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Deconstructing Pedestrian Crossing Decisions in Interactions with Continuous Traffic: An Anthropomorphic Model

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***Abstract*—Increasing attention has been drawn to computational pedestrian behavior models aimed at understanding the interaction mechanisms between pedestrians and vehicles. Nevertheless, existing research lacks exploration of the underlying behavioral mechanisms of pedestrian crossing decisions, which leads to unrealistic modeling results. In particular, when dealing with continuous traffic flow scenarios, the concept of *waiting time* is frequently used to account for all intricate traffic flow effects. Moreover, very few studies considered the time-dynamic nature of crossing decisions. To address these research limitations, this study deconstructs pedestrian crossing decisions at uncontrolled intersections with continuous traffic flow through a cognitive process and proposes an anthropomorphic crossing decision model. Specifically, we propose a novel visual collision cue-based crossing decision-initiation model to characterize time-dynamic crossing decisions. In light of the risk-aversion theory, a traffic gap comparison strategy is put forward to explain and model pedestrian waiting behavior in traffic flow. Two datasets collected from a CAVE-based immersive pedestrian simulator are applied to calibrate and validate the model. The proposed model accurately predicts pedestrian crossing decisions across all traffic scenarios. The modeling performance is significantly enhanced by considering the proposed traffic gap comparison strategy. Moreover, the model accurately captures the timing of crossing decisions. This work concisely demonstrates how pedestrians dynamically adapt their crossings in continuous traffic based on visual collision cues, potentially offering insights into modeling pedestrian-vehicle interactions or serving as a tool to realize anthropomorphic pedestrian crossing decisions in simulators.**

***Index Terms*—Pedestrian-vehicle interaction, Road crossing decision, Anthropomorphic model, Traffic flow.**

I. Introduction

CONSENSUS suggests that automated vehicles (AVs) should be capable of operating on roads mixed with other road users during the transition from manual to fully automated driving [1]. As AV deployment expands from confined low-risk areas to a variety of operational design domains, conflicts with other road users will inevitably increase [2]. Failures in interactions between AVs and other road users may impede large-scale adoption and social acceptance of AVs [3], [4]. This, therefore, leads to the research context of this study, which is to promote safe and smooth interactions in traffic [1], [3], [4]. Pedestrians are generally regarded as the most vulnerable road users in modern transport systems due to their lack of protective equipment and slow movement compared to other road users. However, it is a challenge to determine pedestrians' actions and intentions. Especially at uncontrolled intersections, pedestrian crossing decisions are more unpredictable, and safety problems are more common than on other controlled road sections, as there are no traffic signals to coordinate the interaction process [5]. Below, we review the relevant theories and results to understand pedestrian crossing decisions.

A. Pedestrian crossing decisions research

Pedestrian crossing decision-making is a complex and dynamic process that involves assessing the traffic situation, selecting an appropriate gap, and coordinating motor activity with the continuous perception of road traffic [6]. Some theoretical frameworks have been used to understand pedestrian crossing decision-making from psychological perspectives. According to Ajzen's theory of planned behavior [7], pedestrians' crossing intentions could be determined by their attitudes toward crossing behavior, subjective norms, and perceived behavioral control. [8] found that perceived behavioral control had the most significant impact and was related to pedestrian-perceived risk. [1] applied the situation awareness model to interpret the crossing decision-making and suggested that pedestrians would perceive the traffic and environment, comprehend the situation, and predict future states and events before making crossing decisions, where the perception process was at first. Moreover, according to cumulative prospect theory [9], [10] supposed that pedestrians were risk aversive when making crossing decisions, attempting to minimize their collision risks and time delays. The mentioned theoretical frameworks agree that the perception process, especially pedestrian risk perception, is critical for crossing decision-making. There are various factors that could affect the crossing decision-making, such as traffic factors (e.g., vehicle speed, vehicle yielding behavior, traffic density) [11], environmental factors (e.g., lane number, road width,

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weather condition) [12], and pedestrian characteristics (e.g., age, gender, distraction) [13], while time to collision (TTC), distance, vehicle speed, and traffic density is closely related to pedestrian risk perception [4].

Before pedestrians start the action of crossing, there is usually a period of time called crossing initiation time, reflecting the time-dynamic nature of pedestrian crossing decisions [13]. When consecutive vehicles drive on the road, the crossing initiation time refers to the duration between when the rear end of the previous approaching vehicle passes the pedestrian position and when pedestrians start crossing. However, if only one car approaches pedestrians, the crossing initiation time is defined as the duration between when the vehicle appears in the lane and when pedestrians initiate crossing [14]. According to a cognitive theory, pedestrian crossing initiation time was assumed to be the product of the accumulation process of noisy evidence in the human cognitive system [15], which reflects the efficiency of pedestrian cognitive and locomotor systems. Pedestrian crossing initiation time could be affected by many factors, such as vehicle TTC and speed. For instance, pedestrians might initiate quickly when facing a vehicle with a low speed [13] or with a small time gap from the approaching vehicle [16]. Therefore, existing empirical observations highlight the role of crossing initiation time in pedestrian crossing decisions.

On busy roads, it is common to see pedestrians waiting for opportunities to cross when facing a stream of vehicles. The *waiting time* before pedestrians make crossing decisions has received considerable attention in research. Intuitively, it might be inferred that pedestrians, after an initial failed attempt to cross, were willing to accept higher risk crossing opportunities to reduce time costs. In other words, as the *waiting time* increased, pedestrians might become more inclined to take riskier crossing opportunities [12], [17]. However, a substantial body of evidence suggested that pedestrians who tended to wait were more cautious and less likely to accept risky gaps [18]–[21]. [21] showed that drivers who rejected the bigger traffic gap tended to incur a longer delay. [20] indicated that pedestrians who tended to reject the crossing opportunities would be more cautious and tend to accept longer gaps. Moreover, [18] found that pedestrians who missed the first opportunity to cross the road would not compensate for their loss by making riskier crossing decisions. A meta-study explored these conflicting findings and concluded that insufficient evidence supports a direct relationship between *waiting time* and the propensity to make riskier crossing decisions [22]. [20] suggested that *waiting time* is more likely a consequence of certain factors rather than the primary cause of crossing decisions. Accordingly, to truly understand pedestrian behavior in traffic flow, it is essential to investigate why pedestrians wait in the first place [18], [19], [21].

B. Computational models for pedestrian crossing decisions

Against the research background above, researchers are naturally interested in considering how to use computational models to reproduce realistic pedestrian crossing decisions by involving the above-mentioned theories and factors, which do have real social implications in terms of traffic safety, management, and more [3], [23]. Much attention has been, therefore, drawn to this pressing issue [14], [24], [25]. Existing computational models for pedestrian crossing decisions have been developed based on a wide range of theories and hypotheses, such as discrete choice models [24], data-driven approaches [26], cognitive models [14] and more.

Common observations suggested that pedestrians base their crossing decisions on the gap between two consecutive vehicles at uncontrolled intersections [13]. Early approaches estimated the critical gap as a threshold for crossing decisions, which is the minimum gap that half of the pedestrians would accept, such as Raff's method [27]. More recently, some similar models estimated the critical gap by considering vehicle speed and distance [28]. Compared to critical gap models, some studies assumed that pedestrian crossing gap acceptance behavior follows binomial distribution and proposed crossing decisions models based on Artificial Neural Networks [17] and Logistic Regression [12]. Those models considered more factors, leading to better generalization performance. Moreover, a class of emerging models, namely evidence accumulation models, utilized a cognitive decision-making process, i.e., the drift-diffusion process, and assumed that the pedestrian crossing decisions were the results of the evidence accumulation process. The crossing decisions were determined after the accumulated evidence reached a certain threshold [14], [15]. However, those approaches have not yet bridged several gaps, as identified and discussed below.

Firstly, computational models are scarce that effectively capture pedestrian crossing decisions in continuous traffic scenarios. In such situations, pedestrians usually encounter a stream of vehicles and accept one traffic gap after rejecting some. Consequently, pedestrian crossing decisions often entail complex safety and time efficiency trade-offs [12]. As discussed in Section I-A, [12], [23] developed models that hypothesized that pedestrians tended to accept riskier crossing opportunities as *waiting time* increased. However, some studies have raised doubts about this assumption, suggesting that there is insufficient evidence to substantiate it [18]–[20], [22]. The concept of *waiting time* circumvents why pedestrians choose to wait. It is unreasonable to assume that pedestrians inevitably opt for riskier crossing chances as *waiting time* increases. Instead, we should treat each case on its own merits by delving into the underlying rationale governing crossing decisions and analyzing the factors influencing pedestrians' waiting decisions. Regrettably, this issue remains unsolved.

Furthermore, modeling crossing decisions in traffic flow is challenging, partly because existing pedestrian models lack psychological underpinnings. These models are rarely based on specific psychological theories, i.e., pedestrian visual perception. Instead, these models often rely on external physical factors, like distance and TTC. For example, [28] developed a pedestrian crossing decision model based on the vehicle deceleration distance. [23] applied a minimum TTC as the threshold for pedestrian crossing decisions. However, growing evidence has shown that the impacts of vehicle kinematics on pedestrians are multi-dimensional. For instance, at the same TTCs, a higher vehicle speed induced more pedestrians to cross the street compared to a lower one [13]. Therefore, the TTC or distance may not properly carry the risk information that pedestrians perceive. Moreover, previous research has found that pedestrian crossing decisions were highly correlated with perceived visual cues [1], [14], [29]. Therefore, it is worthwhile to consider the pedestrian visual perception in crossing decisions.

Last but not least, very limited models pay attention to the time-dynamic nature of pedestrian crossing decisions, as discussed in Section I-A. Existing models often make the simplifying assumption that every pedestrian could cross the road immediately after satisfying decision criteria, disregarding the concept of crossing initiation time [24]. However, it is crucial to recognize that crossing initiation time varies and plays an important role in modeling crossing decisions. It should be noted that evidence accumulation models could capture crossing initiation time. Nevertheless, due to the complexity of these models, they focused more on the cognitive process, making it unclear whether they can effectively incorporate additional factors, like vehicle kinematics.

C. Research gaps

The effectiveness of the pedestrian model in applications, such as incorporating pedestrian models into simulation models, depends on the model's behavioral realism. However, in light of the above discussion, there are several research issues in existing crossing decision models that make the models not anthropomorphic, which can be summarized as follows:

- There is a lack of computational models that characterize pedestrian crossing decisions based on the pedestrian visual perception mechanism.
- There is a lack of computational models that concisely consider the time-dynamic nature of crossing decisions and relate them to visual perception cues.
- Why do pedestrians wait for crossing opportunities in the traffic flow? The crossing decision patterns of pedestrians when interacting with the continuous traffic flow remain unclear, resulting in few computational models simulating such crossing decisions.

D. Contribution of this study

This study proposes an anthropomorphic model for simulating pedestrian crossing decisions at uncontrolled intersections with continuous traffic flow. We highlight the importance of visual collision cues and provide a paradigm to reproduce crossing decisions from this aspect. The detailed contributions of this paper are as follows:

- Instead of using traditional vehicle kinematic cues, a visual collision cue model is introduced as the main cue for pedestrian crossing decisions.
- In light of risk aversion theory, a novel gap comparison strategy is proposed to interpret and model pedestrian waiting behavior in continuous traffic flow, instead of using *waiting time*.
- A crossing initiation time model is developed and associated with the visual collision cue model, indicating that every pedestrian has a time-dynamic crossing decision rather than crossing simultaneously.
- Two datasets collected in a highly immersive pedestrian simulator are applied to calibrate and validate the model. Based on the proposed model, a simulation model is established to reproduce pedestrian crossing decisions in a customized traffic scenario.

II. Methodology

A. Deconstructing pedestrian road crossing decisions

In road crossing tasks, pedestrians may be simplified as data processing systems, including perception, comprehension & decision, and response [14], [25]. Firstly, pedestrians perceive the traffic environment to acquire cues, such as vehicle distance, speed, TTC, and more. Based on those cues, pedestrians comprehend traffic situations, like collision risks, and decide whether to cross the road. Finally, pedestrians respond to the task through behaviors such as crossing initiation time, walking speed, and trajectory. According to the deconstructed three-stage cognitive process, we propose a collision cue-based decision-initiation model for characterizing pedestrian crossing decisions (Fig. 1), assuming that crossing decision-making is described by three parts, namely, collision cues perception, decision, and crossing initiation.

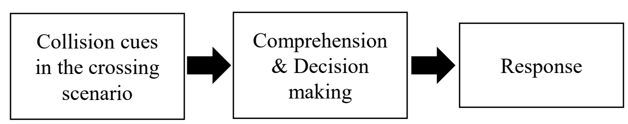

Fig. 1. A simplified cognitive process for pedestrian road crossing decisions.

B. Visual collision cue model

According to psychological theory, when moving through the environment, people rely on their visual perception of the space around them [30]. The road crossing task is a typical case that highly demands pedestrians to use visual cues to evaluate the collision risk from approaching vehicles and guide their movements. Relevant behavioral research has shown that the human visual system is sensitive to changes in some visual cues, which may be the source of collision perception. Specifically, one group of cues may provide reliable collision time information, such as Tau, binocular disparity, and more [31]. Other cues, like visual angle and its first temporal derivative [30], effectively translate motion information into visual cues through images that expand on the retina. Although most daily naturalistic road crossings involve all of the above visual cues (and possibly others), Delucia [30] has suggested that humans may rely on collision time-related cues when the scenarios include robust optical information or occur at a near distance. Conversely, when the optical information in the task is impoverished or occurs at a long distance, the visual angle and its first temporal derivative may play a dominant role. In light of this conceptual framework, we have previously identified that the first temporal derivative of visual angle,

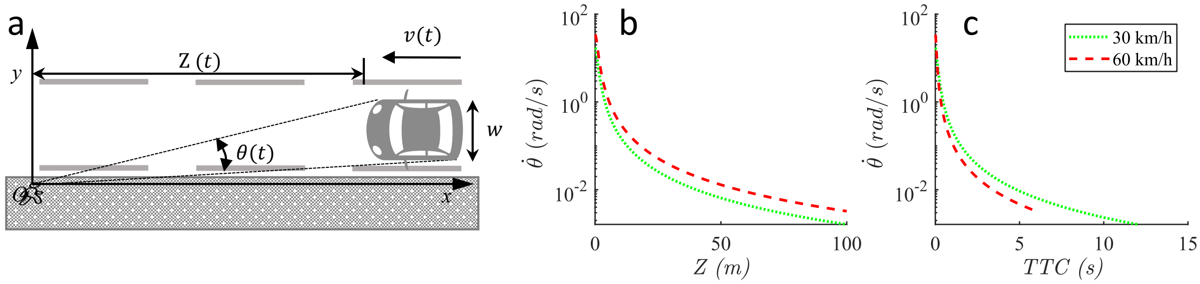

Fig. 2. (a) Visual collision cue model in road crossing scenario. Collision cues are as a (b) function of distance from and speed of the vehicle or (c) TTC from and speed of the vehicle.

$\dot{\theta}$, is a critical collision cue for making crossing decisions at uncontrolled intersections. We have demonstrated that $\dot{\theta}$ not only well explains pedestrian crossing decisions across a wide range of traffic scenarios from two different datasets, but also reasonably characterizes the impacts of vehicle speed and traffic gap on pedestrians [29]. Therefore, in this study, we formalized the pedestrian crossing decision model based on our previous findings. Typically, $\dot{\theta}$ refers to the change rate of the visual angle subtended by an approaching vehicle, θ, (Fig. 2a) [32]. The following equations specify its physical model:

$$\theta = 2\tan^{-1}\frac{w}{2Z} \Rightarrow \dot{\theta}\left(Z,v,w\right) = \frac{wv}{(Z)^2 + w^2/4} \tag{1}$$

where v denotes the vehicle speed, Z and w are the distance to the vehicle and maximum edge of the vehicle's front profile, which could be vehicle width or height. It should be noted that Eq. 1 is a simplified expression of visual angle, assuming that pedestrians stand in front of vehicles and observe approaching vehicles. This simplification is valid in our study because, (1) in our cases, when pedestrians make decisions, vehicles are still far away from them, from 18-88 m. Pedestrians cannot clearly see the length of the vehicle. (2) Our previous study [29] strictly derived the visual angle based on the geometric relationship in Fig. 2a and found no significant pattern differences between that method and this simplified one. To better interpret the collision cue model, an example is shown in Fig. 2. Suppose that a vehicle ($w = 1.95$ m) approaches the pedestrian with two different constant speeds (30 and 60 mph) from 100 m. $\dot{\theta}$ is an approximately inversely exponential function of distance and TTC from the approaching vehicle (Fig. 2b, c), showing that $\dot{\theta}$ increases slowly at long distances and rapidly at close distances, which agrees qualitatively with the observation that pedestrians usually feel safe to cross for long distance or big time gap conditions but not when the vehicle is close [13]. Further, it can be noticed that speed effects vary across distance (Fig. 2b) and TTC dimensions (Fig. 2c). When $\dot{\theta}$ is a function of distance and speed, it increases with speed, which is opposite to the results in Fig. 2c, suggesting that pedestrians may perceive a higher collision threat from the vehicle with higher speed at the same distance. However, the approaching vehicle with a slower speed gives pedestrians a bigger collision threat under the same TTC. The results tie well with the previous experimental observations on pedestrian crossing decisions [13], [33].

C. Decision model

Regarding crossing decisions at uncontrolled intersections, pedestrians typically make crossing decisions by judging and selecting the appropriate gaps between two consecutive vehicles, called gap acceptance [12]. Our previous study has proven that $\dot{\theta}$ is significantly negatively correlated with pedestrian gap acceptance, and a collision cue-based binary choice logit model predicts pedestrian gap acceptance well across different vehicle speeds and traffic gap experimental scenarios [29]. Furthermore, evidence from experimental observations indicated that individuals' judgments toward traffic gaps are not necessarily entirely static over time, especially in traffic streams [18], [19]. Due to certain learning or comparison strategies, pedestrians may tend to compare the current crossing opportunity to previous decisions or opportunities upstream of traffic. If the current crossing opportunity does not satisfy pedestrians' expectations, pedestrians then tend to reject the current opportunity and wait. We, therefore, propose the following strategy for the crossing decisions in the traffic flow:

(i) Pedestrians make decisions mainly based on collision cues, i.e., $\dot{\theta}$, provided by approaching vehicles.

(ii) Risk-aversion strategy: People tend to prefer outcomes with low uncertainty (low risk) to those outcomes with high uncertainty (potentially high risk), even if the average outcome of the latter is equal to or higher in value than the more certain outcome [34]. Therefore, in traffic flow, pedestrians could be unwilling to accept the current gap with a collision cue equal to or greater than the maximum collision cue previously rejected. For example, if pedestrians reject a $\dot{\theta}$ cue with a value of 0.02 rad/s, they would be more likely to reject the same or bigger one upstream of traffic. The rule is given by:

$$X_1 = \begin{cases} 1, & \dot{\theta}_c \geq \dot{\theta}_{mr} \\ 0, & \dot{\theta}_c < \dot{\theta}_{mr} \end{cases} \tag{2}$$

where X_1 is the dummy variable for the rule. $\dot{\theta}_c$ and $\dot{\theta}_{mr}$ represent collision cues for the current gap and maximum rejected gap, respectively. However, if pedestrians find that a gap next to the current gap has a smaller collision cue than the current gap, they may prefer to wait for this gap rather

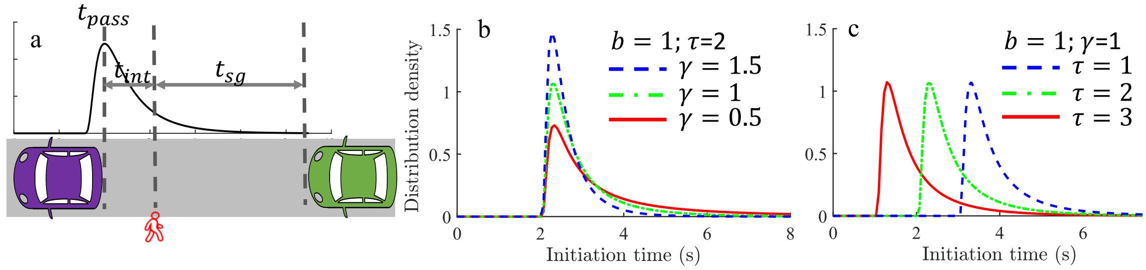

Fig. 3. Illustration of the initiation model. (a) Initiation time t_{int} is the duration between t_{pass} and the time when the pedestrian start crossing. t_{sg} denotes the actual gap to the approaching vehicle when pedestrians initiate. (b) The shapes of the initiation model by changing γ. (c) The positions of the initiation model by changing τ.

than accept a current gap with a greater collision threat, given the rule:

$$X_2 = \begin{cases} 1, & \dot{\theta}_c > \dot{\theta}_f \\ 0, & \dot{\theta}_c \leq \dot{\theta}_f \end{cases} \tag{3}$$

where X_2 is the dummy variable for the decision rule. $\dot{\theta}_f$ represents a collision cue of the gap following the current one. X_1 and X_2 form risk-aversion terms. Therefore, the utility function of the decision model is formulated as:

$$V = \rho_0 \ln(\dot{\theta}) + \rho_1 X_1 + \rho_2 X_2 + \rho_3 \tag{4}$$

where ρ_0 to ρ_3 are estimated coefficients. In this study, every $\dot{\theta}$ only refers to the $\dot{\theta}$ value of the approaching vehicle at the time when the rear end of the previous vehicle just past the pedestrian (Fig. 3a). Regarding the ln transformation, we have previously proven that it can efficiently increase the accuracy of model fitting [29]. Since crossing decisions at uncontrolled intersections are assumed to be a binary choice task, a logistic function is applied [12]. Then, a decision model for crossing tasks in the traffic flow is given by:

$$p(\dot{\theta}, X_1, X_2) = \frac{1}{1 + \exp(-V)} \tag{5}$$

where p is the probability of the gap acceptance. The (5) without the terms X_1 and X_2 degenerates to the model we proposed in [29].

D. Crossing initiation model

In real traffic, the time at which pedestrians start to cross the road is a variable. As illustrated in Fig. 3a, crossing initiation time, t_{int}, is typically defined as the duration between the time when the rear end of the previous car passes the pedestrians' position, t_{pass}, and the time when pedestrians start their movements [13]. The skewed and lognormal-like shape of the distribution of crossing initiation time is similar to those distributions that can be written in a closed mathematical form, such as Ex-Gaussian, Shifted Wald (SW), and Weibull [35]. Considering the similarities of those methods, we only apply the SW distribution instead of trying all of them. The SW distribution is a simple and concise distribution modeling tool, which can fully qualify the crossing initiation time distribution with three parameters: b (deviation around the mode), γ (tail magnitude), and τ (onset of the distribution). Its equation is defined as:

$$\begin{gathered} x \sim \mathrm{SW}(b, \gamma, \tau) \\ \Rightarrow \frac{b}{\sqrt{2\pi(x-\tau)^3}} \cdot \exp\left(\frac{-[b-\gamma(x-\tau)]^2}{2(x-\tau)}\right) \end{gathered} \tag{6}$$

An illustration of the distributional effect that occurs by changing each of the γ and τ parameters is shown in Fig. 3 b and c. The tail becomes heavier as γ decreases, (Fig. 3b). Changes in τ control the position of the distribution (Fig. 3c) [35].

According to our assumptions in Fig. 1, the crossing initiation time model is affected by collision cues, so we define the initiation time model as follows:

$$\begin{gathered} t_{int} \sim \mathrm{SW}(b, \gamma, \tau) \\ \text{with } \gamma = \beta_1 \ln(\dot{\theta}) + \beta_2; \tau = \beta_3 \ln(\dot{\theta}) + \beta_4 \end{gathered} \tag{7}$$

where t_{int} is the crossing initiation time. β_1 to β_4 are estimated coefficients. The idea behind these equations is that the strength of collision cues could affect the distribution pattern of pedestrian initiation time. For a more intensive collision threat, if pedestrians choose to cross, they tend to do so more quickly, so the distribution is concentrated and has a short tail. In contrast, when the collision threat is small, pedestrians tend to start crossing slowly, so the distribution is more likely to have a long tail [14]. Accordingly, the SW model is not only a practical distribution model but also provides notable psychological significance for our decision model. In addition, b is assumed to be a coefficient not influenced by collision cues. Furthermore, since response time data are routinely assumed to be normally distributed in many studies [13], another crossing initiation time model based on the Gaussian distribution is proposed as a comparison to the SW model, defined as the following equations:

$$\begin{gathered} t_{int} \sim \mathcal{N}(\mu, \sigma), \\ \text{with } \mu = \beta_1 \ln(\dot{\theta}) + \beta_2; \sigma = \beta_3 \ln(\dot{\theta}) + \beta_4 \end{gathered} \tag{8}$$

where μ and θ are parameters of the Gaussian model, $\mathcal{N}$.

E. Pedestrian crossing decisions in traffic flow

Finally, a computational model based on the SW distribution (SW-PRD), accounting for pedestrian road crossing decisions in the continuous traffic flow, is then established by employing (5) and (7):

$$
\begin{aligned}
& f_{SW}(t_{\text{int}}) = \sum_{n=1}^{N} P_n \cdot \mathrm{SW}\left(b, \gamma\left(\dot{\theta}_n\right), \tau\left(\dot{\theta}_n\right)\right) \\
& P_n = p\left(\dot{\theta}_n, X_{1,n}, X_{2,n}\right) \cdot \left(1 - P_{n-1}\right) \\
& P_0 = 0
\end{aligned}
\tag{9}
$$

where n is the position number of the gap in the traffic flow. $\dot{\theta}_n$, $X_{1,n}$ and $X_{2,n}$ represent the decision variables for the nth traffic gap. P_n means the recursive probability that pedestrians accept the nth gap, which is calculated based on p and P_{n-1}. Similarly, a road-crossing decision model based on Gaussian distribution (G-PRD) is given by:

$$
f_G(t_{\text{int}}) = \sum_{n=1}^{N} P_n \cdot \mathcal{N}\left(\mu\left(\dot{\theta}_n\right), \sigma\left(\dot{\theta}_n\right)\right) \tag{10}
$$

F. Simulation model

To demonstrate the performance and application capabilities of the proposed decision model, an agent-based simulation model is established to reproduce pedestrian crossing behavior at uncontrolled intersections with traffic flow. The framework of the simulation model mainly includes three parts: the pedestrian decision model, the traffic environment model, and the pedestrian kinematics model. Regarding the traffic environment, a single-lane road with an uncontrolled intersection is considered. A stream of vehicles travels on the lane at a constant speed, wherein the vehicle quantity, speed, and traffic gaps can be customized. Afterward, a basic social force model is applied as a pedestrian kinematics model [36], which considers the driving force of pedestrian i towards the destination, $\overrightarrow{\mathbf{F}}_{id}$, and repulsive force from the boundary of the crosswalk, $\overrightarrow{\mathbf{F}}_{ib}$, given by:

$$
\overrightarrow{\mathbf{F}}_i = \overrightarrow{\mathbf{F}}_{id} + \overrightarrow{\mathbf{F}}_{ib} \tag{11}
$$

Finally, according to the information provided by the traffic environment and kinematics model, each pedestrian's road crossing decision is generated through PRD models. The detailed process of the simulation model is provided in the supplementary file (Appendix. A). A demonstration video of the simulation model is also provided. Please see the attachment.

III. Model calibration and evaluation

This study applied two empirical datasets collected in a CAVE-based highly immersive pedestrian simulator to calibrate and evaluate the PRD models. Dataset one included traffic gap and vehicle speed as experimental variables. While dataset two considered the experimental variable, i.e., the order of traffic gap size in the traffic flow. Therefore, dataset one was important for verifying the model's performance in modeling the impact of vehicle kinematics on pedestrian crossing decisions. Dataset two was applied to verify the model's performance in modeling pedestrian crossing decisions in traffic flow. The following sections provide detailed information on the calibration and validation methods of the two datasets.

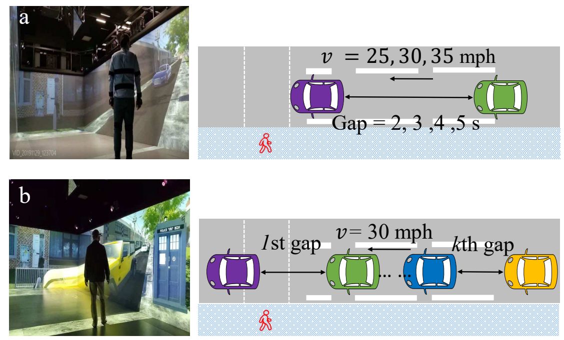

Fig. 4. Schematic diagrams and photos of traffic scenarios in simulated experiments. The crossing scenarios and traffic of the (a) first dataset and (b) second dataset.

A. Empirical data

Dataset one. A virtual road scene with a 3.5 m wide single lane and 1.85 m wide pavement was created in the simulator. Two consecutive vehicles of 1.95 m in width were driven in the middle of the road at the same constant speed. According to vehicle speed compliance statistics in the UK [37], the speed of vehicles in the town is concentrated between 25-35 mph, so three common observed vehicle speeds were selected, namely, 25 mph, 30 mph, or 35 mph. The first vehicle came into view 96 m away from the pedestrian, and the second vehicle maintained a specific time gap behind the first vehicle, i.e. 2 s, 3 s, 4 s, or 5 s (Fig. 4a). Sixty participants were instructed to cross the road between the two cars if they felt comfortable and safe to do so. Otherwise, they could reject the gap. Three experimental blocks were created, and each of the 12 scenarios (4 time gaps $\times$ 3 speeds) was presented in random order and repeated once in each experimental block. Therefore, each participant experienced 72 trials, and 4270 trials of data were obtained in total.

The virtual environment and simulation process mentioned above were designed and controlled by the Unity3D platform. Internal code automatically recorded the positions and velocities of vehicles and participants on each time step. Two main metrics were applied: gap acceptance, u, and crossing initiation time, t_{int}. The gap acceptance data were the binary crossing decisions made by participants, i.e., $u = 1$ means pedestrians accepted the gap, while 0 indicated rejected the gap. The crossing initiation time was defined as described in Section II-D and Fig.3a. For more detailed information about this dataset, please refer to [38].

Dataset two. To explore pedestrians' road crossing decisions in traffic flow, pedestrians were asked to cross a one-lane road with continuous traffic in the simulator (Fig.4b). The size of time gaps between every two consecutive vehicles varied, which provided pedestrians with different opportunities to make crossing decisions (Fig.4b). Four traffic scenarios with different sequences of gap sizes (in seconds) were designed as follows:

- Scenario one: 1 1 1 3 3 3 6 1 1 6;
- Scenario two: 1 1 1 1 3 3 7 1 1 3 8;
- Scenario three: 1 1 1 3 1 3 1 3 5 4 8;
- Scenario four: 2 3 1 1 3 1 1 1 5 4 7;

Among these scenarios, the one-second and two-second time gaps between vehicles were considered dangerous crossing opportunities that very few pedestrians would accept. For the three-second and four-second gaps, decisions were expected to significantly differ between participants due to their heterogeneity (e.g., age and gender). The time gaps longer than four seconds were considered safe gaps that most pedestrians were expected to accept confidently. In all scenarios, a range of compact, midsize, van, and SUV vehicles were driven at 30 mph. Since the types of approaching vehicles were randomly selected, in the analyses here, the width of the vehicle was calculated by averaging the width of all vehicles in the corresponding gap in each scenario. 60 participants completed four crossing tasks in any of the four scenarios and repeated them once more (4 crossing tasks $\times$ 4 scenarios $\times$ 2 repetitions). We, therefore, collected data from 1920 trials. All the trials that participants experienced were in a randomized order. Similar to the first dataset, two main metrics were used: gap acceptance, u, and crossing initiation time, t_{int}. For more detailed information about this dataset, please refer to [19].

B. Data processing and model calibration

With regard to data processing, both datasets were divided into a training set and a validation set. Regarding dataset one, as controlled experimental variables were vehicle speed and time gap size, we separated the training and validation sets by choosing the data from different combinations of experimental variables (As illustrated in Section III-A, there were 12 different combinations). To have enough data in the training and validation sets, data from 10 combinations were grouped into the training set, while the rest of the data belonged validation set. Moreover, in order to make sure the validation data were sufficiently different, the 2 combinations are not adjacent to each other in terms of speed or time gap size. Accordingly, the validation set included data in 4 s 25 mph and 5 s 35 mph conditions, approximately accounting for 23% of the initiation time data and 14% of the gap acceptance data (The data size of the two metrics was not the same as there was no initiation time data for participants who rejected the gap). The remaining data of all other conditions were grouped into the training set. Similarly, with respect to dataset two, the data from traffic scenario four were used as the validation set, accounting for 24% of gap acceptance data and 25% of initiation time data.

A Maximum Likelihood Estimation (MLE) method was used to calibrate the parameters in the models. Firstly, regarding the decision model (5), since it assumes that crossing decisions are drawn from a Bernoulli distribution, its likelihood function is given by:

$$\begin{gathered}\mathcal{L}_1(\omega)=\prod_{i=1}^{n} p\left(\Theta \mid \omega\right)^{u_i}\left(1-p\left(\Theta \mid \omega\right)^{1-u_i}\right)\\ \rho_1,\rho_2,\rho_3,\rho_4 \in \omega \\ \dot{\theta}_i, X_{1,i}, X_{2,i} \in \Theta\end{gathered} \tag{12}$$

where ω includes all the estimated parameters $\rho_1,\rho_2,\rho_3,\rho_4$. Θ denotes $\dot{\theta}_i, X_{1,i}, X_{2,i}$ for the ith trial. n is the size of the dataset. With respect to the initiation models, their likelihood functions are given by the following equations based on (7) and (8):

$$\begin{gathered}\mathcal{L}_2(\Delta)=\prod_{j=1}^{m} \mathrm{SW}\left(t_{int,j}, \dot{\theta}_j \mid \Delta\right)\\ \beta_1,\beta_2,\beta_3,\beta_4,b \in \Delta\end{gathered} \tag{13}$$

$$\mathcal{L}_3(\Delta)=\prod_{j=1}^{m} \mathcal{N}\left(t_{int,j}, \dot{\theta}_j \mid \Delta\right) \tag{14}$$

where Δ is the summary of the estimated parameters of crossing initiation models. $t_{int,j}$ is the jth crossing initiation time data. The data size is m. According to the MLE method, the maximization problem is equivalent to minimizing the negative log-likelihood. Thus, the optimal estimations for parameters are achieved when negative log-likelihood functions are minimized, e.g., $-\ln\left(\mathcal{L}_1(\omega)\right)$. We applied a built-in 'fminuc' function in MATLAB to find the solution to the above minimization problems [39].

C. Model evaluation method

Since no specific existing model simulates pedestrian crossing decisions and initiation time in continuous traffic flow, the proposed model could not be evaluated by comparing it with an existing model. Nevertheless, to demonstrate that the proposed model captures pedestrian crossing decisions correctly, we applied two comparison methods. First, we established two crossing decision models, using the SW and Gaussian distributions as the initiation time model, respectively. At this point, neither model, i.e., SW-PRD (without flow) and G-PRD (without flow), included the decision strategy for traffic flow. We could, therefore, evaluate the difference between the two initiation time models based on two different distributions. The two models were fitted to dataset one that did not include traffic flow scenarios. The estimated parameters are presented in Table I. Secondly, to demonstrate the benefit of the proposed traffic flow decision strategy, we manipulated the SW-PRD model to include the decision strategy and compared it with the G-PRD model without the decision strategy. The two models were fitted to dataset two, which was collected in traffic flow scenarios. The estimated parameters are presented in Table II. In addition, the parameters of the social force model are adopted from [36], provided in Appendix. A.

D. Evaluation criterion

After calibration, the predictions were compared with the validation set to verify the ability of the models. Two evaluation methods were applied to compare the performance of the

TABLE I
CALIBRATION RESULTS OF MODELS BASED ON DATASET ONE

Parameter	SW-PRD (without flow)		G-PRD (without flow)	
	Estimate	95 % C.I.	Estimate	95 % C.I.
β_1	0.03	[-0.19, 0.24]	-0.03*	[-0.05, -0.01]
β_2	4.48*	[3.35, 5.62]	0.15*	[0.07, 0.24]
β_3	-0.20*	[-0.26, -1.78]	-0.21*	[-0.24, -0.18]
β_4	-2.11*	[-2.43, 1.22]	-0.76*	[-0.91, -0.62]
b	6.06*	[4.43, 7.68]	n/a	n/a
ρ_0	-2.14*	[-2.28, -1.98]	-2.14*	[-2.28, -1.98]
ρ_3	-9.95*	[-10.64, -9.26]	-9.95*	[-10.64, -9.26]
LL	-108.43		-176.69	
BIC	252.37		381.79	

Note. LL: log-likelihood of the entire model, C.I.: confidence interval, *: significant at a 5% significance level
With/Without flow: consider/not consider decision strategies for traffic flow

TABLE II
CALIBRATION RESULTS OF MODELS BASED ON DATASET TWO

Parameter	SW-PRD (with flow)		G-PRD (without flow)	
	Estimate	95 % C.I.	Estimate	95 % C.I.
β_1	0.47*	[0.29, 0.66]	-0.05*	[-0.06, -0.04]
β_2	7.36*	[6.15, 8.57]	0.01	[-0.05, 0.07]
β_3	0.04	[-0.02, 0.10]	-0.10*	[-0.13, -0.09]
β_4	-1.41*	[-1.70, -1.13]	-0.59*	[-0.68, -0.50]
b	7.76*	[5.6, 9.90]	n/a	n/a
ρ_0	-2.92*	[-3.16, -2.68]	-3.31*	[-3.55, -3.07]
ρ_1	-1.29*	[-1.56, -1.02]	n/a	n/a
ρ_2	-0.50*	[-0.84, -0.15]	n/a	n/a
ρ_3	-13.23*	[-14.30, -12.16]	-15.50*	[-16.56, -14.46]
LL(Decision model)	-1536.40		-1672.50	
LL(CIT model)	-36.35		-104.03	
BIC	3218.40		3600.40	

Note. LL(Decision model/CIT model): log-likelihoods of decision models /crossing initiation time models

proposed models, namely BIC and K-S test. The BIC is given by:

$$\mathrm{BIC} = k \ln(n) - 2 \ln(L) \tag{15}$$

where k is the number of parameters in the model. n is the size of the dataset. L is the maximum likelihood. The preferred model is the one with the minimum BIC [40]. K-S test is a nonparametric test, which is used to evaluate the goodness-of-fit of the predicted results by quantifying the distance between empirical and predicted distributions [41]. The main equation of K-S test is:

$$D_{n,m} = \sup |\boldsymbol{F}_n(x) - \boldsymbol{F}_m(x)| \tag{16}$$

where sup denotes the supremum function. $\boldsymbol{F}_n(x)$ and $\boldsymbol{F}_m(x)$ are the distribution functions of the observed data and predicted result. n and m represent the size of the samples. The K-S test rejects the null hypothesis, i.e., two samples are drawn from the same probability distribution if $D_{n,m}$ is bigger than the selected threshold. In addition, the R-squared, R^2, and Root Mean Square Error (RMSE) are also used in the model discussion.

IV. RESULTS AND ANALYSIS

In this Section, we first discuss the calibration results of the SW-PRD and G-PRD models. Afterward, the validation results of the two models were compared using the BIC and K-S test. Finally, the model with better performance is compared to two entire datasets, and the reproduced crossing decision patterns are discussed in detail. Additionally, regarding the first dataset, as it does not include the traffic flow scenario, we focus on the impacts of speed and time gap on pedestrian crossing decisions, while the effect of traffic is discussed using the results based on the second dataset.

A. Calibration results

Dataset one. The parameters of the SW-PRD and G-PRD models were calibrated using the first dataset. One thing to note is that as the first dataset did not include traffic flow scenarios, these two models thus did not implement decision strategies in traffic, which means ρ_1 and ρ_2 were not included in the models, and two decision models in the SW-PRD and G-PRD models were the same. The calibration results are shown in Table. I, where the maximum log-likelihood and BIC of the SW-PRD model based on the training set are -108.43 and 252.37, which are significantly better than those of the G-PRD model, i.e., -176.69 and 381.79, indicating that the SW-PRD model can better describe pedestrian crossing initiation time than the G-PRD model on the calibration set. Moreover, it can be found that the effect of ρ_0 is significantly negatively correlated with $\dot{\theta}$ (Est. $= -2.14, \text{C.I.} = [-2.28, -1.98]$), showing that pedestrian crossing gap acceptance decreases as the risk of collision increases. Additionally, the estimated effect of β_3 in the SW-PRD model is significantly correlated with $\dot{\theta}$ (Table. I), suggesting that pedestrian crossing initiation time is negatively related to the collision risk.

Dataset two. The calibration results based on the second dataset are shown in Table. II. As the SW-PRD model implemented the decision strategies in traffic flow, it included ρ_1 and ρ_2. However, the G-PRD model did not. Meanwhile, as both the decision model and initiation time model in the SW-PRD model and the SW-PRD model were different, we calculated the respective log-likelihood of the decision and initiation time models to facilitate the comparison of the results. Again, the SW-PRD model fits data better than the G-PRD model, where the SW-PRD model has larger log likelihoods for both the decision and crossing initiation time models, and its BIC is smaller than that of the G-PRD model. In particular, concerning the SW-PRD model, except for the significant effect of ρ_0 (Est. $= -2.92, \text{C.I.} = [-3.16, -2.68]$), ρ_1 and ρ_2 also significantly affect the pedestrian gap acceptance (Est. $= -1.29, \text{C.I.} = [-1.56, -1.02]; \text{Est.} = -0.50, \text{C.I.} = [-0.84, -0.15]$), consistent with our assumed crossing decision strategies in traffic flow. In addition, although the effect of β_3 in the SW-PRD model is not significant, the positive

TABLE III
VALIDATION RESULTS OF MODELS BASED ON DATASET ONE

Condition	Model	LL	BIC	K-S test score	P value
25 mph 4 s	SW-PRD	-23.08	71.47	0.06	0.56
	G-PRD	-27.28	74.82	0.10	0.08
35 mph 5 s	SW-PRD	-13.19	54.81	0.05	0.31
	G-PRD	-24.83	72.41	0.09	0.02*

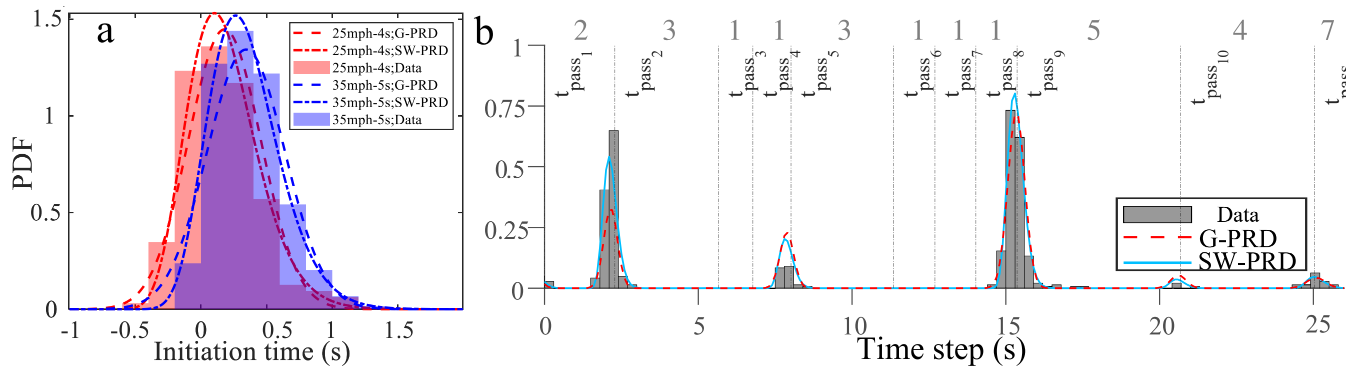

Fig. 5. Validation results. Probability density functions and data based on datasets (a) one and (b) two. The vertical dash-dotted lines in (b) indicate the time when the rear end of the vehicle passes the pedestrian's position. The size of the time gap (in seconds) between every two vehicles is indicated at the top of the diagram.

effect of β_1 reduces the tail magnitude of the distribution of crossing initiation time as $\dot{\theta}$ increases and thus can reduce pedestrians crossing initiation time.

B. Validation results

The calibration results indicate that the SW-PRD model fits the training sets better than the G-PRD model. In this section, the validation sets of two datasets are compared with the predicted results of two models.

Dataset one. Regarding the validation results, as shown in Table. III, the SW-PRD model has better BIC values and K-S scores for all conditions. Specifically, in the 35 mph 5 s condition, the K-S test rejects the null hypothesis and indicates that the results of the G-PRD model are different from the observed data at a 5% significance level. As shown in Fig. 5a, it can be found that the G-PRD model tends to overestimate the initial parts of the data, but the SW-PRD model does not.

Dataset two. The predicted results are compared to the validation set of the second dataset. The log-likelihood of crossing initiation time models of SW-PRD and G-PRD are presented separately for reasons explained previously (Table. IV). Both SW-PRD and G-PRD models accurately capture the timing of pedestrian crossing decisions in the traffic flow, i.e., the peak location of the initiation time distribution (Fig. 5b). The predicted peak shapes of both models are close to the data. However, the SW-PRD model has a relatively better performance than the G-PRD model because the log-likelihood of the crossing initiation time model for SW-PRD is bigger than the value for G-PRD in Table. IV. The overall predictions of the SW-PRD model are closer to the data than those of the G-PRD model. Specifically, the SW-PRD model has a better BIC value and log-likelihood than the G-PRD model (Table. IV). Also, the K-S test supports that the predicted density function of the SW-PRD model is similar to the empirical distribution. In contrast, the predicted result of the G-PRD model is rejected by the K-S test at a 5% significance level (Table. IV). As shown in Fig. 5b, it can be found that consistent with the empirical data, the SW-PRD model predicts a decrease in the gap acceptance from the first 3 s gap (at t_{pass_2}) to the second 3 s gap (at t_{pass_5}). By contrast, the G-PRD model calculates a constant value for both 3 s gaps, resulting in a significant underestimation of gap acceptance in the first 3 s gap. In general, the SW-PRD model has better performance than the G-PRD model on the validation set of dataset two.

TABLE IV
VALIDATION RESULTS OF MODELS BASED ON DATASET TWO

Model	LL	LL(CIT model)	BIC	K-S test score	p value
SW-PRD	-578.37	-11.23	1193.10	0.08	0.10
G-PRD	-707.53	-52.76	1444.10	0.16	0.001*

In the following sections, we discuss the predicted pedestrian crossing decisions in detail by comparing predicted results with the two full datasets to provide a complete understanding of the proposed model. Since SW-PRD performs better on all datasets than G-PRD, the SW-PRD model generates our results in the following sections.

C. Dataset one: Speed and time gap effects

The SW-PRD model predictions of crossing gap acceptance for each speed and time gap condition are compared with the observed data in Fig. 6a. According to the empirical data, crossing gap acceptance increased with vehicle speed and traffic gap, aligning well with previous studies [13], [33]. The SW-PRD model reproduces these decision patterns very well ($R^2 = 0.890$, $RMSE = 0.050$), suggesting that pedestrians might adapt their crossing decisions based on the changes in collision cues.

Fig. 7a shows a comparison between the predicted crossing initiation time and observed data. In line with the literature, [13], the empirical data showed that pedestrian crossing initiation time correlated with vehicle kinematics, i.e., it decreased as traffic gaps and vehicle speeds decreased. This decision pattern can be understood as a distance-dependent phenomenon whereby a reduction in vehicle speed and time gap leads to a reduction in spatial distance, resulting in an increase in the perceived risk of collision [29]. Hence, if pedestrians choose to cross, they tend to do so more quickly. Based on our modeling results, the proposed SW-PRD model captures this pattern with a good fit ($R^2 = 0.890$, $RMSE = 0.050$),

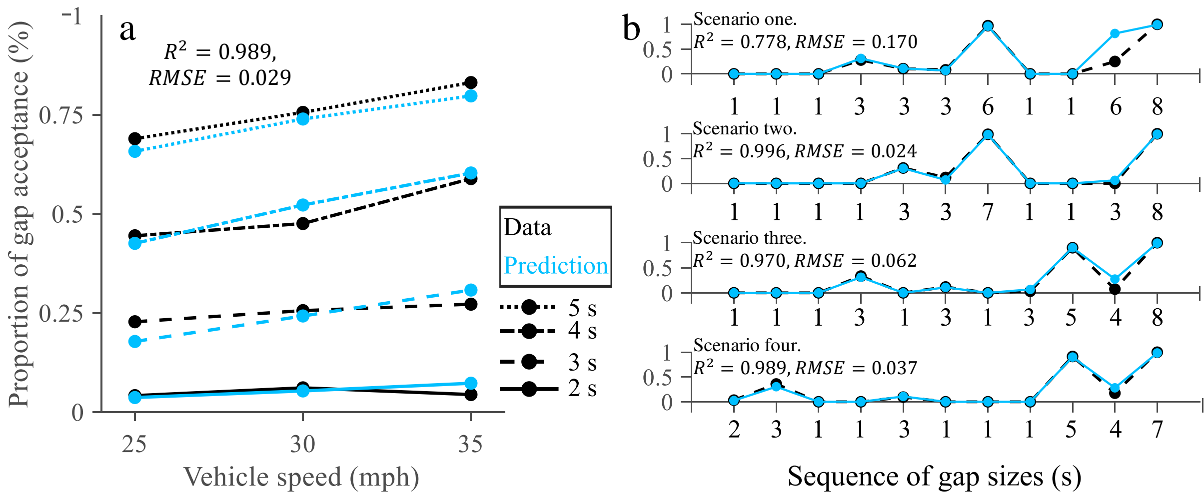

Fig. 6. Predicted gap acceptance of the SW-PRD model for both datasets. The data and the predicted results are represented in black and blue, respectively. (a) For dataset one, the proportion of gap acceptance is plotted as a function of vehicle speed and gap size (Gap sizes are indicated by different line styles). (b) For dataset two, the proportion of gap acceptance for each gap of each traffic scenario is presented.

again indicating that visual collision cues are associated with pedestrian crossing decisions.

Moreover, a more detailed comparison between predictions and data is shown in Fig. A2 in Appendix B. The predicted probability density function of pedestrian crossing decisions is plotted as the function of the initiation time and traffic scenario. It can be found that the SW-PRD model predicts pedestrian crossing behavior qualitatively and quantitatively. It not only describes the pedestrian crossing decisions distributed along the time axis but also captures the variation in the mean crossing initiation time.

D. Dataset two: Impacts of traffic flow

Predicted gap acceptances of the SW-PRD model in the traffic flow are compared to the observed data in Fig. 6b. Firstly, it can be noticed that pedestrians in the traffic flow did not accept gaps of the same size equally. For instance, regarding the 4th gap and the 5th gap in traffic scenario one (The size of both traffic gaps is 3 s), the probability of crossing gap acceptance dropped significantly from 27.9% to 10.5%. When pedestrians faced the 6th gap, the decreasing trend became even stronger. The probability of crossing gap acceptance was 8.1%, more than three times smaller than the value of the 4th gap, showing that a greater proportion of pedestrians tended to reject this crossing opportunity and wait for the next one. Further looking at the predictions, the SW-PRD model reproduces this decision pattern across all traffic scenarios with reasonable goodness-of-fit (Fig 6b), indicating that the proposed decision strategy captures pedestrian waiting behavior in the traffic flow.

Fig.7b plots the predicted crossing initiation time as a function of the time gap and compares it with the observed data. The SW-PRD model fits the crossing initiation time data well ($R^2 = 0.850$, $RMSE = 0.038$). Consistent with empirical observations and similar to the first dataset [16], the SW-PRD model predicts a shorter initiation time as the time gap decreases, again suggesting that pedestrians attempted to compensate for crossing risk in unsafe traffic gaps by initiating faster.

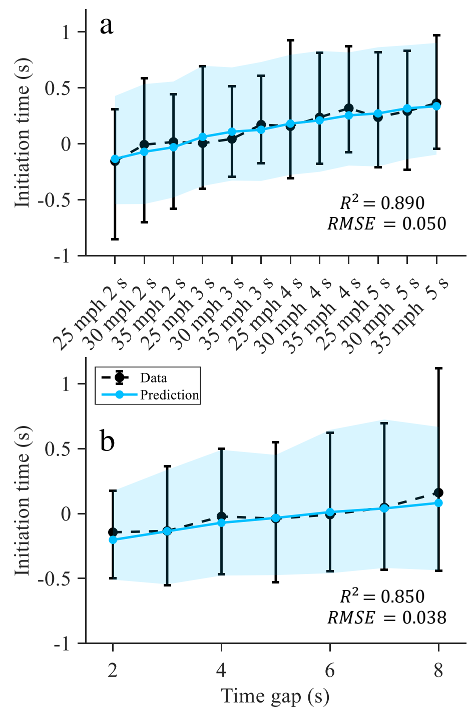

Fig. 7. Predicted crossing initiation time of the SW-PRD model for both datasets. Error bars and the edge of blue areas indicate the 2.5% and 97.5% percentiles of the data and predicted results. (a) For dataset one, the crossing initiation time is plotted as a function of vehicle speed and gap size. (b) For dataset two, the crossing initiation time is a function of gap size.

Furthermore, pedestrian crossing probability and crossing timing corresponding to each traffic gap in the flow are compared with the observed data shown in Fig. A3 in Appendix B. Across all traffic scenarios, the SW-PRD model accurately predicts the level, shape, and location of peaks of the crossing initiation time distribution, showing that the model has a good ability to characterize pedestrian crossing decisions in a continuous flow of traffic.

V. Discussion and Conclusion

This study demonstrates an anthropomorphic approach to simulate pedestrian crossing decisions at uncontrolled intersections with continuous traffic flow. According to the three-stage cognitive process, a crossing decision-initiation model is proposed with three constituent parts: visual collision cue perception, crossing decision, and crossing initiation. The crossing decision model based on visual collision cues characterizes pedestrian crossing decisions in continuous traffic flow in a way that is consistent with pedestrian decision logic and more ecological. The crossing initiation model based on visual collision cues captures the time-dynamics nature of pedestrian crossing decisions on a more fine-grained level. This study provides a paradigm to model pedestrian crossing decisions in complex scenarios, i.e., deconstructing the underlying psychology mechanism, and allows pedestrian agents' road crossing decisions to be more realistic. Following is a summary of the detailed research results.

A. Pedestrians make crossing decisions relying on visual collision cues

In our previous study [29], we showed that the visual collision cue, $\dot{\theta}$, could capture the effects of vehicle kinematics on pedestrian crossing decisions in single gaps and explain why pedestrians tended to rely on distance from vehicles to make crossing decisions [13], [33]. In this study, this finding is formally applied to model crossing decisions and extended to a more complicated traffic scenario, i.e., a continuous flow of traffic. The modeling results support that $\dot{\theta}$ is capable of characterizing the risk perceived by pedestrians, at least at uncontrolled intersections with constant speed traffic.

B. Time-dynamic crossing decisions

Moreover, regarding the second research gap, i.e., pedestrian crossing initiation is time-dynamic and influenced by vehicle kinematics, we relate the proposed crossing initiation time model to $\dot{\theta}$. The modeling results support our hypothesis and show that pedestrians dynamically adjust their initiation time based on perceived collision cues. Both the SW and Gaussian distributions can reasonably describe pedestrian initiation time, whilst the SW distribution has relatively better goodness-of-fit than the Gaussian distribution, which further indicates that the distribution of crossing initiation time is right-skewed. The findings may suggest that pedestrians not only follow the gap-maximization rule when making a crossing decision but are also influenced by certain factors that lead to delays in decision-making. For example, there are inherent neural conduction delays in human sensory and motor pathways that make most people's response times slower than a theoretical optimal. Meanwhile, people often sacrifice speed for improved decision accuracy. The limits in human functions and trade-offs determine that the response time distribution is right-skewed [35].

C. Pedestrian waiting behavior in traffic flow

Notably, to accurately reproduce pedestrian crossing decisions in continuous traffic flow, we propose a traffic gap comparison strategy based on the risk-aversion theory, which is supported by reasonable modeling results. These results provide an explanation for pedestrians' waiting behavior in traffic: (1) Pedestrians may have a reduced tendency to accept the current gap if they see larger gaps upstream of traffic. (2) Pedestrians may have a greater tendency to reject the current gap if they have rejected a gap of that size or larger previously. These results are supported by empirical observations [18], [20], [21]. The novelty of the study is that we show that as *waiting time* increases, pedestrians do not accept riskier crossing opportunities, which may provide an explanation for the non-significant effect of *waiting time* on pedestrian crossing decisions found in [22]. Furthermore, this finding is interesting in that pedestrians waiting for the crossing opportunities in the traffic flow may be caused by their risk-aversion strategy. Accordingly, the increase in *waiting time* is the result of this waiting behavior.

D. Implications and limitations

Regarding the practical implications of this study, there are many possible ways to extend these concepts and models to further improve research in pedestrian-AV interactions. First, the proposed decision model may provide predictive information to help automated driving systems better anticipate pedestrian crossing intentions and initiations. Early work is emerging where researchers are attempting to plan and coordinate the actions of AVs and pedestrians toward common goals by considering the visual collision cue of pedestrians [5]. Another possible application case is future traffic scenarios involving AV platoons and pedestrians, where AV platoons may need to take into account the dynamic pedestrian crossing decisions along the length of the platoon and adopt the decision strategy of each AV. Moreover, there is an urgent need to train and evaluate AVs to perform well also in safety-critical interactions with human road users. However, due to the low frequency of critical traffic scenarios in real life, i.e., the corner case, and safety reasons, both academia and industry have agreed on using simulation methods as a complementary way to validate AVs. Reliable simulation results rely on the behavioral authenticity of simulated road users [23]. Hence, another practical significance of this study is that the model can serve as a module in the virtual testing platforms to realize naturalistic pedestrian road crossing decisions.

This study also holds theoretical significance. Firstly, the proposed crossing initiation time model might inspire more modeling studies on time-dynamic crossing decisions. Meanwhile, these findings offer potential insights into pedestrian behavior research by allowing us to study the influence of specific factors, such as age or gender, on pedestrian crossing initiation using Wald distribution. Secondly, the innovative gap comparison strategy proposed in this study interprets and models pedestrian waiting behavior within traffic flow. It has the potential to shed light on modeling and behavior research, facilitating a deeper understanding and deconstruction of traffic flow effects.

However, several limitations of this study need to be addressed in the future. Since the datasets used are limited, it is uncertain whether the gap comparison strategy is universal among all pedestrians or just some pedestrians are more cautious and tend to wait. The gap comparison strategy has not captured some observations in previous studies, such as pedestrians' riskier crossing decisions. Hence, there may be more intricate patterns in pedestrian decision strategy in traffic flow. Moreover, the Wald distribution we used is just one of many response time models. Other response time models could be used to characterize crossing initiation time alternatively. Also, other approaches, like deep learning or reinforcement learning, may have potential ability. Furthermore, since the results only cover scenarios with single-lane and constant-speed traffic flow, the model cannot be directly generalized to other scenarios without further development [42]. Meanwhile, the model should further consider other factors, such as the lateral movement of vehicles, road width, group size, and pedestrian heterogeneity. Furthermore, compared to simulated tasks, in real traffic, pedestrians could flexibly adjust their behaviors and be affected by many potential factors, and the virtual nature of tasks may also affect the observed behavior. Hence, important future work should apply the model to a reliable naturalistic dataset. Finally, the model is developed based on current theories of human collision perception and does not assert that pedestrians use the applied visual cues and perception strategy. The research conclusions do not support the conclusion that our deconstruction into three stages is the best approach to characterize pedestrian crossing decisions. Rather, this approach provides an intuitive representation of crossing decisions and models pedestrian crossing data better than previous models. As psychological theory is further developed, the model can be improved accordingly.

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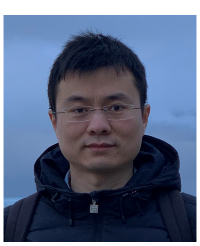

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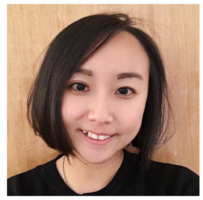

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APPENDIX

A. Simulation model

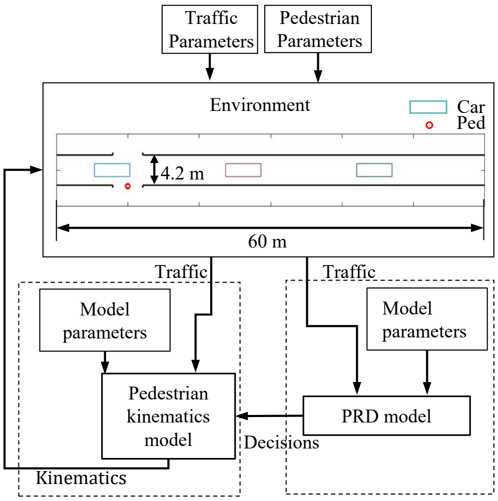

Fig. A1. Structure of the simulation model. The traffic environment contains a single lane (60 m long and 4.2 m wide) and a stream of vehicles (colored rectangles).

In this study, an agent-based simulation model is proposed using the established PRD models for reproducing pedestrian crossing behavior at uncontrolled intersections with traffic flow. The framework mainly includes three parts: the PRD model, the environment model, and the pedestrian kinematics model (Fig.A1). The detailed process of the simulation model is as follows:

(i) Generate the traffic environment using the given traffic and pedestrian parameters.

(ii) Generate a pedestrian agent at a random location on the pavement near the intersection. After that, the pedestrian walks to the edge of the pavement. Since this study focuses on the crossing decisions in the traffic flow, the pedestrian performs the PRD model after the first vehicle has passed him/her (Algorithm. 1).

(iii) The PRD model generates each pedestrian's decision and initiation time through a Monte Carlo sampling method (Algorithm. 2).

(iv) Pedestrians cross the road and walk to the opposite side of the road, driven by the following social force model:

$$\begin{aligned}
\overrightarrow{\mathbf{F}}_i &= \overrightarrow{\mathbf{F}}_{id} + \overrightarrow{\mathbf{F}}_{ib} \\
\overrightarrow{\mathbf{F}}_{id} &= m_i \frac{v_i^d \mathbf{e}_i^d - \mathbf{v_i}}{\tau_i} \\
\overrightarrow{\mathbf{F}}_{ib} &= \left[A_b e^{(r_i - d_{ib})/B_b)} \right] \mathbf{n}_{ib}
\end{aligned} \tag{1}$$

where m_i is the mass of pedestrian agent i, 60 kg. $v_i^d \mathbf{e}_i^d$ is the desire speed vector. $\mathbf{v_i}$ is the actual speed vector. τ_i is the characteristic time, 0.5 s. r_i is the radii of a pedestrian agent, 0.35 m, and d_{ib} is the perpendicular distance between the pedestrian and the boundary. A_b and B_b are constant parameters, equal to 2×10^3 N and 0.08 m. $\mathbf{n}_{ib}$ is the normalized vector perpendicular to the boundary. The simulation model stops when the traffic scenario ends, or all pedestrians cross the road.

A demonstration video of the simulation model is also provided. Please see the attachment.

Algorithm 1 Simulation model of road crossing decision

Input: Model parameters $\rho_0, \rho_1, \rho_2, \rho_3, \beta_1, \beta_2, \beta_3, \beta_4, b$
Output: $\boldsymbol{u}, \boldsymbol{t}_{int}$

$I_r = I$ // Number of remaining participants I_r and total number participants I
for nth gap in traffic N **do**
  $\dot{\theta}_n \leftarrow$ Eq.1
  $X_{1,n}, X_{1,n} \leftarrow$ Eq.2 and Eq.3
  $p_n \leftarrow$ Eq.5
  $P_n = p_n \cdot (1 - P_{n-1}) \leftarrow$ Eq.9
  for ith pedestrian in I_r **do**
    $u_i \leftarrow Binomial(1, P_{n,i})$ // Sampling: crossing decision
    if $u_i == 1$ **then**
      $f(t_{int}) \leftarrow$ Eq.9 or Eq.10 // Caulculate probability density function of crossing decision
      $t_{int,i} \leftarrow$ Algorithm. 2 // Sampling: crossing initiation time
    else
      Continue
    end if
  end for
  $I_r = I_r -$ length($\boldsymbol{t}_{int}$) // Update remaining participants
end for

Algorithm 2 Monte Carlo sampling of the model

Input: $f(t_{int})$
Output: $t_{int,i}$

Initialise $s = 1$
while $s \neq 2$ **do**
  $\pi(x) = f(x)$
  $s \leftarrow$ Uniform$(0, 1)$;
  $y \leftarrow Q(x|y)$ // Arbitrary probability density
  if $u \leq$ min$(\frac{pi(y)Q(x|y)}{pi(x)Q(y|x)}, 1)$ **then**
    $t_{int,i} = y$
    $s = s + 1$
  else
    $s = 1$
  end if
end while

B. Detailed modeling results

Detailed comparisons between modeling results and observations are shown in Fig.A2 and Fig.A3. In Fig.A2, the probability density functions of crossing initiation time are plotted against time gaps and vehicle speeds. While, In Fig.A3, the probability density functions of crossing initiation time are plotted as a function of traffic scenarios and crossing initiation time.

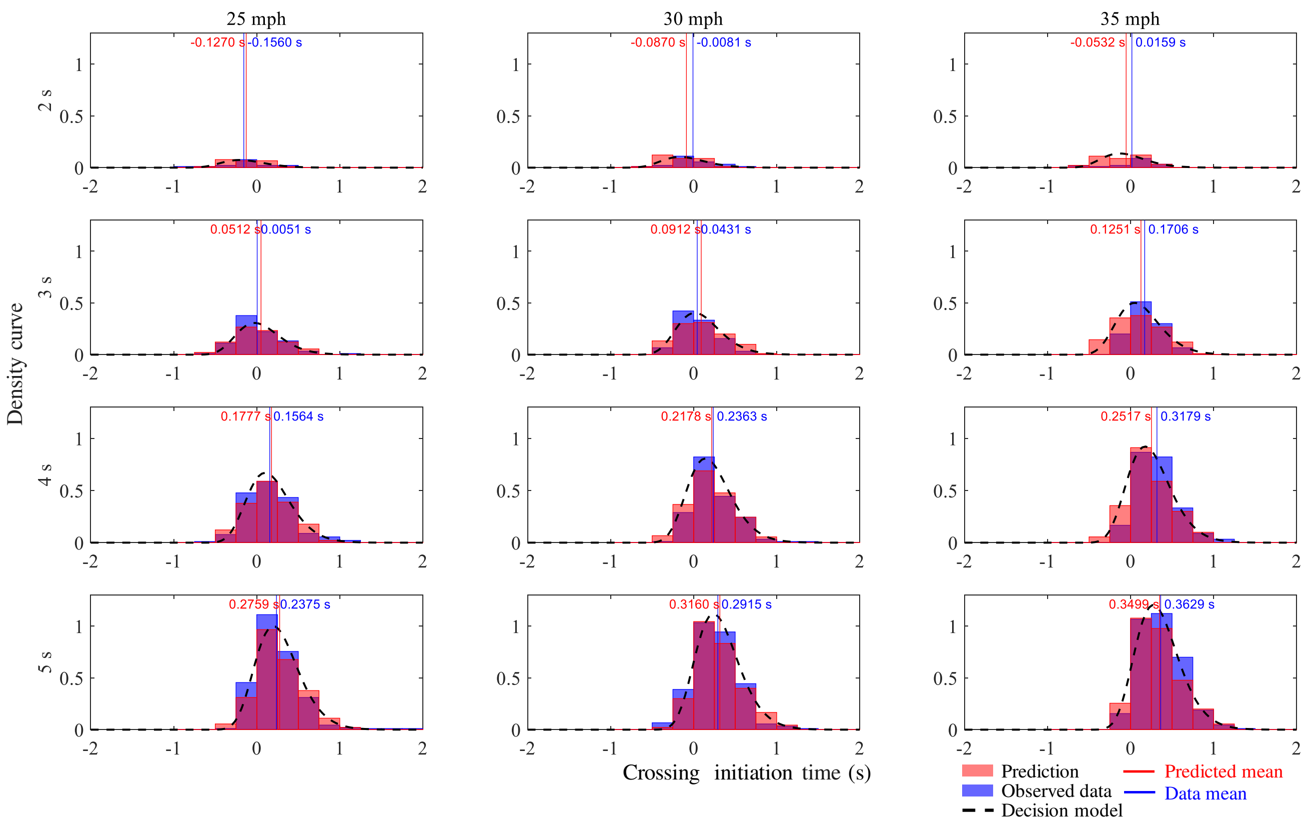

Fig. A2. Predicted density function of crossing initiation time of the SW-PRD model based on dataset one. The predicted results, including density function, samplings and mean values of crossing initiation time, are compared with the observed data in terms of vehicle speed and traffic gap size.

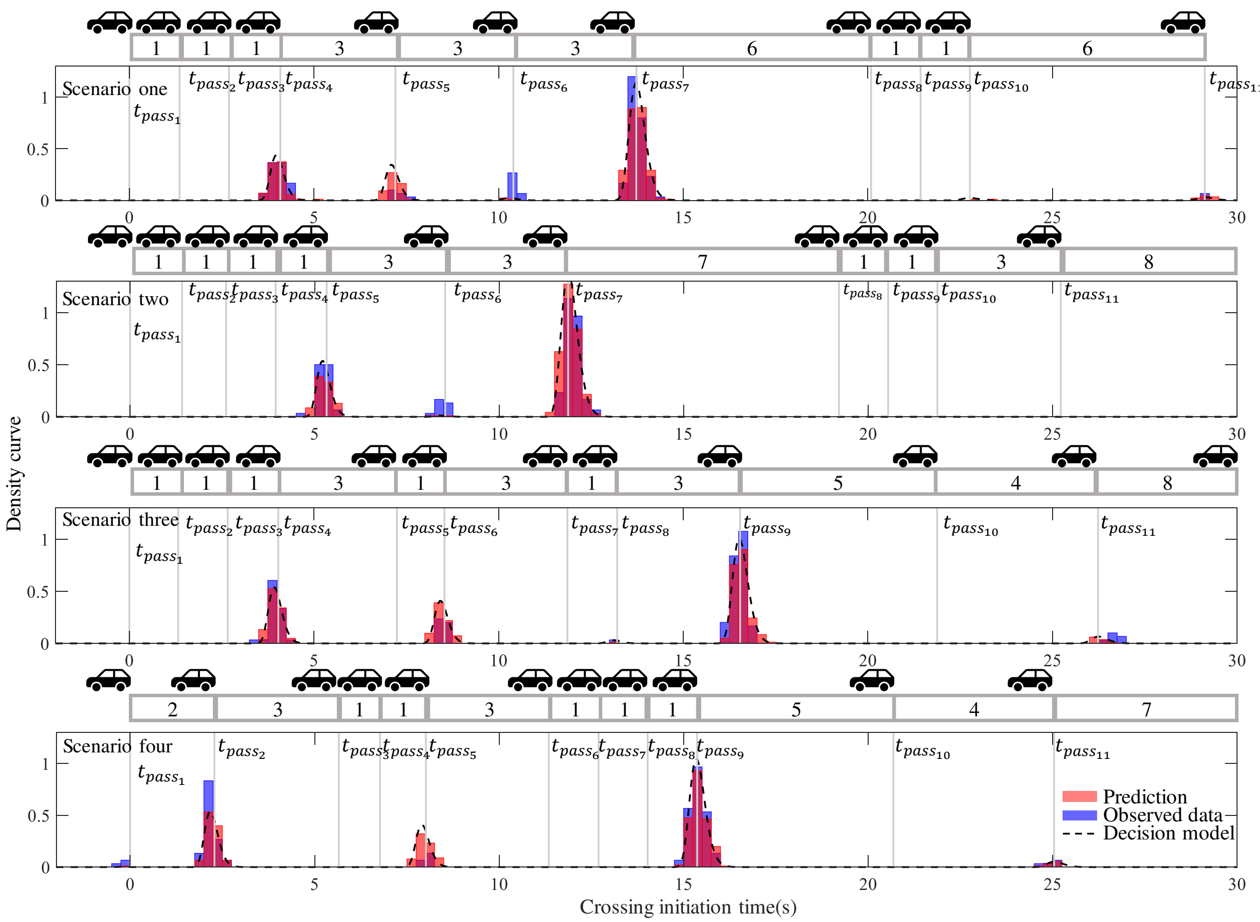

Fig. A3. Predicted density function of crossing initiation time of the SW-PRD model based on dataset two. The predicted density functions and samplings are compared with the observed data. Regarding each traffic scenario, the order of traffic gaps is indicated above each sub-figure. The vertical lines represent the time when the rear end of the related vehicle passes the pedestrian's position, i.e., t_{pass}.