

Modelling bus bunching along a common line corridor considering passenger arrival time and transfer choice under stochastic travel time[☆]

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ABSTRACT

This study examines bus bunching along a common-line corridor, considering crucial factors underexplored in existing literature, such as stochastic travel times, passenger arrival patterns, and passenger transfer behaviours. We first develop a bus motion model that captures the interaction between bus trajectories and passenger movement. Then we formulate a reliability-based passenger arrival time choice and a transfer choice model to characterise passengers' behaviours. Afterwards, the bus motion model and the passenger choice models are integrated, and a Method of Successive Averages type iterative algorithm is developed to obtain stable passenger arrival patterns and transfer choices. Numerical experiments are carried out on a hypothetical network followed by a case with real-world data. Our findings demonstrate that a high transfer demand could amplify the propagation of bus bunching across lines along the common-line corridor. Meanwhile, a 50% increase in transfer demand leads to a 24%–30% rise in headway fluctuation. Furthermore, our results suggest that non-uniform passenger accumulation patterns can restore headway regularity as a result of coordinated passenger movement and bus motions, thus alleviating the persistent deterioration in bus bunching.

1. Introduction

Providing high-quality public transport services is of great importance to achieve sustainable urban mobility. However, as the most representative public transport mode, bus has been criticised for its inefficiency and unreliability, particularly in many metropolitan areas plagued by the bus bunching problem. Bus bunching refers to the phenomenon in which consecutive buses of the same bus line run with headways significantly deviating from predesigned headways. The adverse effects of bus bunching are of concern to both passengers and transit agencies (Newell and Potts, 1964; Osuna et al., 1972; Hollander and Liu, 2008; Verbich et al., 2016). From the passenger's perspective, bus bunching can result in prolonged and unpredicted waiting times at stations, disrupting their travel plans and prompting them to reconsider their route choices. From the operator's perspective, bus bunching causes schedule disruption or headway irregularity, which compels agencies to bear higher costs to maintain service levels or risk losing their bus modal share.

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Table 1
Relevant literature on modelling bus propagation and passenger behaviours.

Literature	Configuration	Bus propagation modelling			Passenger behaviour modelling		
		Bus dwelling time		Bus running time	Arrival	Transfer	Choice behaviours considered
		Boarding	Alighting				
Newell and Potts (1964)	Single line	Yes	No	Deterministic	Uniform	No	No
Daganzo (2009)	Single line	Yes	No	Deterministic	Uniform	No	No
Toledo et al. (2010)	Single line	Yes	Yes	Stochastic	Possion	No	No
Hernández et al. (2015)	Two lines	Yes	Yes	Deterministic	Uniform	No	No
Argote-Cabanero et al. (2015)	Multiple lines	Yes	No	Stochastic	Possion	No	No
Fonzone et al. (2015)	Single lines	Yes	No	Deterministic	Reliability-based	No	No
Nesheli et al. (2016)	Multiple lines	Yes	Yes	Deterministic	Uniform	Yes	No
Schmöcker et al. (2016)	Multiple lines	Yes	No	Deterministic	Uniform	No	Queue-swapping boarding choice
Wu et al. (2017)	Single line	Yes	Yes	Stochastic	Uniform	No	Queue-swapping boarding choice
Sun and Schmöcker (2018)	Single line	Yes	No	Deterministic	Uniform	No	Queue-swapping boarding choice
Gavriilidou and Cats (2019)	Multiple lines	Yes	Yes	Stochastic	Uniform	Yes	No
Seman et al. (2020)	Two lines	Yes	Yes	Deterministic	Uniform	No	No
Laskaris et al. (2019)	Two lines	Yes	Yes	Stochastic	Uniform	No	No
Laskaris et al. (2021)	Two lines	Yes	Yes	Stochastic	Uniform	No	No
Wang et al. (2021)	Single line	Yes	Yes	Stochastic	Possion	No	Logit boarding choice
Estrada et al. (2021)	Two lines	Yes	Yes	Stochastic	Uniform	No	No
Zhang et al. (2022)	Multiple lines	Yes	Yes	Stochastic	Uniform	Yes	No
Chen et al. (2022)	Single line	Yes	No	Stochastic	Uniform	No	No
This paper	Multiple lines	Yes	Yes	Stochastic	Reliability-based	Yes	Logit transfer choice

Resolving the bus bunching problem requires characterising bus motion affected by uncertain traffic conditions and passenger behaviours. Numerous bus motion models have been proposed in existing literature (Hans et al., 2014; Fonzone et al., 2015; Schmöcker et al., 2016; Laskaris et al., 2019, 2021). However, most of them focus on simplified scenarios. Crucial factors, such as common lines, non-uniform passenger arrivals, and passenger transfer behaviours, have been overlooked or not simultaneously considered, leaving their joint impacts on bus bunching underexplored. This study is thus motivated to address the above-identified research gap by developing a methodology composed of a bus motion model and passenger choice models, allowing us to consider the aforementioned crucial factors simultaneously.

The remainder of this paper is organised as follows. Section 2 reviews the literature and points out the research gap. Section 3 describes the problem and lists the main assumptions and notations for the model development. Section 4 formulates the bus motion model and develops the corresponding algorithm for obtaining bus trajectories. Section 5 devises the passenger arrival time and transfer choice models and integrates them with the bus motion model. The resultant model is proposed to be solved by the Method of Successive Averages (MSA). Section 6 conducts extensive numerical experiments to gain insights into the bus bunching problem under different scenarios. Finally, Section 7 concludes this study and sheds light on future research directions.

2. Literature review

This section starts with a comprehensive literature review concerning bus bunching modelling, emphasising the models considering the common-line scenario. Then, it revisits the studies modelling passenger behaviours within the context of bus bunching. Finally, it summarises the literature in Table 1 to identify the research gaps and elaborate on our contributions.

2.1. Bus bunching modelling

The seminal work by Newell and Potts (1964) initially introduced a model to investigate the occurrence and evolution of bus bunching along a single line. Their model captures the dynamics of bus movement by concise equations, relying on several fundamental assumptions, such as the bus dwell time is determined by the headway, and the bus running time between stations is constant. The intrinsic instability of the bus headway along a single line was mathematically proved for the first time. Following Newell and Potts (1964), numerous studies were dedicated to further characterising the dynamics of bus bunching and analysing its adverse effects (Osuna et al., 1972; Hickman, 2001; Daganzo, 2009; Schmöcker et al., 2016; Chen et al., 2022). Although the single-line bus bunching models are theoretically elegant, their capability to replicate realistic bunching phenomena is limited. For example, Chen et al. (2022) found that bus bunching occurs at a lower probability in empirical data than expected. They attributed it to the counteraction effect on links against headway deviations at stations. However, their study does not pay attention to the interaction between multiple bus lines.

It is prevalent that urban bus corridors with high passenger demand are served by more than one bus line, referred to as common lines. Extending single-line bus bunching models to common line models has been an emerging trend in recent years. For example, Hernández et al. (2015) extended the optimal bus holding control method (Eberlein et al., 2001; Sun and Hickman, 2005; Delgado et al., 2012) to a configuration with two common lines. They demonstrated that by considering the interaction between two lines, centralised control can significantly reduce waiting costs compared to scenarios involving independent or no control for each line.

Meanwhile, [Argote-Cabanero et al. \(2015\)](#) introduced common lines in the application of adaptive bus control. Additionally, common lines have also been considered in the studies to enhance timetable adherence (e.g. [Argote-Cabanero et al., 2015](#); [Estrada et al., 2021](#)).

Most of the mentioned extensions expanded the single-line bus bunching model straightforwardly without considering the interaction between multiple bus lines along a common line corridor. Although different bus lines are expected to run separately along the corridor, they can affect each other as a result of the changes in passengers' choices. This is because passengers' choices, including which line to board, when to arrive, and whether and where to transfer, lead to different numbers of boarding and alighting passengers at each stop for each line. These numbers determine a bus's dwell time, which impacts the bus motion. Therefore, considering that delays and bus bunching can affect passengers' choices, the intermediate to spread the impact, it is necessary to take these into account when modelling bus bunching along a common line corridor. To the best of our knowledge, this has only been done in a few studies. For example, [Schmöcker et al. \(2016\)](#) extended the work of [Newell and Potts \(1964\)](#) and examined the impact of common stations on bus bunching when bus overtaking is allowed. [Laskaris et al. \(2019, 2021\)](#) studied holding control measures in two circumstances: (a) two bus lines merge into a common corridor and (b) diverge from a common corridor into two branches. Recently, [Wang and Sun \(2023\)](#) introduced a multi-agent reinforcement learning framework to control multiple bus lines traversing a shared corridor.

2.2. Passenger behavioural modelling in bus bunching

Conventionally, the models that determine passengers' routing behaviour are commonly known as transit assignment models. They can be classified into link-based ([Wu et al., 1994](#); [Kurauchi et al., 2003](#); [Cepeda et al., 2006](#); [Hamdouch and Lawphongpanich, 2008](#)), path-based ([Spiess and Florian, 1989](#); [de Cea and Fernández, 1993](#); [Lam et al., 1999, 2002](#); [Cominetti and Correa, 2001](#); [Szeto et al., 2013](#); [Jiang and Ceder, 2021](#)), and approach-based ([Long et al., 2013, 2015, 2018](#); [Szeto and Jiang, 2014](#); [Jiang and Szeto, 2016](#); [Jiang et al., 2016](#); [Sun and Szeto, 2018](#)) models. To capture travellers' perception error on travel time, stochastic transit assignment models are developed ([Daganzo and Sheffi, 1977](#); [Fisk, 1980](#); [Sun and Szeto, 2018](#); [Nielsen, 2000](#); [Nielsen and Frederiksen, 2006](#); [Liu and Meng, 2014](#)), which can be formulated as a fixed point problem and solved by the method of successive averages (MSA) ([Wu and Lam, 2003](#); [Nielsen and Frederiksen, 2006](#); [Sumalee et al., 2009](#)).

Modelling passenger routing and transfer behaviour considering common lines and can be traced back to the 1970s ([Chriqui and Robillard, 1975](#)). However, most studies focus on modelling passenger behaviours at the planning stage overlooking their interactions with bus bunching (see the literature review in [Gkiotsalitis and Cats, 2021](#)). In reality, the recurrent occurrence of bus bunching could affect passengers' routing and transfer decisions, which in turn influences the propagation of bus bunching. Therefore, it is necessary to incorporate passenger routing and transfer behaviour into bus bunching models.

In the context of bus bunching, studies with passenger behavioural models are quite limited. [Schmöcker et al. \(2016\)](#) and [Wu et al. \(2017\)](#) modelled passenger queue-swapping behaviours when bus bunching occurs. Passengers are assumed to form equilibrium queues for two buses dwelling at one station simultaneously if overtaking is allowed. [Wang et al. \(2021\)](#) focused on the behaviour of boarding choice under the condition of providing passengers with bus crowding information and bus arrival time information. They supposed that some passengers would spontaneously choose to wait for the next bus with low occupancy, leading to more even bus loads if reliable bus crowding information is given.

Passenger arrival time choice is also an important aspect of passenger behaviour. [Fonzone et al. \(2015\)](#) proposed a reliability-based passenger arrival time choice model and generated non-uniform passenger arrivals which can be more realistic in some circumstances ([Ingvardson et al., 2018](#)). The impact of non-uniform passenger arrival patterns on the occurrence and evolution of bus bunching is demonstrated in [Fonzone et al. \(2015\)](#), while most of the bus bunching models in the literature assumed that passengers arrive uniformly to simplify the model development. Nevertheless, their study did not investigate the passenger accumulation process at common line stations under bus bunching, which may affect passengers' waiting time at a station and their arrival time choice.

2.3. Synthesis

A comprehensive comparison between our work and the reviewed literature is presented in [Table 1](#). As discussed, most existing models primarily focus on a single line. Only a few studies have considered common lines but overlook or underestimate the interaction between bus lines as a consequence of passenger choice behaviours. Furthermore, several crucial modelling factors have not been comprehensively addressed in previous modelling efforts, including stochastic travel times, passenger arrival patterns, and transfer behaviours, despite some studies touching upon some of them.

Thus, this study aims to fill the research gap by developing a more general and realistic model that simultaneously captures these crucial factors. Specifically, a common line bus motion model coupled with passenger arrival time choice and transfer choice models is established. To sum up, the main contributions of this study include:

- (1) Developing a bus motion model that explicitly considers passengers' routing behaviour in a common line corridor with stochastic travel times.
- (2) Devising reliability-based passenger arrival time and transfer choice models integrated with the bus motion model to capture interactions between passenger choice behaviours and bus operations.
- (3) Developing an MSA-type iterative framework to obtain steady-state passenger arrival patterns and transfer choices for analysis.
- (4) Revealing common line bus bunching properties resulting from the joint impact of passenger arrival patterns and transfer flows, distinguishing them from the findings in the single-line bus bunching literature.



Fig. 1. Common lines in the real world.

3. Modelling framework

3.1. Problem description and assumptions

We consider a general common-line corridor that includes the overlapping part traversed by multiple bus lines and the upstream and downstream branches of these lines. Fig. 1 shows several real cases in Beijing. Passengers travelling along such a transit network can be divided into three types:

- Type (I) Passengers who can travel by a direct bus line and never transfer.
- Type (II) Passengers who have no direct bus line service and must make a transfer.
- Type (III) Passengers who can choose direct bus lines or make a transfer.

Type (I) passengers refer to those whose origin–destination (OD) stations are served by at least one direct bus line, and they only board buses that take them to their destinations directly. No transfer, regardless of whether it exists or not, is used. Type (II) passengers refer to those whose OD stations are not served by direct bus lines and must make a transfer choice among alternative options. Type (III) passengers refer to those whose OD stations are served by at least one direct bus line and have competitive transfer options. Meanwhile, the following assumptions are made to facilitate modelling bus motion and passenger behaviour.

- (A1) Passengers' OD demand is given.
- (A2) Buses depart at a predefined headway from the terminal.
- (A3) A station only allows one bus to dwell at one time, and skipping or overtaking at a station is not permitted.
- (A4) The bus service time at a bus station is determined by when passengers finish boarding and alighting.
- (A5) Passengers make their choices aiming to maximise their perceived utility.

(A1) is reasonable in that the development of advanced automatic data collection systems and studies on extracting information from automated passenger counting (APC) data make it possible to predict passengers' demand (Tang et al., 2020, 2021). (A2) is a commonly adopted assumption in the literature (e.g., Sánchez-Martínez et al., 2016; Schmöcker et al., 2016; Wu et al., 2017). Determining the departure headway of a bus line is a separate problem in the bus route planning stage and is beyond the scope of our study. (A3) follows the prevalent setting where different bus lines share one berth at a station (Bian et al., 2019; Laskaris et al., 2019, 2021; Schmöcker et al., 2016; Wang et al., 2021). This means that only one bus can serve a station at one time and the first-arrive-first-depart principle is respected. Nevertheless, we would like to acknowledge that allowing multiple buses to serve at a station simultaneously could be observed in real life, but it would bring additional complexity to the problem (Schmöcker et al., 2016; Laskaris et al., 2019, 2021). Hence, we left it for future studies. (A4) is an acceptable assumption (Sun et al., 2014; Wu et al., 2017; Li et al., 2019; Estrada et al., 2021), and related discussion has been given by Sun et al. (2014). According to (A5), passengers' arrival time choices are determined by anticipated risk-averse waiting time following existing studies (Fonzone et al., 2015; Turnquist and Bowman, 1980), while passengers' transfer choices are determined by perceived transfer approach costs, which are made up of in-vehicle travel time and transfer waiting time through the approach they choose.

Based on the preceding description, this study aims to develop a model that can predict and evaluate the status of the bus system under bus bunching, considering stochasticities in both the supply and demand sides, where the former is handled by the bus motion model and the latter is captured by passengers' stochastic choice models. To this end, the main outputs of the model will include passenger arrival time choices, transfer choices, and bus trajectories in terms of bus arrival and departure times at each station.

3.2. Framework

The overall framework is conceptualised in Fig. 2. It is an iterative procedure. Without loss of generality, we begin with the bus motion model from the leftmost block. The key output of the bus motion model is the bus trajectory in the form of bus arrival

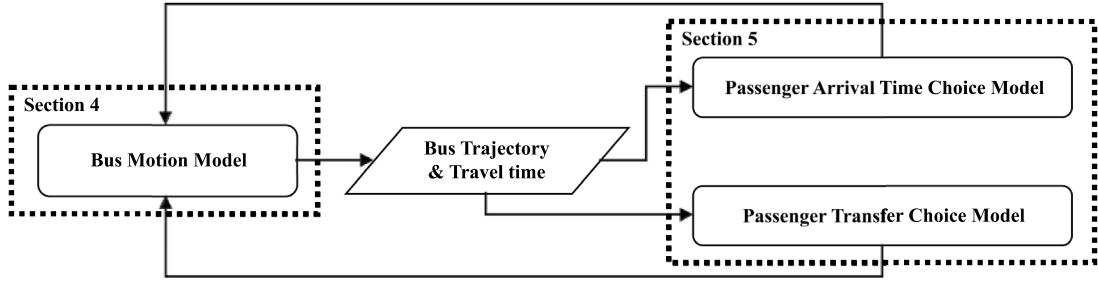


Fig. 2. Overview of the framework.

time, dwell time, and departure time (see Section 4). Meanwhile, based on the bus trajectory outputs at a very fine resolution, the passenger boarding/alighting times, waiting times and transfer times can be obtained. Furthermore, derived from the integration of outputs from multiple simulations, passenger waiting time and transfer time are calculated and, as inputs, passed into the arrival time choice model and transfer choice model, respectively. The former is a continuous choice model extended from Fonzzone et al. (2015). We make it compatible with complicated passenger demand composition at multi-line stations. The latter is a novel discrete transfer choice model (see Section 5) inspired by concepts in approach-based transit assignment formulations. Given passenger arrival time choice and transfer choice, the passenger arrival process at each station and the assignment of transfer passengers can be obtained and passed into the bus motion model to start another iteration.

For the proposed framework, we would like to make the following remarks.

(1) In reality, passengers' transfer choices might also impact passengers' arrival time choices. For example, if passengers want to transfer to a certain line at a specific time, they must also arrive at the departing station before/after a particular time. In our framework, this mechanism is implicitly captured through the bus motion model, based on the widely adopted assumption that passengers' decision is essentially attributed to the utility function related to their perceived travel costs. Hence, there is no directed arrow from the block of the transfer choice model to the block of the arrival time choice model. The same argument applies to the setting where we do not add a direct arrow from an opposite direction.

(2) The iterative procedure, in principle, resembles the procedures of fixed-point problems, which inspires us to develop an MSA-type algorithm. The existence of a fixed point relies on the assumption that the function is continuous and the solution space is bounded, which generally can be catered to (see Sections 4–5). The uniqueness of the fixed-point solution requires the monotonicity of the mapping function, which may not be proved straightforwardly.

(3) Several existing studies in the literature have adopted the stochastic equilibrium for analysing the passengers' choosing behaviours in the context of bus bunching (e.g., Fonzzone et al., 2015; Schmöcker et al., 2016; Wang et al., 2021). In our study, the similarity to a fixed-point problem implies that the framework attains a steady state. We adapt the stochastic equilibrium because we consider a scenario where the factors causing the variation in bus travel time occur recurrently within a certain period, e.g., the persistent random traffic congestion, and an individual makes a decision based on maximising his/her perceived utility.

3.3. Notations

The following key notations are used throughout this paper: (see Table 2).

4. Multi-line bus motion model

This section establishes the bus motion model for analysing the bus bunching problem in a common-line corridor. We first develop formulas for calculating passenger demand under time-varying passenger arrival rates. Then, the calculations for the numbers of boarding and alighting passengers are introduced, followed by calculating the numbers of onboard and waiting passengers at stations. Finally, the formulas are integrated to obtain the bus trajectory represented in terms of bus arrival time, dwell time and departure time at each bus station.

4.1. Demand composition

Passengers heading to different destinations have different intended lines. The total passenger arrival rate at station n , denoted as $A_n(t)$ (pas/min), can be decomposed according to their destinations:

$$A_n(t) = \sum_{\tilde{n} \in \mathcal{N}(n)} \lambda_{n,\tilde{n}}(t), \quad (1)$$

Table 2
Notations.

Indices and sets		
$r \in \mathcal{R}$		Index of a bus line belonging to a set of bus lines
$n_{i,k} \in \mathcal{N}_i^{\mathcal{R}}$		The k th station belonging to the i th set of bus stations served by lines in \mathcal{R}
$m_k^r \in \mathcal{M}^r$		The k th bus belonging to the bus fleet \mathcal{M}^r of line r
$\tilde{\mathcal{N}}(n)$		Set of destination stations for passengers arriving at station n
$\mathcal{N}^r; \mathcal{N}^{\mathcal{R}}$		Set of bus stations served by line r ; set of bus stations served by lines in \mathcal{R}
$b \in \mathcal{B}$		Index of a transfer approach belonging to a set of alternative transfer approaches
$\mathcal{M}(n)$		Set of buses serving station n
$\mathcal{M}^r; \mathcal{M}^{\mathcal{R}}$		Set of buses of line r ; set of buses of set \mathcal{R}
$m^{\mathcal{R}}(m, n)$		Index of the last bus that departs station n before bus m and belongs to a line in \mathcal{R}
$n^r(n)$		Index of the next bus station downstream to station n in bus line r
Parameters		Unit
$\tilde{\lambda}_{\hat{n},\check{n}}$	Average arrival rate of passengers at station \hat{n} heading to station \check{n}	pas/min
$Q_{\hat{n},\check{n}}$	Amount of passenger travel demand from station \hat{n} to station \check{n}	pas
H^r	Predefined bus headway of line r	min
S^r	Service start time of line r	min
β	Average passenger boarding rate	pas/min
α	Average passenger alighting rate	pas/min
Variables		Unit
$a_{m,n}$	Arrival time of bus m at station n	min
$s_{m,n}$	Dwell time of bus m at station n	min
$s_{m,n}^B$	Time consumed for passengers at station n to board bus m	min
$s_{m,n}^A$	Time consumed for passengers on bus m to alight at station n	min
$s_{m,n}^{\tilde{B}}$	Time consumed for passengers at station n to board bus m considering the bus vehicle capacity constraint	min
$d_{m,n}$	Departure time of bus m from station n	min
$t_{m,n}$	Travel time of bus m between station n and the next station	min
$\Lambda_n(t)$	Total passenger arrival rate of at station n at time t	pas/min
$\Lambda_n^{\mathcal{R}}(t)$	Total passenger arrival rate of at station n at time t with intended bus line set \mathcal{R}	pas/min
$\lambda_{\hat{n},\check{n}}(t)$	Arrival rate of passengers heading to station \check{n} at station \hat{n} at time t	pas/min
$q_{n,\mathcal{R}}(t)$	Arrival time choice ratio at station n at time t for passengers with intended bus line set \mathcal{R}	–
$U_{n,\mathcal{R}}^a(t)$	Utility function regarding arriving at station n at t for passengers with intended bus line set \mathcal{R}	–
$W_{n,\mathcal{R}}^r(t)$	Anticipated risk-averse waiting time regarding arriving at station n at t for passengers with intended bus line set \mathcal{R}	min
$\Pi_{m,n}(t)$	Probability that passengers arriving at station n at t can board bus m	–
$E(w_{n,\mathcal{R}}(t))$	Expected waiting time regarding arriving at station n at t for passengers with intended bus line set \mathcal{R}	–
$f_{m,n}(\cdot)$	Perceived probability distribution function of $d_{m,n}$	–
$\pi_{m,n}(t, \tau)$	Probability distribution function of the event that “passengers arriving at t can board bus n and bus n departs at τ ”	–
$U_{b,\mathcal{R}}^t$	Utility function regarding transfer approach b for passengers heading to destinations served by the set of bus lines \mathcal{R}	–
$\alpha_{b,\mathcal{B},\mathcal{R}}$	Proportion of passengers choosing approach b among all alternative transfer approaches \mathcal{B} and heading to destinations served by \mathcal{R}	–
$A_{m,n}$	Number of alighting passengers of bus m at station n	pas
$B_{m,n}$	Number of boarding passengers of bus m at station n	pas
$\tilde{B}_{m,n}$	Number of boarding passengers of bus m at station n constrained by vehicle capacity	pas
$P_{m,n}$	Number of all onboard passengers on bus m when it arrives at station n	pas
$p_{m,\check{n}}$	Number of onboard passengers whose destination is station \check{n} on bus m when it arrives at station n	pas
$\rho_{m,\check{n},n}$	Proportion of transfer passengers heading to destination \check{n} who choose station n as their transfer station	–
$L_{m,n}$	Number of all waiting passengers at station n when bus m departs	pas
$L_{m,n}^{\mathcal{R}}$	Number of waiting passengers with intended bus line set \mathcal{R} at station n when bus m departs	pas
$l_{m,n,\check{n}}$	Number of waiting passengers whose destination is station \check{n} and are waiting at station n when bus m departs	pas
$u_{b,\mathcal{R}}^t$	Expected transfer waiting time via approach b for set of intended transfer lines \mathcal{R}	min
$w_{n,\mathcal{R}}(t)$	The waiting time considering fail-to-board events due to capacity constraints for passengers arriving at station n at t and intending to board a bus belonging to \mathcal{R}	min
$P(r, \mathcal{R})$	Probability of transferring to line r among set of lines \mathcal{R}	–

where $\tilde{\mathcal{N}}(n)$ represents the set of destinations for all arriving passengers at station n ; $\lambda_{\hat{n},\check{n}}(t)$ represents the arrival rate at time t for passengers at station n heading to station \check{n} . The passenger arrival rate, which captures the passenger arrival process over the full period concerned, is obtained by

$$\lambda_{\hat{n},\check{n}}(t) = Q_{\hat{n},\check{n}} \cdot q_{n,\mathcal{R}}(t), \quad (2)$$

where $Q_{\hat{n},\check{n}}$ represents the number of passengers travelling from station n to station \check{n} during the period concerned for the passenger arrival process and $q_{n,\mathcal{R}}(t)$ represents the passenger arrival time choice ratio and is introduced in Section 5.

4.2. Number of boarding passengers

At station n served by a set of lines \mathcal{R} , the passenger boarding demand for bus m of line $r \in \mathcal{R}$ is denoted by $B_{m,n}$, and can be obtained by

$$B_{m,n} = \sum_{\forall \mathcal{R}' \subseteq \mathcal{R}, \mathcal{R}' \ni r} \left(\int_{d_{m^{\mathcal{R}'}(m,n),n}}^{d_{m,n}} A_n^{\mathcal{R}'}(t) dt + L_{m^{\mathcal{R}'}(m,n),n}^{\mathcal{R}'} \right), \quad (3)$$

where $m^{\mathcal{R}'}(m,n)$ refers to the last bus that departs from station n before bus m and belongs to \mathcal{R} , and $L_{m^{\mathcal{R}'}(m,n),n}^{\mathcal{R}'}$ refers to the number of waiting passengers at the station and will be introduced in Section 4.4.

4.3. Number of alighting passengers

In the common-line corridor, passengers alighting from a bus at station n include passengers whose destination is station n and passengers who choose station n as their transfer station. Then, the number of alighting passengers at station n from bus m , denoted by $A_{m,n}$, can be calculated by

$$A_{m,n} = p_{m,n,n} + \sum_{\check{n} \in \check{\mathcal{N}}(n) \setminus \mathcal{N}^r} \rho_{m,n,\check{n}} \cdot p_{m,n,\check{n}}, \quad (4)$$

where $p_{m,n,\check{n}}$ represents the number of onboard passengers heading to station \check{n} on bus m when it arrives at station n ; $\rho_{m,n,\check{n}}$ represents the proportion of onboard passengers heading to station \check{n} on bus m when it arrives at station n who choose station n as their transfer station; and \mathcal{N}^r represents the set of stations served by line r .

4.4. Numbers of onboard passengers and waiting passengers

The number of onboard passengers and the number of waiting passengers heading to station \check{n} when bus m departs from station n are denoted by $p_{m,n^r(n),\check{n}}$ and $l_{m,n,\check{n}}$, respectively. They can be obtained by

$$p_{m,n^r(n),\check{n}} = \begin{cases} p_{m,n,\check{n}} + \int_{d_{m^{\mathcal{R}}(m,n),n}}^{d_{m,n}} \lambda_{n,\check{n}}(t) dt + l_{m^{\mathcal{R}}(m,n),n,\check{n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \cap \mathcal{N}^r \\ (1 - \rho_{m,n,\check{n}}) \cdot p_{m,n,\check{n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \setminus \mathcal{N}^r \end{cases} \quad (5)$$

and

$$l_{m,n,\check{n}} = \begin{cases} 0 & \text{if } \check{n} \in \check{\mathcal{N}}(n) \cap \mathcal{N}^r \\ \int_{d_{m^{\mathcal{R}}(m,n),n}}^{d_{m,n}} \lambda_{n,\check{n}}(t) dt + l_{m^{\mathcal{R}}(m,n),n,\check{n}} + \rho_{m,n,\check{n}} \cdot p_{m,n,\check{n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \setminus \mathcal{N}^r, \end{cases} \quad (6)$$

respectively. $n^r(n)$ stands for the subsequent downstream station of station n traversed by line r .

Then, the total numbers of onboard passengers and waiting passengers can be obtained by

$$P_{m,n^r(n)} = \sum_{\check{n} \in \check{\mathcal{N}}(n)} p_{m,n^r(n),\check{n}} \quad (7)$$

and

$$L_{m,n} = \sum_{\check{n} \in \check{\mathcal{N}}(n)} l_{m,n,\check{n}}, \quad (8)$$

respectively.

4.5. Bus dwell time

Bus dwell time is the maximum between passenger boarding and alighting times. The boarding time can be obtained by calculating the boarding demand clearance time via $\frac{\text{The boarding demand}}{\text{The demand-clearance rate}}$. Specifically, the time consumed for passengers at station n to board bus m is denoted by $s_{m,n}^B$, and is expressed by

$$s_{m,n}^B = \frac{\sum_{\forall \mathcal{R}, \mathcal{R} \ni r} \left(\int_{d_{m^{\mathcal{R}}(m,n),n}}^{a_{m,n}} A_n^{\mathcal{R}}(t) dt + L_{m,n}^{\mathcal{R}} \right)}{\beta - \sum_{\forall \mathcal{R}, \mathcal{R} \ni r} A_n^{\mathcal{R}}(a_{m,n})}, \quad (9)$$

where $a_{m,n}$ represents the arrival time of bus m at station n , and β represents the average boarding rate. The average demand-clearance rate is approximated by $(\beta - \sum_{\forall \mathcal{R}, \mathcal{R} \ni r} A_n^{\mathcal{R}}(a_{m,n}))$.

The time consumed for passengers to alight at station n from bus m , denoted as $s_{m,n}^A$, can be calculated by

$$s_{m,n}^A = \frac{A_{m,n}}{\alpha}, \quad (10)$$

where α represents the average alighting rate of passengers.

As a result, the actual dwell time of bus m at station n , denoted by $s_{m,n}$, is obtained by

$$s_{m,n} = \max \left\{ s_{m,n}^A, s_{m,n}^B \right\}. \quad (11)$$

4.6. Modelling bus capacity constraint

The following modifications should be made to capture the bus capacity constraints in the bus motion model.

1. When determining the number of passengers onboard, the number of passengers who successfully board a bus is constrained by the residual capacity of the bus. Accordingly, the number of passengers at station n who successfully board bus m is given by:

$$\bar{B}_{m,n} = \min \left\{ C_m - P_{m,n} + A_{m,n}, B_{m,n} \right\}, \quad (12)$$

where C_m represents the capacity of bus m ; $P_{m,n}$ represents the number of onboard passengers when bus m arrives at station n ; $A_{m,n}$ represents the number of alighting passengers; $B_{m,n}$ represents the number of passengers who want to board bus m at station n .

2. The formulation for updating the number of passengers onboard and the number of passengers waiting at the station should take into account the proportion of passengers who can successfully board the dwelling bus. Specifically, when bus m departs from station n , the number of onboard passengers and the number of waiting passengers who are heading to station \check{n} are modified as

$$p_{m,n^r(n),\check{n}} = \begin{cases} p_{m,n,\check{n}} + \left(\int_{d_{m^R(m,n),n}}^{d_{m,n}} \lambda_{n,\check{n}}(t) dt + l_{m^R(m,n),n,\check{n}} \right) \cdot \frac{\bar{B}_{m,n}}{B_{m,n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \cap \mathcal{N}^r \\ (1 - \rho_{m,n}) \cdot p_{m,n,\check{n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \setminus \mathcal{N}^r \end{cases} \quad (13)$$

and

$$l_{m,n,\check{n}} = \begin{cases} \left(\int_{d_{m^R(m,n),n}}^{d_{m,n}} \lambda_{n,\check{n}}(t) dt + l_{m^R(m,n),n,\check{n}} \right) \cdot \left(1 - \frac{\bar{B}_{m,n}}{B_{m,n}} \right) & \text{if } \check{n} \in \check{\mathcal{N}}(n) \cap \mathcal{N}^r \\ \int_{d_{m^R(m,n),n}}^{d_{m,n}} \lambda_{n,\check{n}}(t) dt + l_{m^R(m,n),n,\check{n}} + \rho_{m,n} \cdot p_{m,n,\check{n}} & \text{if } \check{n} \in \check{\mathcal{N}}(n) \setminus \mathcal{N}^r, \end{cases} \quad (14)$$

respectively.

3. The bus dwell time is bounded by the passenger boarding time with bus capacity constraint. Hence, the dwell time of bus m at station n , denoted as $s_{m,n}$, is modified by

$$s_{m,n} = \max \left\{ s_{m,n}^A, \min \left\{ s_{m,n}^B, s_{m,n}^{\bar{B}} \right\} \right\}, \quad (15)$$

where $s_{m,n}^{\bar{B}}$ represents the upper bound of the actual boarding time considering the capacity constraint and is calculated by $\frac{\bar{B}_{m,n}}{\beta}$.

4.7. Algorithm for bus trajectory

Based on the equations in the previous subsections, we developed the algorithm for obtaining bus motion trajectories.

The following remarks are made for Algorithm 1.

- (1) The bus arrival times at the first station of each line are initialised in advance. Based on (A2), the arrival times of buses at the first station of line r are determined by:

$$a_{k,n_1^r} = S^r + H^r \cdot (k - 1) \text{ for } k = 1, 2, \dots, |\mathcal{M}^r|, \quad (18)$$

where a_{k,n_1^r} represents the arrival time of the k th bus at station n_1^r , which refers to the first station of line r ; S^r represents the service start time of line r ; H^r represents the predefined headway of line r ; and \mathcal{M}^r represents the bus fleet of line r .

- (2) Algorithm 1 takes passengers' arrival time and transfer choices as input parameters. These will be introduced in Section 5.
- (3) Algorithm 1 runs with one travel time sample generated under a stochastic travel time setting. In the proposed iterative framework in Fig. 4, multiple runs of Algorithm 1 are executed in one iteration based on the same passengers' arrival patterns and transfer choices calculated in the previous iteration but with different stochastic travel times to obtain an average performance of bus operations.

¹ If bus capacity constraints are considered, Eqs. (5)–(6) are modified by Eqs. (13)–(14).

Algorithm 1 Algorithm for obtaining bus trajectories.

Initialisation

for bus station n indexed from upstream to downstream **do**

Get the set of buses serving at station n : $\mathcal{M}(n)$.

for bus $m \in \mathcal{M}(n)$ ordered by their arrival times at n **do**

Calculate the number of alighting passengers, $A_{m,n}$, by Eq.(4).

Calculate bus dwell time, $s_{m,n}$, of bus m at station n by Eq.(9), Eq.(10) and Eq.(11).

Calculate the departure time of bus m from station n by

$$d_{m,n} = a_{m,n} + s_{m,n}. \quad (16)$$

Calculate the number of boarding passengers, $B_{m,n}$, by Eq.(3).

Calculate the numbers of onboard passengers and waiting passengers with Eqs.(5)–(6).¹

if n is not the terminal station, **then**

Get the index of the next downstream station of bus station n : $n^r(n)$.

Generate a sample of stochastic travel times between stations n and $n^r(n)$: $t_{m,n}$.

Calculate the arrival time of bus m at $n^r(n)$:

$$a_{m,n^r(n)} = d_{m,n} + t_{m,n}. \quad (17)$$

end if

end for

end for

5. Passenger choice models

This section develops a continuous logit model for modelling passenger arrival time choice and a discrete logit model for modelling passenger transfer choice. The two models are jointly coupled with the bus motion model in the proposed framework.

5.1. Arrival time choice

The passenger arrival time choice model extends the single-line reliability-based passenger arrival model developed by [Fonzone et al. \(2015\)](#). We generalise it to consider common lines. Denote $q_{n,\mathcal{R}}(t)$ as the arrival time choices ratio at station n at time t for passengers whose intended bus line set is \mathcal{R} . Then, it can be obtained by:

$$q_{n,\mathcal{R}}(t) = \frac{\exp \left\{ U_{n,\mathcal{R}}^a(t) \right\}}{\int_{t_{n,\mathcal{R}}^{\inf}}^{t_{n,\mathcal{R}}^{\sup}} \exp \left\{ U_{n,\mathcal{R}}^a(t) \right\} dt}, \quad (19)$$

where $\left[t_{n,\mathcal{R}}^{\inf}, t_{n,\mathcal{R}}^{\sup} \right]$ is the interval spanning all the perceived bus departure times at station n for passengers considering lines in \mathcal{R} ; $U_{n,\mathcal{R}}^a(t)$ is the utility function associated with the arrival time choice at station n at time t for passengers with the intended line set \mathcal{R} and is defined by:

$$U_{n,\mathcal{R}}^a(t) = \alpha \left(W_{n,\mathcal{R}}(t) \right)^\beta, \quad (20)$$

where we set $\alpha = -1$ and $\beta = 0.55$ following [Fonzone et al. \(2015\)](#) and [Bowman and Turnquist \(1981\)](#); $W_{n,\mathcal{R}}(t)$ represents anticipated risk-averse waiting time of passengers arriving at station n at time t and is computed by:

$$W_{n,\mathcal{R}}(t) = E(w_{n,\mathcal{R}}(t)) + \left(1 - \sum_{m \in \mathcal{M}^{\mathcal{R}}} \Pi_{m,n}(t) \right) c, \quad (21)$$

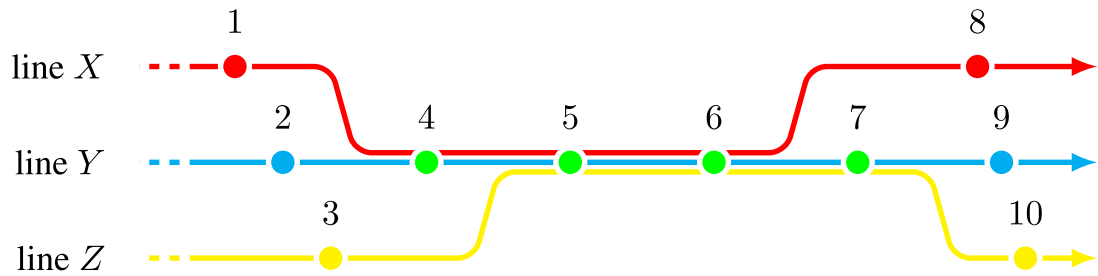
where c is the constant used to represent passengers' aversion to the possibility of missing the last alternative boarding opportunity; $\mathcal{M}^{\mathcal{R}}$ is the union of all bus fleets of lines in \mathcal{R} (i.e., $\mathcal{M}^{\mathcal{R}} = \cup_{r \in \mathcal{R}} \mathcal{M}^r$); and $\Pi_{m,n}(t)$ represents the probability that passengers arrived at station n at time t can board bus m and is obtained by:

$$\Pi_{m,n}(t) = \int_t^{+\infty} f_{m,n}(\tau_m) \prod_{m' \in \mathcal{M}^{\mathcal{R}} \setminus \{m\}} \left[1 - \int_t^{\tau_m} f_{m'}(\tau_{m'}) d\tau_{m'} \right] d\tau_m, \quad (22)$$

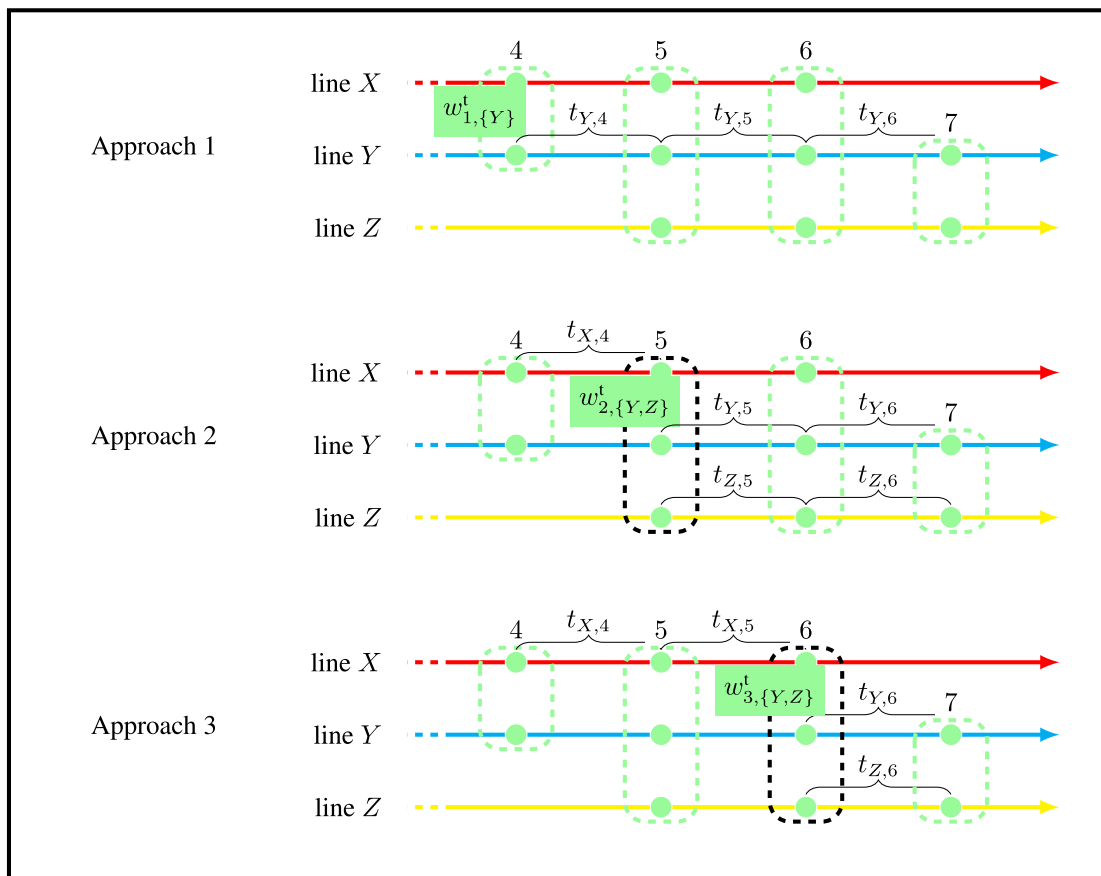
where $f_{m,n}(\cdot)$ represents the perceived probability distribution function of $d_{m,n}$.

$E(w_{n,\mathcal{R}}(t))$ represents the expected waiting time at station n for passengers with intended bus line set \mathcal{R} and choose to arrive at t :

$$E(w_{n,\mathcal{R}}(t)) = \int_t^{+\infty} (\tau - t) \sum_{m \in \mathcal{M}^{\mathcal{R}}} \pi_{m,n}(t, \tau) d\tau, \quad (23)$$



(a) Network structure



(b) Composition of approach costs for passengers from station 4 to station 7

Fig. 3. Demonstration of the concept of transfer approach.

where $\pi_{m,n}(t, \tau)$ is given by:

$$\pi_{m,n}(t, \tau) = f_{m,n}(\tau) \prod_{m' \in \mathcal{M}^R} \left[1 - \int_t^\tau f_{m'}(\tau_{m'}) d\tau_{m'} \right]. \tag{24}$$

The above equation computes the probability distribution function that passengers arrive at station n at time t , board bus m , and bus m departs at τ .

5.2. Transfer choice

Passengers might alter their transfer choices by choosing different bus lines or transfer stations when encountering bus bunching. For bus lines suffering from the recurrent bunching problem, this study assumes that passenger transfer choices eventually attain an equilibrium state for the sake of analysis, and we leave the day-to-day evolutionary process (Cats and West, 2020; Guo and Szeto, 2018) out of the scope of this study.

The discrete logit model is adopted to model passenger transfer choice behaviour. The proportion of passengers heading to destinations served by a set of bus lines \mathcal{R} and choosing transfer approach b among all alternative transfer approaches \mathcal{B} is obtained by:

$$\alpha_{b,\mathcal{B},\mathcal{R}} = \frac{\exp\{U_{b,\mathcal{R}}^t\}}{\sum_{b \in \mathcal{B}} \exp\{U_{b,\mathcal{R}}^t\}}, \quad (25)$$

where $U_{b,\mathcal{R}}^t$ represents the utility function associated with approach b for passengers with intended bus lines \mathcal{R} (i.e., passengers whose destination stations are served by a set of bus lines \mathcal{R}).

The concept of transfer approach is in light of the approach-based transit assignment formulations developed in the literature (Long et al., 2013; Szeto and Jiang, 2014; Jiang et al., 2016). A transfer approach choice generally consists of the line chosen to board and the station chosen to transfer. The transfer choice would be even more complicated in some special scenarios, like permitting multiple buses to dwell simultaneously and overtake (Schmöcker et al., 2016; Wu et al., 2017), or providing bus crowding information (Wang et al., 2021). These occasions are not considered in this study and are left for future research.

The utility function of a transfer approach is determined by the perceived travel cost regarding this approach, which is generally composed of two parts: in-vehicle travel cost and waiting cost at the transfer station. Hence, $U_{b,\mathcal{R}}^t$ can be expressed by

$$U_{b,\mathcal{R}}^t = c_1 \cdot \left[t_{r,b}^{\text{before}} + \sum_{r' \in \mathcal{R}} P(r', \mathcal{R}) \cdot t_{r',b}^{\text{after}} \right] + c_2 \cdot w_{b,\mathcal{R}}^t, \quad (26)$$

where c_1, c_2 are weighting coefficients associated with in-vehicle travel time cost and waiting time cost, respectively; $t_{r,b}^{\text{before/after}}$ represents the travel time before/after transferring within approach b ; $w_{b,\mathcal{R}}^t$ represents the expected waiting time via approach b for lines in \mathcal{R} and we will provide a general formulation for it in Section 5.3 considering capacity constraints; and $P(r, \mathcal{R})$ represents the probability of transferring to line r among \mathcal{R} .

To illustrate the concepts of the transfer approach, an example is shown in Fig. 3(a). We focus on passengers from station 4, which is served by lines $\{X, Y\}$, to station 7, which is served by lines $\{Y, Z\}$. When a bus of line X arrives, there are three alternative approaches (shown in Fig. 3b) for these passengers: Approach (1) waiting for direct bus line Y to station 7; Approach (2) boarding line X first and transferring to line Y or Z via station 5; Approach (3) boarding line X first and transferring to line Y or Z via station 6. The utility functions corresponding to these approaches are formulated as:

$$\begin{aligned} U_{1,\{Y,Z\}}^t &= c_1 \cdot (t_{Y,4} + t_{Y,5} + t_{Y,6}) + c_2 \cdot w_{1,\{Y\}}^t, \\ U_{2,\{Y,Z\}}^t &= c_1 \cdot \{t_{X,4} + P(Y, \{Y, Z\}) \cdot (t_{Y,5} + t_{Y,6}) + P(Z, \{Y, Z\}) \cdot (t_{Z,5} + t_{Z,6})\} + c_2 \cdot w_{2,\{Y,Z\}}^t, \\ U_{3,\{Y,Z\}}^t &= c_1 \cdot \{t_{X,4} + t_{X,5} + P(Y, \{Y, Z\}) \cdot t_{Y,6} + P(Z, \{Y, Z\}) \cdot t_{Z,6}\} + c_2 \cdot w_{3,\{Y,Z\}}^t, \end{aligned} \quad (27)$$

where $P(Y, \{Y, Z\})$ and $P(Z, \{Y, Z\})$ are approximated with $\frac{(H^Y)^{-1}}{(H^Y)^{-1} + (H^Z)^{-1}}$ and $\frac{(H^Z)^{-1}}{(H^Y)^{-1} + (H^Z)^{-1}}$, respectively.

The waiting time of transfer passengers, denoted as $w_{b,\mathcal{R}}^t$, is a key component in Eq. (26) and is calculated by the difference between the departure time of the bus from which the transfer passenger originates and the departure time of the bus to which the transfer passenger intends to board. For instance, passengers travelling from station 4 to station 7 face three options with distinct transfer waiting times, as illustrated in Fig. 3. Passengers choosing Approach 1 encounter no transfer time. The transfer time associated with Approach 2 or Approach 3 regarding the transfer time spent at station 5 or station 6. This transfer time is estimated by the mean value of the simulated headways between the departure of the last bus of line X , and the departure of the first bus of either line Y or line Z . One thing worth noting is that the calculation for transfer time does not consider fail-to-board occasions due to capacity constraints. The additional waiting time arising from the capacity constraint is introduced in Section 5.3.

5.3. Modelling passenger choices considering bus capacity constraint

For passengers arriving at station n at t and intending to board a bus belonging to \mathcal{R} , their waiting time considering possible fail-to-board occasions due to capacity constraints is estimated by:

$$\begin{aligned}
w_{n,\mathcal{R}}(t) &= (d_{m^{[1]},n} - t) + \left(1 - \frac{\bar{B}_{m^{[1]},n}}{B_{m^{[1]},n}}\right) \cdot \frac{\bar{B}_{m^{[2]},n}}{B_{m^{[2]},n}} \cdot (d_{m^{[2]},n} - d_{m^{[1]},n}) \\
&+ \left(1 - \frac{\bar{B}_{m^{[1]},n}}{B_{m^{[1]},n}}\right) \cdot \left(1 - \frac{\bar{B}_{m^{[2]},n}}{B_{m^{[2]},n}}\right) \cdot \frac{\bar{B}_{m^{[3]},n}}{B_{m^{[3]},n}} \cdot (d_{m^{[3]},n} - d_{m^{[1]},n}) \\
&+ \dots \\
&+ c \cdot \prod_{j=1}^{|\mathcal{M}_{n,\mathcal{R}}(t)|} \left(1 - \frac{\bar{B}_{m^{[j]},n}}{B_{m^{[j]},n}}\right) \\
&= (d_{m^{[1]},n} - t) + \sum_{i=2}^{|\mathcal{M}_{n,\mathcal{R}}(t)|} \left\{ \left[\prod_{j=1}^{i-1} \left(1 - \frac{\bar{B}_{m^{[j]},n}}{B_{m^{[j]},n}}\right) \right] \cdot \frac{\bar{B}_{m^{[i]},n}}{B_{m^{[i]},n}} \cdot (d_{m^{[i]},n} - d_{m^{[1]},n}) \right\} \\
&+ c \cdot \prod_{j=1}^{|\mathcal{M}_{n,\mathcal{R}}(t)|} \left(1 - \frac{\bar{B}_{m^{[j]},n}}{B_{m^{[j]},n}}\right),
\end{aligned} \tag{28}$$

where $\mathcal{M}_{n,\mathcal{R}}(t)$ represents the set of buses belonging to \mathcal{R} and arriving at station n after time t ; $m^{[i]}$ is the index of the i th bus sorted by their arriving times at station n in $\mathcal{M}_{n,\mathcal{R}}(t)$. For transfer passengers intending to transfer to a bus belonging to \mathcal{R} at station n from bus m , their expected waiting time is estimated by $w_{n,\mathcal{R}}(d_{m,n})$.

5.4. MSA-type framework

An MSA-type algorithm, detailed in Algorithm 2, is developed to obtain passenger transfer choices and arrival time choices.

Algorithm 2 MSA-type algorithm for updating passenger transfer choices.

Step 0: Initialisation

Set iteration counter $k = 0$. Assign the transfer passengers evenly to all alternative approaches.

Step 1: Update the iteration counter

$k \leftarrow k + 1$.

Step 2: Computer an auxiliary solution

Step 2.1: Run Algorithm 1 to get bus trajectories represented by bus arrival and departure times at stations.

Step 2.2: Compute the mean value of perceived transfer utility functions by Eq. (26): $U_{b,\mathcal{R}}^t$.

Step 2.3: Obtain an auxiliary solution to the transfer choice model via Eq. (25): $\alpha_{b,B,\mathcal{R}}(k)$.

Step 3: Update the new solution for the next iteration

Use the following equation to update the solution.

$$\alpha_{b,B,\mathcal{R}}(k) \leftarrow \left(1 - \frac{1}{k}\right) \cdot \alpha_{b,B,\mathcal{R}}(k-1) + \frac{1}{k} \cdot \alpha_{b,B,\mathcal{R}}(k), \forall b \in B. \tag{29}$$

Step 4: Check termination criteria

if $\alpha_{b,B,\mathcal{R}}(k) - \alpha_{b,B,\mathcal{R}}(k-1) >$ predefined gap then

Back to Step 1.

else

Stop.

end if

For a better understanding of the relationship and the data flows between each model, a detailed flowchart is shown in Fig. 4.

Note that the update of passenger arrival time choices is conducted through an iterative framework. More specifically, in each iteration, random samples of link travel times are generated from the truncated normal distribution of link travel time. These samples are used to generate corresponding sets of bus trajectories. Based on these trajectories, we update the arrival time of passengers at each station. To elaborate on our method, we visualised the primary five steps in Fig. 5. The first step involves collecting the statistical distribution of bus departure times at each stop, corresponding to Step 1 in Fig. 5, which shows the bus departure time statistics for stops 5, 6, and 7. The second step follows the method applied by Fonzone et al. (2015), using a triangular distribution to statistically fit the bus departure time density distribution $f(\tau)$. The third step involves calculating the probability function of passengers arriving at the station at time t and the target bus has not yet departed $\Pi(t)$, using Eqs. (22)–(24). The fourth step calculates the perceived arrival time utility function $U(t)$, using Eqs. (20)–(21). The final step calculates the passenger arrival time preference $q(t)$ through Eq. (19).

6. Numerical experiments

This section first illustrates the theoretical properties of the model via a stylised example. The effects of stochastic travel times, passenger arrival patterns, transfer flows, and vehicle capacity constraints on bus bunching modelling are demonstrated. Afterwards,

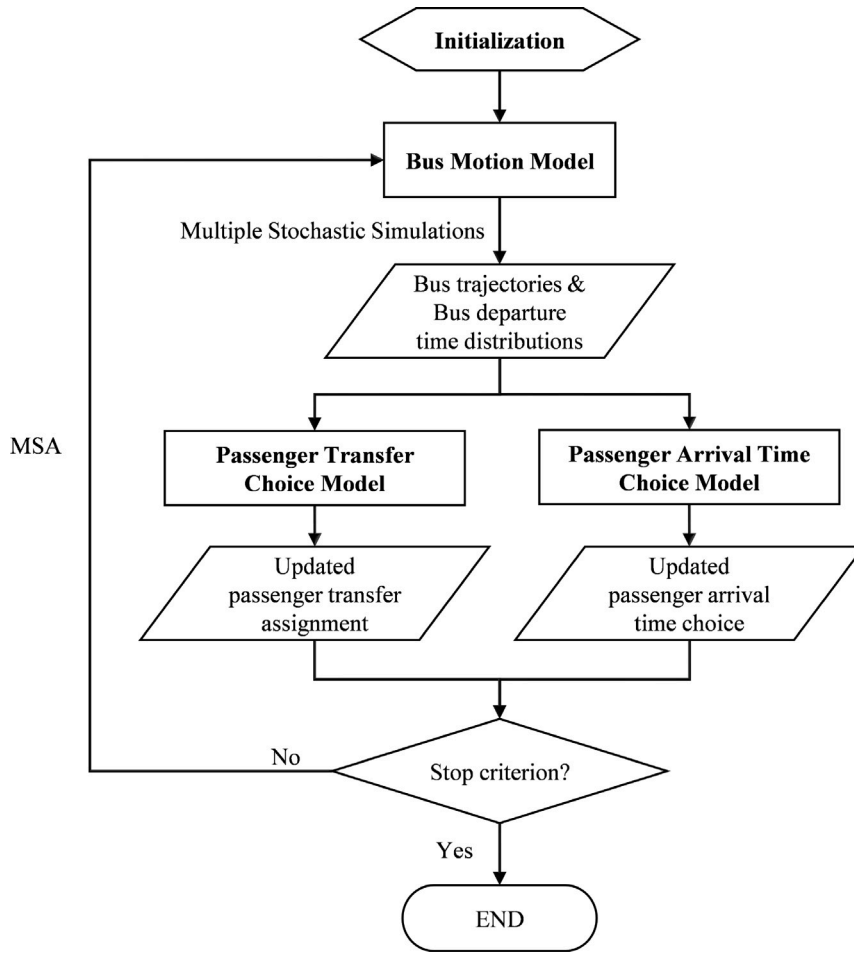


Fig. 4. MSA-type iteration framework.

we conducted a case study using real data in Beijing to examine the characteristics of the bus bunching under uncertain traffic status, varying transfer proportions, and different total travel demand levels.

6.1. Illustrative example

6.1.1. Specifications

We construct an illustrative example with two bus lines shown in Fig. 6. There are 11 stations divided into 5 sets: $\mathcal{N}_1^{(1)} = \{1, 2\}$, $\mathcal{N}_2^{(2)} = \{3, 4\}$, $\mathcal{N}_3^{(1,2)} = \{5, 6, 7\}$, $\mathcal{N}_4^{(1)} = \{8, 9\}$, $\mathcal{N}_5^{(2)} = \{10, 11\}$. The parameters are listed in Table 3. The average boarding (alighting) rate, denoted by β (α), refers to the number of passengers boarding (alighting) per unit of time. The values of the average boarding and alighting rates adopted in Table 3 are set according to Sun et al. (2014). The frequency is set to be the same for both lines (i.e., $H^1 = H^2 = 10$ min). The first bus of line 2 departs 5 min later than the first bus of line 1 (i.e., $t^1 = 0, t^2 = 5$ min). All bus stations have the same average passenger arrival rate (rightward direction in Fig. 6), which is set at 1.5 pas/min. 5 scenarios are designed, and the corresponding settings are listed in Table 4. In the following experiments, two performance indicators widely used in the literature (e.g., Delgado et al., 2012; Hernández et al., 2015; Schmöcker et al., 2016; Wu et al., 2017), namely, the passenger waiting time and the standard deviation of bus headway are adopted to measure the efficiency and reliability of a certain bus line in a scenario setting. For example, $w_{S_1}^{(1)}$ refers to the passenger waiting time for line 1 in Scenario 1, and $\sigma_{S_2}^{(1)}$ refers to the standard deviation of headway of line 1 in Scenario 2. All the numerical experiments are implemented in Python 3.10 on a Windows 10 PC with 8 Intel Core i7-6700 CPU and 16.0 GB RAM.

6.1.2. Effect of stochastic running time on bus bunching

A deterministic benchmark scenario (Scenario 1) and a stochastic scenario (Scenario 2) are designed and compared to demonstrate the effect of the stochastic running time on bus bunching. In Scenario 1, the bus running times between two adjacent bus stations are set to be a constant value of 6 min. In Scenario 2, bus running times follow a truncated normal distribution with

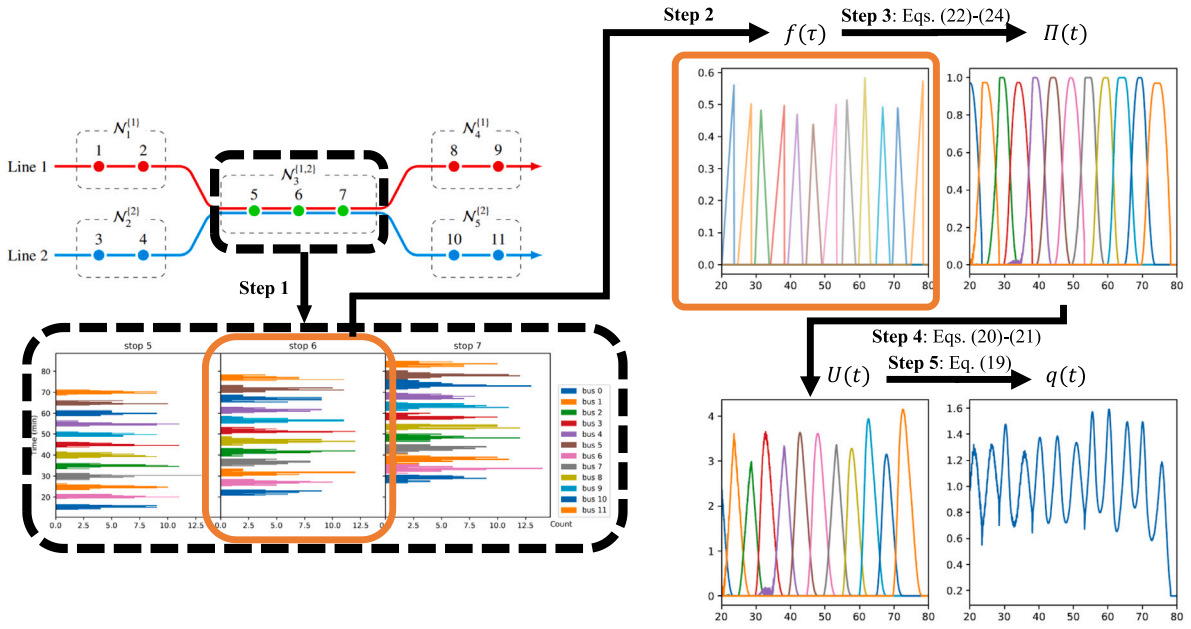


Fig. 5. The process of updating passenger arrival choices.

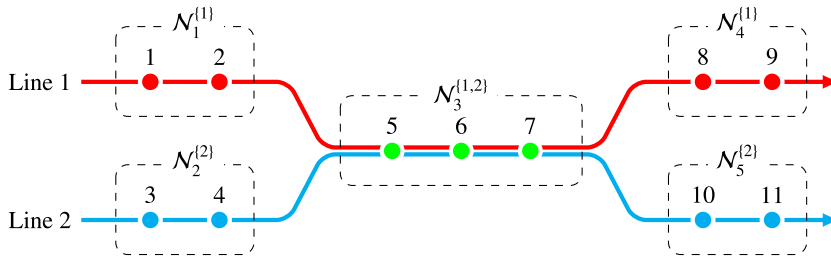


Fig. 6. Routes and stations in the illustrative example.

Table 3
Basic parameter settings.

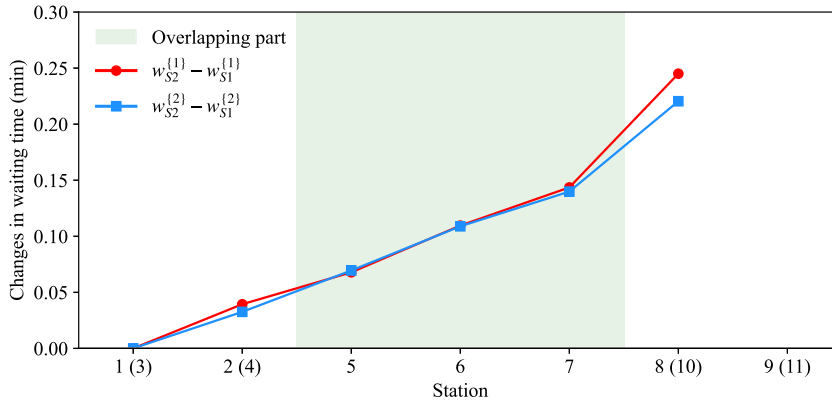
Item		Unit
Average boarding/alighting rate	$\beta = 30, \alpha = 40$	pas/min
Bus departure timetable	$H^1 = H^2 = 10, r^1 = 0, r^2 = 5$	min
Passenger demand	$\bar{\lambda}_n = 1.5, \forall n \in \mathcal{N}^1 \cup \mathcal{N}^2 \setminus \{9, 11\}$	pas/min

Table 4
Experiment configuration settings.

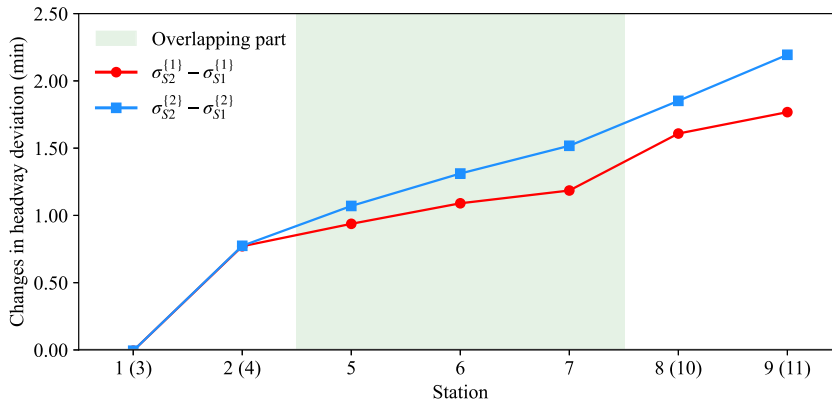
Scenario	Bus running time	Passenger arrival pattern	Transfer demand	Capacity constraint
1	Deterministic	Uniform	No	No
2	Stochastic	Uniform	No	No
3	Stochastic	Reliability-based	No	No
4	Stochastic	Reliability-based	Yes	No
5	Stochastic	Reliability-based	Yes	Yes

the same mean value of 6 min, the standard deviation of 2 min, and shape parameters $(-0.5, 1)$. We conducted 100 simulations in Scenario 2 and recorded the average values of the evaluation indicators.

Fig. 7 plots the changes in the measurements in Scenarios 1 and 2. All indicators under Scenario 2 are higher than those under Scenario 1, evidencing that the fluctuation in the bus running times in Scenario 2 induces bus bunching. A general trend along the bus line can be observed: the further away from the upstream stations to the downstream stations, the larger the gaps of the



(a) Changes in the waiting time



(b) Changes in the standard deviation of headway

Fig. 7. Comparison between Scenario 1 and Scenario 2.

indicators between the two scenarios. Such a trend is similar to what the literature has found in the single-line configuration (Chen et al., 2022; Schmöcker et al., 2016; Newell and Potts, 1964), in which the interaction across parallel lines can hardly be revealed.

For better visualisation, bus trajectories in Scenario 2 are plotted in Fig. 8, where the solid lines represent the simulated bus trajectories and the dashed lines represent the scheduled or expected bus trajectories with regular headways. Stations 5, 6, and 7 in the abscissa are common line stations shared by two lines and are highlighted with a light green background. It can be seen that there is an evident bunching phenomenon on line 1 while the bus trajectories of line 2 are relatively uniform. In other words, while line 1 suffers a bunching problem, line 2 is barely affected.

6.1.3. Effect of passenger arrival pattern on bus bunching

To test the impact of the reliability-based non-uniform passenger arrival patterns, we compare the indicators obtained in Scenario 3 with those in Scenario 2. In Scenario 3, the passenger-perceived bus departure time distribution is set as a triangular distribution with shape parameters $(-1, 2)$ following Fonzone et al. (2015). But distinct from their deterministic experiment setting in Fonzone et al. (2015), the experiments in Scenario 3 are conducted in a stochastic environment (the same as Scenario 2). Worth noting is that passengers with different alternative bus lines arrive in different patterns. For example, Fig. 9 shows the arrival pattern for two types of passengers arriving at station 6. They are (a) passengers who intend to board either line that comes first and (b) passengers who only intend to board line 1. Subfigures (1)-(4) in Fig. 9 represent passengers' perceived bus departure time distributions, passenger boarding probabilities, passenger waiting time functions, and passenger arrival rates, respectively.

Fig. 10 presents the increases in the indicators of Scenario 3 w.r.t. Scenario 2 to illustrate the impact of passenger arrival time choices on bus bunching. Fonzone et al. (2015) stated that the reliability-based arrival pattern of passengers can contribute to bus bunching when the arrival process of passengers does not match the service well because they have imperfect knowledge of bus arrival times. They verified this phenomenon in deterministic experiments where the passengers' perceived distribution of bus

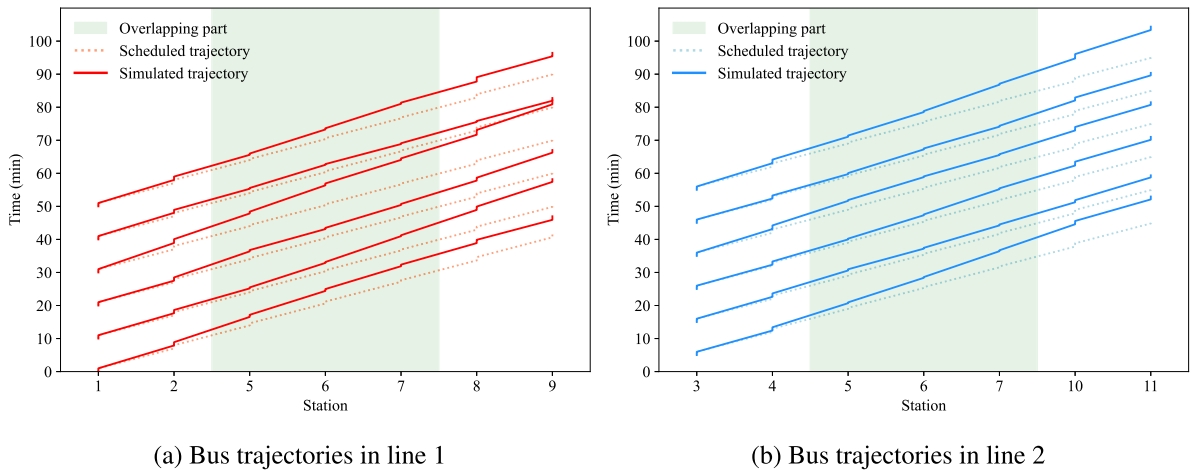


Fig. 8. Bus trajectories in Scenario 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

departure time is predefined as an exogenous input. In our stochastic experiments, similar results are replicated at some stations, such as station 6 of line 1 and station 4 of line 2. More passengers' arrivals miss their expected buses, leading to longer waiting times. However, we can observe that at some stations (such as stations 1, 2, and 8 of line 1, and stations 3 and 6 of line 2), the waiting time is significantly reduced, because more passengers arrive at these stations in proper patterns that are synchronised with the actual bus operation.

Fig. 11 shows the bus trajectories of one simulation in Scenario 3. It can be seen in Fig. 11(a) that the fourth bus on line 1 is delayed, and the delay is aggravated along the line in the overlapping part (green background), but after entering the single-line part its headway gradually recovers. This self-recovery phenomenon was not observed in Scenario 2 with uniform passenger arrival settings.

6.1.4. Effect of transfer demand on bus bunching

Experiments in Scenario 4 are constructed under the same settings as Scenario 3, but transfer passengers are considered. The comparison results are shown in Fig. 12. It is found that both line 1 and line 2 perform worse in terms of headway fluctuations. As for the waiting time, there is a spreading phenomenon in common stations: the waiting time for one line increases but decreases for the other line. A similar spreading phenomenon between two parallel lines was found firstly in Schmöcker et al. (2016) by modelling passengers' queue-swapping behaviours. Fig. 12 shows that the transfer passengers across lines also contribute to the spreading phenomenon. Transfer passenger flow can also be regarded as a special passenger arrival pattern at the transfer station. The accumulation of transfer passengers is highly related to the upstream operation status of their original line. This can explain why the interaction between lines is observed when transfer demand is considered.

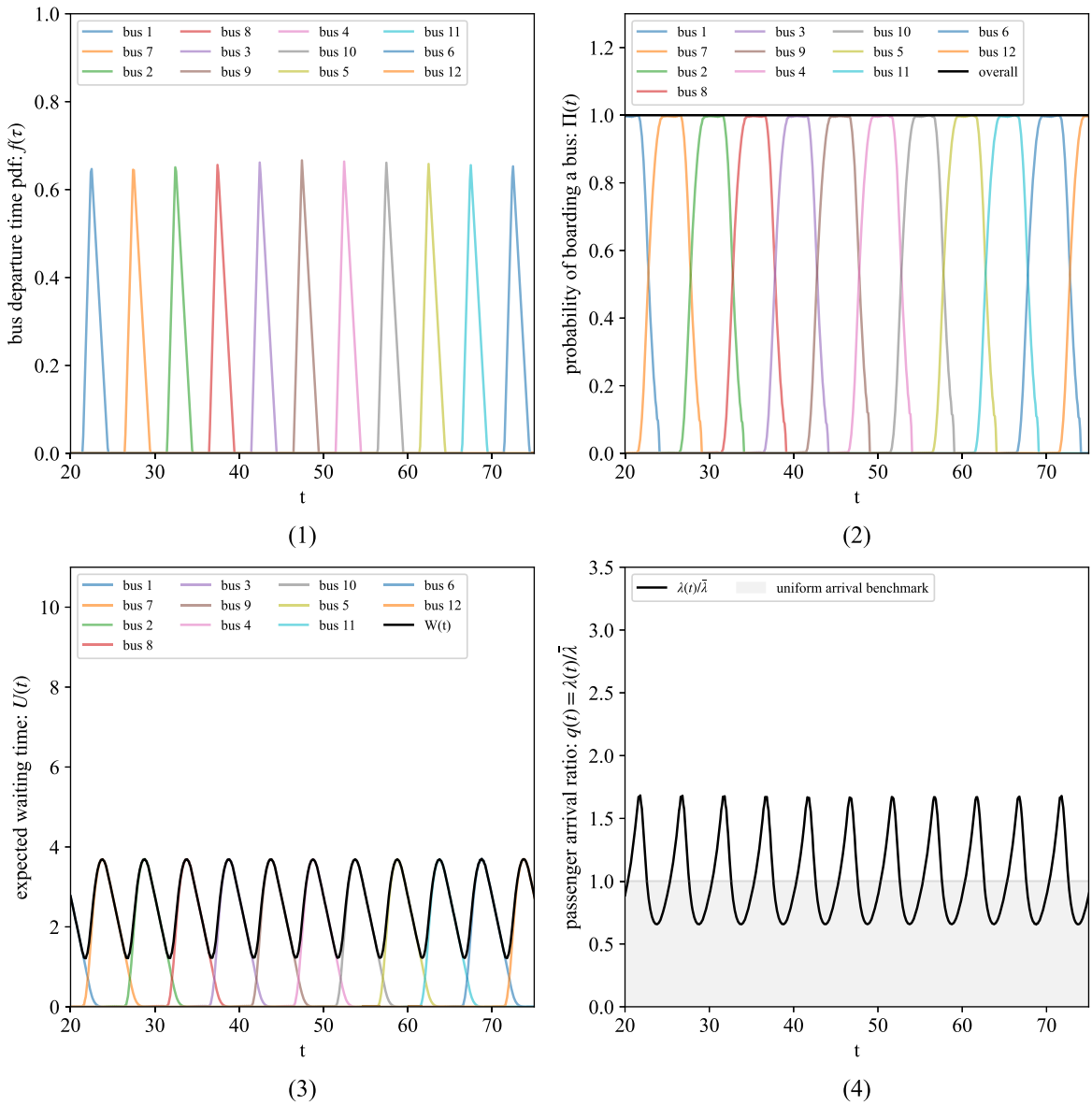
The bus trajectories obtained in a simulation in Scenario 4 are shown in Fig. 13. It can be seen that the fifth bus on line 1 is delayed, and more transfer passengers are brought by line 1 at common stations 5, 6, and 7. As a result, the fifth bus on line 2 is delayed and the bunching problem spreads from line 1 to line 2.

6.1.5. Effect of capacity constraint on bus bunching

To investigate the impact of capacity constraints on bus bunching, Scenario 5 is designed to compare with Scenario 4. The capacity constraint is only activated when a bus is fully occupied and a fail-to-board event occurs. In the conventional procedure of bus service provision design, both passenger demand and available bus models are considered to determine bus service frequency and prevent passengers' boarding failures caused by bus capacity constraints. However, in reality, bus capacity shortages and passenger boarding failures can be observed during peak hours due to unpredictable traffic conditions and bus bunching issues. The purpose of the following experiment in Scenario 5 is to depict such occasions of capacity shortage, with the bus capacity set to be 30 pas.

Fig. 14 shows the changes in the indicators in Scenarios 4 and 5. It can be observed that under capacity constraints, the standard deviations of headway for both line 1 and line 2 decrease in Scenario 5. This suggests that capacity constraints could alleviate the irregularity of bus headway. The improvement is noticeable further downstream along the line. In contrast, the waiting time indicators fluctuate along the line under capacity constraints. A spreading effect through the overlapping section is observed: as the waiting time for one line decreases, the waiting time for the other line increases.

Fig. 15 shows the bus trajectories in Scenario 5, where the bunching problem is significantly suppressed. In Scenario 5, although the trajectory of line 1 deviates from the benchmark schedule of Scenario 1, the headway remains regular along the line. Line 2 in Scenario 5 adheres well to the scheduled trajectory. In contrast, when there is no capacity constraint, the bunching problem tends to worsen along the line, which can be seen from trajectories in Scenario 4.



(a) For passengers can board either line

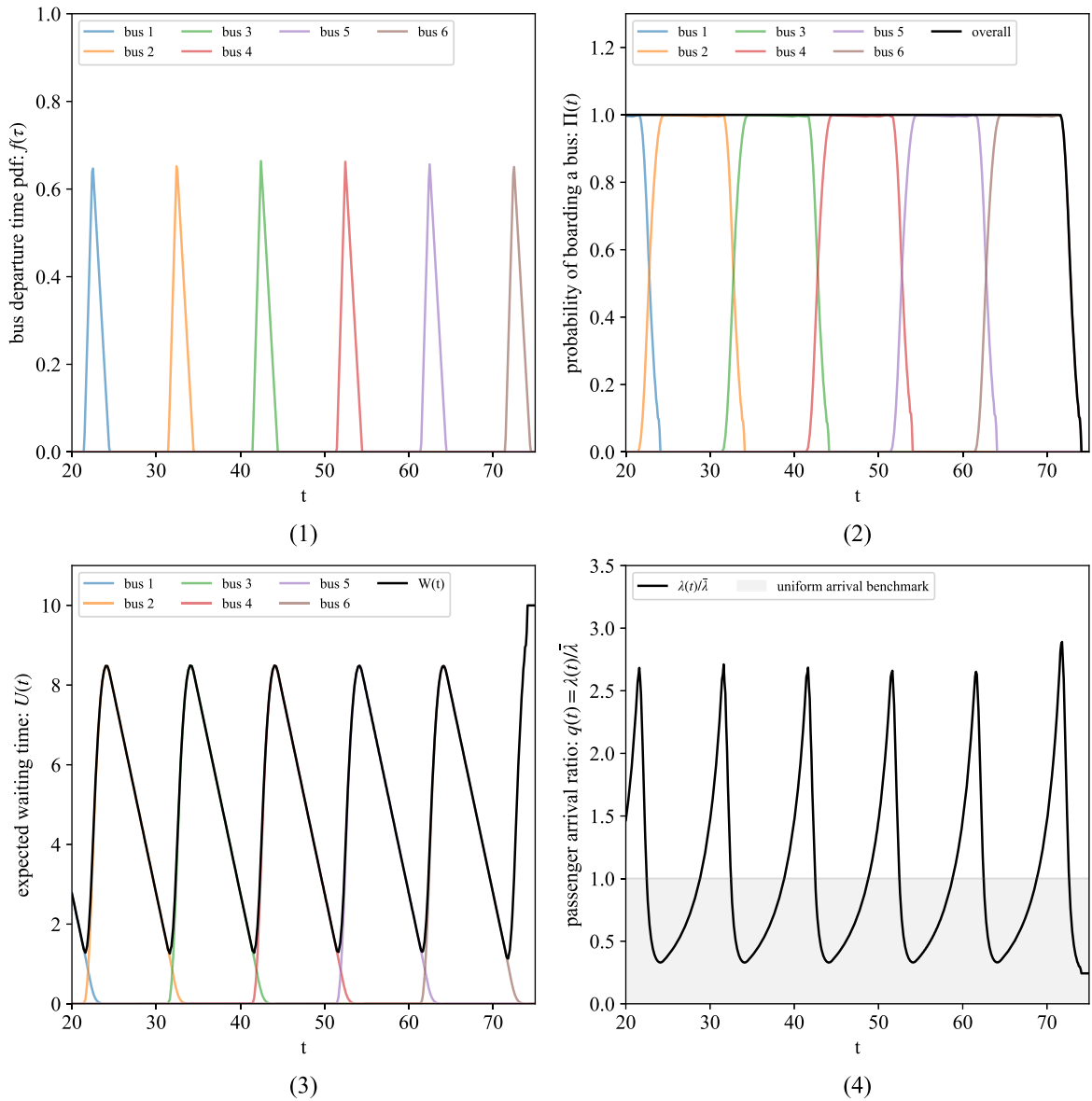
Fig. 9. Passenger arrival pattern.

The mechanism of capacity constraints alleviating the bunching trend has been explained in previous single-line literature (Liang et al., 2021; Wu et al., 2017): appropriate bus capacity constraints serve as an upper limit on the number of boarding passengers, preventing a bus from dwelling too long time at stations with particularly high boarding demand due to bunching. Our experiment with a common-line configuration further confirms that capacity constraints also limit the spreading phenomenon of headway fluctuations from lines with high travel time randomness to lines with low randomness.

6.2. Case study

6.2.1. Real data

The case study is based on three bus lines (line 549, line 601 and line 392) in Beijing as shown in Fig. 16. There are four common stations, marked as n_1, n_2, n_3, n_4 . We focus on one direction operation from n_1 to n_4 . Station n_1 is served by line 549 and line 601, station n_4 is served by line 601 and line 392, and stations n_2, n_3 are served by all three lines. Passenger OD information is extracted from the bus IC card data for the period 8:00–11:00 a.m. in April 2018. Buses are dispatched at start terminals with fixed headways,



(b) For passengers only intending to board line 1

Fig. 9. (continued).

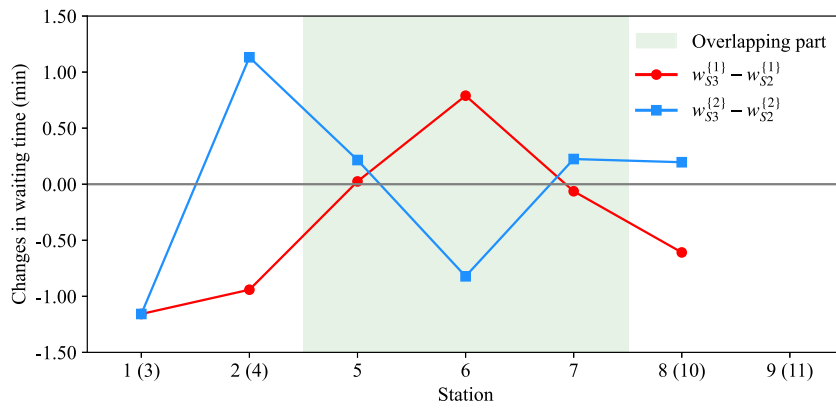
15, 13, and 14 min, respectively, for the three lines. The bus running time between stations is depicted by fitting given GPS data into truncated normal distributions.

To validate the accuracy of our proposed model, we employ the real bus vehicle GPS data and IC card tapping data to compare with the bus trajectories calculated by the model. Fig. 17(a) presents the bus trajectories of bus line 601 in Beijing on April 2nd, 2018, derived from data, while bus trajectories calculated by models are shown in Fig. 17(b). The comparison demonstrates that our model effectively reproduces actual bus operations to a certain extent.

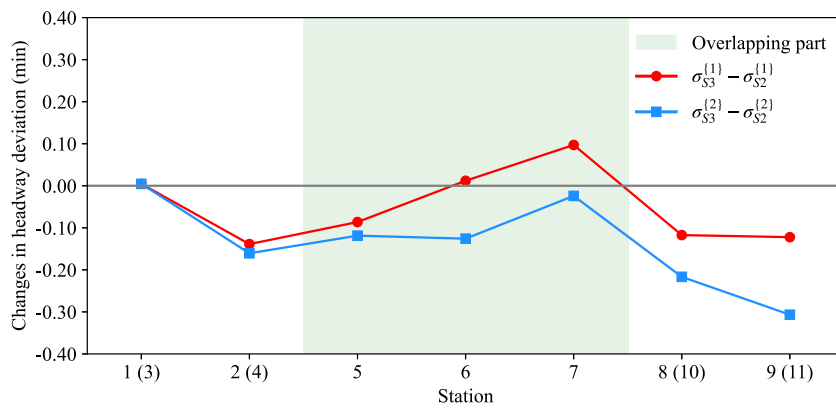
6.2.2. Sensitivity analysis of random traffic status

A sensitivity analysis is conducted to test the impact of uncertain traffic status on bus bunching. The value of the standard deviation of bus running time in the upstream segment of line 549 is adjusted by multiplying a variability ratio, while those of the other two lines remain unchanged. In this sensitivity analysis, a variability ratio varies from 0.5 to 1.5. For each setting of the ratio, 500 simulations were conducted. The performance measures are reported in Fig. 18.

Overall, increasing the running time uncertainty in the upstream segment of line 549 not only leads to worse headway irregularity for itself but also affects the service of other lines to some extent. For instance, as shown in Fig. 18(b), when the standard deviation

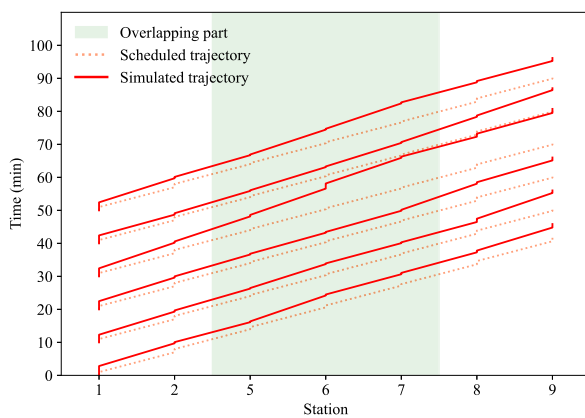


(a) Changes in the waiting time

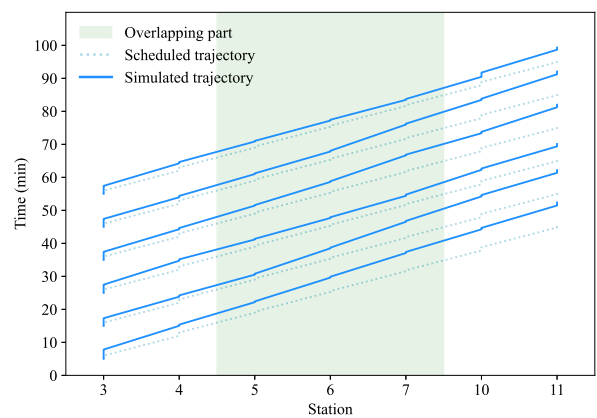


(b) Changes in the standard deviation of headway

Fig. 10. Comparison between Scenario 3 with reliability-based arrival pattern and Scenario 2 with uniform arrival pattern.

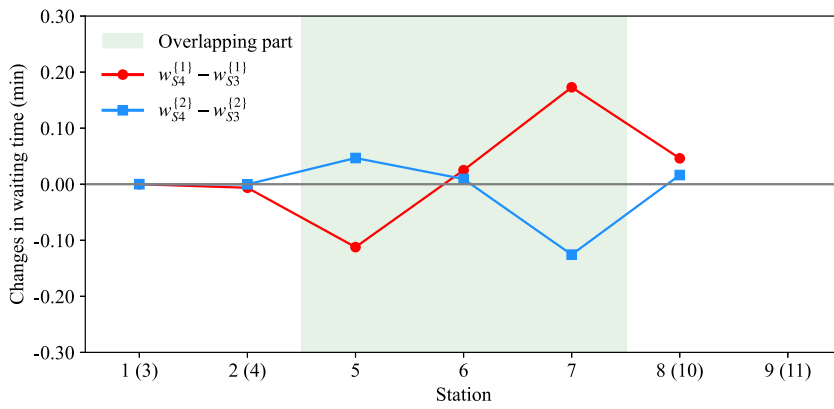


(a) Bus trajectories in line 1

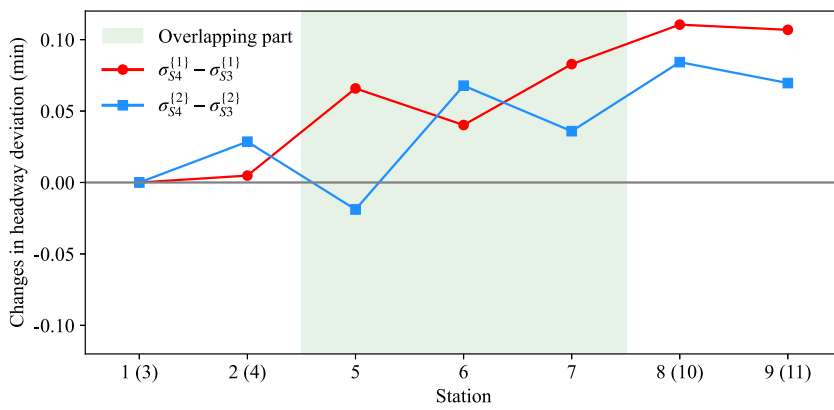


(b) Bus trajectories in line 2

Fig. 11. Bus trajectories in Scenario 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

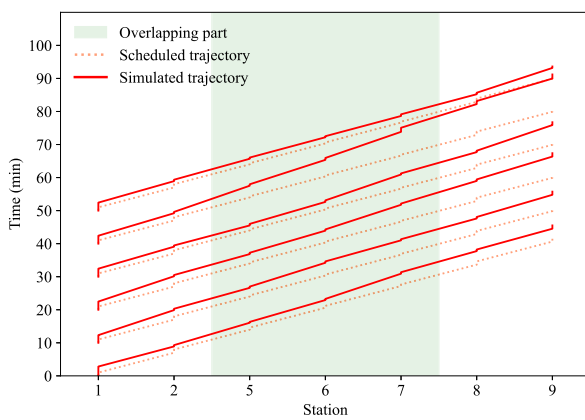


(a) Changes in the waiting time

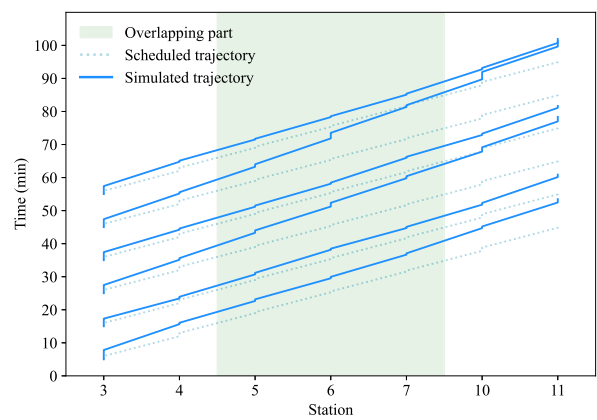


(b) Changes in the standard deviation of headway

Fig. 12. Comparison between Scenario 4 with transfer demand and Scenario 3 without transfer demand.

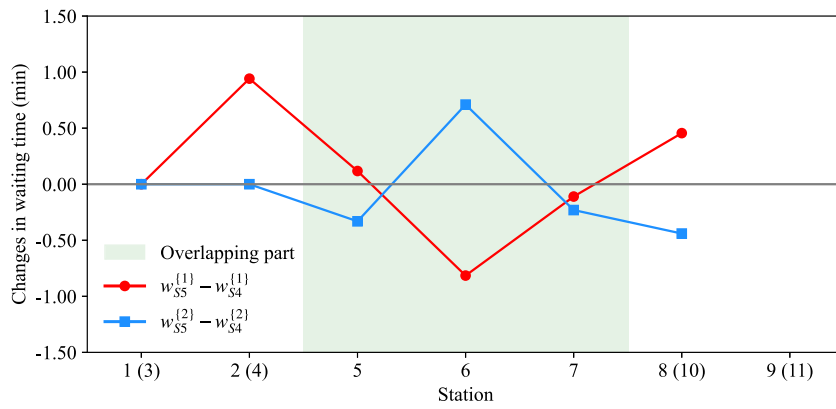


(a) Bus trajectories in line 1

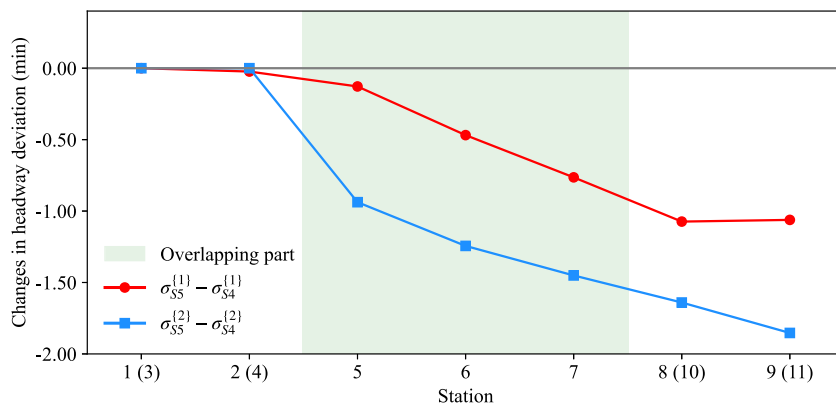


(b) Bus trajectories in line 2

Fig. 13. Bus trajectories in Scenario 4.

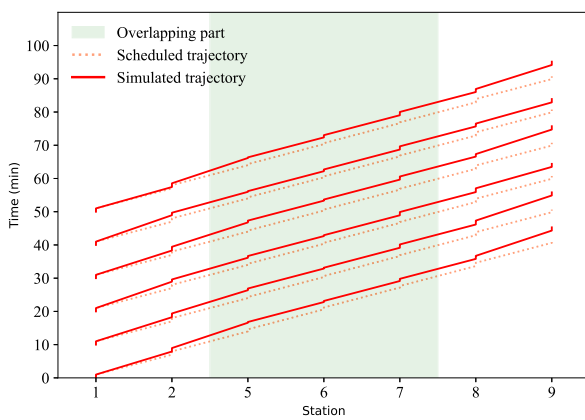


(a) Changes in the waiting time

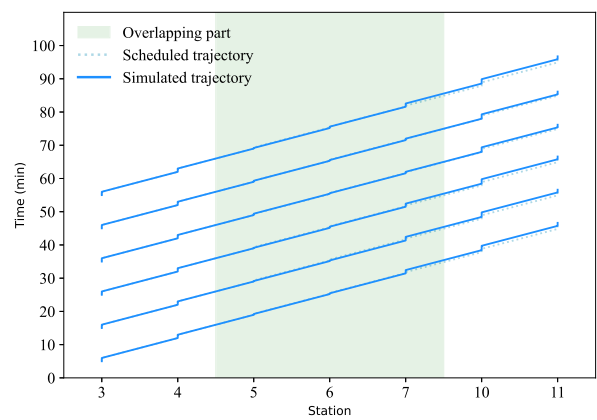


(b) Changes in the standard deviation of headway

Fig. 14. Comparison between Scenario 5 with capacity constraint and Scenario 4 without capacity constraint.

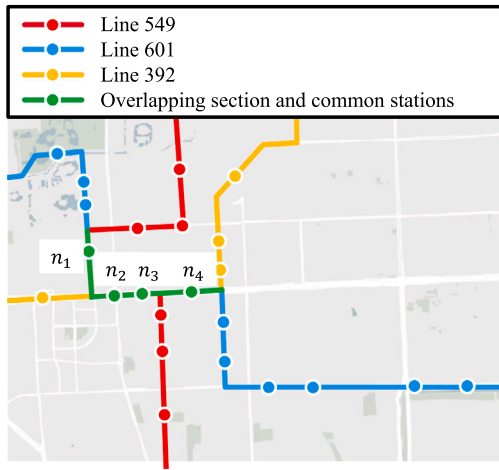


(a) Bus trajectories in line 1

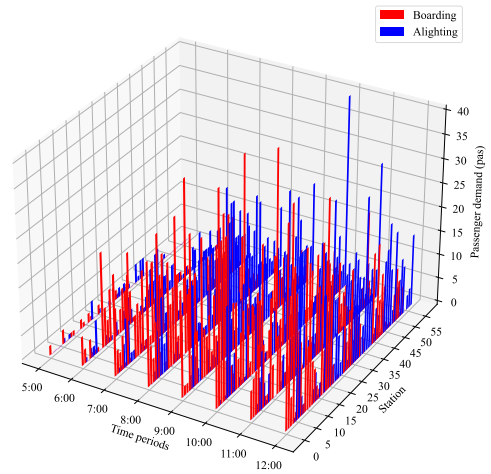


(b) Bus trajectories in line 2

Fig. 15. Bus trajectories in Scenario 5.

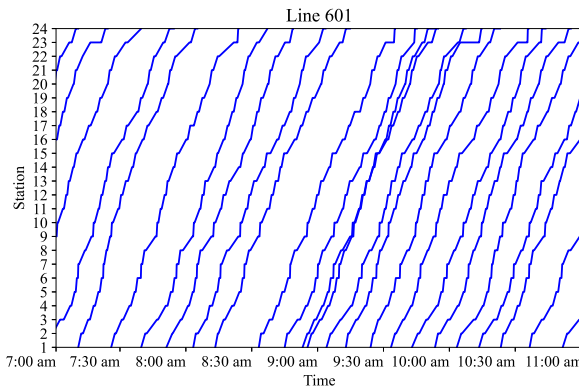


(a) Routes and stations

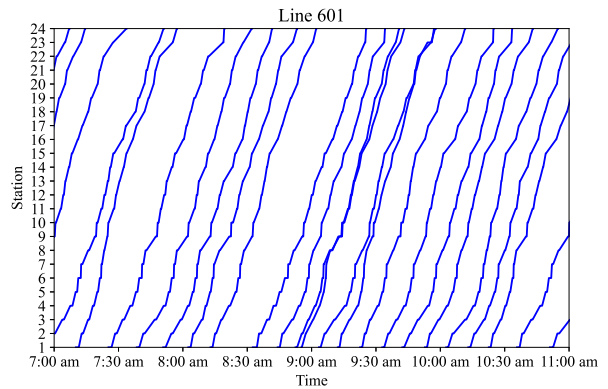


(b) Passenger boarding/alighting demands

Fig. 16. Bus lines 549, 601, and 392.



(a) Trajectories derived from empirical data



(b) Trajectories obtained by model

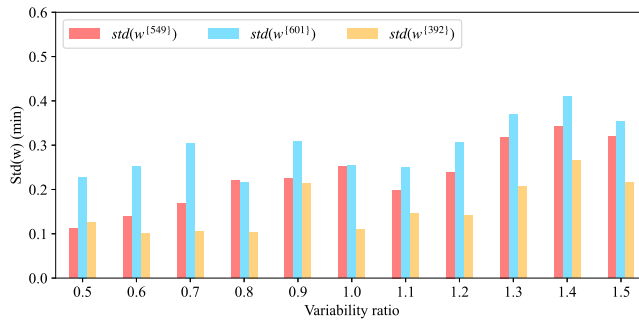
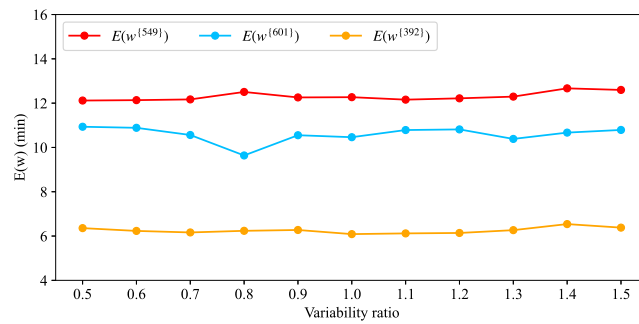
Fig. 17. Comparison of bus trajectories.

of the bus running time of line 549 is enlarged by 1.5 times, its headway fluctuation increases from around 2 min to around 3 min. Meanwhile, the standard deviation of headway for line 392 increases from around 1.4 min to around 1.7 min.

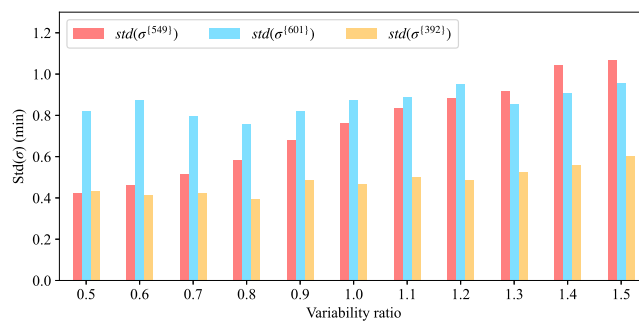
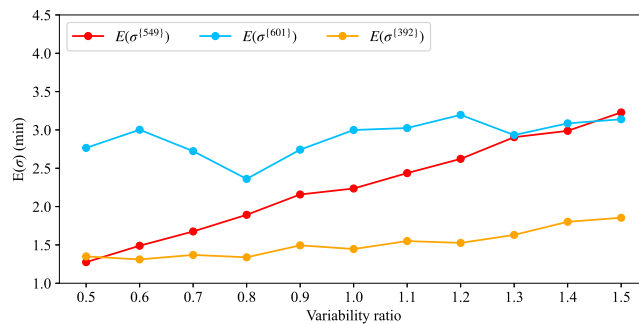
Taking the passengers with OD pair (n_1, n_4) as a representative example for the Type (III) passengers, the convergence process for transfer choices of this passenger group in the MSA algorithm is shown in Fig. 19. Specifically, the transfer approaches represented by Approaches 1, 2, and 3 have been illustrated in Fig. 3. Choosing Approach 1 means waiting for direct bus line 601 when a bus on line 549 arrives at station n_1 . Choosing Approach 2 means boarding line 549 first and seeking to transfer to line 601 or 392 at station n_2 . Choosing Approach 3 means boarding line 549 first and seeking to transfer to line 601 or 392 at station n_3 . It can be seen that after 30 iterations, the passenger transfer choices tend to be stable, indicating that a steady-state solution has been reached.

6.2.3. Sensitivity analysis of transfer demand

In this experiment, the number of potential transfer passengers is adjusted to examine its impact on the bus bunching. The transfer demand variability ratio varies from 0.5 to 1.5, representing that the number of transfer potential transfer passengers changes from 50% to 150% while other setups remain the same. The resultant indicators at different levels of transfer demand are presented in Fig. 20. The bus bunching problem of line 549 is alleviated, measured by headway fluctuations when the proportion of transfer demand increases especially during the range [0.7, 1.2]. Within the same range, a rising trend of headway fluctuations is observed in line 601 and line 392. Notably, when the proportion of transfer demand is enlarged by 1.5 times, the headway fluctuations measured by the standard deviation of headway increase by 30.32% and 24.35% for line 601 and line 392, respectively. This can be explained by the spreading effect that the bunching problem in line 549 spreads to the other two lines more evidently when the proportion of transfer passengers increases.



(a) The average waiting time



(b) The headway deviation

Fig. 18. Sensitivity analysis of random traffic status.

6.2.4. Sensitivity analysis of bus travel demand

To investigate the impact of fluctuating total bus travel demand, a sensitivity analysis is executed by adjusting the travel demand variability ratio from 0.5 to 1.5 in increments of 0.1.

As illustrated in Fig. 21, both the waiting time and the standard deviation remain at nearly similar levels when the travel demand variability ratio ranges within [0.5, 1.1], primarily due to low occupancy. However, when the variability ratio exceeds 1.2, different

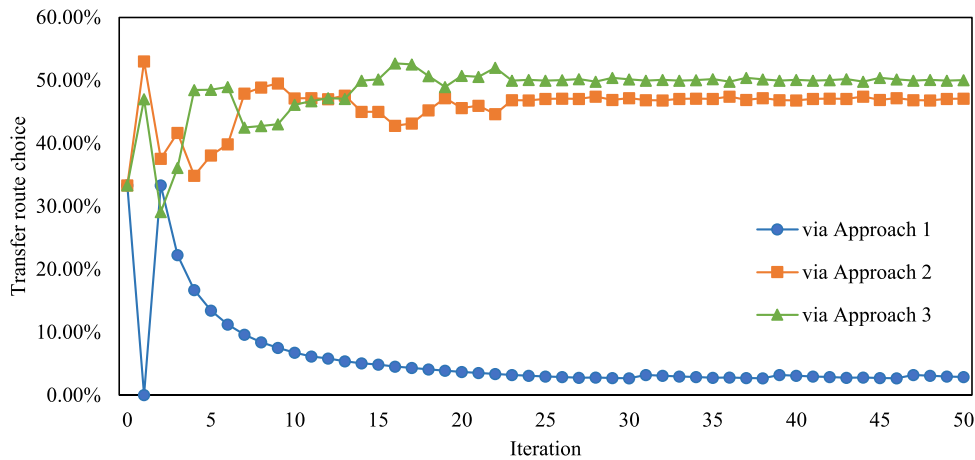


Fig. 19. Convergence of the transfer choice for passengers from station n_1 to station n_4 .

levels of bus travel demand lead to varying waiting times and standard deviations of headway. As demand increases, the waiting time notably becomes longer. Conversely, the standard deviation of headway relatively decreases at high demand levels.

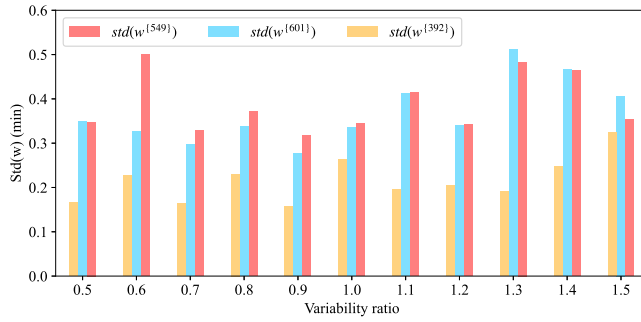
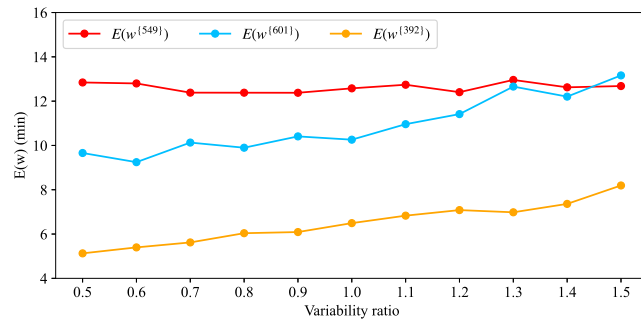
This phenomenon can be explained by examining the counts of bus full-load events presented in Fig. 22. No full-load events are observed when the travel demand variability ratio is under 1.2, indicating the capacity of the three bus lines is adequate. However, when the travel demand ratio exceeds 1.2, capacity constraints become active, which, to a certain degree, restricts the fluctuation in headway and contributes to longer waiting times.

7. Conclusion

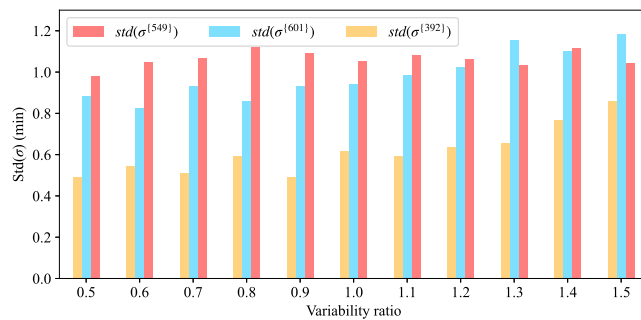
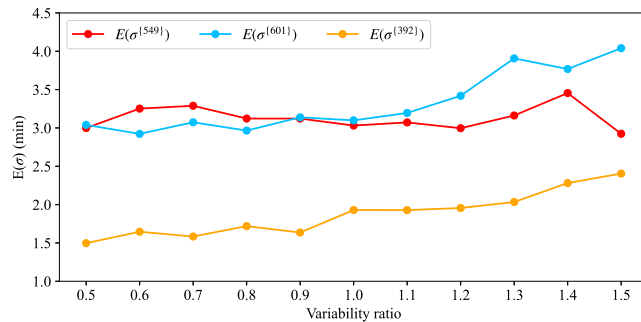
This study establishes a multi-line bus bunching model that integrates both bus motion and passenger choice models. It advances the existing studies that primarily focus on single-line bus bunching by considering the interaction among bus lines running along a common line corridor. For the first time, the joint effect of passengers' arrival time and transfer choices on bus bunching is considered. The stochastic nature of traffic status and passengers' perceived travel time is captured and an equilibrium state is obtained via iteratively applying logit choice models in an MSA-type framework. Numerical experiments are conducted to illustrate the characteristics of bus bunching in the common line configuration and explore the influence of variabilities in travel time, transfer demand, and the total bus travel demand on bus bunching. In summary, the main conclusions of this study can be summarised as follows:

1. Bus bunching can spread across parallel lines through common line stations, even in the absence of queue-swapping behaviours described in the literature (i.e., Schmöcker et al., 2016; Wu et al., 2017).
2. Passenger transfer behaviours amplify the spreading of bus bunching across lines. A high level of transfer demand can partially alleviate the bunching problem of the original line with a side effect that the service irregularity of other parallel lines increases.
3. It has been widely believed that bus bunching, whether it propagates along the original bus line or spreads to parallel lines, demonstrates an inevitable downstream trend. However, this is challenged in this study when passenger arrival rates are time-dependent and in a high variability mode. Unlike previous bus bunching literature assuming a constant rate of passenger arrival (e.g., Newell and Potts, 1964; Schmöcker et al., 2016; Chen et al., 2022), this study supplements the previous theory with a new condition: passenger arrival rates are supposed in a low variability mode or time-independent.
4. The severe deviations in bus headways can be mitigated if passengers' arrival patterns match those obtained from the reliability-based model and are well synchronised with the bus motion under stochastic travel times. This novel insight is attributed to considering the interaction between passenger choices and bus operation. It is distinguished from the existing findings in Fonzone et al. (2015) where passenger arrival patterns are predefined as exogenous inputs to a deterministic bus propagation model.

This study opens several future research directions. Firstly, it can be extended to address more specific situations, such as different bus station configurations (Schmöcker et al., 2016; Wu et al., 2017; Bian et al., 2019) that allow for the simultaneous boarding and alighting of passengers from multiple buses belonging to different lines. Additionally, incorporating multimodal and mode choice would allow for exploring their impacts on bus bunching and might provide valuable insights for bus operations. Furthermore, it would be a promising direction to develop a methodology that guides passenger arrival time and transfer choices by providing passengers with real-time information, including bus arrival time and bus crowding information (Wang et al., 2021). This would enable the provision of route recommendations (e.g., Ceder and Jiang, 2020; Jiang and Ceder, 2021) to passengers, thereby facilitating more reliable and efficient bus operations.

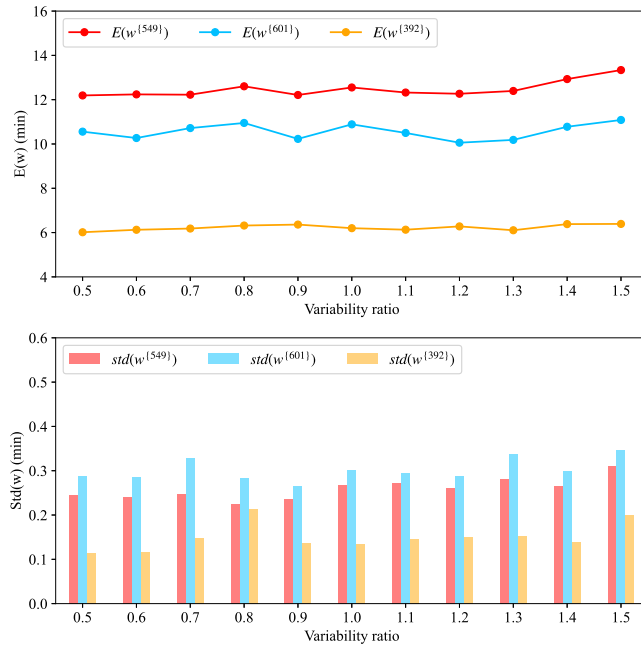


(a) The average waiting time

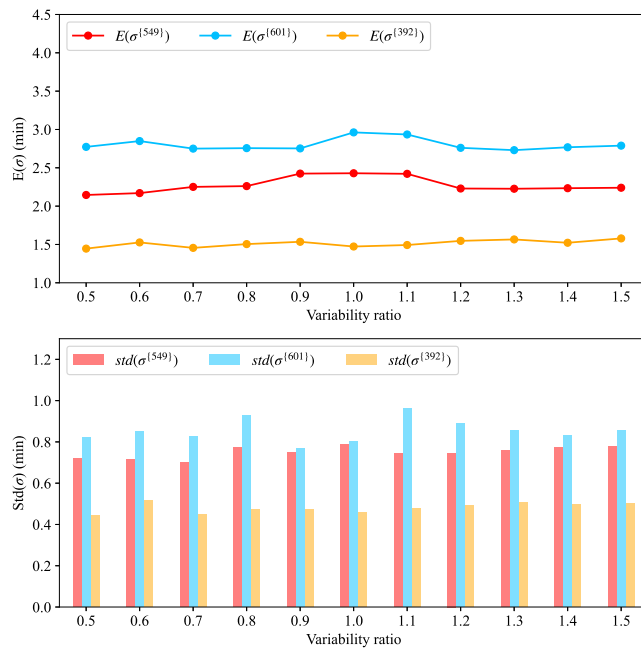


(b) The headway deviation

Fig. 20. Sensitivity analysis of transfer demand.



(a) The average waiting time



(b) The headway deviation

Fig. 21. Sensitivity analysis of bus travel demand.

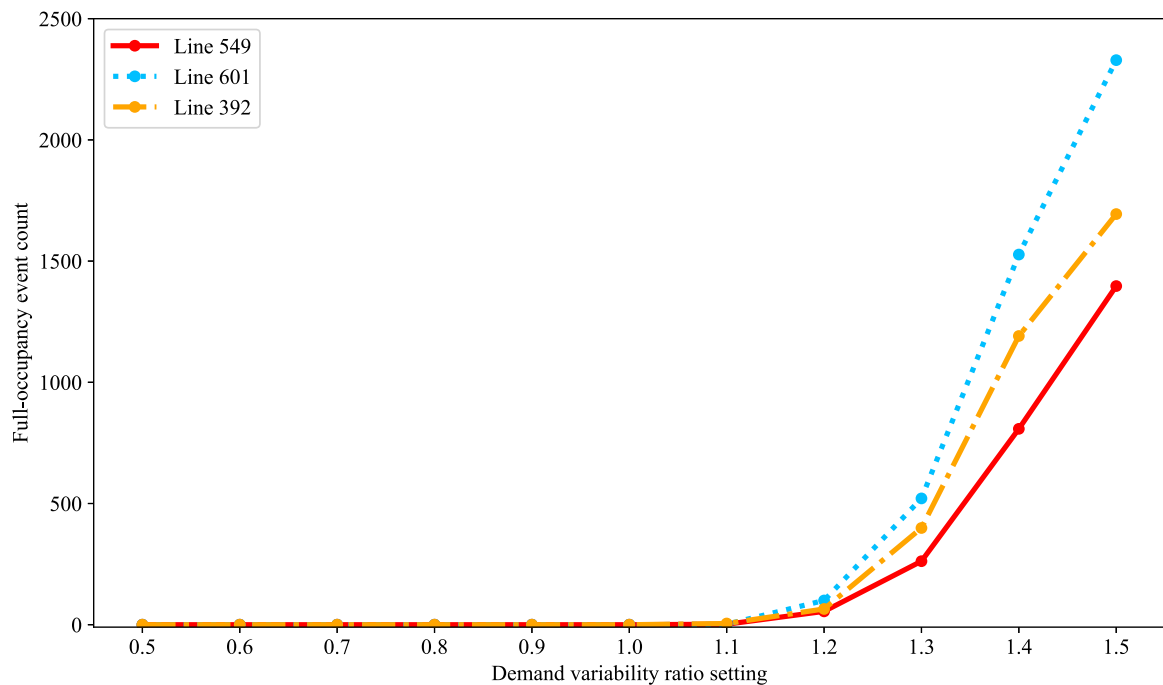


Fig. 22. Full-occupancy event count under varying travel demand levels.

CRedit authorship contribution statement

Zhichao Wang: Methodology, Software, Data curation, Formal analysis, Writing – original draft. **Rui Jiang:** Conceptualization, Methodology, Funding acquisition, Data curation, Supervision, Writing – review & editing. **Yu Jiang:** Conceptualization, Methodology, Formal analysis, Writing – review & editing. **Ziyou Gao:** Resources, Project administration. **Ronghui Liu:** Conceptualization, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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