

This is a repository copy of *Addendum to: Editorial note to: Erwin Schrödinger, Dirac electron in the gravitational field I.*

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/209648/>

Version: Published Version

Article:

Kay, Bernard S. orcid.org/0000-0002-8766-0532 and Gibbons, Gary (2022) Addendum to: Editorial note to: Erwin Schrödinger, Dirac electron in the gravitational field I. *General Relativity and Gravitation*. 134. ISSN 0001-7701

<https://doi.org/10.1007/s10714-022-03026-y>

Reuse

This article is distributed under the terms of the Creative Commons Attribution (CC BY) licence. This licence allows you to distribute, remix, tweak, and build upon the work, even commercially, as long as you credit the authors for the original work. More information and the full terms of the licence here:

<https://creativecommons.org/licenses/>

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



Addendum to: Editorial note to: Erwin Schrödinger, Dirac electron in the gravitational field I

Gary W. Gibbons¹ · Bernard S. Kay²

Published online: 26 October 2022
© The Author(s) 2022

Abstract

An editorial note by one of us in this journal in 2020, argued in favour of the name *Schrödinger–Lichnerowicz formula* for the formula, $g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2 + R/4$, for the ‘square’ of the Dirac operator in curved spacetime since it had been obtained by Schrödinger in 1932 and rediscovered by Lichnerowicz in 1962. However, unfortunately, it overlooked the rediscovery of the formula by Asher Peres in 1963. We briefly recall the context of each of these discoveries and reflect on the naming of mathematical formulae in general and of this formula in particular.

Keywords Schrödinger · History · Dirac equation · Spin connection · Curved spacetime · Schrödinger–Lichnerowicz formula · Square of Dirac operator · Asher Peres

Addendum to: Gen Relativ Gravit

<https://doi.org/10.1007/s10714-019-2625-z>

In an editorial note [1] by one of us (BSK) to the recent (re-)publication [2] of an English translation (by Claus Kiefer) of the 1932 paper [3] of Erwin Schrödinger, it was mentioned that the latter paper of Schrödinger had been (by thirty years!) the first place where the formula $g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2 + R/4$ had been obtained for the ‘square’ $(-i\gamma^\mu\nabla_\mu - m)(i\gamma^\nu\nabla_\nu\psi - m)$ of the Dirac operator.

It was also mentioned in [1] that, around 30 years later, the corresponding formula was rediscovered by André Lichnerowicz. (As stated in [1], this was done in the special case of zero mass and on a Riemannian manifold rather than a spacetime in [4] but actually the formula for the square of the massless Dirac operator also

✉ Gary W. Gibbons
gwg1@cam.ac.uk

Bernard S. Kay
bernard.kay@york.ac.uk

¹ Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

² Department of Mathematics, University of York, York YO10 5DD, UK

appeared in the slightly earlier paper [5] in a Lorentzian context.) Lichnerowicz used the result in [4], to prove the vanishing of the index—i.e. the Hirzebruch \hat{A} -genus—for even-dimensional compact Riemannian manifolds admitting spin structures with non-negative (but not identically zero) scalar curvature. (This was an early application of the Atiyah–Singer index theorem, when applied to the case of Riemannian manifolds with spin structures.) And it was argued in [1] that, of the several names (including “Bochner–Lichnerowicz formula”, “Lichnerowicz–Bochner–Weitzenböck formula” etc. as well as “Schrödinger–Lichnerowicz formula”) that had been used by other authors, the name *Schrödinger–Lichnerowicz formula* seemed the most appropriate.

However, it was unfortunately overlooked in [1] that, also in 1963, Asher Peres rediscovered the same formula in [6] in aid of making the point that the ‘squared’ Dirac equation in an external gravitational field doesn’t contain a gyro-gravitational term analogous to the gyro-magnetic term (see the top of Page 10 in [1]) $\frac{1}{2}\sigma^{ab}F_{ab}$ in the ‘squared’ Dirac equation in an external electromagnetic field. Also, Peres pointed out that it is not analogous to the equation for a classical spinning particle in a curved spacetime where there is a term of form $\frac{1}{2}R_{\alpha\beta\gamma\delta}v^\beta S^{\gamma\delta}$ where v^α is the velocity vector and $S^{\alpha\beta}$ the angular momentum tensor of the classical spinning particle.¹ Actually, in obtaining the formula for the ‘square’ of the Dirac operator, [6] makes reference to the equations in Peres’s earlier paper [7] which also discusses the theory associated with the names of Einstein, Cartan, Kibble, Sciama and Weyl and others (see Footnote 11 in [1]) of gravity with torsion and thus, arguably, Peres deserves to have his name included in that list of names too. (Somewhat ironically though, [6] does not mention that the theory involving torsion discussed in [7] would lead to a gravitational counterpart of gyro-magnetic coupling.)

This unfortunate omission leads us to reflect on the often noted fact that the naming of theorems in mathematics and of results in physics is often the result of historical quirks and accidents and frequently unsatisfactory or unfair in one way or another. In the case of the formula for the ‘square’ of the Dirac operator, we feel that in an ideal world, in which Lichnerowicz and Peres and everyone else would have been aware of Schrödinger’s prior contribution, it would have deserved to be called, simply, the ‘Schrödinger formula’. Indeed not only did Schrödinger obtain it 30 years earlier but it is all the more creditable that, as explained in [1], he managed to do so without a formula for the spin connection. In contrast, both Lichnerowicz and Peres benefited from the fact that, by 1963, the notion of spin connection had (as also discussed in [1]) by then penetrated the realm of the routine in both physics and mathematics. However, there are other reasons for naming formulae than historical priority. And perhaps it is good to append some other names so as to distinguish this particular formula of Schrödinger from the several other formulae that bear his name. If so, though, perhaps it should more justly be known as the *Schrödinger–Lichnerowicz–Peres formula*.

Let us also take the opportunity of this addendum to correct Footnote 10 of [1] which suggested that the square-roots of g in Eq. (74) in [2, 3] were unnecessary. The

¹ It is not really correct, though, to say, as Peres does that “Dirac particles . . . cannot be used to test general relativity.” since of course, say in a coordinate system, the term $g^{\mu\nu}\nabla_\mu\nabla_\nu$ in the ‘squared’ Dirac equation is not the same as the scalar Laplacian.

square roots are in fact needed since, in [3], and unlike in modern usage, the symbol ‘ ∇_k ’ was always taken to mean ([3, Eq. (74)]) $\frac{\partial}{\partial k} - \Gamma_k$ even when it acts on a spinor vector.

Acknowledgements GWG thanks Mike Stone for pointing out to him Schrödinger’s priority over Lichnerowicz and for subsequent discussions.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. Kay, B.S.: Editorial note to: Erwin Schrödinger, Dirac electron in the gravitational field I. *Gen. Relativ. Gravit.* **52**, 3 (2020). <https://doi.org/10.1007/s10714-019-2625-z>
2. Schrödinger, E.: Republication of: Dirac electron in the gravitational field I. *Gen. Relativ. Gravit.* **52**, 4 (2020). <https://doi.org/10.1007/s10714-019-2626-y>
3. Schrödinger, E.: Diracsches Elektron im Schwerefeld I. *Sitzungsber. Preuss. Akad. Wiss., Phys.-Math. Kl.* 105–128 (1932)
4. Lichnerowicz, A.: Spineurs harmoniques. *C. R. Acad. Sci. (Paris) A-B* **257**, 7-9 (1963). <https://gallica.bnf.fr/ark:/12148/bpt6k4007z/t7>
5. Lichnerowicz, A.: Les spineurs en relativité générale. *Ann. Sci. Univ. Clermont Ferrand II Math.*, **8**(2), 171–177 (1962). http://www.numdam.org/item/ASCFM_1962__8_2_171_0
6. Peres, A.: Gyro-gravitational ratio of Dirac particles. *Nuovo Cim.* **28**, 1091–1092 (1963). <https://doi.org/10.1007/BF02782034>
7. Peres, A.: Spinor fields in generally covariant theories. *Nuovo Cim.* **24**, 389–452 (1962). <https://doi.org/10.1007/BF02896552>

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.