Article:

McHardy, J. orcid.org/0000-0003-2441-7110 (2024) Platform business models and strategic price interaction. Transportation Research Part B: Methodological, 182. 102920. ISSN 0191-2615

https://doi.org/10.1016/j.trb.2024.102920

Transportation Research Part B 182 (2024) 102920

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Platform business models and strategic price interaction

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ARTICLE INFO

JEL classification:
D21
D43
L13
R40

Keywords:
Platforms
MaaS
Strategic interaction
Transport network
Pricing

ABSTRACT

Business platforms delivering Mobility-as-a-Service ($MaaS$), alongside providing additional user services, have been shown to bring important price strategic interaction impacts which can be welfare improving. We show that the Integrator platform, whose structure alleviates harmful price externalities with two operators, such that it everywhere welfare dominates the Free-Market, can also limit beneficial competitive forces with more operators, reversing this welfare ordering. This is important since $MaaS$ envisages platforms bringing together *multiple* operators and services in practice. However, calibrations show that where the Integrator platform fails to outperform the Free-Market (with higher operator numbers and lower substitutability), a new platform-based model that we introduce, is optimal. Furthermore, the Integrator is promoted in welfare terms in the presence of non-trivial marginal costs and mid- to high-end market demand elasticities, conditions both consistent with the inclusion in the $MaaS$ journey mix of taxi cabs and other on-demand services which are common-place in $MaaS$ offerings. We also introduce a number of extensions to the modelling framework incorporating additional real network characteristics. Though this can result in the welfare supremacy of the Integrator relative to the Free-Market being undermined, it again tends to be restored under mid- to high-range market elasticity calibrations. Finally, profit incentives tend to neither align across platform providers and operators, nor promote the welfare-best regime, indicating potential for regulatory oversight.

1. Introduction

Mobility-as-a-Service ($MaaS$) is an emerging concept in the field of transport which places emphasis on matching the mobility needs of users combining transport services across multiple operators and modes. The advantages of a unified service meeting a user's travel needs by integrating various transportation elements include better accessibility and inclusivity. Additionally, this approach offers wider benefits like decreasing reliance on private cars and reducing pollution (e.g., see Jittrapirom et al., 2017).[2] Such innovations also have a potential role to play in helping drive the net-zero and levelling-up agendas in countries where regional inequalities in transport infrastructure across second-tier cities hold back economic growth and the transition away from private vehicle use.

With such a fast-developing literature around this relatively new concept, there has been some confusion and inconsistency regarding exactly what $MaaS$ constitutes. The following quote, Hensher et al. (2021, p. 153), adds important further detail about the characteristics of $MaaS$, including what it is not: "... $MaaS$ is an integrated transport service brokered by an integrator through a digital platform. A digital platform provides information, booking, ticketing, payment (as PAYG and/or subscription plans), and

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[1] I am grateful to two anonymous referees for comments and suggestions that have helped improve the paper. The usual caveats apply.

[2] However, whilst the general expectation is that $MaaS$ implementation should be broadly pro-environment, this need not be the case (e.g., see Jang et al., 2021).

https://doi.org/10.1016/j.trb.2024.102920
Received 12 April 2023; Received in revised form 27 February 2024; Accepted 29 February 2024
Available online 2 March 2024

feedback that improves the travel experience. The MaaS framework can operate at any spatial scale (i.e., urban or regional or global) and cover any combination of multi-modal and non-transport-related multi-service offerings, including the private car and parking, whether subsidised or not by the public sector. $MaaS$ is not simply a digital version of a travel planner, nor a flexible transport service (such as Mobility on Demand), nor a single shared transport offering (such as car sharing)".

One aspect of $MaaS$, that until recently, has not received much attention (e.g., see, Polydoropoulou et al., 2020, and a literature review more broadly, Maas, 2022, and Kriswardhana and Eszterglár-Kiss, 2023) concerns what business structures supporting $MaaS$ provision might look like.[3] Whilst König et al. (2016) set out and analyse the relative attractiveness and merits of alternative organisations of $MaaS$ structures, van den Berg et al. (2022) provide insights into the implications of the different pricing structures associated with each business model and their impacts on relative profit and welfare outcomes. Both studies recognise the importance of potential differences across $MaaS$ provision in terms of the organisation of the supply chain for mobility services including which agents are involved in setting prices.

A major contribution of van den Berg et al. (2022) is to conceptualise a variety of $MaaS$ business platform models, principally the Intermediary, Platform (we denote this the Passive platform) and Integrator models, within a framework which permits examination of the impacts of the differing associated strategic incentives in terms of relative regime performance.[4,5] In common with wider practice in transport studies in recent times, they embed their alternative $MaaS$ regimes in an (Economides and Salop, 1992)-type network model, essentially extending the model to allow a new agent which provides a user platform for purchasing combined complementary multi-operator services on the network.[6] This network setting envisages transport operators competing in the provision of horizontally differentiated services that have rival and interchangeable parts, capturing important countervailing strategic interactions often evident in real-world transport networks. Their central result is that the Integrator platform offers to everywhere improve the welfare outcomes on the network relative to a Free-Market case based upon its pricing configuration alone. Moreover, if services are not too highly substitutable, then the Integrator platform also improves on profit. As such, they demonstrate an important potential benefit arising from this platform business model in terms of solving a strategic interaction problem that can be present in multi-operator transport settings under the Free-Market.[7]

However, as is typically the case in models using this framework in a transport setting, the number of agents is assumed fixed at $n = 2$, giving rise to a four-link transport network. McHardy (2022) demonstrates that the countervailing strategic forces present in such a network make the $n = 2$ model susceptible to predictions that do not generalise well to larger networks with more operators. Principally, increasing n (with an associated n^2-link network) can change the balance of the countervailing forces, and in different ways under different regimes, potentially reversing the orderings of these regimes in profit and welfare terms. They also show that allowing a larger number of services in the modelling framework can facilitate the investigation of real-world transport network characteristics which are not possible in the $n = 2$ setting but which can have further implications for the relative performance of different regimes. For instance, the $n = 2$ model assumes rival operators each provide only a single service i.e., one route with unit frequency. In practice, route numbers and/or service frequencies are choice variables for operators which, if included in the analysis, can result in different regimes supporting different network sizes, again with the potential to reorder regime performance rankings.

In this paper we extend the analysis of van den Berg et al. (2022) to the n-operator setting and examine how increasing n beyond two can change the relative performance of the three main $MaaS$ platform business models outlined therein. Given $MaaS$ explicitly concerns the integration of services across *multiple* providers,[8] the potential for non-monotonic impacts of n on relative regime performance suggests this exercise is both relevant and has the potential for important new insights. Indeed, we show that the welfare dominance of the Integrator model over the Free-Market everywhere for $n = 2$, based solely on price effects, does not generalise. This is important because it means that a policy move from the Free-Market to a platform model may not be justified on pure price strategy effects alone.

Our main methodological contribution is to extend the analysis of the business models, introduced in van den Berg et al. (2022) under a two-operator setting, to the n-operator case. We denote this n-operator framework the Base Model. We also introduce a series of extensions to the Base Model, including other common transport network characteristics such as frequency choice, single (domestic) demands, a larger number of components in an origin–destination (OD) journey and generalised cost benefits under the

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3 Until quite recently, the main contributions around $MaaS$ have been focused on categorisation and analysis of existing and pilot $MaaS$ projects (e.g., see for instance König et al., 2016; Ebrahimigharehbaghi et al., 2018), and the $MaaS$ supply chain and wider operating environment, its agents and their functions (e.g., see Kamargianni and Matyas, 2017). More recently, Hörcher and Tirachini (2021, p. 21) observe "... most of the MaaS proposals are not supported with solid theoretical and empirical evidence, and therefore we see substantial room for future contributions in this field".

4 They also consider two other regimes, independent operators and a one-sided regulation case, which relate directly to scenarios outlined in Economides and Salop (1992), but are not central to their findings.

5 According to König et al. (2016) whilst the Intermediary (they term this the reseller) may be most suited to national and international travel, the Integrator has broad suitability across international as well as urban and suburban travel. van den Berg et al. (2022) identify examples of the Intermediary, e.g., Whim, the Platform, e.g., De Lijn (akin to Airbnb, Booking, eBay, Amazon or AliExpress), and, potentially, Transdev, Keolis and Veolia, amongst others, under the Integrator banner.

6 This modelling framework has been widely employed in the transport literature with many examples of work explicitly using it or nested within it (e.g., see Shy, 1996; Lin, 2004; McHardy et al., 2012; Bataille and Steinmetz, 2013; Silva and Verhoef, 2013; Socorro and Viecens, 2013; van den Berg, 2013; Clark et al., 2014; D'Alfonso et al., 2016; van den Berg et al., 2022).

7 See van den Berg et al. (2022) and references therein for an explanation of how other studies address related areas in this field but do not study the price-based issues central to their analysis and findings.

8 Mulley et al. (2023) report examples of MaaS platforms with up to 8 different modes, where some will have more than one operator.

platform regimes. In addition, we introduce and analyse a new platform business model, based on the Multi-operator Ticketing Card (MTC) pricing arrangement, currently permitted under a block exemption in the UK, which is under statutory review and due to expire in 2026.

In the Base Model, we show that the uniform welfare dominance of the Integrator platform over the Free-Market under $n = 2$ is not robust to increasing n, under-performing the latter for higher n at low levels of substitutability. However, a calibration exercise has the Integrator and MTC sharing the status of best-performing regime in welfare terms, favouring the former where n is not too high and the market demand elasticity is in the mid- to high-range of industry estimates. Profit incentives, nevertheless, do not typically align to promote the best welfare-performing regime. Indeed, the Intermediary model is often the most attractive for the platform providers whilst performing poorly from a welfare perspective.

We also show that relative regime performance is impacted under each of our extensions to the Base Model. For instance, in most cases the welfare dominance of the Integrator over the Free-Market is shown to not be robust, even for $n = 2$. However, applying the calibration with mid- to high-range industry elasticity estimates tends to restore the Integrator's welfare dominance, including in the case of non-zero constant marginal cost.[9] This suggests that the inclusion of taxi cabs and other on-demand services (commonplace in $MaaS$ offerings) in the transport mix, with associated non-trivial marginal costs and typically higher-end demand elasticities, would tend to elevate the welfare performance of the Integrator relative to the Free-Market.

In line with van den Berg et al. (2022), our focus is mainly on price implications of the various platform models, ignoring the wider potential benefits of the $MaaS$ platform models. Given the expected benefits of $MaaS$ include improving the user experience via additional services such as journey planning and simplified ticketing (with associated surplus gains), we would expect that their inclusion in the model would unambiguously favour platform- over non-platform regimes. We briefly consider this issue and show that, whilst this can be the case, it need not be so.

The following Section introduces the Base Model, focusing solely on network price effects in a simple two-component journey framework but with n independent operators. It sets out the network strategic interaction problem, identified above, which the platform can help address and which underlies its potential performance advantages over the Free-Market, as indicated in van den Berg et al. (2022). Section 3 derives the equilibria in the three platform regimes, along with a new platform business model. Section 4 analyses the regimes and sets out the key findings under the Base Model. Section 5 considers how a number of extensions to the Base Model can impact the relative performance of the regimes. Section 6 concludes.

2. The base model

Consider a transport network where all users combine an x and y journey component in their OD travel and there are n operators, with operator i offering component pair $\{x_i, y_i\}$, which can be combined in the in-service journey, J_{ii}.[10] Operator j ($j \neq i = 1, \dots, n$) similarly offers a horizontally differentiated component pair $\{x_j, y_j\}$. Hence, passengers can choose to undertake one of n differentiated OD journeys with a single operator. These services are imperfect substitutes, where differentiation of services can be temporal and/or spatial. For instance, passengers can choose between operator 1's bus service, J_{11}, and operator 2's tram service, J_{22}, both providing equally appealing outward (x_i) and return (y_i) journey combinations at different times and/or locations. That is to say, the service times of the two modes and ingress/egress to the bus and tram stops are equally convenient. Journeys where the y_i component represents an onward (rather than return) journey have similar interpretation with associated interchange costs being assumed common across services.[11]

Let P_{ii} be the price for the OD journey J_{ii} with operator i. By the above assumptions, and given throughout we will assume linear demand and, for the most part, constant (zero) marginal cost, the relationship between two rival operators' prices is strategic complements. Operators' prices are increasing in the OD price of a rival operator, i.e., best response functions in prices will be upward sloping: $\frac{\partial P_{jj}}{\partial P_{ii}} > 0$. Therefore, amongst rival operators we have a situation of differentiated Bertrand competition: strategic interaction across the in-service OD prices places downward pressure on those prices, and increasing the number of operators, n, will tend to intensify price competition resulting in lower prices.

If we allow passengers to undertake J_{ij} OD journeys, combining operator i's x and operator j's y components, then we have an additional $n(n-1)$ differentiated cross-service OD journey options. For instance, a passenger might take bus service x_1 from the suburbs to city centre and onward train service y_3 to another city, comprising OD journey J_{13}. This offers a different combination of outward and onward times from, say, the reverse OD journey J_{31}. As before, ingress, egress and interchange generalised costs are assumed symmetric across services, with the latter consistent with a common interchange area. In line with van den Berg et al. (2022), passengers make full OD travel decisions before departure under certainty (e.g., there is no uncertainty around travel needs and departure and arrival times), hence there is no updating of plans after the x component of travel regarding y component choice.

[9] As we will see, fee-charging platform models taking a share of revenue rather than profit are not neutral to the introduction of constant marginal cost.

[10] We extend this two-component journey assumption of the Base Model to a three-component journey in Section 5.3.

[11] The generalised costs associated with ingress, egress and interchange are captured in the parameter α, introduced below, which for the most part we treat as symmetric across OD journeys. This symmetry is relaxed in Section 5.4.

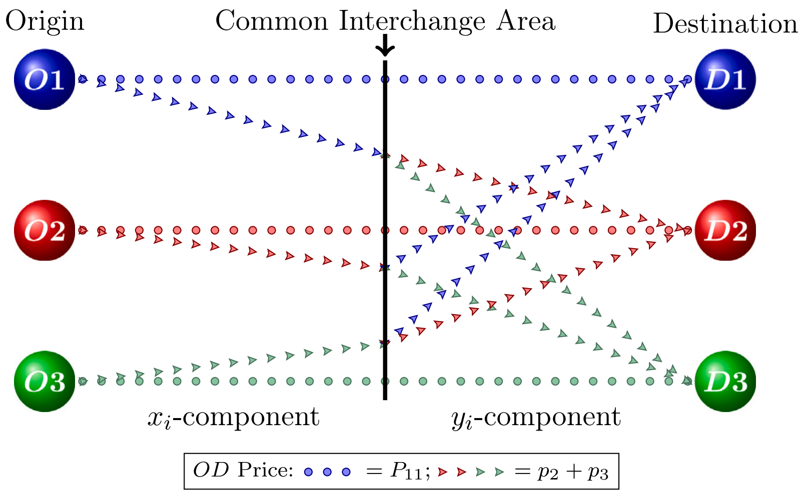


Fig. 1. Free-Market Ticket Configuration with 3 Operators.

Let p_i be the price set by operator i for travel on either one of its component journeys x_i or y_i.[12] Hence, cross-service OD journey J_{ij} has price $P_{ij} = p_i + p_j$. Fig. 1 illustrates the pricing structure under these Free-Market (non-platform) conditions in the case of just three operators.[13] Passengers can take one of three in-service journeys with price P_{ii} (coloured circles) or one of six combinations of cross-component services with OD price $p_i + p_j$ (coloured darts), from three (geographically and/or temporally) distinct origins feeding into three distinct destinations with interchange in each case around some common interchange area.

On the one hand, since the cross-service journeys are substitutes amongst themselves (i.e., J_{12} is a substitute for J_{31}) but also substitutes with respect to the n in-service journeys (i.e., J_{12} is a substitute for J_{33}), their OD prices P_{ij} are strategic complements with respect to all other OD prices: the cross-service journeys add to the downward pressure on OD prices. On the other hand, the x and y component prices have a complementary relationship which is characterised by strategic substitutes. Operators' component prices are decreasing in the price of a rival's component price, i.e., component price best response functions are downward sloping: $\frac{\partial p_j}{\partial p_i} < 0$. Therefore, amongst rival operators we have a situation where strategic interaction places upward pressure across component prices, and increasing the number of operators will tend to intensify these upward price pressures.[14] Since the cross-service journeys are substitutes for in-service journeys, the upward pressure on cross-service prices, through strategic substitutes, to some extent counteracts the downward (strategic complement) pressure on in-service prices.

This is the basis for what we will term the Network Problem. Hence, in transport networks where services have substitute and interchangeable aspects, independent price-setting can involve two countervailing strategic interactions, acting on prices in opposing directions. As a result, Free-Market price-setting in a network can yield in-service prices below, but cross-network prices above, the monopoly level. This can result in welfare outcomes worse than monopoly (e.g., see McHardy, 2022). Essentially, in the presence of monopoly, the socially harmful strategic substitute externalities and socially beneficial strategic complement externalities are both internalised. It can be shown that where the services are not close substitutes, internalising the harmful strategic substitute externalities can have the dominant effect with monopoly improving welfare relative to a Free-Market outcome with n independent operators. Indeed, this is the route through which the gains under platform models in van den Berg et al. (2022) can arise relative to the Free-Market: harmful cross-price strategic substitute externalities are internalised by the platform.

[12] With symmetric demand and constant marginal OD service cost, it is straightforward to show that equilibrium OD price is invariant to the a priori assumption of symmetric x and y component prices.

[13] As we will see, the Free-Market regime and Passive platform stand out as the only cases we consider where there is no capacity to co-ordinate cross-service prices.

[14] This upward pressure on prices through the strategic substitute channel is what gives rise to double marginalisaton. To illustrate, in a Cournot (1838) oligopoly game with outputs as strategic substitutes the aggregate Nash equilibrium output is above the monopoly level. By the same reasoning, in a Cournot (1838) complementary monopoly game with prices as strategic substitutes, the Nash equilibrium composite (aggregate) price exceeds the monopoly level (e.g., see Sonnenschein, 1968, who pointed out this equivalence of the two models). Given strategic complement, as well as strategic substitute, effects are present throughout the paper we adopt this nomenclature rather than double marginalisation which applies to just one of the channels of strategic interaction. Further, just as more independent agents in the Cournot (1838) oligopoly quantity game tends to lead to increases in aggregate quantity, so an increase in the number of independent agents in Cournot (1838) complementary monopoly tends to yield a higher composite price. Where relevant, we do make the distinction between parallel and serial double marginalisation across regimes, but otherwise refer to them collectively under strategic substitutes.

Let operator i's component pair $\{x_i, y_i\}$ be symmetrically differentiated from each of the other $\{x_j, y_j\}$ pairs according to the linear demand system[15]:

$$Q_{ii} = a - bP_i + d\left[\sum_{k\neq i}^{n} P_k + 2\sum_{k\neq i}^{n}(p_i + p_k) + \sum_{m\neq i}^{n}\sum_{k\neq m,i}(p_m + p_k)\right] \tag{1}$$

$$Q_{ij} = a - b(p_i + p_j) + d\left[P_i + \sum_{k\neq i}^{n} P_k + \sum_{k\neq i,j}^{n}(p_i + p_k) + \sum_{k\neq i}^{n}(p_i + p_k) + \sum_{m\neq i}^{n}\sum_{k\neq m,i}(p_m + p_k)\right]$$

We underpin this demand system with a quasi-linear utility function, employed in many transport and industrial economics studies (e.g., see Hackner, 2000; Silva and Verhoef, 2013)[16]:

$$U(\boldsymbol{Q}, M_0) = \alpha\sum_{t}^{N} Q_t - \frac{1}{2}\left[\sum_{t}^{N} Q_t^2 + 2\gamma\sum_{t\neq r}^{N} Q_t Q_r\right] + M_0 \quad (r \neq t = 1, \ldots, N) \tag{2}$$

where $N \equiv n^2$ is the total number of OD services on the network, α is a shift parameter positively impacting the intercept a in each OD demand, $\gamma \in [0, 1]$ represents the degree of substitutability between OD service pairs (with independent services and perfect substitutes under $\gamma = 0$ and $\gamma = 1$, respectively), $\boldsymbol{Q}$ is the N-vector of OD quantities and M_0 is a composite of all other goods. This utility specification incorporates network service density effects and imposes:

$$a \equiv \frac{\alpha}{(1 + \gamma(N - 1))}, \quad b \equiv \frac{1 + \gamma(N - 2)}{(1 - \gamma)(1 + \gamma(N - 1))}, \quad d \equiv \frac{\gamma}{(1 - \gamma)(1 + \gamma(N - 1))} \tag{3}$$

Service density effects arise as, at given prices, additional $\{x, y\}$ pairs bring additional consumer surplus rather than resulting in a pure redistribution of a fixed surplus. This means that adding new operators with new services has two potential channels to impact surplus: through density effects and altering price competition.

Regarding costs, marginal costs in transport settings are often treated as constant (e.g., see Clark et al., 2014), for which there is empirical justification, including the assumption of zero marginal costs (e.g., see Jørgensen and Preston, 2003). With constant marginal cost, it is straightforward to show that the distribution of costs across the x and y component journeys here is, in all cases, irrelevant to the equilibrium OD price, which drives all key results. We have noted the fee-charging platform regimes studied here are non-neutral to the inclusion of constant marginal costs where the fee is a share of revenue rather than profit. We later see that, other than reinforcing one of our central results, the introduction of non-zero constant marginal cost has limited qualitative impact our main findings. However, the inclusion of an extra parameter to incorporate costs adds complexity including non-interior solutions making analytical results hard to obtain. Hence, in line with van den Berg et al. (2022), we assume zero marginal costs in the Base Model.[17]

To summarise, the Base Model is characterised by the following Assumption:

Assumption 1. The Base Model assumes: (i) n independent operators, where each supplies a single x and y component, (ii) OD journeys each have two components, x and y, consumed in fixed proportions, but interchangeable across operators, with full OD journey plans made before departure under information certainty, (iii) a symmetric and linear differentiated demand system, (iv) underpinned by a utility function incorporating service density effects, (v) zero constant operator marginal costs.

These assumptions align with van den Berg et al. (2022), against which we initially seek to make direct comparisons. Unless otherwise indicated, all results are derived under Assumption 1.

3. Regimes

In this Section we present, under the Base Model (Assumption 1), the three main platform regimes in van den Berg et al. (2022) alongside a Free-Market case and a stylised MTC, as introduced earlier.

[15] This demand model imposes symmetry in substitutability across all OD services. Amongst other things, this implies that option J_{ij} is an equally good substitute as J_{km} relative to, say J_{ii}, which shares a common component with the former but not the latter. Whilst one might expect journeys with a shared component to be closer substitutes, the symmetric demand model is standard in applications of Economides and Salop (1992)-style studies, including van den Berg et al. (2022). Incorporating asymmetries would involve additional model complexity, reducing the scope for analytical results. However, as it will not change the direction of the strategic forces at play it is unlikely to materially change the main thrust of findings. A brief robustness check in Appendix F does not reveal qualitative sensitivity here. Finally, it is not obvious that having a shared component necessarily makes journeys closer substitutes. Let J_{ii} comprise an outward 9am and return 11am journey: a short round-trip to the shops. If the individual has a preference for a short trip, ideally in the morning, but a short trip in the afternoon is a good alternative, then J_{jj} providing a 2pm–4 pm round trip might be a better substitute for J_{ii} than J_{ij} which involves a 9am–4 pm round-trip. Also implicit in this demand structure is the assumption that passengers do not suffer uncertainty, for instance, about whether a service will be on time or their own plans will change during the course of the OD journey. Hence, full OD journey selections are made before travelling. There is no updating of journey plans at the mid-way stage.

[16] This is classed as a Spence (1976)-Shubik and Levitan (1980)-type utility function (see Choné and Linnemer, 2020).

[17] We introduce non-linear marginal costs and a fixed cost per $\{x, y\}$ pair in Section 5.1 and then consider the welfare and cost implications of non-zero constant marginal costs on the non-cost-neutral regimes in Section 5.5.

3.1. Free-market

Beginning with the Free-Market regime, operators set their in-service price and cross-service price components simultaneously and independently, with price structure under three operators as illustrated in Fig. 1. This represents the market in a situation where there is no platform or construct for organising cross-service prices. Given the demand symmetry assumed here, each operator receives half of the revenue on each of its cross-service operations. Hence, operator i solves the problem:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + p_i \sum_{j\neq i}^{n}(Q_{ij} + Q_{ji}) \quad (i \neq j = 1, \ldots, n)$$

which results in the equilibrium in- and cross-service prices (where convenient, we exploit symmetry and denote in- and cross-service prices as P and P_x, respectively, with quantities treated accordingly)[18]:

$$P^{FM} = \frac{3\alpha(1-\gamma)}{6+(2n^2-3n-5)\gamma}, \quad P_x^{FM} = \frac{4\alpha(1-\gamma)}{6+(2n^2-3n-5)\gamma} \tag{4}$$

As outlined above, the Free-Market regime experiences both strategic externalities in the Network Problem such that, whilst the in-service price lies below the monopoly level, the cross-service price can exceed the monopoly level for sufficiently low n and γ (see McHardy, 2022).

3.2. Integrator platform

In keeping with van den Berg et al. (2022), pricing in the Integrator model is simultaneous. The platform provider sets the prices of all cross-services. Given symmetry, in equilibrium this price is common across all cross-services and, without loss of generality, we denote it P_x, a priori. The platform takes a share $\phi \in [0,1]$ of all revenues on these cross-service journeys.[19] The operators simultaneously and independently set their in-service prices. Fig. 2 illustrates the price structure under the platform set-up with three operators. Contrasting with the Free-Market case in Fig. 1, where there are multiple cross-service prices made of independently-set single-component prices, there is now (given symmetry) a single cross-service price (yellow circles) set by the platform across all cross-services. We will see later that this control of cross-prices by a single agent will have potentially important implications for the relative performance of the two models.

Each operator i solves the following problem[20]:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + \frac{1}{2}(1-\phi)P_x \sum_{j\neq i}^{n}(Q_{ij} + Q_{ji})$$

The platform provider simultaneously solves the problem:

$$\max_{P_x} \pi_{plat} = \phi P_x \sum_{j\neq i}^{n} Q_{ij} \tag{5}$$

which results in the equilibrium in- and cross-service prices:

$$P^I = \frac{\alpha(1-\gamma)[2+\gamma(n-1)(3+n-\phi)]}{\Delta}, \quad P_x^I = \frac{\alpha(1-\gamma)(2\gamma n^2-3\gamma+2)}{\Delta} \tag{6}$$

where $\Delta \equiv 4 + \gamma^2(n-1)\{[3n^2-6-n(3-\phi)]\} + 2\gamma(2n^2+n-5)$. Analysis of the equilibrium prices yields the following Proposition.

Proposition 1. [21] *Under the Integrator, (i) in- and cross-service prices are equal under $\gamma = 0$, and, (ii) in-service (cross-service) prices are strictly (weakly) falling in the Integrator's share of profit, ϕ: $P^I_{\phi} < 0, P^I_{x\,\phi} \leq 0$.*

The intuition behind this can best be understood with respect to the Network Problem outlined above. We know that in the Free-Market outcome, cross-service prices can exceed monopoly levels, reducing downward pressure on in-service prices and yielding welfare outcomes worse than monopoly. Here, though, the Integrator platform provider is internalising the strategic substitute externality which otherwise raises cross-service prices. It is also taking a share of the associated profit, which alters the balance of the channels through which operators earn their rewards, increasing the relative importance of in-service revenue as ϕ increases. As we have seen above, in setting its in-service price, operator i knows that, taking other prices as given, a reduction in P_{ii} will yield a market share gain relative to other in- and cross-service operations, gaining it profit through its in-service revenues but damaging its profit through lost revenues on its cross-services. The latter effect therefore injects a resistance against incentives for in-service

[18] Base Model equilibria derivations for each regime are reported in Appendix B.

[19] Whilst ϕ close to or equal to 1 lacks plausibility, where possible we produce results across the full parameter set for completeness. Support for ϕ being low and around 2% is found, for example, in van den Berg et al. (2022, p. 212) "Informal statements by people involved in the negotiations reveal such shares have been discussed. This share appears in the discussions between transport operators and MaaS service providers, ..." and "Transport providers in Belgium offer third parties selling their mobile tickets fixed fee of 6 eurocents per ticket or about 2% of the ticket price".

[20] Note, the quantities in the Integrator problem are based on the Base Model quantities in Eq. (1), with component prices, $(p_i + p_j)$, replaced with P_x.

[21] Where appropriate, proofs are reported in *Appendix* A.

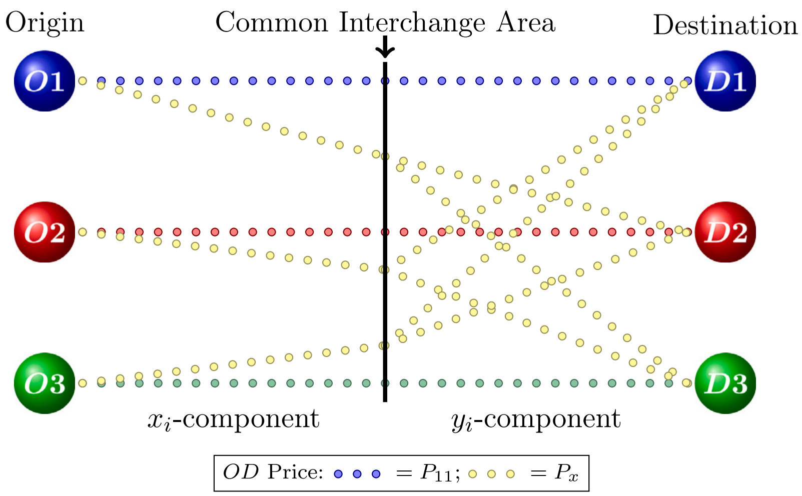


Fig. 2. Platform Ticket Configuration with 3 Operators.

price-cutting. As the Integrator platform provider takes a larger share of the cross-service profit, so it insulates the operator from this resistance that would otherwise result in lower in-service prices and lower prices more generally given all OD services are substitutes. The following result encapsulates this in consumer surplus and welfare terms.

Corollary 1. *Under the Integrator, consumer surplus and welfare are weakly increasing in the Integrator platform provider's share of revenue, ϕ: $W^I_\phi, CS^I_\phi \geq 0$.*

The following Proposition confirms that the lower prices under the Integrator with higher levels of ϕ result in lower aggregate profit, which is not as straightforward as might be expected.

Proposition 2. *Under the Integrator, aggregate profit is weakly decreasing in the Integrator platform provider's share of profit, ϕ: $\pi^I_\phi \leq 0$*

To understand why the above result is not necessarily obvious, note that under the Free-Market for sufficiently low levels of n and γ, cross-service prices exceed the Monopoly level (see McHardy, 2022), and hence reducing cross-network prices can increase profit. However, here the platform prevents cross-service prices exceeding monopoly levels, even with $\phi = 0$, and any further reductions in prices are moving strictly away from monopoly levels reducing aggregate profit.

Note, the Integrator and other platform regimes are likely in practice to have benefits beyond the pricing ones captured here, e.g., reducing the generalised cost of cross-service travel. Though we abstract away from such potential platform benefits in the analysis of the next Section, we explore the possible impact of these considerations on welfare in Section 5.4.

3.3. Passive platform

In this platform regime, the operators set prices as in the Free-Market case, but pay a share ϕ of cross-service revenue to the platform provider. The platform provider combines the operator's cross-service component prices without engaging in strategic price-setting.[22] Operator i solves the problem:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + (1-\phi)p_i \sum_{j\neq i}(Q_{ij}+Q_{ji}) \quad (i \neq j = 1,\dots,n)$$

which results in equilibrium in- and cross-service prices:

$$P^P = \frac{3\alpha}{\Delta_1}(1-\phi)(1-\gamma)\left[1+\gamma(n-1)\left(n+1-\frac{2\phi}{3}\right)\right], \qquad (7)$$
$$P^P_x = \frac{4\alpha}{\Delta_1}(1-\gamma)\left[1-\phi+\gamma\left((1-\phi)n^2+\frac{3\phi}{2}-1\right)\right]$$

where $\Delta_1 \equiv (1-\phi)\left\{6+\gamma(n+1)(8n-11)+2(n-1)\gamma^2\left[n^3+\frac{n^2}{2}+n\frac{(2-\phi)^2}{1-\phi}+\frac{5}{2}\right]\right\} \geq 0$. It is straightforward to show that prices here are not monotonic in ϕ. However, note that under this regime the interior solution and second-order conditions are not met across the full parameter set (even for $n = 2$), specifically for higher levels of ϕ. Given the earlier evidence suggests interest is likely to centre around low levels of ϕ, all future results including the Passive platform are limited under the following assumption:

[22] van den Berg et al. (2022) denote this the "Platform" regime. Since we apply the term platform to distinguish $MaaS$ from non-$MaaS$ regimes, we rename this case the Passive platform, reflecting the passive role that the provider has, taking a share of revenue but not engaging in setting prices.

Assumption 2. To ensure internal solutions and that the second-order condition for the Passive platform is satisfied, let $\phi^P \in \left[0, \frac{1}{3}\right)$.

Like the Free-Market case, the Passive platform is subject to both strategic externalities, whereas the Integrator internalises the strategic substitute externality. However, as a platform regime it has potential benefits such as reducing the generalised cost of cross-service travel relative to the Free-Market, as discussed above, and again we will analyse the potential for this to materialise in welfare in Section 5.4.

3.4. Intermediary platform

In this model the operators set their prices as in the Free-Market case but the cross-service component prices that they set are not the final prices. Rather, they are the prices they charge the platform provider, and the latter takes these input prices and sets the cross-service prices to maximise its profit across all cross-service sales.[23] Hence, we have a two-stage process with (i) operators setting all their prices in the first stage, and, (ii) the platform provider setting the cross-service prices at the second stage, taking as given the operators' first-stage prices. By backward induction, the Intermediary platform provider solves the following problem in stage two[24]:

$$\max_{\boldsymbol{P}_{ij}} \pi_{plat} = \sum_{j\neq i}^{n} (P_{ij} - p_i - p_j) Q_{ij}$$

taking all P_{ii} and p_i as given, where $\boldsymbol{P}_{ij}$ is the $n(n-1)$-vector of cross-service prices. This results in the first-order condition:

$$\frac{\partial \pi_{plat}}{\partial P_{ij}} = Q_{ij} - b[P_{ij} - p_i - p_j] + d \sum_{k\neq i,j}^{n} [P_{ik} - p_i - p_k] + d \sum_{k\neq i}^{n} [P_{ki} - p_i - p_k] + d \sum_{m\neq i}^{n} \sum_{k\neq m,i} [P_{mk} - p_m - p_k] = 0$$

Taking the total differential across all $n(n-1)$ first-order conditions, we have the following optimising Intermediary stage-two responses to a change in stage-one prices[25]:

$$\frac{\partial P_{ij}}{\partial P_{ii}} = \frac{\partial P_{mk}}{\partial P_{ii}} = \frac{d}{2[b - d(n^2 - n - 1)]}, \quad \frac{\partial P_{ij}}{\partial p_i} = \frac{1}{2}$$

Taking these as given in the first stage, operator i solves the following problem:

$$\max_{P_{ii}, p_i} \pi_i = P_{ii} Q_{ii} + p_i \sum_{j\neq i}^{n} (Q_{ij} + Q_{ji}) \tag{8}$$

yielding equilibrium input component price, and final in- and cross-service prices, respectively:

$$p^{IM} = \frac{4\alpha(1-\gamma)}{\Delta_2}(1 + \gamma(n-1)), \quad P^{IM} = \frac{\alpha(1-\gamma)}{\Delta_2}[6 + \gamma(n^2 + 3n - 4)],$$
$$P_x^{IM} = \frac{2\alpha(1-\gamma)}{\Delta_2(1 + \gamma(n-1))}\left[20 + (4n^2 + 7n - 16)(n-1)\gamma^2 + (4n^2 + 28n - 38)\gamma\right] \tag{9}$$

where $\Delta_2 \equiv 12 + \gamma^2(3n-8)(n^2-1) + 2\gamma(2n^2 + 3n - 11)$.

3.5. Multi-operator ticket card

We now briefly introduce an alternative pricing system that currently operates in the UK allowing operators to jointly set cross-service prices under rules permitted by the Public Transport Ticketing Scheme Block Exemption (Competition Commission, 2001), henceforth Block Exemption. This is an interesting regime since, as McHardy (2022) shows in a model akin to our Base Model, it everywhere dominates the Free-Market in welfare terms. The recommended pricing rule set out by the Competition Commission (see Department for Transport, 2013, pp. 22) for cross-service tickets is:

P_x = "Average or median single fares x Estimated [typical] ticket usage x Passenger discount for purchasing a multi-journey ticket" (10)

and hence the cross-service price is set as a discount (agreed by the operators) on the weighted average of the prevailing in-service prices on the network. The simplified characterisation of this pricing structure in McHardy (2022) is employed, whereby we assume: (i) a single-OD journey rather than multi-OD journeys on the card, and, for now (ii) no potential transaction cost benefits of the scheme (e.g., gains from the need to buy only a single cross-service ticket rather than two individual component tickets and associated travel flexibility).[26] Additionally, (iii) we assume a zero discount (ensuring, in the symmetric model, that the cross- and in-service prices are the same). However, we include the MTC as a potential platform product with a passive platform provider

[23] Although the equilibrium results in symmetric cross-service prices, this cannot be imposed for simplicity a priori.

[24] Note, the quantities in the Intermediary problem are based on the Base Model demands in Eq. (1), with component prices, e.g., $(p_i + p_j)$, replaced with P_{ij}.

[25] This yields the two-stage game's closed-loop solution (e.g., see Fudenberg and Tirole, 1991, pp. 132).

[26] The Block Exemption requires average journeys of three or more, but since we do not model here any potential benefits arising from non-price factors, this assumption comes at no further loss of generality.

receiving a share ϕ of cross-service revenues but not being active in setting prices. Operators simultaneously set in-service prices, knowing the cross-service price will be set according to Eq. (10). Hence, operator i now solves the problem[27,28]:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + (1-\phi)(n-1)P_xQ_x$$

taking as given that P_x is the weighted average of the in-service prices across the n operators, resulting in the uniform equilibrium price:

$$P^M = P_x^M = \frac{\alpha[n(2-\phi)+\phi-1](1-\gamma)}{(\gamma n^3 - \gamma n^2 + 2\gamma n\phi - 4\gamma n - 2\gamma\phi - 2n\phi + 2\gamma + 4n + 2\phi - 2)} \tag{11}$$

Proposition 3. *The uniform price under the MTC is weakly decreasing in ϕ.*

Corollary 2. *Consumer surplus and welfare are weakly increasing under an MTC delivered by a passive platform as the share of revenue to the platform provider increases.*

Hence, even in the absence of non-price benefits of the platform, a passive platform delivering the MTC is weakly welfare enhancing (relative to the MTC with no platform), more so for a higher share of the revenue to the platform provider. McHardy (2022) shows that the non-platform MTC (i.e., $\phi = 0$), is strictly superior to the Free-Market in consumer surplus and welfare for $\gamma \in [0, 1)$. In light of Corollary 2, this gives rise to the following result.

Corollary 3. *The platform MTC is strictly superior to the Free-Market in consumer surplus and welfare terms for $\gamma \in [0, 1)$.*

4. Analysis

In this Section we provide analysis of the Free-Market and platform regimes under the Base Model (Assumption 1).

4.1. Free-market versus integrator

We begin by considering aspects of the relative performance of the Integrator regime compared with the Free-Market. In van den Berg et al. (2022) Proposition 1 (under $n = 2$), it is found that the Integrator model is everywhere superior to the Free-Market in consumer surplus and welfare terms. In essence, the Integrator, by internalising the externality that forces cross-service prices up under the Free-Market, benefits consumers and welfare. The following Proposition considers how the price story underlying this result generalises.

Proposition 4. *Whilst under the Integrator with $n \in \{2, 3\}$, in-service (cross-service) prices are weakly (strictly) lower than their equivalents under the Free-Market, the relative size of prices is ambiguous for larger n, and there exist open-intervals of parameter combinations (n, γ, ϕ), with sufficiently high γ (low ϕ), supporting a reversal in the ordering of in- and cross-service prices.*

Corollary 4. *Whilst the Integrator strictly dominates the Free-Market on consumer surplus and welfare terms under $n \in \{2, 3\}$, the reverse ranking arises for $n \geq 4$ on some selection of γ and ϕ.*

Thus the consumer surplus and welfare superiority of the Integrator model over the Free-Market in van den Berg et al. (2022) is not robust to a larger network: the policy prescription that moving from the Free-Market to an Integrator model is welfare enhancing on price-based measures (not including wider potential platform benefits) does not generally hold.

In terms of the intuition behind this result, whilst the Integrator internalises the strategic substitutes externality across component prices (e.g., preventing the cross-service prices exceeding the monopoly level) the negative effects of this externality are most strong when n is small and there is a low level of substitutability between the differentiated services. However, in addition, with the Integrator acting as a single agent setting all cross-service prices, it is able to stifle pro-competitive (strategic complement) effects between cross-service OD prices. Under the Free-Market, for higher n and higher levels of substitutability, these interdependencies amongst the cross-service prices naturally add to the downward competitive pressure on prices across the network. Effectively, the Integrator, which is useful at countering the damaging strategic substitute cross-service price effects with low n and γ, can become relatively harmful by also curtailing the strategic complementarity effects amongst the cross-service OD prices when n and/or γ are not low.

Regarding aggregate profit, it is straightforward to show that neither regime dominates in the case of $n = 2$ or more generally. However, the following Proposition indicates that, regardless of n, if the degree of substitutability is sufficiently low then the Integrator regime has a particular attraction.

[27] For clarity, the second term in the maximand reflects the fact that each operator has an equal share (half) of the revenue on each of its cross-service routes after the platform has taken its cut: $\frac{1}{2}(1-\phi)P_xQ_x$, and each operator offers a component on $2(n-1)$ of these services.

[28] Note, the quantities in the MTC problem are based on the Base Model demands in Eq. (1), with component prices, $(p_i + p_j)$, replaced with P_x.

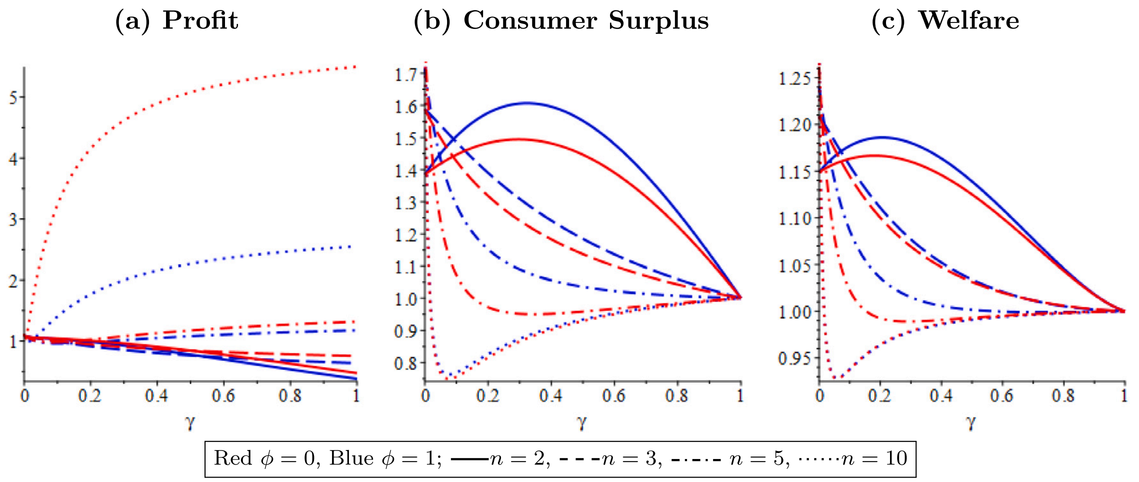


Fig. 3. Profit, Consumer Surplus and Welfare under the Integrator relative to the Free-Market with $\phi \in \{0, 1\}$.

Proposition 5. *For sufficiently low levels of substitutability the Integrator regime strictly dominates the Free-Market in aggregate profit, consumer surplus and welfare terms.*

Fig. 3 reports ratios of aggregate profit, consumer surplus and welfare for the Integrator relative to the Free-Market under $n \in \{2, 3, 5, 10\}$ and $\phi \in \{0, 1\}$, supporting Corollaries 1 and 4 and Propositions 2 and 5. In particular, note that for $n = 10$ at relatively low levels of substitutability, whilst the Integrator can still be relatively attractive in terms of aggregate profit, the Free-Market offers gains in excess of 5% for welfare and 25% for consumer surplus.

In the two-operator case, van den Berg et al. (2022) Proposition 2 reports that operator profit under the Integrator lies below the Free-Market level for sufficiently high levels of substitutability and platform revenue share. However, the following Proposition shows that the reverse can hold for a larger network.

Proposition 6. *Whilst (i) for $n \in \{2, 3\}$, operator profit is strictly higher under the Free-Market than the Integrator for sufficiently high γ, (ii) this ordering is reversed for $n \geq 4$ and $\phi \in [0, \bar{\phi})$, where $\bar{\phi} \in (0, 1)$.*

The intuition underlying this is that when n and γ are small, the Integrator is helping reduce the profit-damaging cross-service price, which under the Free-Market, exceeds the monopoly level in this region. However, for high levels of substitutability and low operator numbers the cross-service price under the Free-Market is not above monopoly levels and the price-reducing effect of the Integrator damages profit. At higher levels of n, the single price-setter on cross-services under the Integrator better insulates the market from the intensity of price competition across the n^2 OD services, protecting profit except for very low γ where there is very little price competition under any regime. If ϕ is not too large then Integrator operator profit is also greater than under the Free-Market. Note, moving to the platform from the Free-Market also weakly improves the platform provider's profit.

Corollary 5. *For sufficiently low substitutability and platform share of cross-service revenue, moving from the Free-Market to the Integrator platform yields a Pareto improvement.*

Fig. 4 reports calibration contours of a Free-Market equilibrium pairing of n and γ fitted to three levels of the average market price elasticity of demand (η), representing extremal and a mid-range level of industry estimates.[29] Hence, if the market operates in line with the Free-Market model and the aggregate elasticity of demand is -0.4, then the market will be at an (n, γ) combination on the red line. The green and blue lines represent equivalent (n, γ) 'real-world' combinations consistent with market elasticities of -0.8 and -1.2 respectively. The cyan (pink) line represents a contour for which welfare is equated under the Free-Market and Integrator with $\phi = \frac{1}{50}$ ($\phi = 1$). Points to the right (left) of this contour yield $\frac{W^{FM}}{W^{I}(\phi)} > 1$ (< 1).

Assuming the market is initially operating according to the Free-Market model with an elasticity of -1.2, if we transition to the Integrator platform while keeping the number of operators constant, the welfare will improve in all scenarios considered because the blue lines are always below the cyan and pink lines. However, if the elasticity is -0.4 (-0.8), and there are enough operators, n, the same transition to the Integrator platform would lead to a reduction in welfare as the red (green) line lies above the cyan

[29] For a derivation of the calibrated market equilibria see Appendix D. The basis for the selection of elasticities in this exercise is long-run price estimates from a variety of studies including: (i) Goodwin (1992) with elasticities for bus -0.6 and rail -1.1, based on an average of numerous other studies; (ii) Small and Winston (1999) U.S. urban (intercity) elasticities for rail -0.6 (-0.7), bus -0.9 (-1.2) and air -0.4; (iii) Rose and Hensher (2014) find demand elasticities for taxi services to be in the range -0.23 to -1.75 with mean estimates in the range -0.5 to -1.0.

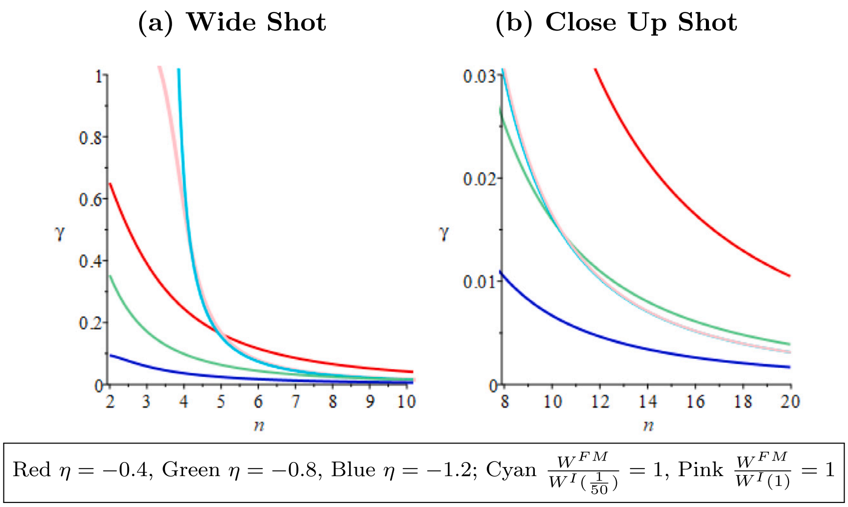


Fig. 4. Calibrated Free-Market Equilibria (n, γ) Combinations for $\eta \in \{-0.4, -0.8, -1.2\}$ and $\frac{W^{FM}}{W^{I}(\phi)} = 1$ contours for $\phi \in \{\frac{1}{50}, 1\}$.

and pink lines once the number of operators reaches 5 (11). The closeness of the cyan and pink lines indicate the relative lack of importance of the share of revenue, ϕ, in the welfare implications of the platform arrangement here.

In terms of the rationale underlying the welfare dominance of the Integrator relative to the Free-Market under the higher elasticity scenarios, we return to the Network Problem. Here, the Free-Market performs badly where harmful strategic interaction, which is internalised under the Integrator, forces up cross-service prices. This has the same effect as a profit-maximising monopolist exploiting its market power and raising prices such that in equilibrium it operates on the elastic part of demand. McHardy (2022) shows that where the Free-Market performs badly, it can yield cross-service prices close to, or above, the monopoly level, meaning equilibrium demand is elastic on cross-services, placing upward pressure on the average industry price elasticity.

4.2. Passive platform v integrator and free-market

Analysis of the equilibrium prices across the Free-Market and the Passive platform models yields the following Proposition.

Proposition 7. *Under the Passive platform, (i) the in- and cross-service prices coincide with their Free-Market equivalents for $\gamma = 0$ and/or $\phi = 0$, otherwise (ii) the in-service (cross-service) price is strictly lower (higher) than under the Free-Market.*

Hence, the Passive platform aids pro-competitive effects on in-service prices but exacerbates the damaging price distortions on the cross-service prices. In terms of intuition, trivially, the Passive platform and the Free-Market regimes are identical under $\phi = 0$ or $\gamma = 0$ in the absence of non-price benefits of the platform. For $\phi > 0$ the operators under the Passive platform are deterred from keeping cross-service prices down as they no longer get all the associated profit, and price reductions on the cross-service journeys feed into greater competition for the in-service prices where they have the full share of profit.

Proposition 8. *Whilst (i) aggregate profit, consumer surplus and welfare are identical under the Passive platform and Free-Market for $\gamma = 0$ and/or $\phi = 0$, (ii) for $\gamma \in (0, 1]$ and $\phi \in (0, \frac{1}{3})$, comparisons of welfare and consumer surplus across the Passive platform and Free-Market are ambiguous and aggregate profit is strictly greater under the Free-Market for $n \in \{2, 3, 4\}$ but otherwise ambiguous.*

Proposition 9. *Under the Passive platform, (i) the in-service (cross-service) price is weakly (strictly) higher than under the Integrator for $n \in \{2, 3\}$, but (ii) this does not generalise unambiguously for $n \geq 4$.*

Corollary 6. *Consumer surplus and welfare are strictly greater under the Integrator relative to the Passive platform for $n \in \{2, 3\}$, but this does not generalise to $n \geq 4$.*

Just as the Integrator fails to unambiguously welfare-dominate the Free-Market moving beyond $n = 2$, its unambiguous dominance of the Passive platform is also limited to small networks.

Fig. 5 reports the ratios of Free-Market and Integrator profit, consumer surplus and welfare relative to the Passive platform for $\phi \in [0, \frac{1}{3})$ (in line with Assumption 2). Notably, the green lines, presenting the ratios of outcomes under the Free-Market relative to the Passive platform (with maximal $\phi = \frac{1}{3}$), are very close to unity. Hence:

Remark 1. The aggregate profit, consumer surplus and welfare outcomes under the Passive platform closely approximate those under the Free-Market across the feasible parameter set.

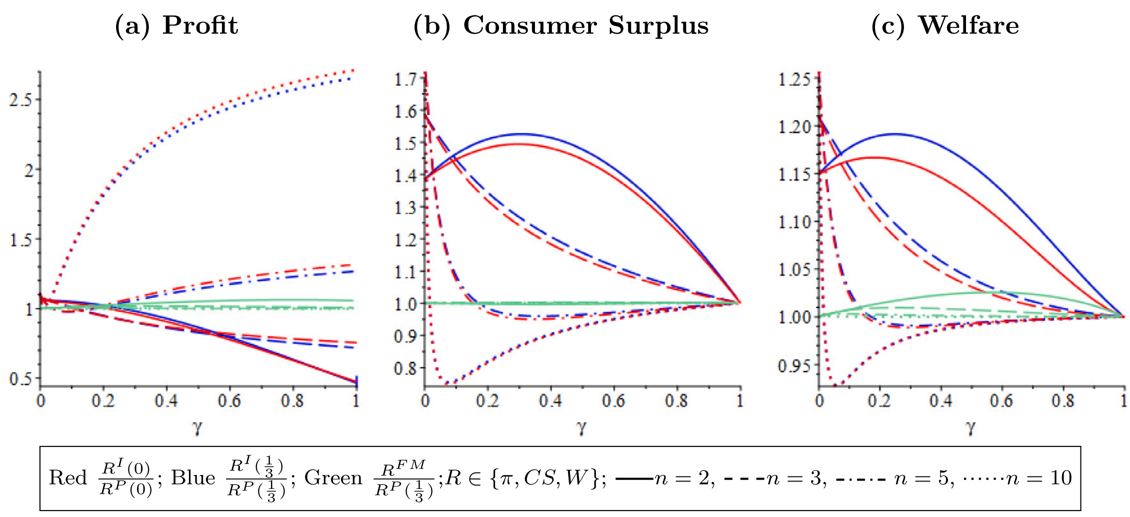


Fig. 5. Profit, Consumer Surplus and Welfare under the Free-Market and Integrator relative to the Passive Platform with $\phi \in \{0, \frac{1}{3}\}$.

Corollary 7. *If price-based benefits favour the Free-Market over the Integrator, but there are significant non-price platform benefits, then the Passive platform, which approximates the Free-Market price outcome but captures non-price platform advantages, may be the optimum business model from a welfare perspective.*

4.3. Intermediary versus Integrator and the Free-Market

Comparison of the equilibrium prices across the Free-Market, Integrator and Intermediary regimes leads to the following results.

Proposition 10. *Under the Intermediary, the (i) in-service price is weakly greater than under the Free-Market and Integrator (with equality for both at $\gamma = 0$, conditional also on $n = 2$ for the Integrator), (ii) cross-service price is strictly greater than under the Free-Market and Integrator.*

Corollary 8. *Consumer surplus and welfare are strictly greater under the Free-Market and Integrator than the Intermediary.*

The intuition here is as follows. The strategic substitute effects, present in the Intermediary (under serial double marginalisation), result in a higher cross-service price than under the Free-Market (where the effect is via parallel double marginalisation, contaminated with strategic complement effects between OD prices), and the Integrator (where these double marginalisation effects are internalised by the platform). Given cross-prices have a strategic complement relationship with in-service prices, the elevation of the former under the Intermediary places additional upward pressure on the latter, relieving price competition intensity and resulting in higher in-service prices.

Proposition 11. *Aggregate profit is strictly greater under the Free-Market and Integrator regimes than the Intermediary for sufficiently low n and γ.*

Corollary 9. *For sufficiently low γ and n, the Free-Market and Integrator regimes strictly dominate the Intermediary in aggregate profit, consumer surplus and welfare terms.*

This presents a potential problem given the existence of Intermediary-type platforms in practice (e.g., see the Introduction), suggesting there may be a role for regulatory oversight in the selection of platform models. We return to this issue in Section 4.5.

4.4. Integrator versus MTC

Having, established the welfare dominance of the MTC over the Free-Market in Corollary 3, we now turn to the Integrator.

Proposition 12. *Under the Integrator relative to the MTC: (i) prices are the same for $\gamma = 0$, otherwise, (ii) prices are strictly lower for $n = 2$, (iii) in-service price ratios are ambiguous for $n \geq 3$, and (iv) the cross-service price is strictly greater for $n \geq 3$.*

Proposition 13. *Consumer surplus and welfare under the Integrator relative to the MTC are: (i) the same for $\gamma = 0$, and otherwise, (ii) strictly greater for $n = 2$, (iii) strictly greater (lower) for $n = 3$ below (above) contours $\phi(\gamma)_{cs}$ and $\phi(\gamma)_w$, respectively, and, (iv) strictly lower for $n \geq 4$.*

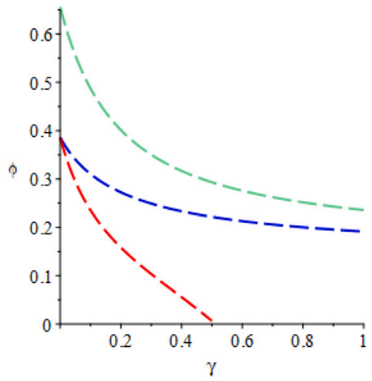


Fig. 6. Integrator and MTC Equality Contours for $n = 3$: Consumer Surplus ($\phi(\gamma)_{cs}$, Blue), Welfare ($\phi(\gamma)_{w}$, Red) and Profit ($\phi(\gamma)_{\pi}$, Green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Hence, whilst the MTC dominates the Free-Market in consumer surplus and welfare terms, the Integrator dominates the MTC on these measures for $n = 2$ and also $n = 3$ below critical $\phi(\gamma)$ contours (see Fig. 6). However, above these contours for $n = 3$, and everywhere for $n \geq 4$, the MTC strictly dominates the Integrator on welfare and consumer surplus. Noting that there is limited evidence of operators selecting into the MTC in the UK under the pricing rule, Eq. (10), we now turn attention to the profit situation.[30]

McHardy (2022) shows that aggregate profit for the non-platform MTC (i.e., $\phi = 0$) exceeds the Free-Market in the case of substitutability and n not being too large. It is straightforward to show this statement is not sensitive to $\phi > 0$. The following Proposition establishes the aggregate profit position with respect to the Integrator.

Proposition 14. *Aggregate profit under the Integrator relative to the MTC is: (i) the same for $\gamma = 0$, and otherwise, (ii) strictly lower for $n = 2$, (iii) strictly lower (greater) for $n = 3$ below (above) contour $\phi(\gamma)_{\pi}$, and, (iv) strictly greater for $n \geq 4$.*

The Integrator/MTC equality contour, $\phi(\gamma)_{\pi}$, is the green line in Fig. 6, with aggregate profit being higher (lower) for the Integrator above (below) the contour. Given (i) for $n = 3$ the green line in Fig. 6 lies everywhere above the blue and red lines, and (ii) more broadly according to Propositions 13 and 14, in the parameter selections where the Integrator improves on consumer surplus and welfare under the MTC it is inferior in aggregate profit, suggest (iii) profit incentives might not support incentive compatibility for the socially best-performing regime of the two.

Fig. 7 illustrates profit, consumer surplus and welfare under the MTC relative to the Integrator for $n \in \{2, 3, 5, 10\}$ and $\phi \in \{0, 1\}$, where the red (green) lines have both regimes with $\phi = 0$ ($\phi = 1$) and blue lines have the Integrator with $\phi = 1$ and the MTC with $\phi = 0$, imitating the case where the MTC is not delivered though a platform. In addition to reinforcing various aspects of the above four results, two further things are notable. First, society tends to prefer the MTC as a platform (blue lines are below the red ones), but this is not incentivised according to aggregate profit. Second, whilst the MTC platform marginally underperforms the Integrator for low n, if γ is low, then for higher n the MTC has the potential to heavily improve relatively on welfare. If in doubt about the long-run equilibrium n and γ in the final market equilibrium on moving to a platform model, this might suggest the MTC as the best-bet platform. We return to this issue with a calibration below.

4.5. Regime incentives

We now turn to the question of which regime would be most favoured from the perspective of a social planner (W) but also the operators (π_{op}) and platform providers (π_{plat}) to shed light on where the various incentives exist for supporting or promoting one regime relative to another.

Beginning with welfare, we know from the earlier results that, the Free-Market (closely approximated by the Passive platform) is inferior to the MTC, but superior to the Intermediary, whilst the Integrator can dominate all other regimes, it can also be inferior to all but the Intermediary.

Fig. 8(a) reports the ratios of welfare under the platform regimes relative the Free-Market under $\phi = \frac{1}{50}$, broadly reflecting the above results, with the Integrator (blue) dominating all other regimes for low n, and the MTC (pink) vying with it for the top rank, otherwise dominating all other regimes. The Integrator also performs relatively less well, in general, compared with the Free Market and MTC at lower levels of substitutability.

[30] Despite the opportunity to coordinate cross-service prices being legally available under the MTC, the evidence does not suggest wide-scale adoption of this pricing rule in practice. Whilst Multi-Operator tickets (of which the MTC is a special case) are available on around three-quarters of admissible services in the UK, they can be priced significantly higher than corresponding in-service tickets (e.g., see TAS, 2020) with a 40% mark-up on in-service prices reported as the norm in Urban Transport Group (2019), who find no counter examples. Wide-spread application of MTC discounted ticket-pricing (even with a zero discount) would not be consistent with these observations.

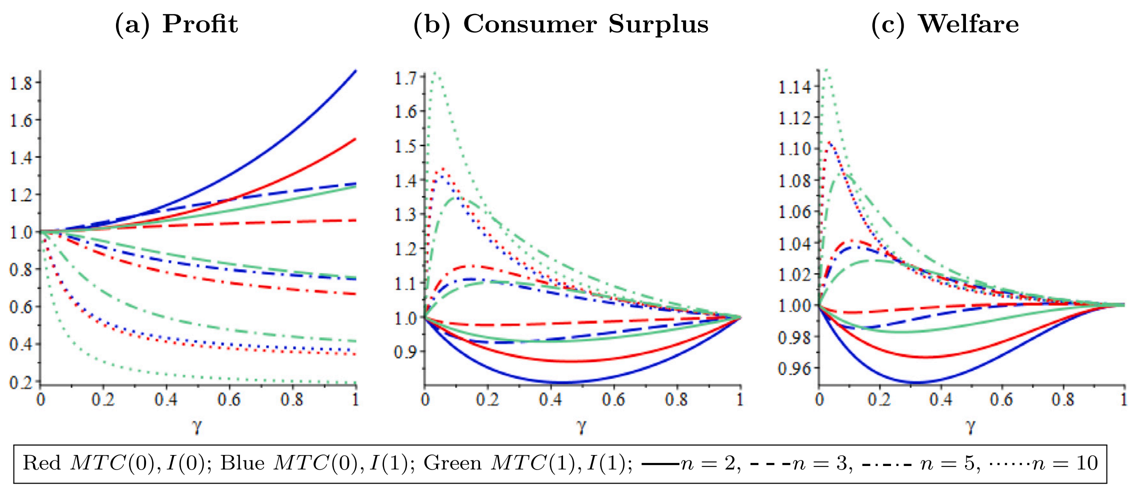


Fig. 7. Profit, Consumer Surplus and Welfare under the MTC relative to the Integrator with $\phi \in \{0, 1\}$.

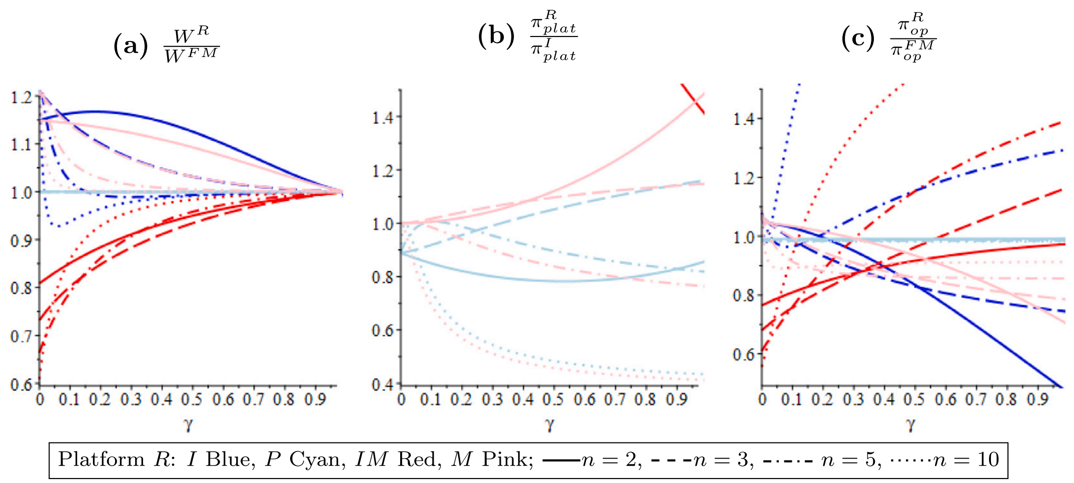


Fig. 8. Welfare and Operator Profit relative to the Free-Market and Platform Profit relative to the Integrator with Platform Regimes $R \in \{I, IM, P, M\}$ and $\phi = \frac{1}{50}$.

Hence, whilst the MTC everywhere dominates the Free-Market, the Integrator does not. However, the calibration exercise demonstrated that the Integrator is likely to dominate the Free-Market, in welfare terms, if the elasticity of demand under the Free-Market is not around the lower market estimate levels. Table 1(A) shows that for calibrations with $\eta \in \{-0.4, -0.8, -1.2\}$, the MTC and Integrator consistently share the top two welfare rankings, with the former (latter) ranked top with higher (lower) n.[31,32] The Passive platform consistently ranks one place below the Free-Market and the Intermediary ranks last in every case.

Remark 2. Under the calibrated Base Model, the Integrator and MTC are the top-performing regimes on welfare with the former (latter) performing best with lower (higher) n.

Turning now to profit incentives, we begin with the choice of the platform provider. Fig. 8(c) reports ratios of the Intermediary (red), Passive (cyan) and MTC (pink) platform provider profits relative to that under the Integrator. The Intermediary has vastly superior platform profits than the other platforms for $n > 2$ (not illustrated due to scaling), and otherwise for γ not too large, where $n = 2$. For very high γ, the Intermediary platform profit falls very marginally below the MTC. Hence, with the fee-charging platforms constrained to take a relatively small share of cross-service revenue, the Intermediary, which adds its own margin to

[31] Note, moving to $\phi = \frac{1}{10}$ has very little impact on the welfare rankings, switching the Integrator and MTC rankings in the single case of $\eta = -0.4$ with $n = 3$. Similarly, it does not materially alter the story in Fig. 8(a).

[32] Table 3 in Appendix E reports the relative size of welfare by regime under each scenario alongside the ranking reported here.

Table 1
Demand Elasticity Calibrated Regime Welfare, Operator Profit and Platform Profit Rankings with $\phi = \frac{1}{50}$.

η	Regime	A. W Rank				B. π_{op} Rank				C. π_{plat} Rank			
		n				n				n			
		2	3	5	10	2	3	5	10	2	3	5	10
	Free-Market	3	3	2	2	1	1	1	2				
	Integrator	1	1	4	4	5	5	2	1	3	4	2	2
−0.4	Intermediary	5	5	5	5	3	3	4	5	1	1	1	1
	Passive	4	4	3	3	2	2	3	3	4	3	3	3
	MTC	2	2	1	1	4	4	5	4	2	2	4	4
	Free-Market	3	3	3	3	1	1	1	1				
	Integrator	1	1	2	2	5	4	3	3	3	3	2	3
−0.8	Intermediary	5	5	5	5	4	5	5	5	1	1	1	1
	Passive	4	4	4	4	2	2	2	2	4	4	4	2
	MTC	2	2	1	1	3	3	4	4	2	2	3	4
	Free-Market	3	3	3	3	1	1	1	1				
	Integrator	1	1	2	2	3	3	2	2	3	3	3	2
−1.2	Intermediary	5	5	5	5	5	5	5	5	1	1	1	1
	Passive	4	4	4	4	4	4	4	4	4	4	4	4
	MTC	2	2	1	1	2	2	3	3	2	2	2	3

the input prices of the operators, is able to generate higher profits, making it the most attractive regime from the perspective of the platform provider.[33] Of course, if the platform faces high fixed costs, this may result in the Intermediary being the only viable platform regime.

Regarding the remaining platform regimes, the MTC platform dominates (second rank, below the Intermediary) for low n, whilst the Passive platform mostly performs relatively poorly. For higher n the Integrator takes the second rank behind the Intermediary. In terms of the calibration, Table 1(C) shows these findings are broadly consistent with fitting the model to real-world elasticities with the MTC being second-best at low n with low elasticity, and with its ranking improving with more elastic demand.[34] Hence, given a choice, the platform provider would opt for the Intermediary model which is the worst-performing from a welfare perspective. However, the choices that are mostly second-best here (MTC and Integrator) are often the highest in the welfare rankings.

Turning to the platform model preferences of the operators, Fig. 8(b) reports ratios of operator profits for all four platform regimes relative to the non-platform Free-Market case. The picture here is very mixed with all but the Passive platform having some scenario with the best performance within which there are a couple of notable points. First, with high n and/or sufficiently low substitutability, the Integrator dominates the Free-Market but can be inferior to the MTC. Second, there is a selection over which the Intermediary dominates (here $n = 5$ with sufficiently high substitutability). According to the calibration in Table 1(B), the Free-Market dominates (trailed by its close approximation, the Passive Platform) for low-medium elasticities, with the MTC and Integrator taking the top ranking for the most elastic case, albeit where neither regime is the top-ranking on welfare.

Remark 3. Under the Base Model, profit incentives for platform model choice tend to be neither aligned across operators and platform providers, nor do they tend to promote the best welfare-performing regime.

5. Network extensions

We now explore the impact on regime rankings of including a number of additional factors which feature in transport networks and their modelling but are currently outside the analysis of our Base Model (Assumption 1). Since a key motivation of this paper is to investigate the robustness of the welfare-dominance result of the Integrator over the Free-Market, our focus in the following sections is primarily on comparing these regimes. This is particularly justified in the case of the Passive platform which closely approximates the Free-Market in price-based outcomes and can be shown to yield results (quantitatively and qualitatively) in line with the Free-Market in each case that follows.[35]

5.1. Frequency selection with two operators

Until now we have been concerned with a model in which additional $\{x, y\}$ service pairs come with new independent 'rival' operators. However, in practice incumbent operators can generally offer a variety of services on a given route at different times and/or offer multiple alternative routes. We now seek to explore this multi-frequency/route possibility in an optimising framework.

[33] Obviously, when taking much larger levels of revenue share in the fee, e.g., $\phi = \frac{1}{3}$, then the Intermediary ceases to be uniformly the most profitable regime for the platform provider. However, whilst existing evidence does not support such high fee levels in practice, the higher fee also fails to align profit and welfare incentives.

[34] Tables 3 and 4 in Appendix E report the relative size of profit by regime under each scenario alongside the ranking reported here.

[35] Adaptations of the calibrations for this Section are detailed in Appendix D.

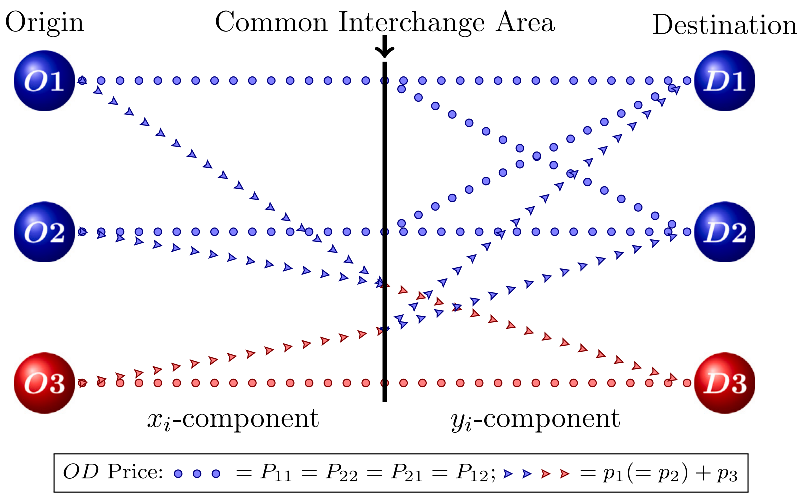


Fig. 9. Free-Market Ticket Configuration with 2 Operators Offering 3 Services.

For brevity, henceforth, we refer to operators selecting frequencies — although route number selection or a combination of frequencies and routes would be analytically equivalent in the stylised framework. Hence, we move from a uni-frequency situation, in which there are n rival operators, each offering one $\{x, y\}$ service pair, to a multi-frequency one, where operator i offers n_i service pairs, on the same geographical route, but at different times.

Before proceeding to the analysis, it is important to understand how moving from a uni- to multi-frequency setting impacts on the strategic forces in the market under our different regimes. Recall that in the case of uni-frequency operators, an additional $\{x, y\}$ service pair brings one new in-service journey and $2(n-1)$ new cross-service journey combinations. However, in the multi-frequency setting an operator's new x and y components can additionally be combined as further in-service journeys with their existing x and y components. Hence, moving from a uni- to multi-frequency setting will change the balance of in- and cross-service journeys, increasing in favour of the former, with implications for the balance of strategic interaction effects under different regimes. For a given network size (overall number of OD services) moving to the multi-frequency case reduces the number of cross-network services and hence also reduces the weight of cross-network pricing, which is the source of the potentially damaging strategic substitute price effects set out in the Network Problem. In the Free-Market case we would expect prices on services converted from cross- to in-service to be lower, reducing potentially harmful effects associated with the strategic substitutability across cross-service pricing. In addition, for a given number of distinct OD services across the network, there are more prices set independently in the uni-frequency case than under multi-frequency. This is because an operator with multiple frequencies will maximise profit setting the in- and cross-service component prices simultaneously and collectively across its n_i $\{x, y\}$ service pairs. With fewer prices being set independently by independent operators, we would expect this to have an anti-competitive upward effect on prices. Hence, we have identified countervailing price effects under the Free-Market from moving to a multi-frequency setting from a uni-frequency network of the same size. Given different regimes have different exposure to each of these two effects, the move to multi-frequency has the potential to alter the relative performance of different pricing regimes.

To illustrate, Fig. 9 presents the pricing structure in a situation with three service frequencies served by two operators. For instance, operator 1 provides temporally differentiated services 1 and 2 and operator 2 provides a further temporally differentiated service, 3, all on a given geographical OD journey. Here there are two independently set in-service prices ($P_{11} = P_{22}$ and P_{33}) and two component prices ($p_1 = p_2$ and p_3) yielding one cross-service price combination ($p_1 + p_3$).[36] Contrast this with the situation where there were three operators, as illustrated in Fig. 1, with three in-service prices and three cross-service price combinations. Further, note that there are now $n_1^2 = 4$ in-service journeys using operator 1: J_{11}, J_{12}, J_{21} and J_{22}.

We now consider the optimising frequency choices of operators under the Free-Market and Integrator regimes. For simplicity we assume there are two rival operators ($n = 2$), but, in order to avoid large (potentially infinite) optimising selections of frequencies, we also need to introduce costs. Let F be the fixed cost for providing a unit of frequency (an $\{x, y\}$ pair), so total fixed cost for operator i with service frequency n_i is $n_i F$. In order to advantage or disadvantage a lower-price regime we also specify a marginal passenger cost for an operator i, which takes value $\bar{c}_i$ in the case the operator has zero passengers and changes at a rate ϵ with each in-service OD passenger journey (on a pro rata basis for each cross-service passenger journey component). Hence, total cost for operator i is[37]:

$$C_i = \left\{\bar{c}_i + \epsilon\left[n_i^2 Q_{ii} + n_i n_j Q_x\right]\right\}\left(n_i^2 Q_{ii} + n_i n_j Q_x\right) + n_i F \tag{12}$$

[36] In the multi-frequency setting under demand symmetry, operators will charge the same price across all their in-service journeys, P_{ii}, and the price of their component in any cross-network service, p_i, will also be common.

[37] For example, see van den Berg et al. (2022), who use this cost specification, albeit in the case of two uni-frequency operators: $n = 2$ and $n_i = 1$.

Table 2
Free-Market and Integrator Multi-Frequency Nash Equilibria with $\alpha = 1$, $\phi = \frac{1}{50}$, $F \in \{0.025, 0.200\}$, $\bar{c}_i \in \{0.05, 0.10\}$, $\epsilon \in \{-0.02, 0.05\}$, $\gamma \in \{0.15, 0.50\}$.

$(\epsilon, \bar{c}_i, \gamma, F)$	Regime	Solution Type	η	$E(n_i + n_j)$	$E(W)$
$(-0.02, 0.10, 0.15, 0.200)$	Free-Market	PSNE $(n_i, n_j) = (2, 2)$	−1.13	4.0	0.92
	Integrator	MSNE	–	3.0	0.94
	Integrator	PSNE $(n_i, n_j) = (1, 2)$	–	3.0	0.98
$(0.05, 0.05, 0.50, 0.025)$	Free-Market	PSNE $(n_i, n_j) = (2, 2)$	−0.60	4.0	0.64
	Integrator	PSNE $(n_i, n_j) = (1, 1)$	–	2.0	0.60

where $\{.\}$ is the marginal cost per full in-service OD passenger journey. Marginal cost is constant under $\epsilon = 0$, whilst $\epsilon < 0$ ($\epsilon > 0$) represents economies (diseconomies) in marginal costs per passenger. In- and cross-service demands are then, respectively[38]:

$$Q_{ii} = a - bP_{ii} + d\left[(n_i^2 - 1)P_{ii} + n_j^2 P_{jj} + 2n_i n_j P_x\right]$$
$$Q_x = a - bP_x + d\left[n_i^2 P_{ii} + n_j^2 P_{jj} + (2n_i n_j - 1)P_x\right]$$

where P_x is selected by the platform provider in the case of the Integrator, and $P_x = p_i + p_j$, in the case of the Free-Market. Operator i, under the Integrator, now solves:

$$\max_{P_{ii}} \pi_i = n_i^2 P_{ii} Q_{ii} + (1-\phi) n_i n_j P_x Q_x - \left\{\bar{c}_i + \epsilon\left[n_i^2 Q_{ii} + n_i n_j Q_x\right]\right\}\left(n_i^2 Q_{ii} + n_i n_j Q_x\right) - n_i F$$

The same objective function applies for operator i in the Free-Market setting with $\phi = 0$ and replacing P_x with $p_i + p_j$. The Integrator platform provider solves the problem:

$$\max_{P_x} \pi_{plat} = \phi n_i n_j P_x Q_x$$

Table 2 reports the payoffs to operators 1 and 2 in the case of the Free-Market and Integrator where each operator has a choice of frequency, n_i.[39] Two different sets of parameterisations are specified to investigate optimum strategic frequency selections with operator passenger cost economies and diseconomies ($\epsilon = -0.02$ and $\epsilon = 0.05$, respectively). The other parameterisations are then specified: (i) without loss of generality ($\alpha = 1$), (ii) based on market evidence ($\phi = \frac{1}{50}$), or, (iii) to ensure that there is a small set of viable frequencies under each regime with Nash equilibria supporting interior solutions for both operators ($\gamma \in \{0.15, 0.50\}$, $F \in \{0.025, 0.200\}$ and $\bar{c}_i \in \{0.05, 0.10\}$).

Beginning with the case of cost economies ($\epsilon = -0.02$) in combination with $\gamma = 0.15$, $F = 0.2$ and $\bar{c}_i = 0.10$, Table 2 reports that, whilst there is a single Pure Strategy Nash Equilibrium (*PSNE*) under the Free-Market with each operator selecting a frequency of $n_i = 2$, it achieves a lower (expected) welfare than either the *PSNE* or Mixed Strategy Nash Equilibrium (*MSNE*) under the Integrator with a smaller (expected) aggregate frequency of $n_i + n_j = 3.0$.[40] In the relevant range, the passenger economies of marginal cost aid the Integrator regime in welfare terms since it achieves higher total quantities relative to the Free-Market with the same frequency configuration, and the operator with $n_i = 2$ has lower marginal cost in the $(1, 2)$ $PSNE$ than the operators in the Free-Market $(2, 2)$ $PSNE$. Even though the strategic interaction in the Integrator regime results in a smaller network than the Free-Market, these cost economies are still working in its favour, such that despite offering a lower number of frequencies, the smaller network under the Integrator, with lower fixed cost exposure, offers a higher welfare outcome than the Free-Market. The approximate aggregate elasticity of demand at the Free-Market Nash equilibrium is −1.13, towards the upper end of our market estimates, and consistent with the Base Model supporting a move from the Free-Market to Integrator on welfare grounds.

On the other hand, in the case of cost diseconomies ($\epsilon = 0.05$) with $\bar{c}_i = 0.05$, $F = 0.025$ and $\gamma = 0.50$, each regime has a single *PSNE* where the Free-Market provides two frequencies per operator, relative to one each for the Integrator, and achieves a higher level of welfare.[41] Note, the approximate aggregate demand elasticity under the Free-Market Nash equilibrium is −0.60, at the lower end of our market estimates, where the Base Model analysis has the Integrator under-performing in welfare relative to the Free-Market. We know from Proposition 4 that with two or three uni-frequency operators and zero costs, the Integrator regime yields lower prices and higher quantities than the Free-Market regime. Cost diseconomies are working against the Integrator here, reversing its profit superiority over the Free-Market. This results in it being unable to profitably sustain the same size network as the

[38] Note, as in the Base Model, symmetry in substitutability is assumed across all services. Whilst greater substitutability might be expected across services sharing a given operator's component relative to one that does not, this need not be the case. Similar reasoning applies to that set out in the Base Model discussion.

[39] Elasticities and expected welfare are reported to 2 d.p. and expected frequencies are reported to 1 d.p.

[40] In the case of the Intermediary, there is no interior Nash equilibrium: the *PSNE* $(1, 3)$ involves a loss-making operator and the *MSNE* involves negative expected profits and associated welfare is below 0.90 in each case. In all cases, the Passive platform payoffs are very similar to the Free-Market yielding the same Nash equilibria frequencies, and welfare very marginally below the Free-Market outcome. In the case of the MTC, the $PSNE$ of $(n_i, n_j) = (1, 2)$, achieves welfare 0.85, and the $MSNE$ has expected n and welfare of 3.5 and 0.99, respectively. Hence, the MTC is the least-well performing under the $PNSE$ and the best under the $MSNE$.

[41] In the case of the Intermediary, the Nash equilibrium is $n_i = 2$ but welfare is also below the Free-Market level at 0.63. In the case of the MTC, the $PSNE$ of $(n_i, n_j) = (1, 2)$, achieves welfare 0.62, and the $MSNE$, has expected n and welfare of 2.2 and 0.58, respectively. Hence, the MTC improves on the Integrator under the $PSNE$ but is the least-well performing under the $MSNE$.

Free-Market such that the latter both provides greater frequency but also covers the larger associated fixed costs yielding a welfare premium over the smaller Integrator network. Contrast this with the cost economies case, above, where the Integrator is still unable to support as large a network as the Free-Market, but where the lower prices, with cost economies, translate into higher welfare.

Remark 4. The welfare dominance of the Integrator platform relative to the Free-Market with two operators is not robust to the introduction of service frequency as a choice variable, but does survive in the case of the elasticity calibration at the high-end of market estimates.

5.2. Single- and dual-component demand

The analysis so far has been based on the assumption that all journeys involve two component parts in fixed proportion, hence ruling out single-trip (or domestic) travel. Yet in practice, many journeys are single-component i.e., do not involve an onward or return trip in the relevant decision period. This is potentially important as the existence of the latter demand type will have differential impacts across regimes depending on whether operators are able to separate single-component prices from dual-component prices. Hence, we now introduce a new traveller type, where demand for operator i's single x and y components are X_i and Y_i, respectively. For simplicity, we treat these symmetrically under p_i:

$$X_i = Y_i = A - Bp_i + D\sum_{j\neq i}^{n} p_j \tag{13}$$

Constrained optimisation of a suitably augmented utility function yields the above linear single-component travel demands where[42]:

$$A = \frac{\mu\alpha}{1+\gamma(n-1)}, \quad B = \frac{1+\gamma(n-2)}{[1+\gamma(n-1)](1-\gamma)}, \quad D = \frac{\gamma}{[1+\gamma(n-1)](1-\gamma)}$$

and the parameter $\mu \in [0,1]$ determines the relative strength of demand for the single- relative to dual-component journeys.[43] Higher values of μ increase the market size of single- relative to dual-component travel.

In the Free-Market setting, operators are not able to separate the single-component prices from their cross-service prices: $P_{ij} = p_i + p_j$. Hence, operator i solves:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + p_i(X_i + Y_i) + \sum_{j\neq i}^{n} p_i(Q_{ij} + Q_{ji}) \tag{14}$$

Under the Integrator however, the operator's prices on the single-component journeys are distinct from the prices of cross-service travel. Under this regime (given symmetry), operator i solves the problem:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + p_i(X_i + Y_i) + \frac{1}{2}(1-\phi)\sum_{j\neq i}^{n}(P_{ij}Q_{ij} + P_{ji}Q_{ji}) \tag{15}$$

whilst the Integrator solves the same problem as before in Eq. (5), with the constraint that $P_{ij} \leq p_i + p_j$ to avoid travellers buying two single-component tickets in place of the Integrator's equivalent cross-service ticket. The two regimes' pricing structures are illustrated in the two-operator case in Fig. 10. Clearly, in the Integrator regime the operators do not have the constraint that when they raise or lower single-component prices (coloured darts), their component of the cross-service price (yellow circles) is raised or lowered accordingly. We would, therefore, expect the absence of this constraint in the Integrator case to impact differentially on the strategic forces at play relative to the Free-Market regime and focus our attention on these two cases.[44]

Fig. 11 illustrates the aggregate profit and welfare ratios for the Integrator relative to Free-Market under two specifications of the relative size of single- versus dual-component demand: $\mu \in \{0.5, 1.0\}$.[45] To avoid cluttering, we present only the $n \in \{2,3\}$ cases, with green lines representing the situation in the absence of single-component demand, replicating the earlier analysis of Section 4.1 for comparison. Since the green lines typically sit above their red and blue counterparts for welfare and profit, respectively, the introduction of the single-component demands is acting to benefit the performance of the Free-Market relative to the Integrator on both measures. In terms of intuition, the single-component demands constrain the Free-Market operators, limiting the damaging externality that otherwise pushes cross-service prices above the monopoly level, hence improving associated profit and welfare. This suggests that an analysis of the gains of the Integrator platform without single-component demands, where they are non-trivially present in the market, will tend to overstate the associated gains of the Integrator relative to the Free-Market since the latter's performance problems are eased by the existence of single-component demands. Hence, the hypothesis of Corollary 4 is not robust to the introduction of single-component (domestic) demands, with the Free-Market dominating the Integrator on welfare at low operator numbers.

[42] We augment our earlier utility function according to: $U(\boldsymbol{Q},\boldsymbol{X},\boldsymbol{Y},M_0) = \alpha\sum_t^N Q_t + \mu\alpha\sum_k^n (X_k + Y_k) - \frac{1}{2}\left[\sum_t^N Q_t^2 + 2\gamma\sum_{t\neq r}^N Q_t Q_r\right] - \frac{1}{2}\left[\sum_k^n (X_k^2 + Y_k^2) + 2\gamma\sum_{k\neq m}^n (X_k X_m + Y_k Y_m)\right] + M_0$, where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are n-vectors of single-component demands for the operators' x and y component services.

[43] Lin (2004), for instance introduces single-component demands in the $n = 2$ case where $A = a$, $B = b = 1$ and $D = d$, effectively setting $\mu = 1$.

[44] We do not report the cases of the Passive and Intermediary Platform regimes as, the former produces extremely similar outcomes to the Free-Market case, and the latter has numerous non-interior solutions over the parameter set.

[45] All solutions here are interior, e.g., $2p^I > P_x^I$.

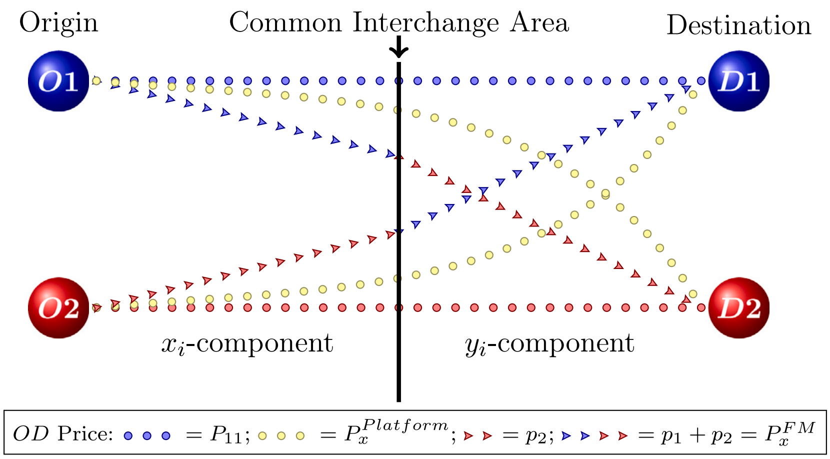


Fig. 10. Free-Market and Platform Ticket Configurations with 2 Operators and Single- and Dual-component Demands.

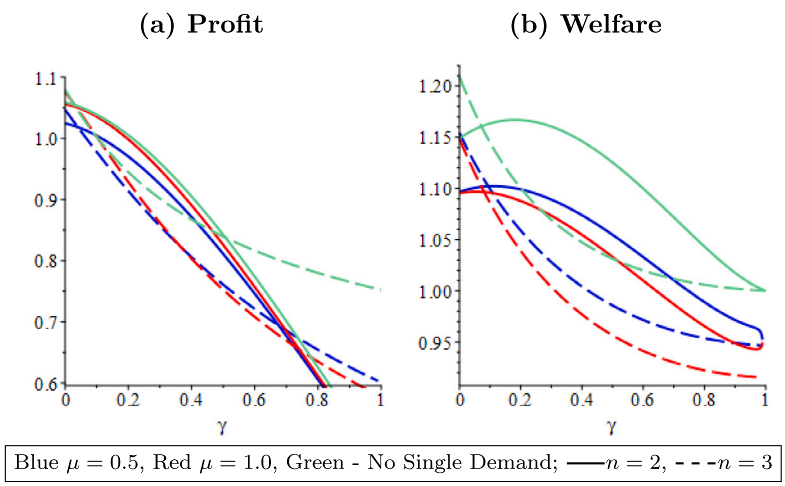


Fig. 11. Profit and Welfare with Single- and Dual-component Demand under the Integrator relative to the Free-Market with $\mu \in \{0.5, 1.0\}$ and $n \in \{2, 3\}$.

A calibration for aggregate elasticity with $n = 2$ ($n = 3$) indicates the following levels of γ at the Free-Market equilibrium for $\eta = -0.4$, $\eta = -0.8$ and $\eta = -1.2$, respectively: 0.71 (0.48), 0.44 (0.23) and 0.19 (0.09). Hence, with reference to Fig. 11, the calibration places the market at γ levels where the Integrator is still welfare superior to the Free-Market for $\eta \in \{-0.8, -1.2\}$, but this does not always hold for $\eta = -0.4$.[46]

Remark 5. The introduction of single-component (domestic) demand can undermine the welfare supremacy of the Integrator over the Free-Market for $n \in \{2, 3\}$. However, the earlier calibration at mid- to high-market elasticity estimates can restore the Integrator's welfare dominance.

5.3. Multiple component journeys

We now consider a situation in which the network has two operators but where all OD journeys involve three components: x, y and z. Therefore, we extend the framework in van den Berg et al. (2022) with two operators to an extra complementary component in each OD journey. For instance, it is not uncommon for a passenger to use a bus to transport them from their home to a train

[46] At these points of comparison for the single- dual-component extension, a basic analysis of the MTC, where the pricing rule is based on single-service OD prices and excludes single-component prices, has the MTC welfare-dominating the Free-Market and Integrator. One interpretation of the MTC could include single-component prices within the pricing rule, Eq. (10). However, this makes the model intractable without further simplifying assumptions, and hence we report the outcome under the simpler interpretation with the pricing rule based on dual in-service OD prices alone, leaving the more complex analysis for future inquiry.

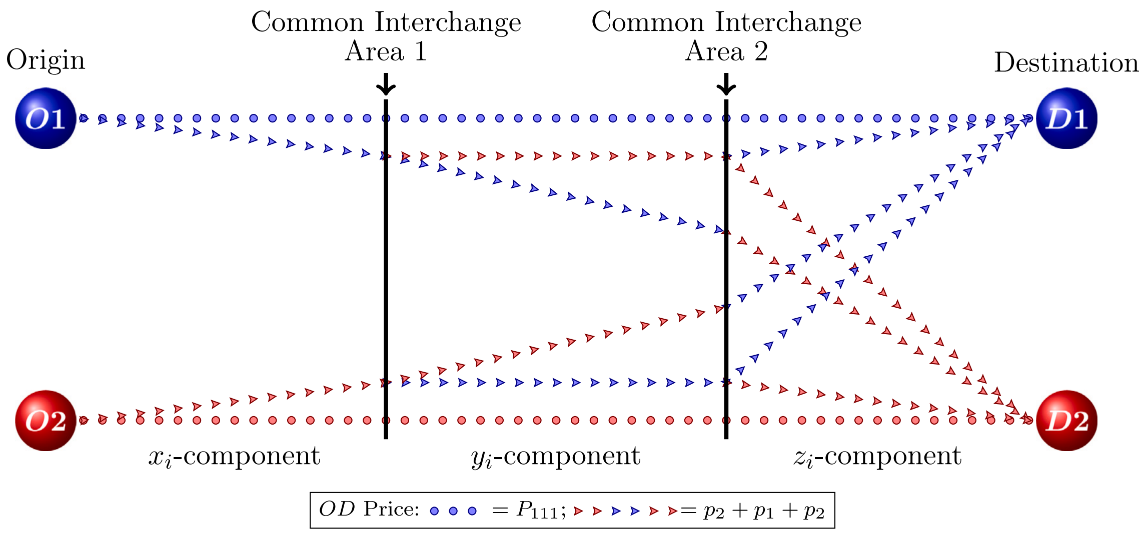


Fig. 12. Free-Market Ticket Configuration with 2 Operators and 3-component OD Journeys.

and thereafter a bus to their final destination. However, this is potentially important as increasing the number of components in OD demand alters the balance of strategic forces across different regimes and so might alter their relative performance.

Fig. 12 illustrates the pricing structure under the Free-Market with two operators and three-component OD journeys. For instance, the blue circle line represents in-service journey J_{111} using operator 1, with OD price P_{111}, whilst cross-service journey J_{212}, illustrated by the red dart, blue dart, red dart sequence, has associated OD price $p_2 + p_1 + p_2$. Clearly, not only are there now more complementary services combined in a single OD journey, compounding the strategic substitute effect with upward price pressure (e.g., increasing the number of complementary components tends to increase the composite price Cournot, 1838), but there are now also more rival OD services with strategic complement downward pressure on prices. A priori, it is not clear which of these effects will dominate in the Free-Market. However, note that under the Integrator platform there is no impact on the number of independently determined cross-service prices as these are jointly set by the platform provider, diminishing the beneficial downward strategic price effects associated with an increase in the number of new OD service combinations.

Updating the demand system in the case of $n = 2$ with three complementary demand components (sub journeys), x, y and z, we have[47]:

$$Q_{iii} = a - bP_{iii} + d\left[P_{jjj} + 9p_i + 9p_j\right]$$
$$Q_{iij} = a - b(2p_i + p_j) + d\left[P_{iii} + P_{jjj} + 7p_i + 8p_j\right]$$
$$Q_{ijj} = a - b(p_i + 2p_j) + d\left[P_{iii} + P_{jjj} + 8p_i + 7p_j\right]$$

where, for instance, P_{iii} is the total price for the in-service journey provided by operator i across its three journey components, and Q_{ijj} is the demand for the three-component journey with two services provided by operator j and one by operator i. Constrained optimisation yields the updated coefficients for the demand system:

$$a \equiv \frac{\alpha(1-\gamma)}{1+6\gamma-7\gamma^2}, \quad b \equiv \frac{1+6\gamma}{1+6\gamma-7\gamma^2}, \quad d \equiv \frac{\gamma}{1+6\gamma-7\gamma^2}$$

In the Free-Market, operator i solves the following problem[48,49]:

$$\max_{P_{iii}, p_i} \pi_i = P_{iii}Q_{iii} + 3p_i(2Q_{iij} + Q_{ijj}) \quad (i \neq j = 1, 2) \tag{16}$$

yielding equilibrium in- and cross-service prices, respectively:

$$P^{FM} = \frac{\alpha(2n+3)(1-\gamma)}{\gamma(n^4 - 2n^2 - 3n - 6) + 4n + 6}, \quad P_x^{FM} = \frac{2\alpha(n+1)(1-\gamma)}{\gamma(n^4 - 2n^2 - 3n - 6) + 4n + 6} \tag{17}$$

47 The demand configurations below follow directly from Fig. 12 with one in-service price and nine single component prices (across the various cross-service journeys) for each operator. Hence, in the case of Q_{iij} the own price component has two operator i single-component prices and one operator j single-component price, with the remaining prices (P_{iii}, P_{jjj}, $(9-2)p_i$ and $(9-1)p_j$) appearing multiplied by d.

48 To understand the configuration of the second part of the maximand, note that for each unit of Q_{iij} (Q_{ijj}) operator i receives $2p_i$ (p_i), and given symmetry there are three equivalent configurations of each of these demands i.e., in equilibrium $Q_{iij} = Q_{iji} = Q_{jii}$.

49 In reality, multiple components of an operator's service offer might be combined with discounts making a more complicated analysis involving, for instance, discounts on in-service prices, P_{iii}, as is applied here (P_{iii} will be at a discount relative to $3p_i$), but also on pairs of component prices for instance in Q_{iij} travel, differentiating these from the single component price, p_i in Q_{ijj} travel.

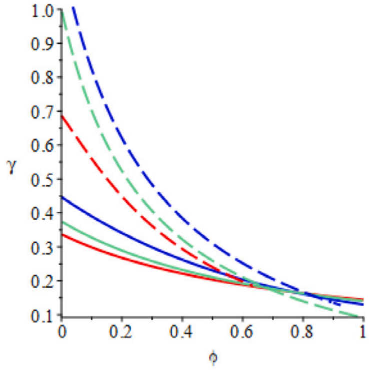


Fig. 13. Free-Market versus Integrator and MTC Profit, Consumer Surplus and Welfare Equality Contours with $n = 2$ with 3 component journeys.

Operator i and the platform provider in the Integrator model, solve the following problems, respectively:

$$\max_{P_{iii}} \pi_i = P_{iii}Q_{iii} + \frac{3}{2}(1-\phi)P_x(2Q_{iij} + Q_{ijj}), \quad \max_{P_x} \pi_{plat} = 3\phi P_x(Q_{iij} + Q_{ijj}) \tag{18}$$

with resulting equilibrium prices:

$$P^I = \frac{\alpha}{\nabla_1}(2 + \gamma(n^2 + n(2-\phi) - 3\phi + 5)(n-1))(1-\gamma), \quad P_x^I = \frac{\alpha}{\nabla_1}(1-\gamma)(2\gamma n^3 - 3\gamma + 2)$$

where $\nabla_1 \equiv (4 + (3n^3 + (\phi - 2)n^2 + (3\phi - 5)n - 2)(n-1)\gamma^2 + (4n^3 + 2n - 10)\gamma)$.

In this Section, we also explicitly include the MTC, given the potential for useful insights. Here, operator i maximises profit in accordance with the Integrator's maximand in Eq. (18), but where, as before, P_x is determined under the MTC pricing-rule according to $\frac{\partial P_x}{\partial P_{iii}} = \frac{1}{n}$.[50] This results in the uniform equilibrium price:

$$P^M = \frac{\alpha(3\phi - 5)(1-\gamma)}{(3n^3\phi - 5n^3 - 30\phi + 42)\gamma + 6\phi - 10}$$

Fig. 13 illustrates the contours for (ϕ, γ) combinations where profit (blue), consumer surplus (green) and welfare (red) are equal under the Free-Market and the Integrator (solid line) and MTC (dashed line), respectively. Combinations above (below) the relevant contour represent outcomes where profit (consumer surplus and welfare) is strictly higher (are strictly lower) under the Free-Market than the Integrator or MTC, accordingly. The parameter combinations below the blue lines and above the red and green lines yield higher profit, consumer surplus and welfare under the Free-Market than the Integrator or MTC platforms.

Hence, whilst in the two-operator setting with two components in each OD journey, the Integrator strictly dominates the Free-Market and offers an apparent improvement in welfare terms, the latter now dominates the former with three-component journeys if substitutability and the share of revenue to the platform are not too small. However, calibrating the model for $\eta = -0.8$ ($\eta = -0.4$) under the Free-Market with $n = 2$ indicates substitutability level $\gamma \approx 0.003$ ($\gamma \approx 0.433$) which, places the market below (above) the lines where consumer surplus and welfare are strictly greater (lower) under the Integrator.[51]

Remark 6. Whilst extending the Base Model under $n = 2$ and demand for two-component journeys to one with three-component journeys results in outcomes where the Free-Market dominates the Integrator in consumer surplus and welfare if γ and ϕ are not too small, a calibration at the elasticity $\eta = -0.8$, restores the welfare supremacy of the Integrator over the Free-Market for $n = 2$.

Similar analysis of the MTC case results in a less positive story.

Remark 7. Extending the Base Model with $n = 2$ and demand for two-component journeys to one with three-component journeys undermines the welfare supremacy of the MTC over the Free-Market for a wide range of parameter values including those under the internal calibrations ($\eta \in \{-0.4, -0.8\}$), with ϕ in the range close to $\frac{1}{50}$.

Hence, whilst a platform model (the Integrator) still welfare dominates the Free-Market under a medium market demand elasticity estimate with this extension to the Base Model, the welfare dominance of the MTC over the Free-Market is not preserved. In terms of intuition, note that, whilst the addition of an extra component in an OD journey adds to the number of independently-set

[50] Although there are now more components and cross-service OD prices, given symmetry, we only need to specify $n = 2$ single-service prices.

[51] The Calibration for $\eta = -1.2$ does not admit an interior solution.

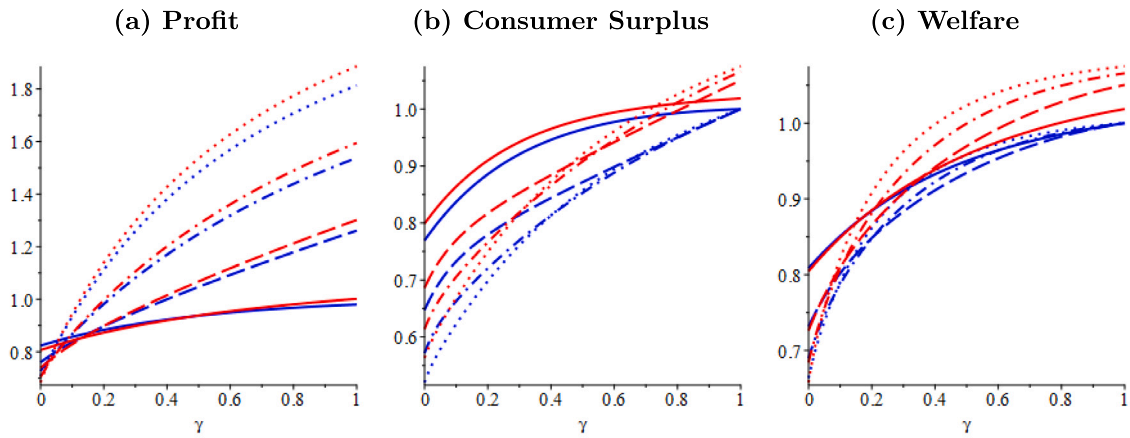


Fig. 14. Aggregate Profit, Consumer Surplus and Welfare under the Intermediary relative to the Free-Market with Free-Market Cross-Service Generalised Cost Penalty $\rho \in \{0, \frac{1}{20}\}$ and $n \in \{2, 3, 4, 5\}$.

complementary prices under the Free-Market, with associated upward, strategic substitute effects, it also results in an increase in the number of cross-service prices, which have downward, strategic complement effects across OD prices. Whilst the former effect is neutralised here in the platform models, with a single agent (the platform provider) or pricing rule (MTC) setting the cross-service prices, this dampens the beneficial strategic complement effects across OD prices, which for lower levels of substitutability, drags down welfare relative to the Free-Market.

5.4. Wider platform benefits

In this Section we briefly explore the implications of the platform offering value-added in the cross-service journey. We do this by allowing the Free-Market provision of cross-service travel to come at an elevated generalised cost, for instance associated with having to engage with two separate operators involving higher transactions costs or having to navigate poorly integrated interchange between cross-services. This analysis is very relevant in the context of platform models, as one of the anticipated benefits of these regimes is that they can improve the provision of cross-service travel, benefitting the passenger. Let the term α in utility be reduced to $\alpha_x = (1-\rho)\alpha$ for cross-service journeys under the Free-Market, where $\rho \in [0, 1]$ is increasing in the size of the cross-service penalty for having to use two uncoordinated operators. In the case of the platform regimes, we allow $\rho = 0$, since the platform eliminates the need to deal with more than one organisation in arranging cross-service travel and potentially results in the platform providing better joined-up cross-service interchange. Hence we need only update the non-platform case, the Free-Market.[52] Implementing the cross-service penalty, ρ yields equilibrium prices under the Free-Market:

$$P^{FM}(\rho) = \frac{\alpha(1-\gamma)\left[3+\gamma(n^2-1)(3-2\rho)\right]}{(6+\gamma(2n^2-3n-5))(1+\gamma(n^2-1))}$$
$$P_x^{FM}(\rho) = \frac{2\alpha(1-\gamma)\{\gamma(n+1)[2n(1-\rho)+3\rho-2]+2(1-\rho)\}}{(6+\gamma(2n^2-3n-5))(1+\gamma(n^2-1))} \tag{19}$$

A priori, it might be supposed that imposing a hit to surplus on cross-service travel under the Free-Market would unambiguously damage the performance of this regime in profit, consumer surplus and welfare terms relative to the platform regimes which do not experience the surplus penalty. However, McHardy (2022) shows that the Free-Market in- and cross-service prices are weakly falling in ρ such that, despite the hit to cross-service surplus due to the additional generalised cost, for sufficiently low levels of γ and n, there is an interval of ρ, over which profit and welfare are increasing in ρ.

In particular, note the case of the Intermediary. Fig. 14 illustrates the ratio of aggregate profit, consumer surplus and welfare under the Free-Market relative to the Intermediary with $\rho = \frac{1}{20}$, and $\rho = 0$ for comparative purposes. Movements from blue to red lines indicate the impact of the introduction of the cross-service generalised cost penalty under the Free-Market. Most notably, whereas Corollary 8 maintains that the Free-Market strictly dominates the Intermediary in terms of consumer surplus and welfare, this conclusion can be undermined by the introduction of cross-service generalised costs which are resolved under the platform. These observations lead directly to the following Proposition.

[52] This is precisely the kind of arrangement that $MaaS$ can deliver, as indicated in the introductory definition, e.g., through feedback that improves the service.

Remark 8. The introduction of a generalised cost penalty on cross-service travel in the Free-Market case: (i) may improve or worsen its performance in aggregate profit and welfare terms relative to the platform regimes, which do not incur the penalty, and, (ii) in particular, can reverse the consumer surplus and welfare rankings in favour of the Intermediary.

5.5. Non-zero constant marginal costs

Up until now, with the exception of Section 5.1, operator costs have been assumed zero. As outlined earlier, zero marginal cost is a common assumption in transport settings and has some basis as a reasonable approximation, especially in the case of public transport where scheduled services run whether or not a given passenger uses the service and where the addition of an extra passenger has negligible impact on running costs. Additionally, models are often neutral to the introduction of non-zero constant marginal costs (henceforth, cost neutrality).[53] We briefly consider the implications of having transport provision where the marginal cost of an additional passenger is non-trivial (e.g., a single passenger using a private cab as part of their OD travel) and where the model does not benefit from cost-neutrality.

It is straightforward to show that the Free-Market and Intermediary regimes are cost-neutral, as are the Integrator, Passive and MTC platforms if the platform provider takes a share, ϕ, of cross-service profit. However, the latter platform regimes are not cost-neutral where ϕ is a share of cross-service revenue. Hence, in the presence of a network where one or more operators incur non-trivial constant marginal costs, the performance of these regimes relative to the cost-neutral regimes may be impacted. It is possible that where ϕ is set as a share of cross-service revenues rather than profit, price negotiations between the platform and operator might be tailored such that the overall remuneration approximates a share of profit in equilibrium, making the regimes cost-neutral. However, we now investigate the impact on key welfare and profit outcomes should this not be the case. For convenience, we label the revenue-based platform models using suffix "$_{rev}$", e.g., Integrator$_{rev}$, I_{rev} and $W^{I_{rev}}$.

It is helpful to introduce constant marginal cost as a proportion τ of α: $c \equiv \tau\alpha$. This additional parameter increases the complexity of market outcome comparisons and, in particular, introduces new non-interior solutions. For instance, interior solutions now require that $c < a$, giving rise to the upper limit $\bar{\tau} \equiv \frac{1}{\gamma n^2 - \gamma + 1}$ (illustrated as the blue lines in Fig. 15(a)). As such, productive progress via analytical results is highly restricted. However, insights do follow from fitting the model according to a calibration based on market demand elasticities. Hence, we identify levels of γ that would be required to achieve the Free-Market outcome with given cost and operator number parameterisations.[54] We then calculate welfare and profit under the resulting parameterisations for the Free-Market as well as under each of the platform regimes to see how they perform should the market move, under the given cost and operator number parameterisations, from the Free-Market to a platform regime.

5.5.1. Welfare

Beginning with welfare, updating the calibration in Fig. 4 to incorporate costs also requires considerations around ensuring interior solutions, resulting in a slightly different approach. In Fig. 15(a), the red (pink) contours represent parameter combinations where the Free-Market and Integrator welfare levels are equated, with and without marginal cost for $\phi = \frac{1}{3}$ ($\phi = \frac{1}{50}$). These contours (based on an underlying continuous function in the relevant range) separate parameter combinations below (above) where welfare is enhanced (reduced) under the Integrator$_{rev}$ relative to the Free-Market by the introduction of constant marginal cost at the corresponding level, τ. The contours become discontinuous where the Integrator$_{rev}$ obtains non-interior solutions. The blue lines indicate maximal, $\bar{\tau}$, feasible cost conditions. Hence, for points below both red and corresponding blue contours, the associated introduction of constant marginal cost is feasible and strictly improves the welfare performance of the Integrator$_{rev}$ relative to the Free-Market.

In terms of the potential for the introduction of $\tau > 0$ to favour welfare under the Free-Market we require parameter combinations above the red/pink contours and below their blue equivalents. Whilst for $n = 2$ there is a limited subset of parameter combinations satisfying these dual requirements with either high τ or γ, as n rises these combinations become highly restricted around high, but not too-high, τ with no solutions for high γ.

It is straightforward to show that there are two crossings of the corresponding blue and red/pink lines for $n \in \{2, 3, 4\}$, with Figs. 15(a) and (b) illustrating the case for the first two of these.[55] Thereafter, for higher levels of n, the red/pink lines become discontinuous before reaching the corresponding blue lines, meaning there is only a lower-γ crossing. Fig. 15(c) reports the lower-γ crossing for different n and $\phi = \frac{1}{3}$ (red) or $\phi = \frac{1}{50}$ (pink) with two notable properties. First, the functions are downward sloping: for higher n the lower-γ limit approaches zero thereby limiting to extremely low levels of substitutability potential cases where the introduction of costs is relatively welfare enhancing for the Free-Market. Second, the red and pink squares broadly coincide: these critical values are highly insensitive to ϕ.

Fig. 15(b) repeats Figure (a) but superimposes calibration contours for (γ, τ) combinations associated with a given n at the Free-Market outcome under the market elasticities of -0.4 (green) and -1.2 (orange). Hence, if the market equilibrium under the Free-Market is consistent with $\eta = -1.2$ and there are 2 operators then the market must be operating on the solid orange line. Note, there is very limited scope in this scenario for the market to be above the red solid line and below the blue solid line whereupon

[53] Specifically, ratios of equilibrium quantities, profits and welfare across cost-neutral regimes do not change with the introduction of constant marginal cost.

[54] This analysis is now based on optimal prices derived from operators paying constant marginal cost, c on each OD unit it services, and pro rata for single-component services (see Appendix C).

[55] The upper-γ crossing for $n = 4$, not illustrated in the Figure, occurs above $\gamma = 0.8$.

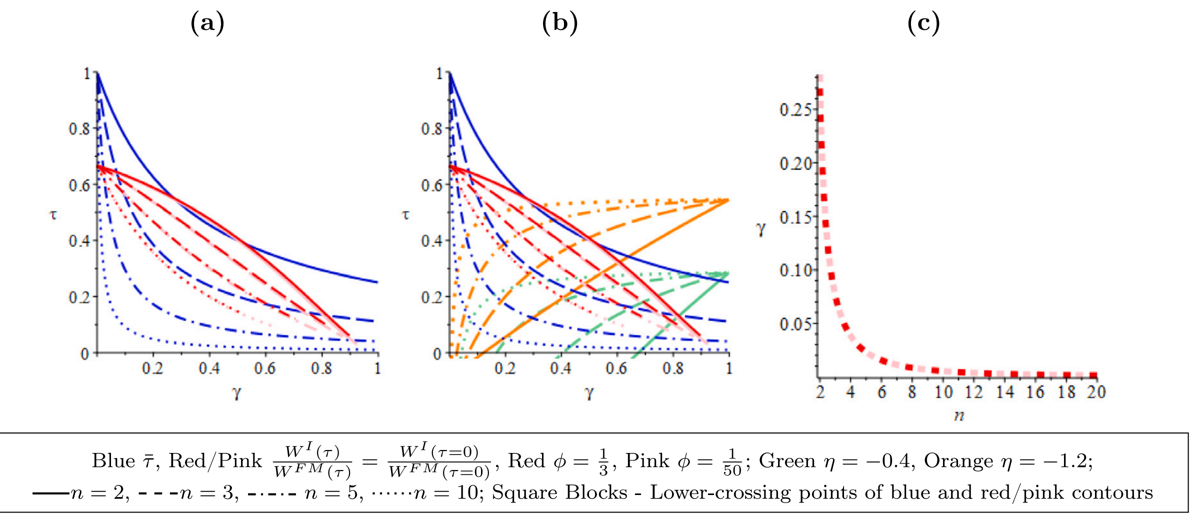


Fig. 15. Upper Limits on Constant Marginal Cost, $\bar{\tau}$, Contours representing Equal Integrator, $\bar{\tau}$, v Free-Market Welfare Ratios with and without Constant Marginal Cost and Lower Blue–Red/Pink Line Crossings with $\phi \in \{\frac{1}{50}, \frac{1}{3}\}$ Calibrated with $\eta \in \{-0.4, -1.2\}$.

introducing non-zero constant marginal costs favours welfare under the Free-Market relative to the Integrator$_{rev}$. For higher n there are no such available outcomes consistent with internal solutions above the corresponding red line that are also in the feasible range. The scope for feasible outcomes above the red lines have more scope under the lower elasticity −0.4 with $n = 2$ but again there are no further instances for higher n. The implication is that, based on the market calibrations, the introduction of non-zero constant marginal costs tends to aid the welfare benefits of the Integrator$_{rev}$ relative to the Free-Market, except in very limited circumstances with $n = 2$, and even more limited for the more elastic demand calibration.

Table 3(A), in Appendix E, which augments Table 1(A) to include welfare rankings under non-zero cost conditions $\tau \in \{\frac{1}{20}, \frac{1}{10}\}$, suggests that the introduction of costs has little impact on the welfare rankings of the Free-Market and Passive$_{rev}$ and Intermediary platforms.[56] The Intermediary continues to perform the least well, often substantially under performing the rest. The Passive$_{rev}$ platform continues to closely approximate the Free-Market, again everywhere one rank lower. The Integrator$_{rev}$ and MTC_{rev} continue to share the number one ranking in all cases, performing particularly well under more elastic demands $\eta \in \{-0.8, -1.2\}$, offering substantial relative welfare improvements over the other regimes. However, notably, the introduction of non-zero costs raises the Integrator$_{rev}$ ranking, in particular at the two higher elasticity calibrations, placing it above the MTC at rank 1.

Remark 9. The introduction of non-trivial constant marginal cost in our calibration raises the ranking of the Integrator$_{rev}$, in particular, to rank 1 above the MTC under the higher elasticity calibrations.

Marginal costs are likely to more closely approximate zero on traditional urban public transport modes (e.g., bus and tram), and marginal costs are likely to be significantly non-zero on modes such as taxi cabs and other on-demand type services. Reported estimates for the latter service types (see footnote 28) also suggest elasticities at the middle to higher end of our calibration. These circumstances are ones which in particular promote the Integrator$_{rev}$ regime, where it most often ranks first and has the biggest relative advantage over the other regimes.

5.5.2. Profit incentives

In terms of the impact of costs on the incentives to pursue different regimes from the point of view of operators and platform providers, we being with the following analytic result.

Proposition 15. *Under the Integrator$_{rev}$ relative to any cost-neutral regime, moving from zero to positive constant marginal cost yields: (i) a decrease in aggregate profit, (ii) an increase in the platform's profit, and, (iii) a decrease in the operator's profit.*

This is reflected in the calibrations reported in Tables 3(B) in Appendix E, where the ranking of Integrator$_{rev}$ operator profit is partially damaged with the introduction of costs. However, despite the regime gaining relative to the cost-neutral regimes in platform profit (Table 4 in Appendix E), consistent with Proposition 15, its ranking here is also partially damaged with the introduction of costs at the expense of the non-cost neutral MTC_{rev}. Overall, the profit story is largely unchanged by the introduction of costs, inasmuch as it still fails to align profit incentives with those of a social planner.

[56] Marginal cost parameterisations beyond $\tau = \frac{1}{10}$ are not included as they result in multiple non-interior solutions. Also, changing the analysis to one with $\phi = \frac{1}{10}$ has very little impact on regime rankings, resulting in a single change, reversing the rankings of the Integrator and MTC platforms in the case of $(n, \eta, \tau) = (3, -0.4, 0)$.

6. Conclusions

Recent developments in technology offering platforms which can tailor integrated door-to-door transport services across multiple operators and modes has generated much interest in terms of their potential to improve passenger mobility, accessibility and pollution especially in second-tier cities. Within this, however, the platform structure has the potential to partially resolve a strategic interaction externality that can otherwise harm the Free-Market performance, damaging both profit and welfare. Indeed, van den Berg et al. (2022) in their analysis of pricing outcomes under $MaaS$ business platform models, conclude, amongst other things, that the Integrator platform offers welfare gains over the Free-Market outcome. The results here are driven by mitigation of this externality with operator numbers restricted to 2.

Extending the network to accommodate n-operators, we show that the Integrator platform can also limit beneficial competitive forces with more operators, undermining its welfare dominance over the Free-Market, especially at higher operator numbers and lower substitutability. This is important since $MaaS$ envisages platforms bringing together *multiple* operators and services in practice. However, calibrations show that where the Integrator platform fails to outperform the Free-Market, a new platform-based model, that we introduce, is optimal. Furthermore, the Integrator is promoted in welfare terms in the presence of non-trivial marginal costs and mid- to high-end market demand elasticities. These conditions are both consistent with the inclusion in the $MaaS$ journey mix of taxi cabs and other on-demand services which are common-place in $MaaS$ offerings. However, profit incentives tend to neither align across platform providers and operators, nor promote the welfare-leading regime, indicating a potential need for policy-maker oversight.

We also introduce a number of extensions to the modelling framework incorporating real network characteristics, including single- and return-trip demand, service frequency as a choice variable and extending the number of components in an OD journey. Where these undermine the welfare supremacy of the Integrator relative to the Free-Market, welfare supremacy tends to be restored under mid- to high-range market elasticity calibrations. Further, whilst it might be thought that the introduction of a cross-service generalised cost penalty under the Free-Market would necessarily worsen its relative profit and welfare outcomes relative to the platforms, this need not be the case. Indeed, it is possible that the penalty can also help resolve a strategic externality under the Free-Market improving its relative performance.

The main takeaways and policy implications are as follows. Although the Free-Market may welfare improve on the Integrator under the multi-operator model and various network extensions, where on-demand or other higher-end elasticity services are likely to play a significant role in the $MaaS$ offering, such platform adoption need not be of concern on price-based grounds. Further, the potential benefits of the MTC as a platform regime lends support to the continuation of the Block Exemption which legally underpins it in the UK (and is under statutory review) and towards consideration of rolling out similar provisions beyond the UK. Whilst the mismatch of profit and welfare incentives cautions against a Laissez-faire policy approach, and the current work has provided some clues about how such policy might be oriented, much remains to be done in terms of future research around the design of a responsible regulatory framework for guiding the development of this growing $MaaS$ platform market.

Finally, this fast-moving topic is generating new avenues for enquiry, including demand-side management of $MaaS$ resources using auctioning (see Xi et al., 2023). Here the auction is overseen by a regulator, but it poses an interesting question as to how a private platform provider might be able to exploit an auction approach to pricing and how this would result in terms of profit and welfare relative to a welfare-maximising regulator. In addition, there is scope to move this predominantly price-based modelling into other important dimensions where $MaaS$ has been studied elsewhere, including, for instance, congestion (e.g., see Di and Ban, 2019) and agent-based modelling (e.g., see Djavadian and Chow, 2017), bringing these into the price-based business model set-up. Indeed, like van den Berg et al. (2022), the model presented here incorporates a number of simplifications which abstract away from potentially important market features, in particular under the assumption of demand symmetry. Interesting questions remain about how the model could be adapted to account for various types of asymmetry beyond those considered here. For instance, a passenger may need to plan flexibly, adapting to a late-running first-component journey, or to a mid-journey change in second-component travel requirements. Accommodating such issues may require a Hotelling (1929) linear city type differentiation, departing from the assumption of all services being symmetrically differentiated. The feasibility of catching an earlier second-component journey, and hence its attractiveness relative to a later one, may be affected by the potential late-running of a given first-component journey. We might expect regimes to accommodate uncertainties and flexibilities differently, depending on whether they operate on a platform. This observation motivates the analysis of $MaaS$ business platforms from a modelling perspective quite distinct from the one adopted here.

CRediT authorship contribution statement

Jolian McHardy: Writing – review & editing, Writing original draft, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

None.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author used ChatGPT in order to check latex and Maple maths code, search industry stylised facts, check alternative definitions of concepts and suggest wording simplifications. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

Appendix A. Proofs to Propositions

A.1. Proof to Proposition 1

(i) This follows directly from observation, setting $\gamma = 0$ in Eq. (6). (ii) First, note that for interior solutions $\Delta > 0$. Then, we can write the partial derivative $\frac{\partial P^I}{\partial \phi} = -\frac{(n-1)}{\Delta}[\alpha(1-\gamma) + n\gamma^2 P^I]$, which is strictly negative for interior solutions in the relevant range. Similarly, $\frac{\partial P_x^I}{\partial \phi} = -\frac{n\gamma^2(n-1)P_x^I}{\Delta}$, which is weakly negative for interior solutions in the relevant range.

A.2. Proof to Proposition 2

It is straightforward to see, using a maths program, that $\frac{\partial \pi^I}{\partial \phi} \leq 0$ has solutions on the full parameter set with equality for $\gamma \in \{0, 1\}$, and no solutions in the relevant range for the reverse inequality, completing the proof.

A.3. Proof to Proposition 3

The partial derivative of the uniform MTC price with respect to ϕ is: $\frac{\partial P^M}{\partial \phi} = -\frac{(1-\gamma)\gamma n^2(n-1)^2}{[\gamma n^3 - \gamma n^2 + 2(2-\phi)(1-\gamma)n - 2(1-\phi)(1-\gamma)]^2} \leq 0$ where the denominator is non-negative and the numerator is zero for $\gamma \in \{0, 1\}$.

A.4. Proof to Proposition 4

Let $H^R \equiv \frac{R^{FM}}{R^I(\phi=0)}$, where $R \in \{P, P_x\}$, which are continuous in (n, γ). Under $n \in \{2, 3\}$, $H^P = 1$ has a single solution at $\gamma = 0$, there are no solutions for $H^{P_x} = 1$, and there exist a parameter combinations, e.g., $\phi = 0$, $n \geq 4$ and $\gamma > \frac{2}{2n^2-8n+3}$, where $H^R < 1$ which, given continuity of H^R and R^I which are non-increasing in ϕ (by Proposition 1), also means there are non-empty intervals for $\phi > 0$ where $H^R < 1$, completing the proof.

A.5. Proof to Proposition 5

Let $H^{CS} \equiv \frac{CS^{FM}}{CS^I}$, $H^W \equiv \frac{W^{FM}}{W^I}$, and $H^\pi \equiv \frac{\pi^{FM}}{\pi^I}$. Since $\lim_{\gamma\to 0} H^{CS} = \frac{5+4n}{9n}$, $\lim_{\gamma\to 0} H^W \approx \frac{(20n+7)}{27n}$ and $\lim_{\gamma\to 0} H^\pi = \frac{8n+1}{9n}$, there is a half-open set on γ: $\gamma \in [0, \bar{\gamma})$ where H^{CS}, H^W and H^π are all strictly below unity for all selections of n and ϕ in the relevant range, completing the proof.

A.6. Proof to Proposition 6

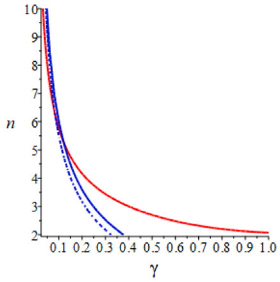
(i) This follows directly from observing Fig. 3(a), noting $\pi^I_{op} \leq \pi^I$. (ii) Let $H = \frac{\pi^{FM}}{\pi^I(\phi=0)}$. Note that H is continuous in the relevant range. For $n \geq 4$, solving for $H = 1$ yields two contours, $\gamma_1 \equiv \frac{2}{2n^2-8n+3}$ and $\gamma_2 \equiv RootOf(-8 + (10n^5 - 28n^4 + 7n^3 + 10n^2 - 11n + 24))_{Z^3} + (8n^4 - 2n^3 + 14n - 52)_{Z^2} + (-8n + 36)_Z$. Solving for $\gamma_1 - \gamma_2 > 0$ we find this is strictly satisfied for all $n \geq 4$, and $\lim_{n\to\infty} \gamma_u = 0$ ($u \in \{1, 2\}$). Finally, between the two contours $H > 1$, whilst on the outside of the two contours $H < 1$. Hence, at $\phi = 0$, yielding the maximal operator profit under the Integrator, the latter dominates the Free-Market on operator profit for $n \geq 4$ and $\gamma > \gamma_1$, and given continuity of H, ensures $H < 1$ is also satisfied on these terms in an open interval of $\bar{\phi} > \phi > 0$, where the lower limit of the open interval is $\bar{\phi} < 1$, completing the proof.

A.7. Proof to Proposition 7

Using a solve command in a maths program shows the full parameter set satisfies $P_x^P \geq P_x^{FM}$ ($P^P \leq P^{FM}$), with equality at either $\gamma = 0$ or $\phi = 0$ and strict inequality otherwise.

A.8. Proof to Proposition 8

(i) First, trivially, the two models coincide at $\phi = 0$ and additionally yield the same prices under $\gamma = 0$. (ii) Otherwise, simple animated plots over the domains of ϕ and γ and over a selection of n clearly show ambiguity in the relative performance of the regimes. Using a solve command in a maths program reveals aggregate profit comparisons to unambiguously favour the Free-Market for $n \in \{2, 3, 4\}$, which does not generalise to locally higher levels of n.



Blue $\frac{\pi^M}{\pi^I(\phi)} = 1$, Red $\frac{\pi^M}{\pi^{FM}} = 1$; —— $\phi = 0$, - - - $\phi = 1$

Fig. 16. Equality Contours for Aggregate Profit under the Intermediary relative to the Free-Market and Integrator for $\phi \in \{0, 1\}$.

A.9. Proof to Proposition 9

(i) Performing a 3D plot of $H \equiv \frac{P^P}{P^I}$ and $H_x \equiv \frac{P_x^P}{P_x^I}$ for $\gamma \in [0,1]$ and $\phi \in (0, \frac{1}{3})$, reveals H_x everywhere strictly greater than one and the same for H except at $\gamma = 0$ where $H = 1$. (ii) Noting that the ratio relative to unity is otherwise ambiguous completes the proof.

A.10. Proof to Proposition 10

Let $H^R \equiv \frac{P^{IM}}{P^R}$ and $H_x^R \equiv \frac{P_x^{IM}}{P_x^R}$, where $R \in \{FM, I\}$. Solutions for $H^R \geq 1$ and $H_x^R > 1$ are across the entire parameter set with no solutions in the parameter set for the reverse inequalities. $H^R = 1$ under $\gamma = 0$, and also $H^{FM} = 1$ for $n = 2$ completing the proof.

A.11. Proof to Proposition 11

The proof is easiest to communicated with reference to Fig. 16 which plots three contours in (γ, n)-space where ratios of aggregate profit across regimes are equal to unity. The red contour represents the full set of (γ, n) combinations for which aggregate profit under the Intermediary is equal to that under the Free-Market. The blue lines capture the equivalent for the case of the Intermediary and Integrator with the solid (dashed) line in the case that the Integrator regime involves zero (full) payment of revenues to the Integrator, $\phi = 0$ ($\phi = 1$). (γ, n) combinations below (above) the contours represents cases where aggregate profit is lower (higher) for the Intermediary relative to the other regime.

A.12. Proof to Proposition 12

The proofs follow directly from examination of solve results for price ratios equal to one and strictly greater than one in a maths program.

A.13. Proof to Proposition 13

Let $H^{CS} \equiv \frac{W^I}{W^M}$ and $H^W \equiv \frac{W^I}{W^M}$. It is straightforward to who that H^{CS} and H^W are continuous in the relevant range. (i) Follows trivially from setting $\gamma = 0$ and simplifying: $H^{CS}(\gamma = 0) = H^W(\gamma = 0) = 1$. (ii) This follows directly from Proposition 12. (iii) For $n = 3$ solving for $H^{CS} = 1$ and $H^W = 1$ yields contours $\phi(\gamma)_{cs}$ and $\phi(\gamma)_w$, illustrated in Fig. 6 by the red and blue lines, respectively. Noting H^{CS} and H^W are less (greater) than unity above (below - but above $\gamma = 0$) the contours completes the proof. (iv) For $n \geq 4$ and $\gamma > 0$ there are no solutions for $H^{CS} = 1$ or $H^W = 1$ and there exists parameterisations for which $H^{CS} < 1$ or $H^W < 1$.

A.14. Proof to Proposition 14

Let $H^\pi \equiv \frac{\pi^I}{\pi^M}$. It is straightforward to who that H^π is continuous in the relevant range. (i) Follows trivially from setting $\gamma = 0$ and simplifying: $H^\pi(\gamma = 0) = 1$. (ii) For $n = 2$ and $\gamma > 0$ there are no solutions for $H^\pi = 1$ and there exist parameterisations for which $H^\pi < 1$. (iii) For $n = 3$ solving for $H^\pi = 1$ yields contour $\phi_\pi(\gamma)$, illustrated in Fig. 6 by the green line. Noting H^π is greater (less) than unity above (below - but above $\gamma = 0$) the contour completes the proof. (iv) For $n \geq 4$ and $\gamma > 0$ there are no solutions for $H^\pi = 1$ and there exist parameterisations for which $H^\pi > 1$.

A.15. Proof to Proposition 15

Given all cost-neutral regimes have common zero to non-zero cost profit ratios it suffices to select any one of these regimes, here the Intermediary. (i) Let $H \equiv \frac{\pi^{I_{rev}}(\tau=0)}{\pi^{IM}(\tau=0)}$ and $H^t \equiv \frac{\pi^{I_{rev}}(\tau)}{\pi^{IM}(\tau)}$. Note, $H^t - H$ is continuous in the relevant range. Given $H^t - H = 0$ has no solutions in the relevant range and there exist feasible parameter combinations for which $H^t - H < 0$ completes the proof. (ii) Let $H \equiv \frac{\pi^{I_{rev}}_{plat}(\tau=0)}{\pi^{IM}_{plat}(\tau=0)}$ and $H^t \equiv \frac{\pi^{I_{rev}}_{plat}(\tau)}{\pi^{IM}_{plat}(\tau)}$. Note, $H^t - H$ is continuous in the relevant range. Given $H^t - H = 0$ has no solutions in the relevant range and there exist feasible parameter combinations for which $H^t - H > 0$ completes the proof. (iii) This follows directly from (i) and (ii).

Appendix B. Base model regime equilibrium derivation

Given symmetry, for the Integrator and MTC we can summarise the single- and cross-service demands for optimisation as:

$$Q_{ii} = a - bP_{ii} + d[vP_{kk} + nvP_x], \quad Q_x = a - bP_x + d[P_{ii} + vP_{kk} + (nv-1)P_x] \quad (k \neq i)$$

where $v \equiv (n-1)$, P_x is the common cross-service price and P_{kk} is a representative single-service price of an operator other than i. The **Integrator** problem can then be summarised by:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + v(1-\phi)P_xQ_x, \quad \max_{P_x} \pi_{plat} = nv\phi P_xQ_x$$

with F.O.C.s, respectively:

$$Q_{ii} - bP_{ii} + vd(1-\phi)P_x = 0, \quad nv\phi(Q_x + P_x(-n + d(nv-1))) = 0$$

Solving simultaneously, recognising symmetry, $P_{ii} = P_{kk}$, yields Eq. (6). The $\boldsymbol{MTC}$ problem can be summarised as:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + v(1-\phi)P_xQ_x$$

where P_x is a function of P_{ii} in accordance with $\frac{\partial P_x}{\partial P_{ii}} = \frac{1}{n}$.[57] This yields the F.O.C.:

$$Q_{ii} + P_{ii}(-b + dv) + v(1-\phi)\left(\frac{Q_x}{n} + P_x\left(-\frac{b}{n} + d\left(1 + \frac{nv-1}{n}\right)\right)\right) = 0$$

Recognising $P = P_x$ (i.e. zero discount on cross-service prices) and yields Eq. (11).

Similarly, for the Free-Market and Passive problems, the demand system can be simplified according to:

$$Q_{ii} = a - bP_{ii} + d[vP_{kk} + 2v(p_i + p_k) + 2v(n-2)p_k], Q_{ik} = a - b(p_i + p_k) + d[P_{ii} + vP_{kk} + (2v-1)(p_i + p_k) + v(n-2)(p_k + p_k)]$$

The **Passive** platform problem can then be summarised by:

$$\max_{P_{ii}, p_i} \pi_i = P_{ii}Q_{ii} + 2v(1-\phi)p_iQ_{ik}$$

with F.O.C.s:

$$Q_{ii} - bP_{ii} + 2dv(1-\phi)p_i = 0, \quad 2vdP_{ii} + 2v(1-\phi)(Q_{ik} + p_i(-b + d(2v-1))) = 0$$

Solving simultaneously, recognising symmetry, $P_{ii} = P_{kk}$ and $p_i = p_k$, yields Eq. (7). The solution for **Free-Market** prices, Eq. (4), follows directly from setting $\phi = 0$.

For the **Intermediary** problem things are a little more complicated. We can summarise the single- and cross-service demands for optimisation as:

$$Q_{ii} = a - bP_{ii} + d[vP_{kk} + 2vP_{xi} + v(n-2)P_{xk}], \quad Q_{xi} = a - bP_{xi} + d[P_{ii} + vP_{kk} + (2v-1)P_{xi} + v(n-2)P_{xk}]$$
$$Q_{xk} = a - bP_{xk} + d[P_{ii} + vP_{kk} + 2vP_{xi} + (v(n-2) - 1)P_{xk}]$$

where P_{xi} and P_{xk} are the cross-service prices for OD journeys involving a single operator i component and no operator i component, respectively. Solving by backward induction, the stage-two platform problem can be written:

$$\max_{P_{xi}, P_{xk}} \pi_{plat} = 2vQxi(P_{xi} - p_i - p_k) + v(n-2)(P_{xk} - 2p_k)$$

with F.O.C.s:

$$2vQ_{xi} + 2v(P_{xi} - p_i - p_k)(-b + d(2v-1)) + 2dv^2(n-2)(P_{xk} - 2p_k) = 0 \tag{20}$$

$$2dv^2(n-2)(P_{xi} - p_i - p_k) + v(n-2)Q_{xk} + v(n-2)(P_{xk} - 2p_k)(-b + d(v(n-2) - 1)) = 0 \tag{21}$$

[57] See McHardy (2022) for the derivation of $\frac{\partial P_x}{\partial P_{ii}}$.

Taking the total derivatives of these F.O.C.s w.r.t. P_{ii} and p_i, yields Eq. (8). Turning to stage one, the problem for operator i can be written:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + 2vp_iQ_{xi}$$

with F.O.C.s given by:

$$Q_{ii} + P_{ii}\left(-b + 2vd\frac{\partial P_{xi}}{\partial P_{ii}} + vd(n-2)\frac{\partial P_{xk}}{\partial P_{ii}}\right) + 2vp_i\left(d + (-b + d(2v-1))\frac{\partial P_{xi}}{\partial P_{ii}} + vd(n-2)\frac{\partial P_{xk}}{\partial P_{ii}}\right) = 0 \tag{22}$$

$$vdP_{ii}\left(2\frac{\partial P_{xi}}{\partial p_i} + (n-2)\frac{\partial P_{xk}}{\partial p_i}\right) + 2vQ_{xi} + 2vp_i\left((-b + d(2v-1))\frac{\partial P_{xi}}{\partial p_i} + vd(n-2)\frac{\partial P_{xk}}{\partial p_i}\right) = 0$$

Recognising symmetry ($P_{ii} = P_{kk}$, $P_{xi} = P_{xk}$ and $p_i = p_k$), and solving the F.O.C.s in Eqs. (20) and (22) simultaneously, yields Eq. (9).

Appendix C. Equilibrium price solutions with constant marginal cost

The introduction of constant and common marginal cost, $c = \tau\alpha$, (with fee-taking platforms appropriating a share, ϕ of revenue, rather than profit), means in all regimes the new equilibria follow the same optimisation process as before but amending operator i's profit in each regime by $-c\left(Q_{ii} + \frac{1}{2}\sum_{j\neq i}^{n} Q_{ij} + Q_{ji}\right)$, assuming, without loss of generality given the symmetry of the model, that the marginal cost of passenger on a single component is $\frac{c}{2}$. This results in the following, revised, equilibrium prices:

$$P^{FM}(\tau) = \frac{\alpha[\gamma(\tau(2n^2-3n-2)-3)+3(1+\tau)]}{\gamma(6+(2n^2-3n-5))}, \quad P_x^{FM}(\tau) = \frac{\alpha[\gamma(\tau(2n^2-3n-1)-4)+4(1+\tau)]}{\gamma(6+(2n^2-3n-5))} \tag{23}$$

$$P^{I}(\tau) = \frac{\alpha[(n-1)(2n^2\tau - n(2\tau+1)+\phi-2\tau-3)\gamma^2 + ((2\tau+1)n^2+(-\phi+2)n+\phi-4\tau-5)\gamma+2\tau+2]}{(4+\gamma^2(n-1)(-6+3n^2+n(\phi-3))+\gamma(4n^2+2n-10))} \tag{24}$$

$$P_x^{I}(\tau) = \frac{\alpha[2+\gamma[\gamma(3+n^3\tau-(\tau+2)n^2-n\tau)+2n^2+n\tau-5]]}{(4+\gamma^2(n-1)(-6+3n^2+n(\phi-3))+\gamma(4n^2+2n-10))}$$

$$\begin{aligned}P^{P}(\tau) = {}& \alpha((\tau(\phi-2)n^3+\tau(\phi+1)n^2+((-4\tau-3)\phi+5\tau+3)n+2(-1+\phi)(\phi-\tau-3/2))(n-1)\gamma^2+(((4\tau+3)\phi-5\tau-3)n^2\\ &+(-2\phi^2+(-2\tau+2)\phi+3\tau)n+2(\phi-(5\tau)/2-3)(-1+\phi))\gamma+3(\tau+1)(-1+\phi))/(2(n-1)((-1+\phi)n^3+(-\phi/2+1/2)n^2\\ &+(\phi-2)^2n-(5\phi)/2+5/2)\gamma^2+8(n+1)(n-11/8)(-1+\phi)\gamma+6\phi-6)\end{aligned}$$

$$\begin{aligned}P_x^{P}(\tau) = {}& -2((\tau n^4-\tau(\phi+3/2)n^3+((\tau+2)\phi-(3\tau)/2-2)n^2+\tau(\phi+3/2)n-3\phi+\tau/2+2)\gamma^2+((-2\phi+2\tau+2)n^2-\tau(\phi+3/2)n\\ &+5\phi-(3\tau)/2-4)\gamma-2\phi+\tau+2)\alpha/(2(n-1)((-1+\phi)n^3+(-\phi/2+1/2)n^2+(\phi-2)^2n-(5\phi)/2+5/2)\gamma^2\\ &+8(n+1)(n-11/8)(-1+\phi)\gamma+6\phi-6)\end{aligned}$$

$$\begin{aligned}P^{IM}(\tau) = {}& 3(((n^3\tau+(-(7\tau)/3-1/3)n^2-n+(4\tau)/3+4/3)\gamma^2+((\tau+1/3)n^2\\ &+(\tau+1)n-4\tau-10/3)\gamma+2\tau+2)\alpha)/(12+(3n^3-8n^2-3n+8)\gamma^2+(4n^2+6n-22)\gamma)\end{aligned}$$

$$\begin{aligned}P_x^{IM}(\tau) = {}& (((n-1)(8/3+n^3\tau+(-2\tau-2/3)n^2+(\tau/6-7/6)n)\gamma^3+(((5\tau)/3+2/3)n^3+(-(11\tau)/6-1/6)n^2\\ &+(-(11\tau)/6-17/2)n+\tau+9)\gamma^2+(((2\tau)/3+2/3)n^2+((4\tau)/3+14/3)n-(5\tau)/3-29/3)\gamma+(2\tau)/3+10/3)\alpha)/(4+(n^3-8/3n^2-n+8/3)\gamma^2\\ &+(4/3n^2+2n-22/3)\gamma)(1+(n-1)\gamma)\end{aligned}$$

$$P^{M}(\tau) = P_x^{M}(\tau) = \frac{\alpha[\gamma(n^3-n^2\tau+(\phi-2\tau-2)n-\phi+\tau+1)+n(-\phi+2\tau+2)+\phi-\tau-1]}{(\gamma(n^3-n^2+n(2\phi-4)-2\phi+2)+n(-2\phi+4)+2\phi-2)} \tag{25}$$

Appendix D. Calibration

D.1. Base model

For the calibration, we assume that the market is currently operating in accordance with the Free-Market regime. Hence, equilibrium prices are given by Eq. (23). In order to fit these to market demand elasticities, we need to establish an expression for an average price elasticity of demand across the network under the Free-Market. We do this by first taking the total demand at a given combination of in-and cross-service prices ($Q_{tot}^{FM} \equiv nQ^{FM} + n(n-1)Q_x^{FM}$) and partially differentiate this with respect to price – allowing in – and cross-service prices to change equally — hence approximating the effect on total demand with a simultaneous change in all prices. This results in $\frac{\partial Q_{tot}^{FM}}{\partial price} = n^2(-b+d(n-1)(1+n))$. To get an expression for the approximate weighted average market elasticity we multiply this by weighted average price $P_{av}^{FM} \equiv \frac{nP^{FM}+n(n-1)P_x^{FM}}{nP^{FM}Q^{FM}+n(n-1)P_x^{FM}Q_x^{FM}}$ and divide by Q_{tot}^{FM}. Hence, the approximate market-wide elasticity is found from $\eta = \frac{\partial Q_{tot}^{FM}}{\partial price}\frac{P_{av}^{FM}}{Q_{tot}^{FM}}$. Fitting a specific elasticity and specifying τ, yields a $\gamma(n)$ contour, associating a level of substitutability at a given level of n such the Free-Market equilibrium results in the fitted elasticity.

D.2. Base-model extensions

Frequency selection with two operators

With the Free-Market Nash equilibrium having 2 operators $n_i = 2$, the calibration is based on there being 16 services with 8 single-service and 8 cross-service. Hence, the elasticity in Appendix D.1 can be updated with $Q^{FM}_{tot} \equiv 8Q^{FM} + 8Q^{FM}_x$, $\frac{\partial Q^{FM}_{tot}}{\partial price} = 16(-b + 15d)$ and weighted average price $P^{FM}_{av} \equiv \frac{8P^{FM}Q^{FM} + 8P^{FM}_x Q^{FM}_x}{8Q^{FM} + 8Q^{FM}_x}$.

Single- and Dual-component Demand

Total market demand is now $Q^{FM}_{tot} \equiv nQ^{FM} + n(n-1)Q^{FM}_x + n(X^{FM} + Y^{FM})$. Given X^{FM} and Y^{FM} are single component journeys with price $\frac{1}{2}P^{FM}_x$, the weighted average two-component price is $P^{FM}_{av} \equiv \frac{nP^{FM}Q^{FM} + n(n-1)P^{FM}_X Q^{FM}_x + \frac{1}{2}P^{FM}_x n(X^{FM}+Y^{FM})}{nQ^{FM} + n(n-1)Q^{FM}_x + \frac{1}{2}n(X^{FM}+Y^{FM})}$ and $\frac{\partial Q^{FM}_{tot}}{\partial price} = n^2(-b + d(n-1)(n+1)) + n(-B + (n-1)D)$, with the elasticity in Appendix D.1 being updated accordingly.

Multiple Component Journeys

There are now $N = n^3 = 8$ OD services, 2 single- and 6 cross-service, such that $Q^{FM}_{tot} \equiv 2Q^{FM} + 6Q^{FM}_x$, $\frac{\partial Q^{FM}_{tot}}{\partial price} = 8(-b + 7d)$, and $P^{FM}_{av} \equiv \frac{2P^{FM}Q^{FM} + 6P^{FM}_X Q^{FM}_x}{2Q^{FM} + 6Q^{FM}_x}$, with the elasticity in Appendix D.1 being updated accordingly.

Non-zero Constant Marginal Costs

The calibration here follows directly from Appendix D.1, interpreting prices and quantities as those under the Free-Market in terms of τ.

Appendix E. Welfare and profit incentives with marginal cost

See Tables 3 and 4

Appendix F. Asymmetric demand robustness check

Though a standard assumption in this literature, demand symmetry of the type employed here restricts the substitutability of all bilateral journey pairings to be the same. In practice, this may not always be accurate, and, although, in theory it is unlikely to be of first-order importance, and a priori, we may not always be able to say which way, if any, such an asymmetry might apply (e.g., see the discussion in footnote 14), we undertake a brief robustness check. Moving away from symmetry and imposing a lower substitutability between some journeys could impact regime rankings in two main ways: (i) by lowering the average substitutability on the network (a scale effect), it might favour regimes that would have performed better at lower symmetric substitutability, and (ii) by changing the power, though not the direction, of strategic effects in one regime relative to another. We wish to test for the latter, as the former scale effects do not derive from asymmetries.

To try and control for (i) and hence test for (ii), we differentiate bilateral substitution parameters, with γ between journeys sharing an origin or sharing a destination (e.g., J_{12} and J_{13}), and ω otherwise (e.g., J_{12} and J_{21}). For simplicity we assume $\gamma > \omega$, hence journeys sharing an origin or sharing a destination are better substitutes with each other relative to those services that do not. With the addition of an extra parameter, modelling complexity is dramatically increased, and progress requires us to focus the analysis. We select the case of $n = 3$ for four reasons. First, it is straightforward to show, using basic combinatorics, that this is the only case where the number of journeys related under γ is the same as the number related under ω, representing the case of maximal difference in the network relative to symmetry.[58] Second, this exact half-and-half characterisation, allows for a simple exercise to minimise effect (i) by setting the symmetric substitutability value at $\gamma = \gamma_0$, and the asymmetric $\gamma = \bar{\gamma}$ and $\omega = \underline{\gamma}$, where $\gamma_0 = \frac{1}{2}(\bar{\gamma} + \underline{\gamma})$. Hence, if there are no (ii) effects, results will be unchanged as the average γ on the network is the same under the symmetric and asymmetric models. Third, the complexity of the functions quickly escalates with n. With $n = 3$ there are already 36 journey pairings under γ in the utility function (e.g., in the right summation in brackets in symmetric utility function, Eq. (2)). Finally, $n = 3$ is quite relatable in terms of an OD journey with interchangeable bus, train and taxi options, each mode with a single operator.

Setting $\alpha = 1$, without loss of generality, solving a modified asymmetric utility function under constrained optimisation, we have the following single- and cross-network demands, respectively, replacing Eq. (1):

$$
\begin{aligned}
Q_{11} = {} & (2P_{11}\gamma^2 + 3P_{11}\gamma\omega + 2P_{11}\omega^2 + 2P_{12}\gamma^2 - P_{12}\gamma\omega - 2P_{12}\omega^2 + 2P_{13}\gamma^2 - P_{13}\gamma\omega - 2P_{13}\omega^2 + 2P_{21}\gamma^2 - P_{21}\gamma\omega - 2P_{21}\omega^2 - 2P_{22}\gamma^2 \\
& - P_{22}\gamma\omega + 2P_{22}\omega^2 - 2P_{23}\gamma^2 - P_{23}\gamma\omega + 2P_{23}\omega^2 + 2P_{31}\gamma^2 - P_{31}\gamma\omega - 2P_{31}\omega^2 - 2P_{32}\gamma^2 - P_{32}\gamma\omega + 2P_{32}\omega^2 - 2P_{33}\gamma^2 - P_{33}\gamma\omega \\
& + 2P_{33}\omega^2 - 2\alpha\gamma^2 + 5\alpha\gamma\omega - 2\alpha\omega^2 - 3P_{11}\gamma - 3P_{11}\omega + P_{12}\gamma + P_{13}\gamma + P_{21}\gamma + P_{22}\omega + P_{23}\omega + P_{31}\gamma + P_{32}\omega + P_{33}\omega - \alpha\gamma \\
& - \alpha\omega - P_{11} + \alpha)/(1 - 8\gamma^3 + 12\gamma^2\omega + 12\gamma\omega^2 - 8\omega^3 - 6\gamma^2 - 3\gamma\omega - 6\omega^2 + 3\gamma + 3\omega)
\end{aligned}
$$

[58] For all other n, there is an imbalance in the number of OD journey-pair substitutability types, favouring either γ or ω, whereupon the set-up more closely resembles the symmetric case with the common substitutability parameter γ or ω respectively.

Table 3
Demand Elasticity Calibrated Regime Welfare and Operator Profit Rankings and as a Percentage of the Free-Market with $\phi = \frac{1}{50}$.

η	τ	Regime	A. Welfare								B. Operator Profit							
			W Rank				W as a Percentage of W^{FM}				π_{op} Rank				π_{op} as a Percentage of π^{FM}			
			n				n				n				n			
			2	3	5	10	2	3	5	10	2	3	5	10	2	3	5	10
		Free-Market	3	3	2	2	100.0	100.0	100.0	100.0	1	1	1	2	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	4	4	108.6	104.9	99.8	93.2	5	5	2	1	69.4	81.0	92.6	102.1
	0.00	Intermediary	5	5	5	5	97.0	90.9	84.0	76.2	3	3	4	5	94.0	91.3	84.2	72.7
		Passive$_{rev}$	4	4	3	3	99.9	100.0	100.0	100.0	2	2	3	3	95.9	93.9	92.3	91.1
		MTC$_{rev}$	2	2	1	1	106.4	104.8	103.7	102.9	4	4	5	4	83.2	82.6	82.1	81.7
		Free-Market	3	3	3	2	100.0	100.0	100.0	100.0	1	1	1	2	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	4	107.6	105.0	101.0	95.0	5	5	4	1	53.4	69.7	87.1	103.0
−0.4	0.05	Intermediary	5	5	5	5	97.7	92.4	86.2	78.7	3	2	3	5	94.7	94.1	88.6	77.0
		Passive$_{rev}$	4	4	4	3	99.9	99.9	100.0	100.0	2	3	2	3	95.4	93.2	91.4	90.2
		MTC$_{rev}$	2	2	1	1	105.3	103.8	102.8	102.1	4	4	5	4	79.6	79.5	79.4	79.4
		Free-Market	3	3	3	2	100.0	100.0	100.0	100.0	1	1	1	2	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	4	105.8	104.6	101.9	97.0	5	5	5	1	26.5	49.0	75.0	101.3
	0.10	Intermediary	5	5	5	5	98.3	94.0	88.7	81.7	2	2	2	4	95.4	97.4	94.2	83.1
		Passive$_{rev}$	4	4	4	3	99.9	99.9	99.9	99.9	3	3	3	3	94.8	92.3	90.4	89.1
		MTC$_{rev}$	2	2	1	1	104.1	102.8	102.0	101.4	4	4	4	5	75.1	75.6	76.0	76.2
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	2	115.4	111.1	105.8	100.4	5	4	3	3	88.3	89.5	90.5	91.0
	0.00	Intermediary	5	5	5	5	92.3	83.7	75.3	68.1	4	5	5	5	88.7	80.1	70.8	61.7
		Passive$_{rev}$	4	4	4	4	99.9	100.0	100.0	100.0	2	2	2	2	95.6	93.8	92.3	91.1
		MTC$_{rev}$	2	2	1	1	111.6	110.7	109.8	109.0	3	3	4	4	93.7	90.3	87.7	85.7
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	2	115.7	111.7	106.5	101.1	5	4	3	3	79.8	83.8	87.1	89.5
−0.8	0.05	Intermediary	5	5	5	5	93.6	85.2	76.8	69.3	4	5	5	5	90.1	82.2	72.8	63.1
		Passive$_{rev}$	4	4	4	4	99.9	99.9	99.9	99.9	2	2	2	2	95.1	93.1	91.4	90.2
		MTC$_{rev}$	2	2	1	1	110.6	109.4	108.3	107.5	3	3	4	4	91.1	87.8	85.3	83.4
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	1	2	115.3	111.8	107.1	101.7	5	5	4	3	67.2	75.0	81.9	87.0
	0.10	Intermediary	5	5	5	5	94.8	86.7	78.6	70.9	3	4	5	5	91.4	84.5	75.3	65.0
		Passive$_{rev}$	4	4	4	4	99.9	99.9	99.9	99.9	2	2	2	2	94.7	92.4	90.5	89.1
		MTC$_{rev}$	2	2	2	1	109.5	108.0	106.8	106.0	4	3	3	4	88.1	85.0	82.7	80.9
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	2	116.3	116.9	114.4	111.2	3	3	2	2	98.8	96.3	94.1	92.4
	0.00	Intermediary	5	5	5	5	84.9	77.7	70.4	64.2	5	5	5	5	80.7	72.9	64.9	58.0
		Passive$_{rev}$	4	4	4	4	100.0	100.0	100.0	100.0	4	4	4	4	95.4	93.7	92.2	91.1
		MTC$_{rev}$	2	2	1	1	114.3	116.4	116.9	116.7	2	2	3	3	99.3	96.5	93.5	91.2
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	2	2	118.5	117.7	114.7	111.2	4	4	4	3	94.8	92.4	90.9	89.8
−1.2	0.05	Intermediary	5	5	5	5	87.3	79.3	71.5	65.0	5	5	5	5	83.3	74.7	66.2	58.7
		Passive$_{rev}$	4	4	4	4	99.9	99.9	99.9	99.9	3	3	2	2	94.9	93.0	91.4	90.2
		MTC$_{rev}$	2	2	1	1	113.7	114.8	114.8	114.5	2	2	3	4	97.5	94.1	90.9	88.5
		Free-Market	3	3	3	3	100.0	100.0	100.0	100.0	1	1	1	1	100.0	100.0	100.0	100.0
		Integrator$_{rev}$	1	1	1	2	120.1	118.3	115.0	111.2	4	4	4	3	87.8	86.7	86.4	86.4
	0.10	Intermediary	5	5	5	5	89.4	80.9	72.9	66.0	5	5	5	5	85.6	76.6	67.7	59.6
		Passive$_{rev}$	4	4	4	4	99.9	99.9	99.9	99.9	3	2	2	2	94.5	92.3	90.5	89.2
		MTC$_{rev}$	2	2	2	1	113.0	113.2	112.8	112.3	2	3	3	4	95.3	91.5	88.2	85.8

$$
\begin{aligned}
Q_{12} = \alpha(&2P_{11}\gamma^2 - P_{11}\gamma\omega - 2P_{11}\omega^2 + 2P_{12}\gamma^2 + 3P_{12}\gamma\omega + 2P_{12}\omega^2 + 2P_{13}\gamma^2 - P_{13}\gamma\omega - 2P_{13}\omega^2 - 2P_{21}\gamma^2 - P_{21}\gamma\omega + 2P_{21}\omega^2 + 2P_{22}\gamma^2 \\
&- P_{22}\gamma\omega - 2P_{22}\omega^2 - 2P_{23}\gamma^2 - P_{23}\gamma\omega + 2P_{23}\omega^2 - 2P_{31}\gamma^2 - P_{31}\gamma\omega + 2P_{31}\omega^2 + 2P_{32}\gamma^2 - P_{32}\gamma\omega - 2P_{32}\omega^2 - 2P_{33}\gamma^2 - P_{33}\gamma\omega \\
&+ 2P_{33}\omega^2 - 2\alpha\gamma^2 + 5\alpha\gamma\omega - 2\alpha\omega^2 + P_{11}\gamma - 3P_{12}\gamma - 3P_{12}\omega + P_{13}\gamma + P_{21}\omega + P_{22}\gamma + P_{23}\omega + P_{31}\omega + P_{32}\gamma \\
&+ P_{33}\omega - \alpha\gamma - \alpha\omega - P_{12} + \alpha)/(1 - 8\gamma^3 + 12\gamma^2\omega + 12\gamma\omega^2 - 8\omega^3 - 6\gamma^2 - 3\gamma\omega - 6\omega^2 + 3\gamma + 3\omega)
\end{aligned}
$$

Solving for all 5 regimes, employing the above demand system in line with the broad optimisation strategy in Appendix B, Table 5 reports the welfare relative to the Free-Market case and rankings for 3 substitution specifications allowing the comparison of the symmetric and asymmetric models. To maximise the signals coming through channel (ii), we allow a substantial 0.25 point difference between $\underline{\gamma}$ and $\bar{\gamma}$ in each case. The subset of substitutability values employed is justified on a number of grounds. First, larger differences and selections above $\gamma = 0.5$ result in corner solutions, i.e., the full network is no longer functional, for $n = 3$. Second, Fig. 4(a) shows that all three aggregate market elasticities result in γ in the range analysed here, $\gamma \in [0, 0.5]$. Specifically, for $n = 3$, γ under the Free-Market calibrated elasticities $\{-0.4, -0.8, -1.2\}$ are, respectively $\{0.390, 0.171, 0.059\}$. Hence, for any calibration under the full range of market estimates, γ does not fall below 0.059 or rise above 0.390, with the latter well below our upper parameterisation of $\gamma = 0.5$. The above figures also serve to justify the use of the term "substantial" in characterising the variation across $\underline{\gamma}$ and $\bar{\gamma}$, as the size of the asymmetry more than encompasses the difference between contiguous elasticity calibrations (e.g., the asymmetry is across a γ gap of 0.25 whilst the γ gap between the −0.4 and −0.8 (−0.8 and −1.2) elasticity calibrations is 0.112 (0.219). Despite the broad range of scenarios and some very small starting-point quantitative variations in welfare ratios under symmetry (e.g., very small relative welfare differences between the MTC and the Integrator, and the Passive and Free-Market regimes), the regime rankings, under the introduction of substantial asymmetries, are unchanged throughout. This is not to say that asymmetries cannot change rankings. With non-zero quantitative effects there is no reason to believe they will

Table 4
Demand Elasticity Calibrated Regime Platform Profit Rankings and as a Percentage of the Intermediary with $\phi = \frac{1}{50}$.

η	τ	Regime	π_{plat} Rank n				π_{plat} as a Percentage of π_{plat}^{IM} n			
			2	3	5	10	2	3	5	10
		Integrator_{rev}	3	4	2	2	48.9	66.6	36.1	25.6
	0.00	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	3	3	3	38.5	67.6	36.1	23.6
		MTC_{rev}	2	2	4	4	58.8	71.5	34.2	21.6
		Integrator_{rev}	3	4	4	2	67.1	77.5	39.2	26.6
−0.4	0.05	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	3	2	3	48.5	77.8	39.7	25.0
		MTC_{rev}	2	2	3	4	83.1	87.9	39.2	23.6
		Integrator_{rev}	3	3	4	2	99.9	95.0	43.8	28.1
	0.10	Intermediary	2	2	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	3	3	63.8	92.7	45.2	27.1
		MTC_{rev}	1	1	2	4	123.8	112.6	46.5	26.2
		Integrator_{rev}	3	3	2	3	32.9	78.3	51.5	40.0
	0.00	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	2	26.1	73.7	51.2	40.0
		MTC_{rev}	2	2	3	4	34.9	80.9	51.2	38.4
		Integrator_{rev}	3	3	3	3	42.0	87.0	54.1	40.8
−0.8	0.05	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	2	31.4	80.4	53.5	40.9
		MTC_{rev}	2	2	2	4	45.3	91.7	54.8	39.9
		Integrator_{rev}	3	3	3	3	55.2	98.1	57.3	41.9
	0.10	Intermediary	1	2	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	2	38.4	88.7	56.4	41.9
		MTC_{rev}	2	1	2	4	60.1	105.4	59.2	41.6
		Integrator_{rev}	3	3	3	2	21.5	86.0	68.2	57.2
	0.00	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	4	18.3	77.6	64.6	55.7
		MTC_{rev}	2	2	2	3	21.6	86.7	68.3	56.7
		Integrator_{rev}	3	3	3	2	27.4	94.4	70.8	58.0
−1.2	0.05	Intermediary	1	1	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	4	21.9	83.3	66.6	56.4
		MTC_{rev}	2	2	2	3	27.6	95.8	71.5	58.0
		Integrator_{rev}	3	2	3	3	35.5	104.7	74.0	59.1
	0.10	Intermediary	1	3	1	1	100.0	100.0	100.0	100.0
		Passive_{rev}	4	4	4	4	26.5	90.0	68.9	57.1
		MTC_{rev}	2	1	2	2	35.8	106.9	75.2	59.4

Table 5
Asymmetric Demand Robustness Check: Welfare Ranking and Ratios for Regimes ($R \in \{I, M, P, IM\}$) Relative to Free-Market with $\phi = \frac{1}{50}$.

$\{\underline{\gamma}, \gamma_0, \bar{\gamma}\}$	Regime	Symmetric ($\gamma = \omega = \gamma_0$)		Asymmetric ($\omega = \underline{\gamma},\ \gamma = \bar{\gamma}$)	
		Rank	$\frac{W^R}{W^{FM}}$	Rank	$\frac{W^R}{W^{FM}}$
	Integrator	1	1.1321	1	1.1149
	MTC	2	1.1267	2	1.0857
$\{0, 0.125, 0.25\}$	Free-Market	3	1.0000	3	1.0000
	Passive	4	0.9998	4	0.9992
	Intermediary	5	0.8149	5	0.8788
	Integrator	1	1.1084	1	1.0885
	MTC	2	1.0801	2	1.0603
$\{0.125, 0.25, 0.375\}$	Free-Market	3	1.0000	3	1.0000
	Passive	4	0.9997	4	0.9989
	Intermediary	5	0.8669	5	0.9178
	Integrator	1	1.0522	1	1.0523
	MTC	2	1.0507	2	1.0398
$\{0.25, 0.375, 0.5\}$	Free-Market	3	1.0000	3	1.0000
	Passive	4	0.9997	4	0.9986
	Intermediary	5	0.9049	5	0.9454

not change rankings under the right circumstances, e.g., bringing a reversal of ranking between two regimes which occurs under symmetry at a given γ, to a slightly lower or higher level of γ under asymmetry. However, it does suggest an absence of evidence for a dramatic re-ordering of regimes via the introduction of this asymmetry which would make the broad pattern of findings in the symmetric analysis fundamentally unreliable and flawed.

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