Article:

Wang, B. (2024) Towards a welfare model of trade and multinational firms with oligopolistic competition. International Journal of Economic Theory, 20 (1). pp. 120-155. ISSN 1742-7355

https://doi.org/10.1111/ijet.12393

Received: 27 September 2022 | Revised: 4 September 2023 | Accepted: 29 November 2023

DOI: 10.1111/ijet.12393

ORIGINAL ARTICLE

International Journal of Economic Theory IJET WILEY

Towards a welfare model of trade and multinational firms with oligopolistic competition

Bingxue Wang

Department of Economics, University of Sheffield, Sheffield, UK

Correspondence
Bingxue Wang, Department of Economics, University of Sheffield, 9 Mappin Street, Sheffield S1 4DT, UK.
Email: bx.emily.wang@gmail.com

Funding information
Economic and Social Research Council

Abstract

This paper constructs a general equilibrium model in a world with two-symmetric countries. It explains welfare gains from international trade and horizontal Foreign Direct Investment (FDI) in the economy with firm heterogeneity and variable markups stemming from oligopolistic competition. My model shows that the pro-competitive effects of trade and horizontal FDI happen because trade openness induces an increase in product market competition that reduces markups and toughens selection, increasing aggregate productivity. The most significant contribution of the paper is that multinational firms, via horizontal FDI, produce the most significant welfare gains through the toughest selection and lowest markups.

KEYWORDS

firm heterogeneity, horizontal FDI, oligopolistic competition, welfare gains

JEL CLASSIFICATION

D60, F12, F13

1 | INTRODUCTION

It has been a long-standing challenge for economists to identify the size and sources of gains from international trade and Foreign Direct Investment (FDI) (e.g., Helpman et al., 2004; Melitz & Ottaviano, 2008). Whether the welfare effects of trade and FDI are significant is a contentious argument in the trade literature. However, in recent decades, a line of research incorporating firm heterogeneity in trade models has uncovered a new source of welfare gains. Rich data sets have emerged, which allow us to look at firm heterogeneity and from which we have begun to see the potential importance of intraindustry differences. Firm heterogeneity arises empirically in the form where there are a small number of large firms that tend to be exporters, and they export a small fraction of their production within an industry (e.g., Aw et al., 2000; Bernard et al., 1995; Clerides et al., 1998). These models with firm heterogeneity (e.g., Bernard et al., 2003; Melitz, 2003) show that international trade, through an increase in competition expelling the least efficient firms from the market, can increase an industry's aggregate productivity and enhance welfare. In my paper, like the above literature, firm heterogeneity is included in the theoretical models with oligopolistic competition to examine the welfare effect of international trade and multinational firms via horizontal FDI, which is novel for heterogeneous firms when mapping trade and FDI onto theoretical framework because of oligopolistic competition among firms.

There are several arguments around the welfare effect of trade. For example, Melitz (2003) and Bernard et al. (2003) found welfare-enhancing properties of the trade when they introduced firm heterogeneity and imperfect competition into the economic environment in their theoretical models. Bernard et al. (2003) applied Bertrand competition in a Ricardian framework as a technique to explain the same basic kinds of trade-induced reallocations, where market shares are reallocated from low to highly productive firms so that sectoral efficiency is increased. The framework delivers an endogenous distribution of markups through competition between domestic and foreign firms of the same variety. A fundamental assumption in the model is an exogenously fixed total number of varieties produced and consumed. Melitz (2003) develops a dynamic industry model to investigate the welfare gains from trade. His framework, based on the work of Dixit and Stiglitz (1977), features monopolistic competition, increasing returns to scale (IRS), and a love for variety. He found a fundamental mechanism, the "competition channel," through which trade positively impacts aggregate productivity and welfare. One limitation in his model is that firms' markups are exogenously fixed since a symmetric elasticity of substitution between varieties holds.

Recent work in the literature on the welfare effects of trade suggests, however, that welfare gains from trade are similar across different types of models and the selection effect may not be so relevant, and their gains could be relatively small (Arkolakis et al., 2012). In my model, I introduce firm heterogeneity like Melitz (2003) with a constant elasticity of substitution (CES) utility function, assuming a continuum of imperfectly substitutable varieties, or product lines, introduced by firms with different productivities. However, a significant distinction from Melitz (2003) is that my model focuses on oligopolistic competition with several perfect substitutable producers in each product line. In contrast, Melitz (2003) has only one firm within each variety in the monopolistic market. That is to say, the critical assumption of my model is that: I assume there is a continuum of varieties and several potential firms for each variety and that they perform as Cournot firms while there is Bertrand competition across homogeneous products in Bernard et al. (2003).

Apart from the study focused on welfare effects from trade, some studies are devoted to gains from multinational production (MP) (e.g., Ahn, 2014; Ramondo, 2014; Ramondo & Rodríguez-Clare, 2010, 2013). Ramondo and Rodríguez-Clare (2010) point out that many possible modes exist for gains from a country's openness: international trade, MP, and diffused technology. Research dealing with welfare effects has given much attention to trade but less to the activity of MP. By 2007, sales of multinational firms were almost twice as high as exporters in the global world, according to the United Nations Conference on Trade and Development in 2007, which means that multinational firms have increasingly become significant channels for countries to exchange goods, capital, ideas, and technology. Ramondo and Rodríguez-Clare (2013) investigate the welfare effects of FDI, building a model with a setting in which trade and MP are competitive and complementary methods to serve a foreign market. Specifically, there are three ways: "horizontal" FDI, in which trade and FDI are competitive methods serving the foreign market; "vertical" FDI, in which foreign affiliates import intermediate goods from the home country; and an export platform, in which firms choose another country as a platform to serve a particular market. They quantify the overall gains from the country's openness and conclude that the welfare gains from trade in their model are twice as high as in trade-only models due to the domination of complementarity forces. In contrast, the gains from MP in their model are slightly lower than gains in MP-only models due to the domination of substitution forces. It is significant as my results also indicate that the scenario of all multinational firms gains the most significant welfare. Ramondo (2014) studies the gains of multinational firms from another point of view. She evaluates the importance of welfare gains from only FDI with foreign firms as the only way to serve a foreign market by building a model combined with Lucas (1978) and Eaton and Kortum (2002). In Ramondo and Rodríguez-Clare (2013) and Ramondo (2014), they all consider horizontal FDI as a substitutable way to serve the foreign market, similar to mine. Garetto (2013) investigates the gains from FDI applied to the Eaton and Kortum (2002) type of model, emphasizing only vertical FDI and quantifying the welfare gains from intrafirm trade, approximately 0.23% of consumption per capita.

However, other literature exploring the welfare effect of multinational activities draws the opposite conclusion. Reis (2001) builds a theoretical framework with asymmetric countries to explain that foreign investment by multinational firms may decrease national welfare since the returns may be repatriated, which indicates that the transfer of capital returns to foreigners would reduce the national welfare. In addition, Yang (2015) builds a theoretical model with domestic oligopolies and suggests that if foreign firms switch production strategy from FDI to exporting, it leads to a fall in domestic welfare because of the decrease in overall cost efficiency. My model focuses on "horizontal FDI," like, Ethier (1986), which offers a competitive (substitutable) way to serve a foreign market with exporters, like, Helpman et al. (2004). I examine the welfare effect of horizontal FDI through a simple and analytically tractable version of the model in which the number of oligopolistic firms per product line is exogenously fixed. Furthermore, I assume all operating producers are multinational firms, compared with the scenarios where all potential firms are non-exporters and all operative firms are exporters with non-zero export fixed costs.

Theoretical frameworks and quantitative analysis in the trade field dealing with welfare effects are characterized mainly by perfect or monopolistic competition (e.g., Arkolakis et al., 2012; Eaton & Kortum, 2002; Ramondo, 2014; Ramondo & Rodríguez-Clare, 2013). However, some evidence shows that large, highly productive firms with substantial market power prevail in modern economies, as shown by Impullitti et al. (2018). For example, empirical work by Bernard et al. (2007) documents that international trade is a rare activity: just 4% of 5.5 million firms operated as exporters in the United States in 2000, and around 96% of the total US exporters account for the top 10% of exporting firms. Regarding trade models, standard frameworks with firm heterogeneity cannot embody strategic competition among large firms. So, in my paper, I will explore the welfare effect of trade and FDI in an economy with heterogeneous firms and oligopolistic competition featuring variable markups. I focus on the welfare gains from international trade and MP when the economy consists of oligopolistic firms with heterogeneity in productivity and market power. The welfare analysis of trade and horizontal FDI comes from the response of the market structure to lowering trade barriers; this is a new channel of welfare gains from variable markups, as previous models focused on monopolistic competition with invariant markups.

I assume a global economy consists of two entirely symmetric countries with the same set of operative product lines and productivity distribution. My model economy has a continuum of potential imperfectly substitutable product lines, known as varieties, with different productivities. Moreover, in each variety, a small number of identical firms produce perfectly substitutable goods competing in an oligopolistic market and generating variable markups, contrasting with Melitz (2003). So individual firms are characterized by "large in the small but small in the large," which means firms are infinitesimal compared with the whole economy but large within their variety (e.g., Neary, 2003, 2010). The firms interact strategically with a small number of identical direct competitors, domestic and foreign rivals, in a Cournot competition game (e.g., Brander & Krugman, 1983) and confront indirect competitors among firms within other varieties. The model framework is similar to Impullitti and Licandro (2018), and a small number of identical firms within each variety enter a market and draw their productivity from the bounded Pareto distribution conditional on paying sunk costs. If a firm enters successfully, it plays Cournot games with its direct competitors within each variety. Notice that, to investigate the welfare effect of multinational firms, first, I begin with a simple and basic framework in which the number of firms n within each variety is exogenous and consider three scenarios where all potential firms are non-exporters, all operating producers are exporters with a zero and non-zero fixed cost of exporting, and all active manufactures are multinational firms. Then, I compare welfare gains from these three cases and find that multinational firms arrive at the highest gains caused by the "pro-competitive effect with lowest markups" and "toughest selection with highest productivity cutoff."

This paper is structured into four additional sections after the introduction. In Section 2, I conduct a review of the literature. In Section 3, I provide the assumptions and set the basics of my model to present and solve a simple version of three scenarios. They include all non-exporters, all exporters with zero export fixed cost, and all multinational firms as a new competitive way to serve in a foreign market separately. Notice that I consider an extra case where all are exporters with a non-zero fixed cost of exporting to conduct a comparison with multinational firms. In Section 4, I investigate the welfare gains from three scenarios, represented by the case in which all are exporters with a zero export fixed cost. Then, I compare the welfare effect from three scenarios and explain the gains when the country opens to trade and the welfare effect of international trade. Section 5 provides the numerical simulation analysis. Finally, Section 6 concludes that horizontal FDI in the open economy could produce the highest welfare gains. References and Appendices are attached at the end.

2 | LITERATURE REVIEW

My work brings together and develops the literature on international trade (e.g., Impullitti & Licandro, 2018; Impullitti et al., 2018; Krugman, 1980; Melitz, 2003) and MP (e.g., Helpman et al., 2004; Ramondo, 2014; Ramondo & Rodríguez-Clare, 2013). It closely relates to three significant strands of the literature. First, the literature on firm heterogeneity for international trade and multinational firms via horizontal FDI and constant markups in the monopolistic market comprised the first strand of literature. That is, how they explain the selection effects due to firm heterogeneity is rational for international trade and multinational firms in the monopolistic environment. Then, by considering an environment consisting of oligopolistic competition with variable markups, I review the welfare effect of trade and FDI in this environment. My aim for the paper is to extend these papers with oligopolistic competition resulting in variable markups and investigate the welfare gains from international trade multinational firms. Here, the way I introduce horizontal FDI is like the way incorporated by Helpman et al. (2004) for simplicity. Third, I compare my paper with the literature regarding the pro-competitive for international trade and multinational firms and their welfare effects (e.g., Ramondo, 2014; Ramondo & Rodríguez-Clare, 2013).

Observations informed the first branch of trade literature since the 1970s that world trade flows were predominantly between developed countries with similar technologies or factor endowments. At the forefront of this literature is Krugman's (1980), "new trade theory," which considers firms within an international industry under monopolistic competition. He assumes that consumers love variety and that an economy exhibits internal economies of scale. Welfare gains from trade in this model come purely from the variety channel since consumers love variety, and there would be more of it available to consumers when a country opens to trade. Melitz's (2003) "new new trade theory" extends Krugman (1980) to take into account firm heterogeneity. He develops a dynamic model of entry and exit based on Hopenhayn (1992), with CES utility, IRSs, and monopolistic competition featuring constant markups. He finds welfare gains will increase as trade results in a reallocation of market shares from less to more productive firms by expelling the least productive firms from the market.

In terms of introducing multinational firms, Helpman et al. (2004) extend Melitz (2003) by introducing horizontal FDI: a firm's affiliate replicates the production process of its parent firm in its domestic facilities elsewhere in the world. It is assumed that firms can serve the foreign market either by export or FDI, and there are different fixed costs for both exporters and multinational firms. They are the fixed cost of exporting f_X, and the fixed cost of creating a new plant in the foreign country f_I, separately. In addition to those fixed costs, a variable iceberg cost τ exists for the exporters only, which means $\tau > 1$ unit of goods should be produced to deliver per unit of the good at the destination. Under particular conditions, among different costs, they find that the most productive firms will serve the foreign markets through FDI; the middle productivity firms will serve the foreign markets via exports, and the less productive firms will remain domestic only. In this paper, I include horizontal FDI and introduce related fixed costs of planting a new company in a foreign country as Helpman et al. (2004). I combine horizontal FDI with a simplified version of Impullitti and Licandro (2018), which embraces oligopolistic competition within each variety and assumes the number of firms in each product line is exogenous, including all operating firms are exporters with a zero fixed cost of exporting. Moreover, I set up three scenarios where all operating firms are non-exporters, all potential firms export, and all operative firms are with FDI activities. Then, I emphasize comparing the welfare gains in these three cases. While different from Helpman et al. (2004), my contribution is that I incorporate multinational firms in an environment with variable markups generated by oligopolistic competition.

Another strand of literature states variable markups stemming from strategical interaction between firms with oligopolistic competition and firm heterogeneity, represented by Impullitti and Licandro (2018) and Impullitti et al. (2018, 2022). Impullitti and Licandro (2018) incorporate endogenous growth into the model with cost-reducing innovation and consider a simple version of the model and a general version. The difference is that the simple model considers exporters only, while the general version includes non-exporters and exporters simultaneously. They find welfare gains generated through the selection and pro-competitive effects, which are absent in Melitz (2003) where markups are constant because of only one firm within each variety and demonstrate that growth can magnify the gains from selection, as market share reallocations can affect the productivity level and its growth rate. In contrast, Melitz (2003) combines a zero cutoff profit (ZCP) condition and a free entry (FE) condition to derive the productivity threshold at equilibrium. Besides, he assumes the mass of entrants can respond endogenously to changes in trade costs, potentially taming or even offsetting the loss of varieties due to selection. According to Impullitti and Licandro (2018), selection has a starker negative welfare effect through the loss of varieties. They conclude that trade liberalization generates tougher firm selection, increasing average firm productivity, as in Melitz (2003). From the perspective of the model-solving technique, Impullitti and Licandro (2018) consider firms' exit conditions (EC) and labor market clearing (MC) conditions to constrain the equilibrium productivity cutoff. They did not consider the FE condition in the simplified version of the model but assumed the number of varieties is endogenous depending on firms' productivity thresholds. Moreover, selection incentivizes firms to innovate, leading to a higher innovation-driven growth rate. This can potentially boost the gains from selection, leading to further welfare improvements since market share reallocations can affect both the productivity level and its growth rate. Impullitti et al. (2018) devise a more sophisticated entry structure, which allows markups to vary with firm productivity through an endogenous number of firms, associated closer with empirical evidence of large markup dispersion across firms. They adopt innovation without knowledge spillovers and demonstrate that innovation still has a nonnegligible contribution to the gains from trade. Impullitti et al. (2022) build a similar economic environment but assume a discrete number of firms compared with the real number of firms of Impullitti et al. (2018). The assumption is also suggested by Eaton et al. (2012), which emphasizes the importance of an integer number of firms, and Eaton et al. (2012), which explains that granularity, a finite number of firms within sectors, accounts for around 20% of the sectoral variation in export intensity, especially in highly export-intensive sectors. Impullitti et al. (2022) find that trade induces an increase in market concentration, which is a significant source of gains from trade. The gains of welfare through the increase in market concentration are conducted through two mechanisms: increasing returns of manufacture and a scale effect on innovation. In the numerical analysis, market concentration is the main driver of welfare gains, specifically, less from increasing returns and more from the scale effect on innovation. My paper is built with a similar economic environment as theirs, including heterogeneous firms under oligopolistic competition, but extends horizontal FDI to the framework as in Helpman et al. (2004), aiming to explore the welfare gains from MP.

The third strand of the literature shows some results regarding the pro-competitive effect on trade and FDI (e.g., Arkolakis et al., 2019; Eckel & Neary, 2010; Edmond et al., 2015; Li & Miao, 2020), especially the welfare effect of multinational firms (e.g., Ahn, 2014; Ramondo, 2014; Ramondo & Rodríguez-Clare, 2013). For example, Edmond et al. (2015) estimate the pro-competitive gains from international trade in a quantitative model with variable markups from the endogenous market structure and find that openness reduces markup distortions by up to one-half and significantly reduces productivity losses. However, Li and Miao (2020) consider a nexus of the use of imported input and markups in the theoretical framework to explain the increase of market power. They find that the selection effect of importers, cost-reducing from imported intermediates and industry firm turnover are the main causes for the increased markup in the market. In contrast, my paper explains that trade liberalization reduced the variable markup through the channel of "pro-competitive" because of oligopolistic competition; furthermore, we contribute to it by adding that multinational firms via horizontal FDI exert the highest pro-competitive effect compared with exporters. Arkolakis et al. (2019) explore the gains from trade liberalization in a body of models with monopolistic competition, firm-level heterogeneity, and variable markups. They conclude that pro-competitive effects of trade are elusive as welfare effects from trade liberalization for the model with variable markups are slightly lower than the model with constant markups. However, among these papers, MP is absent. Ramondo and Rodríguez-Clare (2013) explore the welfare effects of trade and FDI and capture key dimensions of the interaction between these two flows. One way of their interaction is that international trade and MP via horizontal FDI behave as competitive methods to serve a foreign market (e.g., Helpman et al., 2004; Markusen & Venables, 2000). Much attention has been paid to quantifying the gains from single mechanisms in isolation, especially trade in goods and, to a lesser extent, FDI. They build on the Ricardian model of international trade developed by Eaton and Kortum (2002) and incorporate MP into the model by allowing a country's technologies to be used for production abroad. They quantify the overall gains from openness and conclude that the welfare from trade is twice as high as in trade-only models, while the gains from MP are slightly lower than gains in MP-only models. Compared with their model, I apply the CES utility function in a monopolistic market, abandon the constant returns to scale in the perfectly competitive market and introduce oligopolistic competition for several firms within each variety. Another deviation is that I compare the welfare gains from three individual scenarios in which all firms are non-exporters, all operating firms export, and all potential firms via horizontal FDI. Ahn (2014) builds a theoretical model and examines the pro-competitive channel of horizontal FDI on welfare gains with firm heterogeneity. The paper finds welfare gains for the source country while welfare decreases for the host country due to the production reallocation leading to the increase of domestic firms in the home country and a decrease in the host country. Like this paper, I consider firm heterogeneity and find the largest pro-competitive effect of horizontal FDI while including oligopolistic competition.

Therefore, my paper contributes to the above three strands of literature from firm heterogeneity, variable markups through oligopolistic competition, and the welfare effect of multinational firms. In this paper, like, Melitz (2003) and its extension of Helpman et al. (2004), I set different types of fixed costs, for example, the fixed costs of production for non-exporters, exporters, and multinational firms, separately. My model also applies variable iceberg transportation costs to trade via exports.

3 | THE MODEL

I now outline the theoretical model, beginning with a simple benchmark case with restricted entry and autarky. I then extend this to the case of exporting with no associated fixed cost and analyze the case where all viable firms export. Key to solving and analyzing the model in Melitz (2003) is the ZCP condition and FE condition. He pins down these conditions, assuming the mass of producing firms in equilibrium is $M > 1$. In contrast, here, entry is restricted, but there is no entry cost, a continuum of varieties has endogenous mass $M \in (0, 1)$ and the productivity cutoff threshold is pinned down with an EC and labor MC condition, as in Impullitti and Licandro (2018). Notice, that Melitz (2003) uses a price index to arrive at the welfare effect while Impullitti and Licandro (2018) do not since they apply a different inverse demand function to deal with the model including the differentiated good according to X (see Equation 2).

3.1 | Preferences and demand

Consider an economy with two identical countries; each is composed of a continuum of individuals of measure 1. In each country, there are two types of final goods. Good 1 is a homogeneous good, while the other goods are differentiated. More precisely, in the differentiated sector, there is a continuum of varieties, $v \in V$, where V is the set of all potential varieties in the differentiated sector. The utility is featured by a standard Cobb–Douglas utility function with homogeneous goods and composited goods characterized by a CES utility function. The following utility function shows preferences across goods:

$$U = (1-\gamma)\ln Y + \frac{\gamma}{\alpha}\ln\left(\int_{v\in V} x(v)^{\alpha}\, dv\right), \tag{1}$$

where Y is the consumption of the homogeneous good and $x(v)$ is the consumption of heterogeneous variety v. Notice that equivalent to V, a continuum of variety of endogenous mass $M \in (0, 1)$ is assumed. $\alpha = \frac{\sigma-1}{\sigma}$ controls for the elasticity of substitution (σ) between any two varieties in the composite sector, where $\alpha \in (0, 1)$.[1] Let

$$X = \left(\int_0^M x(v)^{\alpha}\, dv\right)^{\frac{1}{\alpha}}. \tag{2}$$

Consumers maximize utility subject to the following budget constraints:

$$Y + \int_{v\in V} p(v)x(v)\, dv = E, \tag{3}$$

[1]Note that the utility function presented here, Equation (1), is a monotonic transformation of the following utility function $U = Y^{(1-\gamma)}X^{\gamma}$.

where the homogeneous good, Y, is considered to be the numéraire, E denotes total expenditure, and consumers are endowed with a unit flow of labor. The consumers spend a fraction γE of their income on composite goods and $(1-\gamma)E$ on homogeneous goods. Solving this utility maximization problem yields the following inverse demand function for each variety, v:

$$p(v) = \frac{\gamma E}{X^{\alpha}} x(v)^{\alpha-1}, \tag{4}$$

where $Y = (1-\gamma)E$.

3.2 | Production and firm behavior

There is a unique production factor, labor L. The homogeneous goods Y are treated as numéraire. It is produced in perfect competition using a linear technology that involves using one unit of labor to obtain one unit of output. Given that good Y is the numéraire (its price equals 1) and perfect competition, implying a unit wage: $w = 1$. Besides, the homogeneous goods Y are also assumed to be freely traded with no additional costs, which implies that wages are equal across countries in the presence of homogeneous goods in both countries. Each of the differentiated goods is also produced with a linear technology whose cost function is given by

$$C(z) = l(z) = z^{\frac{\alpha-1}{\alpha}} q(z) + \lambda_d, \tag{5}$$

where z denotes the firm productivity (which varies across varieties and is assigned to the firm following the mechanism explained below), $q(z)$ is firm output, $C(z)$ is the total cost, and $l(z)$ is the total amount of labor used in production. To produce output, the firm must incur a fixed cost, λ_d.

In this simple setup, I assume there is a restricted entry. Before entry, firm productivity is unknown. More precisely, the productivity, z, for all firms producing a given variety in a sector is drawn from a continuous Pareto productivity distribution with a distribution function given by the following expression:

$$G(z) = 1 - \left(\frac{\underline{z}}{z}\right)^k = 1 - z^{-k}, \quad z \geq 1, \;\; k \geq 1 \tag{6}$$

with lower productivity bound, $\underline{z} = 1$, and shape parameter, k, measures the inverse of the heterogeneity, which implies a high k is more homogeneous, like, Chaney (2008).

In the restricted entry model, I assume that a firm generates a specific variety with their direct competitors participating in Cournot competition in each sector h. Before entry, the productivity of a variety with identical firms is unknown, and firms within each new variety enter until expected profits are zero, which means the number of firms within each variety n is the same for all of these varieties. It allows strategic interaction but abstracts from nontrivial complications associated with firm number endogeneity. These firms observe their variety's productivity draw and stay if the expected profit is positive with $n-1$ rivals in their variety. Otherwise, the firm (and all $n-1$ rivals) will exit. Notice that

n identical firms always exit simultaneously within a variety, which means a variety with n firms disappears if only one firm within the variety chooses to exit. Firms in a surviving variety compete à la Cournot.

Finally, I assume firms face the risk of death at each point in time, as in Melitz (2003). However, here, rather than firms facing this risk individually, I assume that variety v in sector h faces this risk collectively. Hence, all firms in variety v of sector h face a bad shock that leads to exit with probability, δ. Moreover, this probability is common across varieties in all sectors and across the history of any variety.

The following subsections set the benchmark case where all viable varieties are traded only in a firm's domestic market. I then introduce the case where all firms export without fixed export costs. I finally generalize the model to include exporting fixed costs.

3.3 | Autarky

In the autarkic scenario, the firms will be able to serve only their domestic market, and I assume there are only non-exporters in a two-country economy with symmetry. A firm manufacturing a nonexported variety with productivity level z will maximize its profits subject to the inverse demand function in Equation (4), taking the quantities their competitors produce as given since firms produce the same variety play a symmetric Cournot game and behave noncooperatively. The firm's problem is

$$\max_{q_a} \pi_a = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_a + q_a)^{\alpha-1}}_{p_a} q_a - z^{\frac{\alpha-1}{\alpha}} q_a - \lambda_d, \tag{7}$$

where subscript a indicates that firms produce a domestic, nonexported variety; that is, firms only serve the domestic market. Here, q_a is the firm's production and $\hat{x}_a$ is the production of its direct competitors within a variety with the same productivity level, z. Total output or consumption for a variety in the domestic country is therefore $x_a = q_a + \hat{x}_a$. Since labor is numéraire, the first-order condition for a firm is

$$\frac{\gamma E}{X^\alpha}((\alpha - 1)(\hat{x}_a + q_a)^{\alpha-2} q_a + (\hat{x}_a + q_a)^{\alpha-1}) = z^{\frac{\alpha-1}{\alpha}}. \tag{8}$$

Given symmetry, the equilibrium is such that $x_a = nq_a$, hence Equation (8) can be written

$$\underbrace{\frac{\gamma E}{X^\alpha} x_a^{\alpha-1}}_{p_a} \left(\frac{(\alpha - 1)q_a}{nq_a} + 1 \right) = z^{\frac{\alpha-1}{\alpha}}. \tag{9}$$

Simplifying and rearranging Equation (9), I have

$$p_a(z, n) = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_a} \tag{10}$$

and

$$x_a(z,n) = \left(\frac{\gamma E}{X^{\alpha}}\right)^{\frac{1}{1-\alpha}} \theta_a^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}}, \tag{11}$$

where θ_a represents the inverse of the markup of a domestic firm that cannot export:

$$\theta_a \equiv \frac{n-1+\alpha}{n}. \tag{12}$$

Let $\bar{z}$ and e denote, respectively, the average productivity and the expenditure per heterogeneous firm:

$$\bar{z} = (1/M)\int_0^M z_v \, dv, \tag{13a}$$

$$e \equiv \frac{\gamma E}{nM}. \tag{13b}$$

The following expression shows a domestic firm's variable production cost[2]:

$$l_a(z,n) - \lambda_d = z^{\frac{\alpha-1}{\alpha}} q_a(z,n) = e\theta_a\left(\frac{z}{\bar{z}}\right), \tag{14}$$

where l_a is the labor of a domestic firm devoted to producing goods for their market.

3.3.1 | Entry and exit

There is a unit mass of potential varieties $M \in (0, 1)$. That is to say, nonoperative new varieties $1 - M$ can be identified as produced by n firms and trying to enter the economy at zero cost, which associated productivity z is jointly drawn for each of them from a bounded Pareto distribution $G(z)$ with lower productivity bound $z = 1$. Moreover, an exogenous death shock δ can cause all firms to exit the market.

To establish the productivity cutoff point, I begin with equilibrium profit for a domestic firm, with productivity z given average productivity $\bar{z}$. Using Equations (10), (11), and (14), firm profit can be expressed as

$$\pi_a(z/\bar{z}) = p_a q_a - l_a = e(z/\bar{z})(1-\theta_a) - \lambda_d. \tag{15}$$

With restricted entry and no entry cost, the EC requires

$$\pi_a\left(z_a^*/\bar{z}_a\right) = e_a\left(z_a^*/\bar{z}_a\right)(1-\theta_a) - \lambda_d = 0. \tag{16}$$

[2]See Appendix A.1.

The productivity cutoff in the domestic case is z_a^*, such that if $z \geqslant z_a^*$ all firms with productivity z stay in the market, and otherwise, they all leave the market. This productivity cutoff is therefore described by the following condition, rearranging Equation (16):

$$e_a(1-\theta_a)\left(z_a^*/\bar{z}_a\right) = \lambda_d. \tag{17}$$

Note, in equilibrium, I obtain the following:

$$\bar{z}_a\left(z_a^*\right) = \int_{z_a^*}^{\infty} z\mu(z)\,dz, \tag{18}$$

where

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G\left(z_a^*\right)} & \text{if } z \geqslant z_a^*, \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

It follows that I can express $\bar{z}_a$ in terms of z_a^*[3]:

$$\bar{z}_a = \frac{k}{(k-1)} z_a^*. \tag{20}$$

Using Equation (20) in Equation (17) I have the following expression for the EC under autarky EC_a:

$$e_a\left(z_a^*\right) = \frac{\lambda_d}{(1-\theta_a)z_a^*/\bar{z}_a} = \frac{k\lambda_d}{(k-1)(1-\theta_a)} \quad (EC_a). \tag{21}$$

Note that under the Pareto distribution, the EC_a condition is independent of the productivity cutoff, and it is a horizontal line in the graph of (e, z). This can be explained as $z_a^*/\bar{z}_a$ is a constant and only related to the shape parameter of Pareto productivity distribution, k.

I now set about deriving the Labor MC condition under autarky, MC_a, which will yield a relationship between e_a and z_a. I begin with a stationarity condition. In the steady state, I assume that the total number of domestic firms in the market remains constant over time. For this to happen, the following conditions must hold:

$$(1-M)\left(1-G\left(z_a^*\right)\right) = \delta M. \tag{22}$$

This condition states that the exit flow, δM, equals the entry flow defined by the number of potential new varieties, $(1-M)$, times the probability of surviving, $1-G(z_a^*)$. Consequently

[3]See Appendix A.2.

$$M\left(z_a^*\right) \equiv \frac{1 - G\left(z_a^*\right)}{1 + \delta - G\left(z_a^*\right)} = \frac{z_a^{*-k}}{z_a^{*-k} + \delta}. \tag{23}$$

Note, Equation (23) describes a decreasing relationship between M and the productivity cutoff z_a^*, given $M \in (0, 1/(1+\delta))$. Since this relationship between M and z is not dependent on a specific market or trading conditions in this model, it gives rise directly to the following general result:

Lemma 1. *The equilibrium mass of produced varieties, M, is strictly decreasing in the productivity cutoff, z^*, for $M \in \left(0, \frac{1}{(1+\delta)}\right)$.*

Lemma 1 is in line with Impullitti and Licandro (2018), which considers the same stationarity condition and results in the decreasing relationship between the mass of operative variety and the productivity threshold. On the other hand, it is distinguished from Melitz (2003) and Atkeson and Burstein (2010), in which the number of entrants is responsible endogenously for the varies in costs.

I now set out the labor MC condition, which can be written as

$$nM \int_{z_a^*}^{\infty} (e\theta_a(z_a/\bar{z}_a) + \lambda_d)\mu(z)\, dz + (1-\gamma)E = 1. \tag{24}$$

The first element on the left-hand side of Equation (24) indicates the total amount of labor devoted to the differentiated sector. In contrast, the second element is the total amount of labor devoted to the homogeneous sector. Since

$$\int_{z_a^*}^{\infty} \mu(z)\, dz = \int_{z_a^*}^{\infty} (z_a/\bar{z}_a)\mu(z)\, dz = 1 \tag{25}$$

after integrating overall varieties and using Equations (13b) and (25) in Equation (24), I have

$$nM\left(z_a^*\right)\left(e_a\theta_a + \lambda_d + \frac{e_a}{\gamma}(1-\gamma)\right) = 1.$$

Rearranging I obtain the following condition:

$$e_a\left(z_a^*\right) = \frac{\frac{1}{nM\left(z_a^*\right)} - \lambda_d}{\theta_a + \frac{1-\gamma}{\gamma}} \quad (MC_a). \tag{26}$$

Equation (23) established that $M(z_a^*)$ is a decreasing function of z_a^*. Hence, Equation (26), which is the MC condition under autarky, establishes an increasing relationship between e and z_a^*. That is to say, MC_a condition describes an increasing line in the graph of (e, z_a^*), with the decreasing number of $M(z_a^*)$.

Note, to obtain the equilibrium productivity threshold, the following is assumed $\lambda_d \geqslant \bar{\lambda_a}$,[4] where

$$\bar{\lambda_a} = \frac{1+\delta}{n\left(1+\frac{k}{k-1}\frac{\theta_a+\frac{1-\gamma}{\gamma}}{1-\theta_a}\right)}.$$

This assumption can be derived from the consideration of $(EC_a) \geqslant (MC_a)$ at the $z_{\min} = 1$. It is because there are two lines in the graph of (e, z_a^*), a horizontal line of (EC_a) and an increasing line of (MC_a), separately. Only by constricting the condition of $(EC_a) \geqslant (MC_a)$ at the $z_{\min} = 1$, I can solve the unique intersection of two lines. At the extreme case, I can derive $z_{\min} = z_a^* = 1$ and $(EC_a) = (MC_a)$.

The productivity cutoff under autarky is obtained by equating the EC_a and MC_a. Setting (EC_a) equal to (MC_a), using Equations (21) and (26) and substituting $M(z_a^*)$ using Equation (23) I get

$$\frac{k\lambda_d}{(1-\theta_a)(k-1)} = \frac{\frac{z_a^{*-k}+\delta}{nz_a^{*-k}} - \lambda_d}{\theta_a + \frac{1-\gamma}{\gamma}}.$$

Rearranging gives us the following expression for the autarky productivity threshold:

$$z_a^* = \left(\frac{n\lambda_d\left(1+\frac{k}{k-1}\frac{\theta_a+\frac{1-\gamma}{\gamma}}{1-\theta_a}\right)-1}{\delta}\right)^{1/k}. \tag{27}$$

Lemma 2. *Under the assumption $\lambda_d \geqslant \bar{\lambda_a}$, there exists a unique interior solution (e_a, z_a^*) of the intersection of MC_a and EC_a, with $M(z_a^*)$ determined by Equation (23).*

Proof. See Appendix A.3. □

The technique of seeking the equilibrium productivity cutoff threshold is similar to Impullitti and Licandro (2018), which emphasizes the case where all are exporters. In my paper, I incorporate the scenario where all are domestic firms to extend the discussion. Specifically, they pin down EC and labor market condition (MC) to derive equilibrium variables in steady state, productivity cutoff, and average expenditure per firm in a case in which all are exporters in two-symmetric countries. However, here I investigate the scenario where all are domestic firms with similar methods and find a unique interior solution of

[4] $(EC_a) \geqslant (MC_a)$ at the minimum $z_a^* = 1$,

$$\frac{k\lambda_d}{(1-\theta_a)(k-1)} \geqslant \frac{\frac{z_a^{*-k}+\delta}{nz_a^{*-k}} - \lambda_d}{\theta_a + \frac{1-\gamma}{\gamma}}.$$

equilibrium productivity cutoff and average expenditure per firm for sector h, under a certain constraint on the fixed production costs, λ_d.

3.4 | Trade openness

In this section, I analyze the scenario in which all the domestic firms can serve the foreign market via exports. It implies that the cost of opening a plant in a foreign market is so large that no firm would like to do this, compared with the scenario where all are multinational firms via FDI in Section 3.6. This will allow us to compare the welfare implications of opening up to trade and MP in welfare analysis. When the firm serves the foreign market via exports, it must bear a transportation cost, τ $(\tau \geq 1)$, of the "iceberg" type, which means τ units of a good must be shipped to obtain one unit at the destination. For exporting, I consider two subcases: the scenario where there is no fixed exporting cost and then one where exporting firms bear a fixed cost of $\lambda_x \leqslant \lambda_d$.

Under the Cournot assumption, firms behave noncooperatively and maximize profits subject to the inverse demand function in Equation (4), taking competitor output in their variety as given. As a result, exporters compete simultaneously in domestic and foreign markets, which are referred to by subindices d_x and f_x, respectively, and they treat each market as segmented.

Let q_{d_x} and q_{f_x} denote the quantities sold by a domestic firm in the domestic and foreign markets, respectively, and p_{d_x} and p_{f_x} denote the associated prices. Hence, the total quantity produced by a firm is given by $q_x = q_{d_x} + \tau q_{f_x}$. Note, this includes $(\tau - 1)q_{f_x}$ which is the amount of output that the firm needs to produce to bear the "iceberg" transportation costs. Let $\hat{x}_{d_x}$ and $\hat{x}_{f_x}$ denote the quantities sold by a firm's competitors in the domestic and foreign markets, respectively. Note, in the domestic market, under exporting, a domestic firm's rivals include domestic competitors and foreign firm exporters of the same variety. Let x_{d_x} and x_{f_x} denote the total quantities sold in the separate domestic and foreign markets for a traded variety, v. If all firms export, there will be $2n$ firms serving any traded variety in the domestic and foreign markets. Firms, in the "all exporting" scenario, solve the problem:

$$\max_{q_{d_x}, q_{f_x}} \pi_x = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_x} + q_{d_x})^{\alpha-1}}_{p_{d_x}} q_{d_x} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_x} + q_{f_x})^{\alpha-1}}_{p_{f_x}} q_{f_x} - z^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - \lambda_d, \tag{28}$$

which yields the following first-order conditions for the domestic and foreign markets, respectively:

$$\frac{\gamma E}{X^\alpha}\Big((\alpha - 1)(\hat{x}_{d_x} + q_{d_x})^{\alpha-2} q_{d_x} + (\hat{x}_{d_x} + q_{d_x})^{\alpha-1}\Big) = z^{\frac{\alpha-1}{\alpha}}, \tag{29}$$

$$\frac{\gamma E}{X^\alpha}\Big((\alpha - 1)(\hat{x}_{f_x} + q_{f_x})^{\alpha-2} q_{f_x} + (\hat{x}_{f_x} + q_{f_x})^{\alpha-1}\Big) = \tau z^{\frac{\alpha-1}{\alpha}}. \tag{30}$$

Given competition is of the Cournot type, each firm takes as given the output of its competitors in variety v of sector $h, \hat{x}_x$, but also aggregate expenditure, E, and aggregate consumption of heterogeneous varieties X in each sector. Country symmetry implies $E_{d_x} = E_{f_x} = E, X_{d_x} = X_{f_x} = X$. Further, since all firms producing the same variety are identical, denote the aggregate output of firms in given variety under "all exporting," $x_x = x_{d_x} = x_{f_x} = n(q_{d_x} + q_{f_x})$, and associated price $p_x = p_{d_x} = p_{f_x}$.

Applying symmetry, $x_{d_x} = x_{f_x} = x_x = n(q_{d_x} + q_{f_x})$, to the ratio between Equations (29) and (30):

$$\frac{\frac{\gamma E}{X^\alpha}\left((\alpha-1)(\hat{x}_{d_x} + q_{d_x})^{\alpha-2}q_{d_x} + (\hat{x}_{d_x} + q_{d_x})^{\alpha-1}\right)}{\frac{\gamma E}{X^\alpha}\left((\alpha-1)(\hat{x}_{f_x} + q_{f_x})^{\alpha-2}q_{f_x} + (\hat{x}_{f_x} + q_{f_x})^{\alpha-1}\right)} = \frac{z^{\frac{\alpha-1}{\alpha}}}{\tau z^{\frac{\alpha-1}{\alpha}}}$$

from which it follows:

$$q_{d_x} = \frac{(\alpha-1) + (1-\tau)n}{\tau(\alpha-1) + (\tau-1)n} q_{f_x}. \tag{31}$$

It is useful to refer, henceforth, to the following:

$$\beta \equiv \frac{q_{f_x}}{q_{d_x}} = \frac{\tau(n+\alpha-1) - n}{n+\alpha-1-n\tau}, \tag{32}$$

where

$$\frac{\partial \beta}{\partial \tau} = \frac{(2n+\alpha-1)(\alpha-1)}{(n+\alpha-1-n\tau)^2} < 0.$$

Analysis of β reveals several notable properties. First, $\beta \in [0, 1]$, hence, the quantity supplied by a domestic firm to the foreign market is smaller or equal to that it supplies in the domestic market. This comes from the fact that the marginal cost of a domestic firm serving the foreign market is, under an "iceberg" cost, weakly greater than one of its foreign competitor firms in the overseas market with equality when $\tau = 1$, so transportation costs are zero and the firm supplies the same output in both markets.

Second, from Equation (32), it follows that the prohibitive level of trade costs, associated with $\beta = 0$, is given by

$$\bar{\tau} = \frac{n}{n+\alpha-1}. \tag{33}$$

Hence, for $\tau \in (1, \bar{\tau})$ then $\beta > 0$. In the extreme case, variable trade costs are at the prohibitive level $\tau = \bar{\tau}$, then $q_{f_x} = 0$ and $\beta = 0$. For $\tau > \bar{\tau}$, the markup in foreign markets turns negative, and firms will not export.

Applying symmetry to the first-order conditions of the firm in Equations (29) and (30), I obtain the equilibrium (symmetric) price in the domestic and foreign markets:

$$p_x(z, n) = \frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_x}} = \frac{\tau z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_x}}, \tag{34}$$

where

$$\theta_{d_x} = \frac{2n + \alpha - 1}{n(1 + \tau)}, \quad \theta_{f_x} = \tau\theta_{d_x}, \tag{35}$$

and θ_{d_x} and θ_{f_x} represent the inverse of the markups of a domestic firm in the domestic and foreign markets, respectively. Note also that firms in this model will, accordingly, charge a higher markup on their domestic sales for $\tau > 1$. This is the result of the fact that the marginal cost of serving the foreign market under $\tau > 1$ is higher, while the equilibrium prices in the domestic and foreign markets are the same.

The derivatives of $\theta_{d_x}, \theta_{f_x}$ with respect to $\tau \in (1, \bar{\tau})$ are

$$\frac{\partial\theta_{d_x}}{\partial\tau} = \frac{\partial\frac{2n+\alpha-1}{n(1+\tau)}}{\partial\tau} = -\frac{(2n + \alpha - 1)}{n(1 + \tau)^2} = -\frac{\theta_{d_x}}{(1 + \tau)} < 0,$$

$$\frac{\partial\theta_{f_x}}{\partial\tau} = \frac{\partial(\tau\theta_{d_x})}{\partial\tau} = \tau\frac{\partial\theta_{d_x}}{\partial\tau} + \theta_{d_x} = \frac{-\tau\theta_{d_x} + \theta_{d_x}(1 + \tau)}{(1 + \tau)} = \frac{\theta_{d_x}}{(1 + \tau)} > 0.$$

Hence, lowering the trade cost, τ leads to an increase in θ_{d_x} because the domestic market becomes more competitive due to the penetration of foreign firms, as in Impullitti and Licandro (2018). This pro-competitive effect, consistent with the empirical evidence, will have significant implications for our welfare analysis. Besides, a reduction in trade costs, τ, induces higher markups on a firm's foreign sales, $1/\theta_{f_x}$, since exporters enjoy the benefit of cost reduction in their shipment while domestic firms do not. The claim can be found in Impullitti et al. (2018) as well.

On the basis of Equation (32), I can express the ratio of total production to total consumption of a domestic firm within an exported variety (in line with Brander & Krugman, 1983), as Φ:

$$\Phi \equiv \frac{q_{d_x} + \tau q_{f_x}}{q_{d_x} + q_{f_x}} = \frac{(1 - n - \alpha)(1 + \tau^2) + 2n\tau}{(1 - \alpha)(1 + \tau)} = \frac{1 + \tau\beta}{1 + \beta} \geq 1. \tag{36}$$

Φ captures the losses related to trade due to the iceberg cost, which means the cost of importing goods that could be otherwise produced locally. It is straightforward to see that Φ is hump-shaped in τ and strictly greater than one, for $\tau \in (1, \bar{\tau})$, and equals one in the extreme cases of free trade, $\tau = 1$, and the prohibitive trade cost, $\bar{\tau}$. Intuitively, when the iceberg trade costs are at a prohibitive level, export sales, q_{f_x}, are zero, a special case of autarky: no share of production is wasted in transportation, implying $\Phi = 1$. A reduction in the iceberg cost results in firms having incentives to export and reduce domestic sales. Consequently, the losses associated with trade costs become positive and Φ increases above one. At the other extreme of free trade, there is no waste in transportation, that is, $\Phi = 1$, and any increase in trade cost increases Φ above one.

I define the inverse of an exporting firm's average markup and derive it as an expression in terms of Φ using Equations (34) and (36):

$$\theta_x \equiv \frac{q_{d_x}\theta_{d_x} + q_{f_x}\theta_{f_x}}{q_{d_x} + q_{f_x}} = \Phi\theta_{d_x}. \tag{37}$$

Note that θ_x is a weighted average of the respective inverse markups for a firm in the domestic and foreign markets. The firm's average markup captures the ratio of total revenue to variable production costs. When iceberg trade costs are prohibitive, $\bar{\tau} = n/(n + \alpha - 1)$, $q_{f_x} = 0$ and this average collapses to $\theta_x = \theta_{d_x} = (n + \alpha - 1)/n$. When trade is costless ($\tau = 1$), I find, $\theta_x = \theta_{d_x} = (2n + \alpha - 1)/2n$, since $\theta_{f_x} = \theta_{d_x}$. I also identify that θ_x is decreasing in τ for $\tau \in (1, \bar{\tau})$:

$$\frac{\partial\theta_x}{\partial\tau} = -\frac{2(\tau - 1)(2n - 1 + \alpha)^2}{n(1 + \tau)^3(1 - \alpha)} = -\frac{2n(\tau - 1)\theta_{d_x}^2}{(1 - \alpha)(1 + \tau)} < 0. \tag{38}$$

This means trade liberalization decreases a firm's average markup on total sales. Intuitively, with regard to decreasing variable transportation cost τ, I can explain that the decrease in the markup of an exporting firm in the domestic market is sufficiently strong to offset the increase in the foreign market. This leads to an overall pro-competitive effect of trade liberalization.

Using Equations (4) and (34), I have that equilibrium output in a country for a given variety under "all exporting," and zero exporting fixed cost is

$$x_x(z, n) = \left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}} \theta_{d_x}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}}. \tag{39}$$

The following expression is an exporting firm's variable production cost[5]:

$$l_x(z, n) - \lambda_d = z^{\frac{\alpha-1}{\alpha}}(q_{d_x}(z, n) + \tau q_{f_x}(z, n)) = e\theta_x(z/\bar{z}), \tag{40}$$

where l_x is the labor of an exporting firm devoted to the production of goods for both domestic and foreign markets.

3.4.1 | Entry and exit

Analogous to the case under autarky, the cutoff productivity z, with all firms exporting and zero exporting fixed cost, is determined by the EC. Using Equations (4), (39), and (40), firm profit in the all exporting with no exporting fixed cost case is

$$\begin{aligned} \pi_x(z/\bar{z}) &= p_x(q_{d_x} + q_{f_x}) - z^{\frac{\alpha-1}{\alpha}}(q_{d_x} + \tau q_{f_x}) - \lambda_d \\ &= e(1 - \theta_x)(z/\bar{z}) - \lambda_d. \end{aligned} \tag{41}$$

Equation (41) defines the operating profits of the firm as a function of two endogenous variables, $\bar{z}$ and e. Consider the case of the variety whose firms just break even in the market

[5]This follows directly from the derivation in Appendix A.1 under autarky with θ_x replacing θ_a.

and denote their productivity with z_x^*. The condition defining this productivity threshold is then given by

$$\pi_x\left(z_x^*/\bar{z}_x\right) = e_x(1-\theta_x)\left(z_x^*/\bar{z}_x\right) - \lambda_d = 0. \tag{42}$$

Note that in equilibrium, I obtain the following:

$$\bar{z}_x = \int_{z_x^*}^{\infty} z\mu(z)\,dz, \tag{43}$$

where

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G\left(z_x^*\right)} & \text{if } z \geqslant z_x^*, \\ 0 & \text{otherwise.} \end{cases} \tag{44}$$

Similar reasoning that produced Equation (20) under autarky results in the following holding under all exporting with no exporting costs:

$$\bar{z}_x = \frac{k}{(k-1)} z_x^*. \tag{45}$$

Using Equation (45) in Equation (42) gives the following expression for the EC under all exporting with no exporting fixed cost:

$$e_x\left(z_x^*\right) = \frac{\lambda_d}{(1-\theta_x)z_x^*/\bar{z}_x} = \frac{k\lambda_d}{(k-1)(1-\theta_x)} \quad (EC_x). \tag{46}$$

Note that under the Pareto distribution, the EC_x condition is independent of the productivity cutoff. I now use the stationarity condition together with the labor market condition to find another relationship between e_x and z_x^*.

Analogous to the case of autarky, preserving a steady-state number of firms in the context of entry and exit of varieties, the following condition must hold $(1-M)(1-G(z_x^*)) = \delta M$ and hence

$$M\left(z_x^*\right) \equiv \frac{1-G\left(z_x^*\right)}{1+\delta-G\left(z_x^*\right)}. \tag{47}$$

Note, as per Lemma 1, Equation (47) describes a decreasing relationship between M and the productivity cutoff z_x^* for $M \in \left(0, \frac{1}{(1+\delta)}\right)$.

In line with the case of autarky, the labor MC condition can be written as

$$e_x\left(z_x^*\right)=\frac{\frac{1}{nM\left(z_x^*\right)}-\lambda_d}{\theta_x+\frac{1-\gamma}{\gamma}}\quad (MC_x). \tag{48}$$

Above, I established $M(z_x^*)$ is a decreasing function of z_x^*. Equation (48) implies an increasing relationship between e_x and z_x^*.

Analogous to the case of autarky, to obtain an equilibrium productivity cutoff, the following is assumed $\lambda_d \geqslant \bar{\lambda}_x$ where

$$\bar{\lambda}_x=\frac{1+\delta}{n\left(1+\frac{k}{k-1}\frac{\theta_x+\frac{1-\gamma}{\gamma}}{1-\theta_x}\right)}. \tag{49}$$

Equation (49) can be derived from the consideration that at the min $z=1,(EC_x)\geqslant(MC_x)$. Equating EC_x and MC_x and manipulating in the same way as with autarky, the productivity cutoff is given by

$$z_x^*=\left(\frac{n\lambda_d\left(1+\frac{k}{k-1}\frac{\theta_x+\frac{1-\gamma}{\gamma}}{1-\theta_x}\right)-1}{\delta}\right)^{1/k}. \tag{50}$$

Similar to Lemma 2 in the case of autarky, here, in the scenario in which all are exporters with zero export fixed cost, I can claim that, under the assumption of the fixed production costs, $\lambda_d \geqslant \bar{\lambda}_x$, for $\tau\in(1,\bar{\tau})$, there exists a unique interior solution of average expenditure per exporting firm e_x and equilibrium productivity cutoff z_x^* for sector h.

It is useful to note the following result:

Lemma 3. $\frac{\theta+\frac{1-\gamma}{\gamma}}{1-\theta}$ *is strictly increasing in* θ *for* $\theta\in(0,1)$, *and hence,* z^* *is strictly increasing with respect to its associated* θ *in that interval.*

In Lemma 3, the component related to the variable markups in the productivity threshold at equilibrium can be used to identify how the change in markups will affect the productivity threshold and the welfare implications for firms.

The following Proposition 1 sets out some comparable properties of the model across the different scenarios considered thus far.

Proposition 1. *(i)* $\alpha\leq\theta_a\leq\theta_{d_x}\leq\theta_x\leq\theta_{f_x}\leq 1$ *and hence firm markups are weakly highest under autarky and weakly lowest for exported goods, and (ii) productivity cutoffs satisfy* $z_x^*\geq z_a^*$ *and hence the selection is greater under all exporting.*

Proof. See Appendix A.4. □

Inspection of Equations (37), (47), and (50) give rise to the following Proposition 2, which establishes some properties of the all-exporting equilibrium with respect to trade liberalization.

Proposition 2. *Under all firms exporting with zero fixed exporting cost, a reduction in* τ *(trade liberalization) (i) reduces the domestic markup for a domestic firm within a variety,* $1/\theta_{d_x}$*, (ii) increases its markup in the foreign market,* $1/\theta_{f_x}$*, (iii) triggers a reduction in the average markup* $1/\theta_x$*, (iv) raises the equilibrium productivity cutoff* z_x^**, and (v) reduces the number of operative varieties* $M(z_x^*)$.

For an exogenous number of firms, n, within a variety for each country in the basic setup, a reduction in τ decreases exporters' markups on domestic sales, increases markups on foreign sales, and decreases the average markup on total sales. In other words, although the export markup increases, it is not sufficiently strong enough to offset the pro-competitive effect on the domestic market since the average markup decreases when considering trade liberalization. Notice that the pro-competitive effect of trade operating through the increased θ_{d_x}, decreases the markup on domestic sales. This results in increased θ_x, which decreases the average markup of exporters due to a reduction in trade cost, τ. In conclusion, when the number of firms is exogenous, there is an overall pro-competitive effect of trade in an economy with oligopolistic competition. The statement is in line with Impullitti et al. (2018), although they consider an endogenous number n within each variety and pick up the different entry strategies compared with my model. Specifically, in their paper, a more sophisticated entry structure, where the entry strategy is focused on a particular product line, is applied to pin down the number of firms in each variety. Therefore, different varieties could produce different markups. Since there exist pro-competitive effects and selection effects of trade liberalization in the global economy, which lead to an increase in the equilibrium productivity cutoff z_x^* and a decrease in the mass of operative varieties, as shown in Equation (47). Here, different from the Melitz model, the number of entrants can respond endogenously to changes in trade costs, which tame or even offset the loss of varieties due to selection. While our model does not allow this, selection can result in the decrease of operative varieties, therefore producing a negative welfare effect as stated in Impullitti and Licandro (2018).

3.5 | Costly trade

Augmenting the all exporting model to include a fixed exporting cost, λ_x, under the assumption of $\lambda_d + \lambda_x \geqslant \bar{\lambda_x}$, where $\bar{\lambda_x}$ is given by Equation (49), the relative productivity cutoff threshold, Equation (50) becomes

$$z_x^{*'} = \left(\frac{n(\lambda_d + \lambda_x)\left(1 + \frac{k}{k-1} \frac{\theta_x + \frac{1-\gamma}{\gamma}}{1-\theta_x}\right) - 1}{\delta} \right)^{1/k}. \tag{51}$$

The following Proposition 3 follows straightforwardly, given Equation (51) is strictly increasing in λ_x.

Proposition 3. $z_x^{*'} > z_x^*$ *for* $\lambda_x > 0$, *and hence the higher productivity cutoff threshold and associated selection effect under all exporting with zero fixed costs relative to autarky, which occurs through the "pro-competitive effect" of trade liberalization is further enhanced and increased in the fixed cost of exporting.*

The derived equilibrium productivity cutoff, the case where all operating firms are exporters with non-zero fixed costs of exporting, increases with the "pro-competitive" effect of trade liberalization and fixed export costs under the constraints of total fixed costs. As stated in Proposition 2, trade liberalization generates a selection effect through a "pro-competitive effect" if there is no fixed cost of exporting. When I consider the fixed export costs, there would be a higher equilibrium productivity cutoff; that is, there are two channels inducing a firm's selection effect, the "pro-competitive effect" and "fixed export cost." Compared with Melitz (2003), there are constant markups, but the fixed cost of exporting can explain the self-selection of producers into the export market. While Impullitti and Licandro (2018) introduce the exporters only assumed there are no fixed exporting costs. However, my model complements them by suggesting that the "procompetition effect" via variable markups and fixed export costs affects the selection effect.

3.6 | Multinational production

This part will address the case when the firm serves the foreign market through horizontal FDI, as in Helpman et al. (2004). It means that a multinational firm based in the domestic market bears an additional fixed cost of λ_m in the foreign market, which includes the costs of planting a subsidiary in the foreign country and duplicating some of the overhead production costs involved in λ_d. I assume $\lambda_m \leqslant \lambda_d$. Similar to the case in which all firms export, domestic multinational firms producing the same variety with productivity level z compete in two separate Cournot games simultaneously in both domestic and foreign markets via FDI, which are referred to by subindices d_m and f_m, respectively, in each sector h. Where q_{d_m} denotes domestic consumption and production of the domestically produced good, q_{f_m} denotes foreign consumption of the domestically produced good via FDI. Therefore, a domestic multinational firm will produce $q_m = q_{d_m} + q_{f_m}$ and consumers will consume $x_m = n(q_{d_m} + q_{f_m})$ for a particular domestic variety since n is the number of firms within a variety domestically from each country playing a symmetric Cournot game in each market, domestic and foreign. $\hat{x}_{d_m}$ and $\hat{x}_{f_m}$ denote the production of competitors in the domestic and foreign markets via FDI, where firms take their competitors' strategies as given. So, in the case in which all are multinational firms, the firm's problem is as follows:

$$\max_{q_{d_m}, q_{f_m}} \pi_m = \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{d_m} + q_{d_m})^{\alpha-1}}_{p_{d_m}} q_{d_m} + \underbrace{\frac{\gamma E}{X^\alpha}(\hat{x}_{f_m} + q_{f_m})^{\alpha-1}}_{p_{f_m}} q_{f_m} - z^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - \lambda_d - \lambda_m. \tag{52}$$

Equation (52) shows the profit function of a domestic multinational firm manufacturing a variety with productivity level z, which means the profits of multinational firms domestically that serve the foreign market via FDI, consist of both domestic and foreign parts, according to the cost function Equation (5) and the inverse demand functions Equation (4). The first-order conditions for domestic sales, q_{d_m}, and foreign sales, q_{f_m}, of multinational firms are, respectively:

$$\frac{\gamma E}{X^{\alpha}}\left((\alpha-1)(\hat{x}_{d_m}+q_{d_m})^{\alpha-2}q_{d_m}+(\hat{x}_{d_m}+q_{d_m})^{\alpha-1}\right)=z^{\frac{\alpha-1}{\alpha}}, \tag{53}$$

$$\frac{\gamma E}{X^{\alpha}}\left((\alpha-1)(\hat{x}_{f_m}+q_{f_m})^{\alpha-2}q_{f_m}+(\hat{x}_{f_m}+q_{f_m})^{\alpha-1}\right)=z^{\frac{\alpha-1}{\alpha}}. \tag{54}$$

Since I focus on the symmetric equilibrium, the total consumption of the domestic multinational firms in a variety via FDI is $x_m = n(q_{d_m} + q_{f_m})$. Given symmetry across countries, the consumption of multinational firms domestically within a variety are the same and equal to x_m, that is, $x_{d_m} = x_{f_m} = x_m$. All multinational firms producing the same variety are identical and "all via FDI."

Using the symmetry implied above and Equations (53) and (54), I can derive

$$q_{d_m} = q_{f_m}. \tag{55}$$

It indicates that domestic and foreign consumption of domestically produced goods via FDI are equal. Domestic multinational firms within a variety have the same production on domestic and foreign sales.

Applying Equation (55) and symmetry to Equations (53) and (54), I have

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}\left(\frac{(\alpha-1)q_{d_m}}{n(q_{d_m}+q_{f_m})}+1\right)=z^{\frac{\alpha-1}{\alpha}}, \tag{56}$$

$$\underbrace{\frac{\gamma E}{X^{\alpha}}x_m^{\alpha-1}}_{p_m}\left(\frac{(\alpha-1)q_{f_m}}{n(q_{d_m}+q_{f_m})}+1\right)=z^{\frac{\alpha-1}{\alpha}}. \tag{57}$$

The equilibrium price of domestic multinational firms within a variety of both domestic and foreign markets then follows straightforwardly:

$$p_m(z,n)=\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{d_m}}=\frac{z^{\frac{\alpha-1}{\alpha}}}{\theta_{f_m}}, \tag{58}$$

where

$$\theta_m=\theta_{d_m}=\theta_{f_m}=\frac{2n+\alpha-1}{2n}. \tag{59}$$

θ_{d_m} and θ_{f_m} represent the inverse of markups of multinational firms within a variety charged in the domestic and foreign markets via FDI. Obviously, since there are no transport/trade costs under FDI, τ does not appear in θ_{d_m} and θ_{f_m}. In other words, variable trade costs have no relationship with markups of domestic multinational firms within a variety since multinational firms domestically within a variety do not suffer iceberg transportation costs but have to pay fixed cost λ_m to establish a plant in the foreign market. Hence, in terms of variable costs, the FDI case is analogous to trade where $\tau = 1$, which means markup of the exporter and multinational firm on domestic sales.

According to Equations (4) and (58), I have that equilibrium consumption of domestic multinational firms for a given variety under "all are multinational firms":

$$x_m(z, n) = \left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}} \theta_{d_m}^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}}. \tag{60}$$

The following expression is a multinational firm's variable production cost[6]:

$$l_m(z, n) - \lambda_d - \lambda_m = z^{\frac{\alpha-1}{\alpha}}(q_{d_m}(z, n) + q_{f_m}(z, n)) = e\theta_{x_m} z/\bar{z}, \tag{61}$$

where l_m is the labor cost of production of goods produced by a multinational firm domestically for both domestic and foreign markets.

3.6.1 | Entry and exit

As before, the cutoff productivity, z, with all firms undertaking FDI, is determined by the EC. Using Equations (58), (60), and (61), firm profit in the variety of "all are multinational firms" with an extra planting fixed cost in the foreign market is

$$\begin{aligned} \pi_m(z/\bar{z}) &= p_m(q_{d_m} + q_{f_m}) - z^{\frac{\alpha-1}{\alpha}}(q_{d_m} + q_{f_m}) - \lambda_d - \lambda_m \\ &= e(1 - \theta_{d_m})(z/\bar{z}) - \lambda_d - \lambda_m. \end{aligned} \tag{62}$$

Equation (62) defines the operating profits of the multinational firm as a function of two endogenous variables, $\bar{z}$ and e. Consider the case of the variety whose multinational firms just break even in the market and denote their productivity with z_m^*. The condition defining this productivity cutoff is then given by

$$\pi_m\left(z_m^*/\bar{z}_m\right) = e_m(1 - \theta_{d_m})\left(z_m^*/\bar{z}_m\right) - \lambda_d - \lambda_m = 0. \tag{63}$$

Note that in equilibrium I obtain that $\bar{z}_m = \int_{z_m^*}^{\infty} z\mu(z)\,dz$, where

$$\mu(z) = \begin{cases} \frac{g(z)}{1 - G\left(z_m^*\right)} & \text{if } z \geqslant z_m^*, \\ 0 & \text{otherwise.} \end{cases}$$

Similar reasoning that produced Equation (20) under autarky results in the following holding under all firms engaging with FDI:

[6]This follows directly from the derivation in Appendix A.1 under autarky with θ_m replacing θ_a.

$$\bar{z}_m = \frac{k}{(k-1)} z_m^*. \tag{64}$$

Using Equation (64) in Equation (63) gives the following expression for the EC under all undertaking FDI:

$$e_m\left(z_m^*\right) = \frac{\lambda_d + \lambda_m}{(1-\theta_{d_m}) z_m^* / \bar{z}_m} = \frac{k(\lambda_d + \lambda_m)}{(k-1)(1-\theta_{d_m})} \quad (EC_m). \tag{65}$$

Note that under the Pareto distribution, the EC_m condition is independent of the productivity cutoff.

I now use the stationarity and labor market conditions to find another relationship between e_m and z_m^*. As before, deriving a steady-state number of firms in the context of restricted entry and exit of varieties requires $(1-M)(1-G(z_m^*)) = \delta M$ to hold, and hence

$$M\left(z_m^*\right) \equiv \frac{1-G\left(z_m^*\right)}{1+\delta-G\left(z_m^*\right)}. \tag{66}$$

Note that Equation (66) describes a decreasing relationship between M and the productivity cutoff z_m^* for $M \in (0, 1/(1+\delta))$.

Following the same manipulations as was the case under autarky, the labor MC condition under all firms engaging with FDI can be written as

$$e_m\left(z_m^*\right) = \frac{\frac{1}{nM\left(z_m^*\right)} - \lambda_d - \lambda_m}{\theta_{d_m} + \frac{1-\gamma}{\gamma}} \quad (MC_m). \tag{67}$$

Since I have established $M(z_m^*)$ is a decreasing function of z_m^*, then Equation (67) implies an increasing relationship between e_m and z_m^*.

The equilibrium productivity threshold is obtained by equating the EC_m and MC_m. Note that the EC_m condition is independent of the productivity cutoff under the Pareto distribution.

In line with the case of "all exporting," to obtain an equilibrium productivity threshold for multinational firms, the following is assumed $\lambda_d + \lambda_m \geqslant \bar{\lambda}_m$, where

$$\bar{\lambda}_m = \frac{1+\delta}{n\left(1 + \frac{k}{k-1} \frac{\theta_m + \frac{1-\gamma}{\gamma}}{1-\theta_m}\right)}. \tag{68}$$

Equation (68) can be derived from the consideration that at the min $z = 1$, $(EC_m) \geqslant (MC_m)$. In the case of the multinational firm, the domestic firm sets up production in the foreign country at a fixed cost $\lambda_m \leqslant \lambda_d$ and enjoys production at the same marginal cost as its foreign rivals in the same variety (there is no iceberg cost for serving the foreign market). The relevant productivity cutoff threshold can then be derived straightforwardly in the same way as under autarky:

$$z_m^* = \left(\frac{n(\lambda_d + \lambda_m)\left(1 + \frac{k}{k-1}\frac{\theta_m + \frac{1-\gamma}{\gamma}}{1-\theta_m}\right) - 1}{\delta} \right)^{1/k}. \tag{69}$$

Similar to the case of "all exporting" and Proposition 1 in the autarky, in the case in which all are multinational firms, I can state that, under the assumption of the fixed production costs, $\lambda_d + \lambda_m \geqslant \bar{\lambda_m}$, there exists a unique interior solution of average expenditure per multinational firm e_m and equilibrium productivity cutoff z_m^* for sector h. The solution is the intersection of MC_m and EC_m with $M(z_m^*)$ determined by Equation (66).

Now, I collect together (from Propositions 1 and 3) and complete comparisons across all scenarios regarding firms' inverse of markups and equilibrium productivity cutoffs.

Proposition 4. *(i) $\alpha \leq \theta_a \leq \theta_{d_x} \leq \theta_x \leq \theta_m \leq \theta_{f_x} \leq 1$ and hence firm markups are weakly highest under autarky and weakly lowest for exported goods, but the average markup is the lowest for multinational firms, and (ii) productivity cutoffs satisfy $z_m^* \geq z_x^{*\prime} \geq z_x^* \geq z_a^*$ and hence the selection is greatest under all multinational firms. If $\lambda_m > 0$ and/or $\tau > 1$ then $z_m^* > z_x^*$ and if $\lambda_m > \lambda_x$ then $z_m^* > z_x^{*\prime}$.*

Proof. See Appendix A.5. □

$\theta_{d_m} = (2n + \alpha - 1)/2n$, it is the largest one between θ_a, θ_x, θ_{d_m}, and leads to the case "all via FDI" has the highest equilibrium productivity threshold. This can also be explained with the calculation in the case "all exporting," since the derivative of z_x^* with respect to θ_x is positive, which means $z_a^* < z_x^* < z_m^*$ because of $\theta_a < \theta_x < \theta_{d_m}$, larger selection effect happens in the economy with all multinational firms. That is to say, in an economy with all multinational firms, the productivity cutoff threshold would be larger than it would be with the economy under all exporters, and these properties can be employed in the welfare analysis in Section 4.

I compare three scenarios as follows: for a given exogenous number of firms within each variety with productivity z, exporters charge lower markups on their domestic sales compared with domestic firms. Moreover, multinational firms possess lower markups on their domestic sales than exporters, as shown above. The reason is that multinational firms have the most competitive pressure, forcing them to reduce their markup on domestic sales, which indicates a stronger pro-competitive effect than the case where all operating firms are exporters. The pro-competitive effect of trade operates through decreased average markups of firms in the case all potential firms are exporters compared with the markup of domestic firms. Proposition 4 is similar to Impullitti et al. (2018) for domestic firms and exporters in the same economic environment, but this study does not include FDI. This statement indicates that multinational firms and trade both exert pro-competitive effects but that it is stronger for multinational firms. This could have important implications for policy-making regarding incentivising firms to be multinational firms to benefit from their higher associated pro-competitive effect.

4 | WELFARE ANALYSIS

This section will identify and decompose the welfare benefits from autarky, trade openness, and multinational firms via FDI and compare the overall gains. In the following analysis, I generalize the notation related to the three scenarios.

4.1 | Decomposition of welfare effects

In line with Impullitti and Licandro (2018), I decompose the welfare effects of trade into three different channels. Notice that love-for-variety exists in the setup model, which means the positive welfare effects through greater selectivity may be offset by the reduction in the mass of potential entrants, M, produced by the same process. Indeed, since Lemma 1 applies across all scenarios, selection always results in a reduction in the equilibrium mass of varieties. $(1 - M)$ is the mass of potential entrants and is bounded above by one. Here, I focus on the equilibrium aggregate welfare effect deriving from the given utility function, and I now decompose aggregate steady-state welfare gains arising through three channels, as follows[7]:

$$U = \underbrace{\gamma \frac{1-\alpha}{\alpha} \ln M\bar{z}}_{\text{Productivity/LFV}} + \underbrace{\gamma \ln \theta enM}_{\text{Consumption}} + \underbrace{(1-\gamma)\ln \frac{(1-\gamma)}{\gamma} enM}_{\text{Homogeneous good}}. \tag{70}$$

There are three different channels: the first two are related to composite goods consumption, while the third is associated with the homogeneous sector. The first term indicates the net effect of the productivity gains from selection effects, which are increased with larger average productivity $\bar{z}_x$, and the welfare losses through Love-For-Value (LFV) caused by fewer varieties, M. The second term is associated with the consumption of the composite goods γE and the oligopolistic distortions in these sectors. I derived $\theta_d = \theta_x/\Phi$, which represents the pro-competitive effect of trade with the cross-hauling effect, as measured by Φ. The third channel measures utility from homogeneous goods consumption.

In line with Impullitti and Licandro (2018), welfare gains from selection operate through $\bar{z}$, M, and e, all of which depend on z^*. Differentiating Equation (70) with respect to z^*, the selection gains can be collected into two sources[8]:

$$\begin{aligned}\text{Selection gains} &= \underbrace{\bar{\gamma}\, \frac{1-\alpha}{\alpha}\left[\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right]}_{\text{Productivity/LFV}} + \underbrace{\left\{\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M}\right\}\frac{\partial M}{\partial z^*}}_{\text{Fixed Cost}} \\ &= \underbrace{\frac{\bar{\gamma}}{z^*}\frac{1-\alpha}{\alpha}\left[1 - \frac{k\delta}{z^{*-k}+\delta}\right]}_{\text{Productivity/LFV}} + \underbrace{\left\{\frac{n\delta k\lambda_d z^{*k-1}}{(1+\delta z^{*k})^2(1-nM\lambda_d)}\right\}}_{\text{Fixed Cost}}.\end{aligned} \tag{71}$$

Proposition 5. *Selection produces (i) unambiguous welfare gains through the fixed cost channel, and (ii) for sufficiently small values of the exogenous death rate δ, the Productivity/LFV trade-off results in positive welfare gains as well:*

[7]See Appendix A.6 for proof.

[8]See Appendix A.7 for a derivation.

$$\delta < \frac{z^{*-k}}{k-1}. \tag{72}$$

Proof. See Appendix A.8. □

It means that when there is a sufficiently small enough value of the exogenous death rate δ, the Productivity/LFV trade-off is positive welfare gains. The second component represents the change in labor allocated to the production of the composite good (excluding the fixed costs). Selection forces some firms to exit the market, reducing the resources needed to cover fixed production costs. These resources are allocated to surviving firms, leading to more production and consumption.

As the statement shown above, I can illustrate that the productivity effect is always positive and independent of δ, but the welfare effect for the reduced mass of variety is negative and increases in δ. However, I can restrict the condition about the value of δ for which the total effect regarding the trade-off between productivity and LFV is positive. This parameter constraint can also be explained with literature assessing the welfare gains from selection, where the probability of firm death could be very small, like, Arkolakis et al. (2012). In addition, they also suggest a classical method to measure aggregate welfare gains from trade liberalization, which examines the welfare effect when two statistics are pinned down: the share of expenditure on domestic goods and the elasticity of imports with respect to variable trade costs. However, unlike their work, I calculate the welfare effect by accessing the utility function with both homogeneous sectors and heterogeneous industries.

4.2 | Welfare comparison

As represented by the welfare analysis regarding the scenario where all operating firms are exporters, it is similar to the case where all are domestic firms and all potential producers are multinational firms. Combining the previous equilibrium productivity cutoff threshold calculated in Section 4.1 and comparing those three scenarios, in the condition of $\delta < \frac{z_m^{*-k}}{k-1}$, I have that

Proposition 6. *Under the certain condition for exogenous death rate δ, welfare gains from the case in which all multinational firms are larger than the scenario where all are exporters, and all are non-exporters in the economy.*

Since $z_a^* < z_x^* < z_m^*$ found in Proposition 4 and I have proved that selection gains always increase with z^* in "Fixed cost" Channel and rise with z^* as well in the Productivity/LFV channel when the condition $\delta < z_m^{*-k}/(k-1)$ holds, as shown above, like, Impullitti and Licandro (2018). That is to say, I extend Impullitti and Licandro (2018) to multinational firms, under the certain condition for exogenous death rate δ, welfare gains from all multinational firms are the largest, compared with the scenario where all are exporters and the environment in which all are domestic firms. The fundamental reason for the result is the largest "pro-competitive effect" via lower markups and the toughest selection if all potential firms are MP.

5 | NUMERICAL SIMULATION ANALYSIS

This section will investigate the model's main properties through numerical simulations. Our main aim is to explore how the markups of non-exporters, exporters, and multinational firms vary in response to the variation of iceberg transportation trade costs, where iceberg-type trade costs are reduced from the prohibitive level to the lowest theoretical value 1 with no transportation costs. In addition, I also explain how the selection effect works through numerical analysis based on the theoretical framework and the welfare effect for different firms, considering the different number of firms in each product line. Notice that I compare welfare effects through three global economies of each scenario similar in all characteristics except for the iceberg-type cost and fixed costs for different firms.

Through our theoretical model, I have parameters $\{\alpha, \tau, \lambda_d, \lambda_x, \lambda_m, k, \gamma, \delta, n\}$ needed to match with empirical research, and these parameters have been taken from the literature. The value for shape parameter k of entry distribution, which follows Pareto, is pinned down as 1.14, which is in the range of 1.06 estimated by Luttmer (2007) for US firms and 1.39 estimated by Head et al. (2014) with French data exporting to Belgium. I set $\alpha = 0.32$ from Impullitti and Licandro (2018), which implies an elasticity of substitution 1.48. They indicate the elasticity of substitution sits in the median microelasticity 3.1 (Feenstra et al., 2018) and macroelasticity close to one. The setting of death rate δ is 0.09 to match the average enterprise annual death rate for manufactures in 1998–2004 using Census 2004 data. I set $\gamma = 2/3$ to represent the aggregated share of composited goods to be in line with Rauch (1999), which finds that the differentiated goods account for 64.6% and 67.1% of total US in manufacturing with the chosen aggregation scheme. I pin down the value of fixed production cost λ_d as 0.01 like Impullitti and Licandro (2018) and $\lambda_x = 0.0022$, which is the value in Impullitti and Licandro (2018). The $\lambda_m = 0.005$ is set according to our assumption $\lambda_m > \lambda_x$ in the theory. The number of firms $n = 1.2$, close to the setting of numerical analysis of Impullitti et al. (2018, 2022), which consider the similar economic environment with variable markups stemming from the oligopolistic competition, firm heterogeneity, and different FE conditions. Finally, I set the variable transportation cost $\tau \in (1, 1.2)$ to represent the lack of barriers and the prohibitive trade cost, like, Impullitti and Licandro (2018). In addition, Anderson and Van Wincoop (2004) estimate that 170% of the tax equivalent of trade cost is accounted for by 21% of transportation costs, which contains both directly measured freight costs and tax equivalent of the time value of goods in transit with US data.

Figure 1 shows that trade liberalization has a pro-competitive effect on exporters only, while the markups of non-exporters and multinational firms are unaffected by trade liberalization. It can be identified through Equations (12), (35), (37), and (59), where the inverse of the markups for non-exporters θ_a and multinational firms θ_m only relate to n, the number of firms per variety and α associated with elasticity of substitution. In contrast, the inverse of the markups for exporters θ_{d_x} and θ_{f_x} in the domestic and foreign markets are also relative to variable iceberg-type costs τ, according to Equations (35) and (37), its properties as shown in Proposition 1, the panel 1 shows. Since the number of firms n per product line is exogenously fixed with $n = 1.2$ no matter with the τ. We know that $1/\theta_m < 1/\theta_x < 1/\theta_a$ from Proposition 4, which implies that multinational firms via horizontal FDI have the lowest markups compared with exporters and domestic firms, shown as panels 2–4. Notice that although exporting firms increase their markups on foreign sales, it is not enough to offset the decreased makeups in their domestic sales. Therefore, average markups of exporters decrease when iceberg-type trade costs decline, like, Impullitti and Licandro (2018). Panel 2 in the figure indicates that trade liberalization

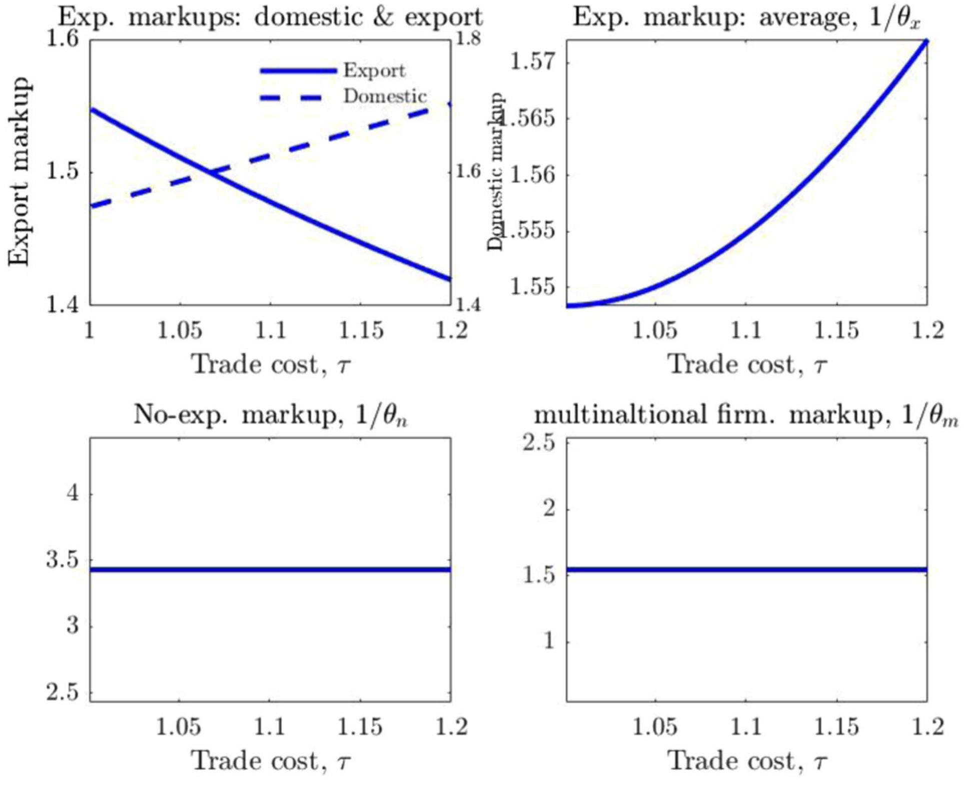

FIGURE 1 Trade liberalization. [Color figure can be viewed at wileyonlinelibrary.com]

produces the pro-competitive effect on exporters from a qualitative perspective, which is in line with Proposition 1.

Figure 2 illustrates the changes in selection effect and Productivity/LFV of utility in response to the different number of firms in each variety. It shows that the selection effect happens by comparing productivity thresholds for non-exporters, exporters, and multinational firms due to firm heterogeneity within firm number 7.5. From Figure 2a, we explain that the productivity threshold z_m^* with a red line for multinational firms is the highest compared with the productivity threshold for exporters with benchmark iceberg transportation cost 1.06 with a green line and prohibitive trade costs 1.12 with the blue line and the lowest productivity threshold for non-exporters with black cord. Eaton et al. (2011) document the efficiency of firm heterogeneity when they examine the French manufacturing firms' trade data. In addition, Yeaple (2009) also claims the significance of heterogeneous multinational firms using US firm-level data and finds that country characteristics will affect the structure of multinational firms' activity. Figure 2b shows that Productivity/LFV of utility is the largest for multinational firms, then exporters with lower iceberg trade costs, followed by the higher iceberg transportation costs, the lowest value is the non-exporters, within firm number 3, which is consistent with Proposition 6. That is, for heterogeneous firms, within firm number 3, the multinational firms have the most significant Productivity/LFV of utility compared with exporters and non-exporters through the mechanism of "pro-competitive effect" and "selection effect."

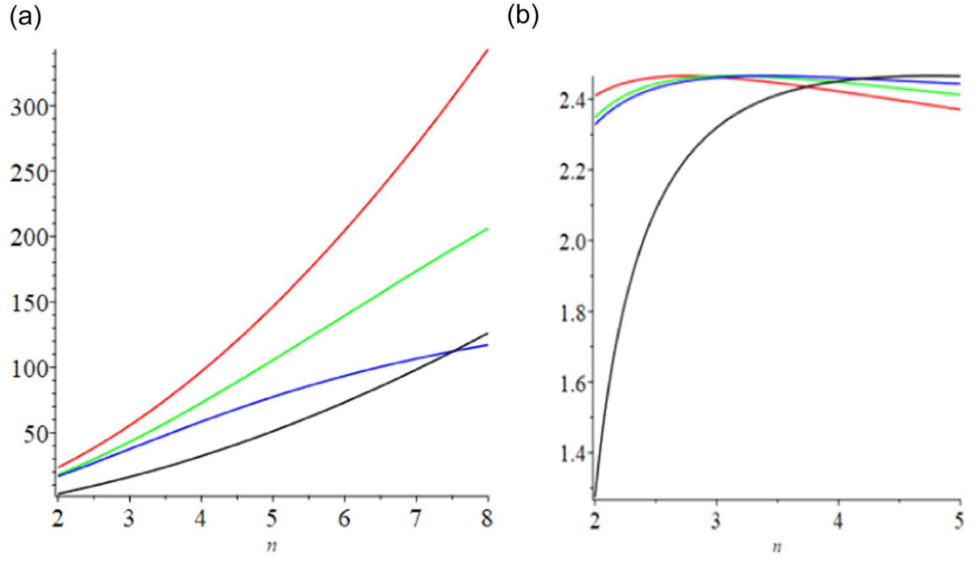

FIGURE 2 Selection effect and welfare. z_m (red), z_x ($\tau = 1.06$) (green), z_x ($\tau = 1.12$) (blue), and z_a (black). (a) Productivity and (b) Productivity/LFV part of utility. [Color figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSION

In this paper, I have explored welfare gains from trade and FDI like Ethier (1986) in an economy with firm heterogeneity and variable markups under oligopolistic competition. I have shown that trade liberalization increases product market competition by reducing markups, like, Impullitti and Licandro (2018). The pro-competitive effect operates through decreased average markup of firms in a scenario where all operating firms are exporters compared with the markups of domestic firms in the environment where all are domestic firms. Specifically, trade liberalization decreases an exporter's markup in domestic sales while increasing the markup in a foreign market, but it decreases the average markup on total sales. Therefore, although the export markup increases, that is not sufficient to offset the pro-competitive effect on the domestic market since the average markup decreases when there is a reduction in iceberg transportation costs. It is in line with the findings of Impullitti and Licandro (2018).

The most important contribution of my paper is that I incorporate MP via FDI Helpman et al. (2004) as an alternative way for firms to serve a foreign market. To explore their effect on welfare, my designed model features Cournot competition in each heterogeneous variety. I consider horizontal FDI and find that MP generates the highest competitive pressure, forcing a reduction in markup, indicating a more substantial pro-competitive effect than trade openness. The pro-competitive channel of FDI can also be found in Ahn (2014), which built a theoretical model with firm heterogeneity and without oligopolistic competition. The economy in the case of "all multinational production" would collapse to an economy of "all exporters" when I consider there are no variable trade costs and the same fixed exporting costs as multinational firms' fixed costs for setting up a plant in a foreign market.

In my model, with firm heterogeneity and variable markups stemming from oligopolistic competition, I conclude that exporters and multinational firms both generate a pro-competitive effect through variable markups from trade openness. Multinational firms produce the most significant welfare due to the lowest markups and toughest selection. However, all three separate scenarios are compared because of the assumption of an exogenous number of firms in each variety and no interaction between domestic firms, exporters, and multinational firms in an economic environment. This is a limitation of the paper and generates a challenge for further research, which would consider the FE condition in the model (e.g., Impullitti et al., 2018, 2022). In future research, I would assume an endogenous mass of product lines and consider that domestic firms, exporters, and multinational firms can coexist in the same industry. I also wish to introduce R&D into the model to resolve the welfare effects of trade and FDI.

ACKNOWLEDGMENTS

I would like to thank the Economic and Social Research Council (ESRC) of UK Research and Innovation (UKRI) for its financial support. In addition, I have benefited from valuable comments by Dr. Antonio Navas, Dr. Jolian McHardy, Prof. Sai Ding, and Dr. Wei Jin. I am especially thankful to Prof. Omar Licandro and Prof. Sarah Brown for their insightful feedback.

CONFLICT OF INTEREST STATEMENT

The author declares that there is no conflict of interest.

REFERENCES

Ahn, J. (2014) A welfare analysis of unilateral trade and FDI liberalization. *Canadian Journal of Economics/Revue canadienne d'économique*, 47(3), 758–774.

Anderson, J.E. & Van Wincoop, E. (2004) Trade costs. *Journal of Economic Literature*, 42(3), 691–751.

Arkolakis, C., Costinot, A., Donaldson, D. & Rodríguez-Clare, A. (2019) The elusive pro-competitive effects of trade. *The Review of Economic Studies*, 86(1), 46–80.

Arkolakis, C., Costinot, A. & Rodriguez-Clare, A. (2012) New trade models, same old gains? *American Economic Review*, 102(1), 94–130.

Atkeson, A. & Burstein, A.T. (2010) Innovation, firm dynamics, and international trade. *Journal of Political Economy*, 118(3), 433–484.

Aw, B., Chung, S. & Roberts, M. (2000) Productivity and turnover in the export market: micro-level evidence from the Republic of Korea and Taiwan (China). *World Bank Economic Review*, 14(1), 65–90.

Bernard, A., Eaton, J., Jensen, J. & Kortum, S. (2003) Plants and productivity in international trade. *American Economic Review*, 93(4), 1268–1290.

Bernard, A., Jensen, J. & Lawrence, R. (1995) Exporters, jobs, and wages in U.S. manufacturing: 1976–1987. *Brookings Papers on Economic Activity: Microeconomics*, 1995(1995), 67–119.

Bernard, A., Jensen, J., Redding, S. & Schott, P. (2007) Firms in international trade. *Journal of Economic Perspectives*, 21(3), 105–130.

Brander, J. & Krugman, P. (1983) A 'reciprocal dumping' model of international trade. *Journal of International Economics*, 15(3–4), 313–321.

Chaney, T. (2008) Distorted gravity: the intensive and extensive margins of international trade. *American Economic Review*, 98(4), 1707–1721.

Clerides, S., Lach, S. & Tybout, J. (1998) Is learning by exporting important? Micro-dynamic evidence from Colombia, Mexico, and Morocco. *The Quarterly Journal of Economics*, 113(3), 903–947.

Dixit, A. & Stiglitz, J. (1977) Monopolistic competition and optimum product diversity. *The American Economic Review*, 67(3), 297–308.

Eaton, J. & Kortum, S. (2002) Technology, geography, and trade. *Econometrica*, 70(5), 1741–1779.

Eaton, J., Kortum, S. & Kramarz, F. (2011) An anatomy of international trade: Evidence from French firms. *Econometrica*, 79(5), 1453–1498.

Eaton, J., Kortum, S.S. & Sotelo, S. (2012) *International trade: Linking micro and macro*. National Bureau of Economic Research. Technical Report.

Eckel, C. & Neary, J.P. (2010) Multi-product firms and flexible manufacturing in the global economy. *The Review of Economic Studies*, 77(1), 188–217.

Edmond, C., Midrigan, V. & Xu, D.Y. (2015) Competition, markups, and the gains from international trade. *American Economic Review*, 105(10), 3183–3221.

Ethier, W.J. (1986) The multinational firm. *The Quarterly Journal of Economics*, 101(4), 805–833.

Feenstra, R.C., Luck, P., Obstfeld, M. & Russ, K.N. (2018) In search of the Armington elasticity. *Review of Economics and Statistics*, 100(1), 135–150.

Garetto, S. (2013) Input sourcing and multinational production. *American Economic Journal: Macroeconomics*, 5(2), 118–151.

Head, K., Mayer, T. & Thoenig, M. (2014) Welfare and trade without Pareto. *American Economic Review*, 104(5), 310–316.

Helpman, E., Melitz, M. & Yeaple, S. (2004) Export versus FDI with heterogeneous firms. *American Economic Review*, 94(1), 300–316.

Hopenhayn, H. (1992) Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 60(5), 1127–1150.

Impullitti, G. & Licandro, O. (2018) Trade, firm selection and innovation: the competition channel. *Economic Journal*, 128(608), 189–229.

Impullitti, G., Licandro, O. & Rendahl, P. (2018) *Technology, market structure, and the gains from trade*. CEPR Working Paper DP12375.

Impullitti, G., Licandro, O. & Rendahl, P. (2022) Technology, market structure and the gains from trade. *Journal of International Economics*, 135, 103557.

Krugman, P. (1980) Scale economies, product differentiation and the pattern of trade. *American Economic Review*, 70(5), 950–959.

Li, Y. & Miao, Z. (2020) *Globalization, import penetration and market power*. Mimeo, McGill University. Technical Report.

Lucas, R.E. (1978) On the size distribution of business firms. *The Bell Journal of Economics*, 9(2), 508–523.

Luttmer, E.G. (2007) Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics*, 122(3), 1103–1144.

Markusen, J.R. & Venables, A.J. (2000) The theory of endowment, intra-industry and multi-national trade. *Journal of International Economics*, 52(2), 209–234.

Melitz, M. (2003) The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695–1725.

Melitz, M. & Ottaviano, G. (2008) Market size, trade, and productivity. *The Review of Economic Studies*, 75(1), 295–316.

Neary, J. (2003) Globalization and market structure. *Journal of the European Economic Association*, 1(2–3), 245–271.

Neary, J. (2010) Two and a half theories of trade. *World Economy*, 33(1), 1–19.

Ramondo, N. (2014) A quantitative approach to multinational production. *Journal of International Economics*, 93(1), 108–122.

Ramondo, N. & Rodríguez-Clare, A. (2010) Growth, size, and openness: a quantitative approach. *American Economic Review*, 100(2), 62–67.

Ramondo, N. & Rodríguez-Clare, A. (2013) Trade, multinational production, and the gains from openness. *Journal of Political Economy*, 121(2), 273–322.

Rauch, J.E. (1999) Networks versus markets in international trade. *Journal of International Economics*, 48(1), 7–35.

Reis, A. (2001) On the welfare effects of foreign investment. *Journal of International Economics*, 54(2), 411–427.

Yang, Y. (2015) Domestic R&D, foreign direct investment and welfare. *Review of Development Economics*, 19(4), 908–924.

Yeaple, S.R. (2009) Firm heterogeneity and the structure of US multinational activity. *Journal of International Economics*, 78(2), 206–215.

How to cite this article: Wang, B. (2024) Towards a welfare model of trade and multinational firms with oligopolistic competition. *International Journal of Economic Theory*, 1–36. https://doi.org/10.1111/ijet.12393

APPENDIX A

A.1 | Equation (14)

I seek to obtain an expression for X in the case that all are domestic firms. Using Equations (2) and (11) I have

$$X = \left[\int_0^M \left(z^{\frac{1}{\alpha}}\left[\frac{\gamma E}{X^\alpha}\theta_a\right]^{\frac{1}{1-\alpha}}\right)^\alpha dv\right]^{\frac{1}{\alpha}}.$$

Rearranging and using Equation (13a):

$$X^\alpha = \left(\frac{\gamma E}{X^\alpha}\theta_a\right)^{\frac{\alpha}{1-\alpha}}\int_0^M z_v dv = \left(\frac{\gamma E}{X^\alpha}\theta_a\right)^{\frac{\alpha}{1-\alpha}} M\bar{z}$$

Rearranging the above terms, I focus on the case that all are domestic firms and identify their aggregate composite goods in the differentiated sector:

$$X^{\frac{\alpha}{1-\alpha}} = (\gamma E\theta_a)^{\frac{\alpha}{1-\alpha}} M\bar{z}. \tag{A1}$$

Then, I derive a domestic firm's variable production cost, based on Equation (5) and using Equations (11), (13b), and (A1), and symmetry $x_a = q_a + \hat{x}_a = nq_a$:

$$\begin{aligned}
l_a(z, n) - \lambda_d &= z^{\frac{\alpha-1}{\alpha}} q_a(z, n)\\
&= z^{\frac{\alpha-1}{\alpha}}\frac{q_a}{x_a}x_a = z^{\frac{\alpha-1}{\alpha}}\frac{q_a}{x_a}\left(\frac{\gamma E}{X^\alpha}\right)^{\frac{1}{1-\alpha}}\theta_a^{\frac{1}{1-\alpha}} z^{\frac{1}{\alpha}}\\
&= \frac{q_a}{x_a}\frac{(\gamma E)^{\frac{1}{1-\alpha}}}{(\gamma E\theta_a)^{\frac{\alpha}{1-\alpha}}}\theta_a^{\frac{1}{1-\alpha}}\frac{z}{M\bar{z}}\\
&= \frac{q_a}{nq_a}\gamma E\theta_a\frac{z}{M\bar{z}}\\
&= \frac{\gamma E}{nM}\theta_a\frac{z}{\bar{z}}\\
&= e\theta_a\frac{z}{\bar{z}}.
\end{aligned}$$

A.2 | Equation (20)

From the Pareto distribution $G(z)$, I have $1 - G(z) = \left(\frac{1}{z}\right)^k$, $g(z) = kz^{-k-1}$, $zg(z) = kz^{-k}$. Since I have defined $\bar{z}_a(z_a^*) = \int_{z_a^*}^{\infty} z\mu(z)\,dz$, combined with the definition of $\mu(z)$ and $\bar{z}_a$, then

$$\bar{z}_a\left(z_a^*\right) = \int_{z_a{}^*}^{\infty} z\mu(z)\,dz = \int_{z_a{}^*}^{\infty} \frac{g(z)}{1 - G(z_a{}^*)} z\,dz$$
$$= \int_{z_a{}^*}^{\infty} \frac{kz^{-k-1}}{z_a{}^{*-k}} z\,dz = \frac{k}{z_a{}^{*-k}} \int_{z_a{}^*}^{\infty} z^{-k}\,dz$$
$$= \frac{k}{z_a{}^{*-k}} \frac{1}{1-k} z^{1-k}|_{z_a{}^*}^{\infty}$$
$$= \frac{k}{k-1} z_a{}^*.$$

A.3 | Lemma 2

With $e_a'(z_a^*) = 0$ from the EC_a condition, Equation (21), $e_a'(z_a^*) > 0$ under MC_a in Equation (26), and the condition on $\bar{\lambda}_a$ ensuring $EC_a \geq MC_a$ at the minimum $z = 1$, completes the proof.

A.4 | Proposition 1

(i) From Equation (7), $\theta_a \geq \alpha$ follows directly from inspection, with equality at the limiting case of $n = 1$. Let

$$\Omega^{dxa} = \frac{\theta_{d_x}}{\theta_a} = \frac{2n + \alpha - 1}{(n + \alpha - 1)(1 + \tau)},$$

where $\Omega_\tau^{dxa} = \frac{\alpha - 1}{[n + \alpha - 1]^2(1 + \tau)} < 0$. Given the relevant range of τ, with $\Omega^{dxa}(\tau = 1) = \frac{2n + \alpha - 1}{2(n + \alpha - 1)} > 1$ and $\Omega^{dxa}(\tau = \bar{\tau}) = 1$, establishing $\theta_a \leq \theta_{d_x}$. Given $\theta_{f_x} = \tau\theta_{d_x} \geq \theta_{d_x}$, by definition of the average $\theta_{d_x} \leq \theta_x \leq \theta_{f_x}$. Finally, given $\frac{\partial\theta_{f_x}}{\partial\tau} = \frac{2n + \alpha - 1}{n(1 + \tau)^2} > 0$ and $\theta_{f_x}(\tau = \bar{\tau}) = 1$, completes the proof.

(ii) This follows directly from Lemma 3 and Proposition 1(i).

A.5 | Proposition 4

(i) Let $\Omega^{mx} = \frac{\theta_m}{\theta_x} = \frac{(1 + \beta)(1 + \tau)}{2(1 + \tau\beta)}$. Noting, $\Omega^{mx}(\tau = 1) = 1$ and $\Omega^{mx}(\tau = \bar{\tau}) = \frac{(n + \alpha - 1 + 2\beta n + n) - \beta(1 - \alpha)}{(n + \alpha - 1 + 2\beta n + n) - (1 - \alpha)} \geq 1$ given $\beta \in [0, 1]$ establishes $\theta_m \geq \theta_x$. Let $\Omega^{mdx} = \frac{\theta_m}{\theta_{d_x}} = \frac{(1 + \tau)}{2}$. Noting, $\Omega^{mdx}(\tau = 1) = 1$ and $\Omega^{mdx}(\tau = \bar{\tau}) > 1$ establishes $\theta_m \geq \theta_{d_x}$. Finally, $\Omega^{mfx} = \frac{\theta_m}{\theta_{f_x}} = \frac{(1 + \tau)}{2\tau}$. Noting, $\Omega^{mfx}(\tau = 1) = 1$ and $\Omega^{mfx}(\tau = \bar{\tau}) < 0$ establishes $\theta_m \leq \theta_{f_x}$.

(ii) This follows from Lemma 3 and the recognition that the fixed cost under multinational firms further enhances z_m^* such that under $\lambda_m > 0$ (in addition to $\tau > 1$) guarantees $z_m^* > z_x^*$ and $\lambda_m > \lambda_x$ guarantees $z_m^* > z_x^{*'}$.

A.6 | Equation (70)

Applying Equation (A1) to the case where the markup is that in the domestic market under any given regime, I have

$$X = (M\bar{z})^{\frac{1-\alpha}{\alpha}} \gamma E \theta_d.$$

Substituting into the aggregate utility function, Equation (1), for a given country (domestic), and using Equation (13b) and $Y = (1-\gamma)E$, I have

$$\begin{aligned} U &= \gamma \ln X + (1-\gamma)\ln Y \\ &= \gamma \frac{1-\alpha}{\alpha} \ln M\bar{z} + \gamma \ln \theta_d \gamma E + (1-\gamma)\ln(1-\gamma)E \\ &= \underbrace{\gamma \frac{1-\alpha}{\alpha} \ln M\bar{z}}_{\text{Productivity/LFV}} + \underbrace{\gamma \ln \theta_d enM}_{\text{Consumption}} + \underbrace{(1-\gamma)\ln \frac{(1-\gamma)}{\gamma} enM}_{\text{Homogeneous good}}. \end{aligned} \tag{A2}$$

A.7 | Equation (71)

This is derived using $\bar{z}$ from Equation (20), M from Equation (23), and e from Equation (26). Starting from $M = (1+\delta z^{*k})^{-1}$ and $e = \frac{1+\delta z^{*k} - n\lambda_d}{nT}$, where $T = \theta_d + \frac{1-\gamma}{\gamma}$,

$$\frac{\partial e}{\partial z^*}\frac{1}{e} = \frac{k\delta z^{*k-1}}{nT}\frac{nT}{1+\delta z^{*k} - n\lambda} = \frac{k\delta z^{*k-1}}{1+\delta z^{*k} - n\lambda} \quad \text{and} \quad \frac{\partial M}{\partial z^*}\frac{1}{M} = \frac{-\delta k z^{*k-1}(1+\delta z^{*k})^{-2}}{(1+\delta z^{*k})^{-1}} = \frac{-\delta k z^{*k-1}}{(1+\delta z^{*k})}.$$

Hence, $\frac{\partial e}{\partial z^*}\frac{1}{e} + \frac{\partial M}{\partial z^*}\frac{1}{M} = \frac{k\delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{*k} - n\lambda)}[1+\delta z^{*k} - 1 - \delta z^{*k} + n\lambda] = \frac{kn\lambda\delta z^{*k-1}}{(1+\delta z^{*k})(1+\delta z^{k*} - n\lambda)}$.

Finally, noting that given $M = (1+\delta z^{*k})^{-1}$, the denominator can be written $(1+\delta z^{*k})^2(1-nM\lambda_d)$, completing the proof.

A.8 | Proposition 5

(i) This follows from the inspection of Equation (26), where, given the numerator is positive, for $e > 0$, requires $1 - \lambda_d nM > 0$, which ensures $\{\cdot\} > 0$, completing the proof.

(ii) The inequality in Equation (72) follows directly from setting the term $[\cdot] > 0$ in Equation (71) and rearranging for δ, completing the proof.