.

Note that TeamLTL has no separation between closed and open formulae, and has no features to encode trace assignments. Thus, when φ is a formula of some team based logic $\mathcal{L}$ and ψ is a formula of a hyper logic $\mathcal{L}'$ without free variables, we say that φ and ψ are equivalent, if the equivalence $T \models \varphi \Leftrightarrow \emptyset \models_T \psi$, holds for all sets of traces T. The notations $\mathcal{L} \leq \mathcal{L}'$ and $\mathcal{L} \equiv \mathcal{L}'$ are then defined in the obvious way, by restricting $\mathcal{L}'$ to formulae without free variables.

Theorem 11. $\mathrm{TeamLTL}^l(\otimes) \equiv \mathrm{PBC}(\forall\mathrm{HyperLTL})$

Proof. Let T be an arbitrary team and φ an arbitrary $\mathrm{TeamLTL}^l(\otimes)$-formula. By [12, Theorem 10], we may assume that φ is in the form $\bigotimes_{i\in I} \alpha_i$, where I in an index set and α_i are LTL-formulae. We let $\alpha_i(\pi)$ denote the HyperLTL-formulae obtained from α_i, by replacing every proposition symbol p by p_π. We obtain the following chain of equivalences:

$$\begin{aligned} T \models \bigotimes_{i\in I} \alpha_i &\iff \text{there is } i \in I \text{ such that } T \models \alpha_i \\ &\iff \text{there is } i \in I \text{ such that } t \models \alpha_i \text{ for all } t \in T \\ &\iff \text{there is } i \in I \text{ such that } \emptyset \models_T \forall\pi\alpha_i(\pi) \\ &\iff \emptyset \models_T \bigvee_{i\in I} \forall\pi\alpha_i(\pi), \end{aligned}$$

where the first equivalence follows from the semantics of $\otimes$, the second equivalence holds by the flatness of α_i, the third equivalence is due to the semantics of $\forall$ and Remark 10, and the final equivalence follows from the semantics of $\vee$.

For the converse direction, consider an arbitrary PBC($\forall$HyperLTL)-sentence ψ. As noted above, ψ is equivalent to a sentence $\bigvee_{i\in I}\bigwedge_{j\in J}\forall\pi\varphi_{i,j}(\pi)$, where $\varphi_{i,j}(\pi)$, for every pair i and j, is a HyperLTL-formula with π as the only free variable. Now by an argument similar to the proof of Proposition 7, $\emptyset \models_T \bigvee_{i\in I}\bigwedge_{j\in J}\forall\pi\varphi_{i,j}(\pi)$ if and only if $\emptyset \models_T \bigvee_{i\in I}\forall\pi\bigwedge_{j\in J}\varphi_{i,j}(\pi)$. Equivalently then by the definition of the semantics of the disjunction, we may fix $i' \in I$ such that $\emptyset \models_T \forall\pi\bigwedge_{j\in J}\varphi_{i',j}(\pi)$. By the definition of the universal quantifier then we get that the previous is equivalent with $\emptyset[\pi \mapsto t] \models_T \bigwedge_{j\in J}\varphi_{i',j}(\pi)$ for all $t \in T$.

Now by Remark 10, the previous holds if and only if $t \models \bigwedge_{j\in J} \varphi_{i',j}$ for all $t \in T$, which is equivalent to $T \models^l \bigwedge_{j\in J} \varphi_{i',j}$, due to the flatness property of TeamLTL. Finally, by the semantics of the Boolean disjunction, the previous is equivalent with $T \models \bigovee_{i\in I} \bigwedge_{j\in J} \varphi_{i,j}$. □

As a corollary we get that TeamLTL$^l(\ovee)$ is subsumed by the universal fragment of HyperLTL, which follows from Theorem 11 and the observations made in the proof of Proposition 7.

Corollary 12. TeamLTL$^l(\ovee) \leq \forall^*$HyperLTL

Note that another consequence of Theorem 11 is that $\forall$HyperLTL is strictly less expressive than PBC($\forall$HyperLTL), as the former is equivalent with LTL [5] and thus has the flatness property, while the latter is equivalent with TeamLTL($\ovee$), which does not satisfy flatness. This stands in contrast to the unrestricted universal fragment $\forall^*$HyperLTL, which by Proposition 7 is equivalent to its positive Boolean closure.

Theorem 13. *left-dc*–TeamLTL$^l(\sim) \equiv$ BC(QHyperLTL) $\equiv$ BC($\forall$HyperLTL)

Proof. Let φ be an arbitrary left-dc–TeamLTL$^l(\sim)$-formula. Now by the quasi-flat normal form $T \models^l \varphi$ if and only if $T \models^l \bigovee_{i\in I} \big(\alpha_i \wedge \bigwedge_{j\in J} \exists\beta_{i,j}\big)$. Equivalently, by the semantics of $\ovee$, we may fix an index $i' \in I$ such that $T \models^l \alpha_{i'} \wedge \bigwedge_{j\in J} \exists\beta_{i',j}$. By the semantics of the logic, flatness, and the interpretation of the shorthand $\exists$, the previous evaluation is equivalent with that $t \models \alpha_{i'}$ for all $t \in T$ and for all $j \in J$ there is a $t_j \in T$ such that $t_j \models \beta_{i',j}$. By Remark 10 the previous holds if and only if $\emptyset[\pi \mapsto t] \models_T \alpha_{i'}(\pi)$ for all $t \in T$ and for all $j \in J$ there is a $t_j \in T$ such that $\emptyset[\pi \mapsto t_j] \models_T \beta_{i',j}(\pi)$. Equivalently, by the semantics of $\forall$ and $\exists$, we have that $\emptyset \models_T \forall\pi\alpha_{i'}(\pi)$ and $\emptyset \models_T \bigwedge_{j\in J} \exists\pi\beta_{i',j}(\pi)$, which, finally by the semantics of $\vee$ and $\wedge$, holds if and only if $\emptyset \models_T \bigvee_{i\in I} \big(\forall\pi\alpha_i(\pi) \wedge \bigwedge_{j\in J} \exists\pi\beta_{i,j}(\pi)\big)$.

On the other hand, let ψ be an arbitrary sentence of BC(HyperLTL). Now we get the following chain of equivalences, where $Q_{i,j} \in \{\exists, \forall\}$:

$$\begin{aligned}
\emptyset \models_T \psi \iff & \emptyset \models_T \bigvee_{i\in I} \big(\bigwedge_{j\in J} Q_{i,j}\pi\varphi_{i,j}\big) \\
\iff & \emptyset \models_T \bigvee_{i\in I} \big(\forall\pi\alpha_i \wedge \bigwedge_{j\in J} \exists\pi\varphi_{i,j}\big) \\
\iff & \text{there is } i \in I \text{ such that } \emptyset \models_T \forall\pi\alpha_i, \text{ and for all } j \in J \\
& \text{it holds that } \emptyset \models_T \exists\pi\varphi_{i,j} \\
\iff & \text{there is } i \in I \text{ such that } \emptyset[\pi \mapsto t] \models_T \alpha_i \text{ for all } t \in T, \text{ and for all} \\
& j \in J \text{ there exists } t_j \in T \text{ such that } \emptyset[\pi \mapsto t_j] \models_T \varphi_{i,j} \\
\iff & \text{there is } i \in I \text{ such that } t \models \alpha_i \text{ for all } t \in T, \text{ and for all } j \in J \\
& \text{there exists } t_j \in T \text{ such that } t_j \models \varphi_{i,j}
\end{aligned}$$

$$\iff \quad \text{there is } i \in I \text{ such that } T \models \alpha_i, \text{ and for all } j \in J \text{ it holds that } T \models \exists\varphi_{i,j}$$

$$\iff \quad \text{there is } i \in I \text{ such that } T \models \alpha_i \wedge \bigwedge_{j \in J} \exists\varphi_{i,j}$$

$$\iff \quad T \models \bigovee_{i \in I} \big(\alpha_i \wedge \bigwedge_{j \in J} \exists\varphi_{i,j}\big),$$

where the first equivalence is due to the normal form for a Boolean closure, the second equivalence is holds because the universally quantified conjuncts can equivalently be evaluated simultaneously, the third equivalence follows from the semantics of $\wedge$ and $\vee$, the fourth equivalence holds by the semantics of $\forall$ and $\exists$, the fifth equivalence holds by Remark 10, the sixth equivalences is due to flatness and the definition of the shorthand $\exists$, the seventh equivalence holds by the semantics of $\wedge$, and the final equivalence follows from the semantics of $\ovee$.

The other equivalence in the theorem is a direct consequence of Remark 8. $\square$

4 Conclusion

In this article we explored the connections in expressivity between extensions of linear temporal logic under set-based team semantics (TeamLTL^l) and fragments of linear temporal logic extended with trace quantifiers (HyperLTL). We showed that TeamLTL^l, when extended with the Boolean disjunction, corresponds to the positive Boolean closure of the one variable universal fragment of HyperLTL. Furthermore we considered a fragment of TeamLTL^l extended with the Boolean negation, where the formulae are restricted to not to contain the Boolean negation on the left-hand side of the 'until' operator (U) or under the 'always going to' (G) operator. We showed a correspondence between that fragment and the Boolean closure of the one variable universal fragment of HyperLTL. From our results it follows that the logics considered are all true extensions of LTL. Decidability of the model checking and satisfaction problems for the team based logics was shown in [12], and by our correspondence results (and the translation implied by the proofs of the theorems), decidability of the problems extends to the hyperlogics in question as well. See Table 1 for a summary of the results.

It is fascinating to see that the restriction to left downward closed formulae in the latter correspondence on the team logic side disappears on the hyperlogic side. This hints at that the fragment considered is intuitive. It is an open question whether the downward closed fragment of TeamLTL$^l(\sim)$ is TeamLTL$^l(\ovee)$, or if some restricted use of the Boolean negation could be allowed and still maintain downward closure. Another open question is whether an analogous correspondence exists for the full logic TeamLTL$^l(\sim)$, or even for some lesser restriction of the logic than the left-downward closed fragment.

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TeamLTL^l	$\overset{[12]}{\equiv}$	LTL	$\overset{[5]}{\equiv}$	$\forall$HyperLTL	$\overset{[8]}{\equiv} *$	$\text{FO}[\leq]$
$\wedge$ [12]		$\wedge\dagger$		$\wedge$ $\dagger$		$\parallel\!\mid *$
$\text{TeamLTL}^l(\otimes)$	$\overset{\text{Thm. 11}}{\equiv}$	PBC($\forall$HyperLTL)	$\overset{\text{Cor. 12}}{\leq}$	$\forall^*$HyperLTL	$\overset{\ddagger}{<}$	$\text{FO}[\leq, \text{E}]$
$\wedge$ [12]		$\wedge\dagger$				$\vee$**
$\overset{\text{left-dc-}}{\text{TeamLTL}^l}(\sim)$	$\overset{\text{Thm. 13}}{\equiv}$	BC($\forall$HyperLTL)	$\overset{\text{Thm. 13}}{\equiv}$	$\text{BC}(\text{HyperLTL}^1)$	$\overset{\ddagger}{<}$	$\text{FO}[\leq]$

Table 1. Expressivity hierarchy of the logics considered in the paper. For the definition of left downward closure, we refer to the next section. Note that the equivalence in $*$ is over traces, whereas the other relations are over sets of traces. †: Follow by transitivity from the other results. ‡: Follow by slightly modifying the proof of [7, Lemma 2]. The one variable case does not require a equal time predicate, as only one trace can be specified at a time. **: Follows by a straightforward EF game argument.

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