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RESEARCH ARTICLE

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Validation of fibre stress utilization model for modified ring spun yarns

Muhammad Zubair[a], Fayyaz Ahmad[b], Zakariya Zubair[c], Abdul Jabbar[a,d] and Jiya Baig[b]

[a]School of Engineering and Technology, Department of Textile Engineering, National Textile University, Faisalabad, Pakistan; [b]School of Science, Department of Applied Science, National Textile University, Faisalabad, Pakistan; [c]School of Engineering and Technology, Department of Materials, National Textile University, Faisalabad, Pakistan; [d]School of Design, University of Leeds, Leeds, UK

ABSTRACT

To model the fibre stress utilization in modified ring spun yarns, we developed an analytical formula from the experimental data. The development of empirical formulae is carried out by using two different techniques, *i.e.*, Cubic Spline and Artificial Neural Network methods. The experimental data of stress-strain curves of fibre and yarn has a large variation. To cope this variability, we used the smoothing spline technique to find the best-fit curve with respect to a reasonable smoothness. The best nonlinear smooth fitting can be used to extrapolate the experimental data beyond the breaking point. The modified ring spun yarns (compact, SIRO and SIRO-compact) with 20/1, 30/1 and 40/1 English count, produced from viscose staple fibre, were used to predict fibre stress utilization up to the yarn break by extrapolating the mean stress-strain curves of fibre and yarn by using the artificial neural network. Moreover, a new distribution function of fibre distribution in yarn has been proposed and successfully implemented for the prediction of fibre stress utilization in yarn. The new formulation helps to compute the fibre stress utilization in the yarn analytically. The validation of the proposed methodology is presented by comparing the numerical results with the experimental data. The predicted fibre stress utilization was in good agreement with the experimental fibre stress utilization for all types of modified ring spun yarns. It has been observed that SIRO-compact yarn exhibits improved fibre stress utilization as compared to SIRO and compact yarns. Moreover, the new distribution functions Gamma and Gaussian distribution were introduced in parallel with the Dirac delta function. In previous similar studies on ring, rotor and air-jet spun yarns, the proposed model can only predict the fibre stress utilization before the breakage point whereas the modified model, in this study, can predict the fibre stress utilization up to the breaking point.

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Fibre stress utilization; artificial neural network; cubic spline; Gaussian quadrature method; modified ring spun yarns

1. Introduction

Fibres are the fundamental unit, or the building blocks used in the manufacturing of yarns and fabrics. Textile fibre is a hair like material mainly produced from natural or synthetic sources which is a raw material for manufacturing of textile yarn. Textile yarn is an assemblage of fibres, twisted or laid together to form a continuous strand that can be used for manufacturing of textile fabric. Textile woven fabrics produced from staple spun yarns have many applications in weaving, knitting and embroidery due to their reasonable strength, elongation, and flexibility. A wide range of mathematical concepts have been used in textile technology to predict different yarn terms and parameters (Rani & Mercy 2018). The mathematics that we use in textile technology is known as textile mathematics, and it has many applications in spinning, weaving, and knitting. In mathematics, some problems can be solved analytically and numerically. Analytic means 'exact' while numerical means 'approximated'. An analytical solution involves framing the problem in a well-understood form and calculating the exact solution. A numerical solution means making assumptions at the solution and testing whether the problem is solved well enough to stop. It is hard to guarantee the accuracy and convergence in a numerical solution. We prefer the analytical method in general because it is faster and gives an exact solution. Sometimes, the analytical solution is unknown, and we have to work with the numerical approach. Algorithms and models expressed with analytical solutions are often more efficient than equivalent numerical implementations. We can use the word 'transparent' for analytical solutions because these are presented as math expressions, they offer a clear view into how variables and interactions between variables affect the results.

Fibre strength of an individual fibre is an important parameter which is useful for the prediction of yarn strength. Fibre strength is very often the dominating characteristics in textile processes. This can be seen from the fact that the nature produces many different fibres, most of which are not usable for textiles because of their inadequate strength. The fabric strength of all types of woven, knitted, non-woven and braided fabrics is dependent on fibre strength. The first theoretical approach regarding to yarn strength was done by R. R. Sullivan (Gegauff, 1907; Rani & Mercy, 2018; Sriprateep & Pattiya 2009). K. Sriprateen and A. Pattiya presented a method for modelling

CONTACT Fayyaz Ahmad fayyaz.ahmad@ntu.edu.pk; Abdul Jabbar a.jabbar@leeds.ac.uk

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the tensile behaviour of multi-filament twisted yarn using computer aided design approach for tensile strength (Sriprateep & Pattiya, 2009; Sullivan 1942). S. Das and A. Gosh used simulated annealing to solve Frydrych's approach to explore best combination of parameters that can produce yarn with the required strength (Das & Ghosh, 2015; Sullivan 1942). Neckar and D. Das presented a theoretical model of tensile behaviour of staple spun yarns with a single distribution function which describes briefly about fibre stress utilization in yarn (Das & Ghosh, 2015; Neckář & Das, 2017). Zubair *et al.* validated the theoretical model through some experimental data of fibre strength utilization before the breakage point of yarns (Zubair, Eldeeb, et al., 2017; Zubair, Neckář, et al., 2017; Zubair, Neckar, et al., 2017). The model validation was limited to ring, rotor and air-jet spun yarns (Zubair, Eldeeb, et al., 2017; Zubair, Neckář, et al., 2017; Zubair, Neckar, et al., 2017; Zubair et al., 2020). The model prediction was good in those studies, but the limitation was its inability to predict the yarn strength up to the breaking point, which is often required by yarn producers (spinners). In this study, the numerical methods and complex distribution function are introduced to provide a solution to this limitation. The fibre stress utilization is predicted up to yarn breaking strength using an analytical approach with a simplified distribution function. The model is validated on modified ring spun yarns produced using compact, SIRO and SIRO-compact technologies.

2. Materials and methods

The viscose fibres of 1.4 dtex linear density and cut length of 38 mm were used to produce three different types of yarn with three yarn linear densities: 29.53 tex (20/1 N_e), 19.69 tex (30/1 N_e) and 14.76 tex (40/1 N_e) using compact, SIRO and SIRO-compact ring spinning technologies.

The yarn linear density of all types of yarn was measured according to the standard test procedure ASTM-D 1907. A lea of one hundred twenty yards of five specimens for each type of yarn were prepared on lea making machine and each sample was weighed in grams on a weighing balance. The yarn count in the tex and indirect cotton system was determined from the mean of five samples.

The yarn diameter was calculated using the relation, yarn diameter [mm] $= \frac{0.9}{\sqrt{\mathrm{Ne}}}$, where N_e is yarn count in the indirect cotton system. Twist per meter were measured on twist testing machine according to standard procedure ASTM-D 1422. Twist in meter for ten samples were measured by the untwist and retwist method for each type of yarn and mean value of ten specimens was recorded. The twist angle β_d of surface fibre along the yarn axis was determined from the relation $\tan \beta_d = \pi DZ$, where D is yarn diameter in mm and Z is yarn twist per meter.

The nominal and actual yarn linear density, yarn twist per meter, yarn diameter and twist angle are recorded in Table 1.

All the fibres were tested for stress-strain behaviour using single-fibre tensile testing equipment in agreement with the ASTM-D 3822 standard test method. A total of twenty measurements of fiber specimens were made for fiber tensile testing. The gauge length was 10 mm and tensile force was applied at a rate of 10 mm/min. Due to the large variability in the fibre testing data, mean stress-strain curves from 20 individual curves of fibres stress-strain curves were obtained up to the break point by extrapolation in Matlab.

All types of yarns SIRO, compact and SIRO-compact were tested for tensile stress–strain behaviour using the Instron-4411 tensile tester in compliance with the ASTM-D 2256 standard test method. The yarns were tested for force and elongation at corresponding speeds depending upon the yarn type and the gauge length being 500 mm for all types of yarns. The tensile force application time was kept from 17 to 22 s. A total of twenty tests were performed on each specimen of the yarns.

There was a large variability in the yarn testing data curves due to the breakage of yarn at different points. This phenomenon happens because of the random distribution of fibre elements in yarn helical structure. To cope variability, we extrapolated the data curves by using the smoothing spline technique to find the best fit curve with respect to a reasonable smoothness. In mathematics, a spline is a special function called piece wise function, that is frequently used to build an interface that allows a user to design and control the shape of complex curves and surfaces having some data variability (Birkhoff, 1969). The most common spline is cubic spline established from piece-wise cubic third-order polynomials which transit through a set of 'n' control points.

We employ two equations, Equations (1) and (3), to estimate fiber stress utilization involving single and double integrals. For a precise estimation, the integrals must be accurately approximated. Our stress-strain data varies in length due to varying breakdown points. We extrapolated each stress-strain curve to its breaking point using the cubic spline method. After this extrapolation, we compute an average curve from the aligned curves. This average curve is in numerical form, and for empirical representation, we use an artificial neural network (ANN). The ANN model has input and output layers, with the input layer having three nodes. With six weights and three biases, the ANN offers a reliable analytical approximation for our average stress-strain curve. We plug this ANN approximation into Equations (1) and (3) to estimate fiber stress utilization. The single and double integrals are approximated using trapezium and Gaussian quadrature techniques. The procedure for the mean curves for fiber and yarn stress-strain up to the breaking point and the fiber stress utilization is mentioned in Figure 1.

When all fibre and yarn stress-strain data curves are extrapolated such that their upper maximum and lower minimum limit points are considered then we can find the mean curve for fibre and yarns from the extrapolated data curves, as shown in Figure 2. All the yarn specific stress-strain curves lie under the fibre specific stress-strain curves.

3. Results and discussions

3.1. Theoretical model

To predict the fibre stress utilization coefficient, the ratio between fibre stress and yarn stress of staple yarns is an important parameter of yarn to know the yarn strength. Yarn structure provides information about the practical

Table 1. Yarn specification.

No.	Spinning techniques	Yarn linear density (tex) Nominal	Actual	Yarn twist (tpm)	Yarn Diameter(mm)	Twist angle (β_d) [degree]
1	Compact	29.53 (20/1 N_e)	28.0 (21.09/1 N_e)	655	0.23	24.20
2	SIRO	29.53 (20/1 N_e)	29.74 (19.87/1 N_e)	652	0.23	23.80
3	SIRO compact	29.53 (20/1 N_e)	29.53 (20/1 N_e)	640	0.23	24.04
4	Compact	19.68 (30/1 N_e)	18.98 (31.11/1 N_e)	756	0.20	25.36
5	SIRO	19.68 (30/1 N_e))	19.68 (30.01/1 N_e)	745	0.20	25.07
6	SIRO Compact	19.68 (30/1 N_e)	19.20 (30.76/1 N_e)	742	0.20	24.98
7	Compact	14.76 (40/1 N_e)	13.85 (42.64/1 N_e)	895	0.16	25.31
8	SIRO	14.76 (40/1 N_e)	14.76 (40.01/1 N_e)	878	0.16	25.22
9	SIRO compact	14.76 (40/1 N_e)	14.85 (39.77/1 N_e)	890	0.16	24.80

Note: Values in parentheses represent count in indirect cotton system.

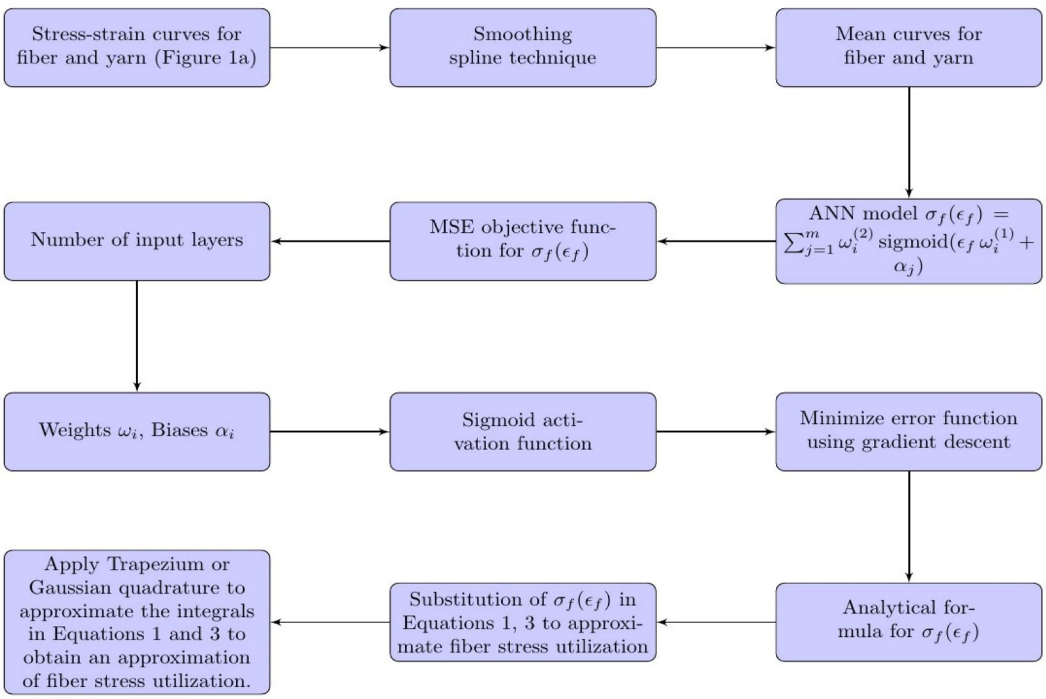

Figure 1. Prediction of mean fiber/yarn stress-strain and fiber stress utilization.

understanding of the coefficient of fibre stress utilization of staple spun yarns. Fibre lies in yarn making helical structural model as shown in Figure 3. The coordinates of fibre elements in a yarn before and after deformation are shown in Figure 4.

We have two mathematical models to compute fibre strength utilization coefficient by using a single distribution function on the basis of general and varying fibre angles in the yarn helical structure. A theoretical model was presented with the concept of a single inclination angle for every fibre in yarn helical structure (Neckář & Das, 2017), and fibre/yarn stress as function of fibre/yarn strain is given below.

$$\varphi_1(\varepsilon_Y) = \frac{\sigma_y(\varepsilon_Y)}{\sigma_f(\varepsilon_Y)} = \frac{2}{\sigma_f(\varepsilon_Y)\tan^2\gamma_D}\int_0^{\gamma_D}\sigma_f(\varepsilon_f)\tan\gamma\, d\gamma \tag{1}$$

For the lowest fibre and yarn strain (0-0.1), we have a relation between fibre and yarn strain

$$\varepsilon_f = \varepsilon_Y\left(\cos^2\gamma - \eta_r\sin^2\gamma\right) \tag{2}$$

A partially generalized theoretical model was presented on the basis of practical concept of various inclination angles denoted by $\theta \in (\theta_v, \frac{\pi}{2})$ for every fibre in yarn helical structure (Neckář & Das, 2017). The second model consists of random character of fibre inclination in yarn, depends upon the probability density function ν of fiber inclinations θ and $\theta_v = \arcsin\frac{1}{\sqrt{(1+\eta)}}$ shown below.

$$\psi(\epsilon_y) = \frac{\sigma_y(\epsilon_y)}{\sigma_f(\epsilon_y)} = \frac{2}{\sigma_f(\varepsilon_Y)\tan^2\gamma_D}\int_0^{\gamma_D}\left[\int_0^{\theta_v}\sigma_f(\varepsilon_f)\cos^2\theta\, u(\theta)\, d\theta\right]\frac{\sin\gamma}{\cos^3\gamma}\, d\gamma \tag{3}$$

Both the integral equations (1) and (3) must have analytical formulae to solve them. Before that, only numerical approximations can be made to find the solution of that integrals due to the term $\sigma_f(\varepsilon_f)$ involved in these equations. Now we can derive an analytical formula for fibre stress as function of fibre strain, $\sigma_f(\varepsilon_f)$ through artificial

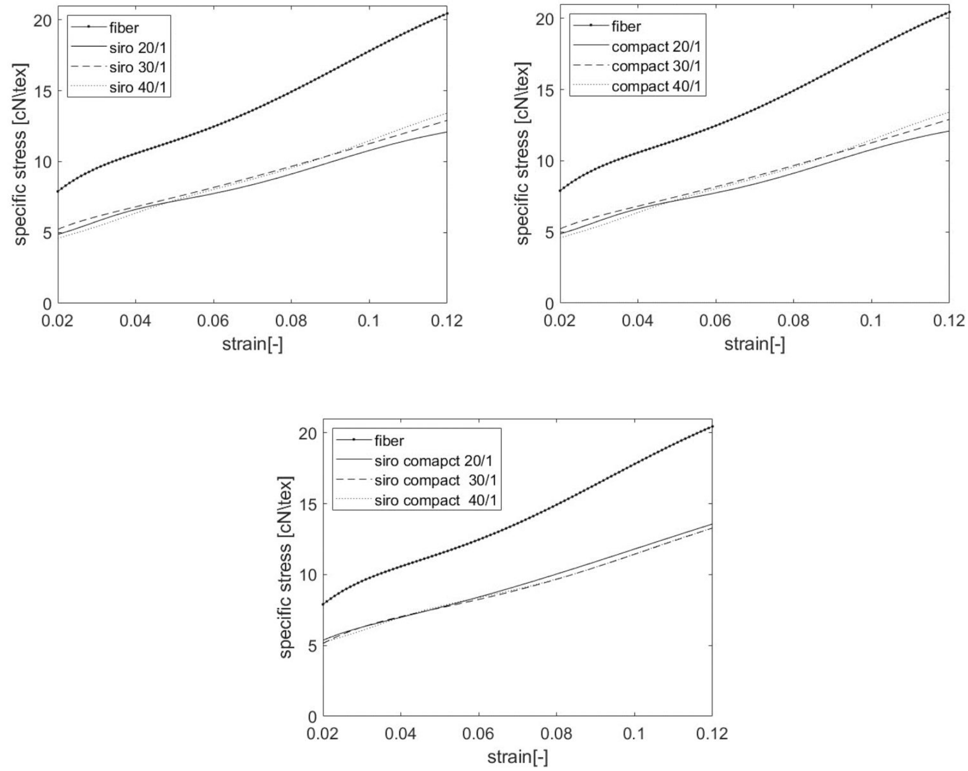

Figure 2. Fibre and yarn mean stress-strain curves.

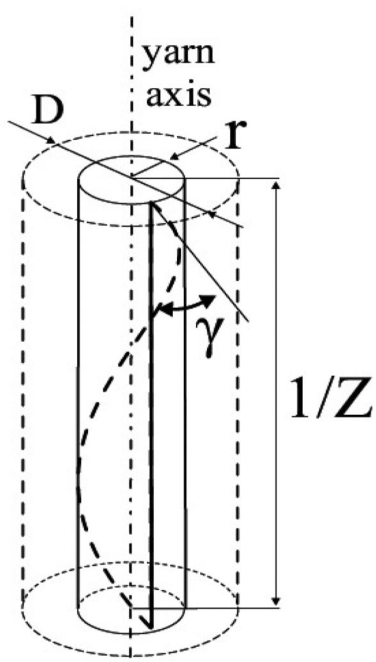

Figure 3. Yarn helical structure. Where, *D* is yarn diameter and *r* is radius of yarn, γ is angle of inclination of fibre along the yarn axis and '1/*Z*' is height of coil of a fibre in yarn.

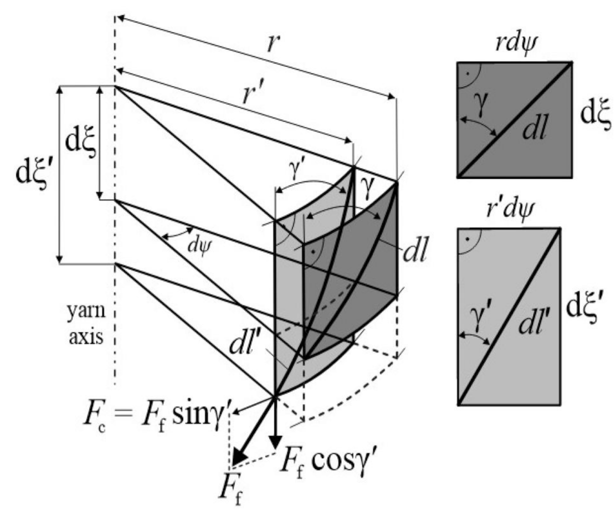

Figure 4. Co-ordinates of fibre element in a yarn before and after deformation.

neural network (ANN) model by using the back propagation (BP) algorithm. This algorithm follows the approach of gradient descent method to minimize the error in the objective function. In this way, this algorithm leads us towards the required solution of the problem by stabilizing the weights assigned to the input layer and hidden layers, which helps to minimize the error function. We can compute the numerical values of $\sigma_f\ (\varepsilon_f)$ but to evaluate the integral analytically, we need some empirical formula. The empirical formula for $\sigma_f\ (\varepsilon_f)$ is constructed by using the artificial neural network (ANN). The objective function is the sum of squared deviation of the ANN computed values of $\sigma_f\ (\varepsilon_f)$ and the experimental values of $\sigma_f\ (\varepsilon_f)$. The gradient of the objective function is defined as:

$$\vec{\nabla}\ \mathrm{obj} = \left[\frac{\partial\ \mathrm{obj}}{\partial\ \alpha_j}, \frac{\partial\ \mathrm{obj}}{\partial\ \omega_j}, \frac{\partial\ \mathrm{obj}}{\partial\ \nu_j} \right] \tag{4}$$

Here 'obj' presents an objective function. We minimize the error and make it sufficiently small according to prescribed tolerance. Let the input values assigned to the input layer are $x_1, x_2, \ldots, x_n$ with their corresponding weights $\omega_1, \omega_2, \ldots, \omega_n$ and biases $\alpha_1, \alpha_2, \ldots, \alpha_n$. According to BP-Algorithm, we assume objective function for the term $\sigma_f\ (\varepsilon_f)$ used in equations (1) and (3) and defined in equation (2), so that, analytical approach can be used for both single and double integral equations for the coefficient of fibre stress utilization in yarn. While, in the previous studies (Zubair, Eldeeb, et al., 2017; Zubair, Neckář, et al., 2017; Zubair, Neckar, et al., 2017), only numerical value was used for that term,

$$\sigma(x_n) = \sum_{j=1}^{m} \nu_j \phi(x_n \omega_j + \alpha_j) \tag{5}$$

In this summation, $m = 3$ and ϕ is sigmoidal function used as an activation function in BP-algorithm.

$$\phi(t) = \frac{1}{1 + \exp(-t)} \tag{6}$$

where, $t = x_n \omega_j + \alpha_j.$

A sigmoidal activation function is a real function with a mapping of the form $f_c: R \rightarrow (0,1)$ and is defined as: $f_c(x) = \frac{1}{1+ e^{-cx}}$ where c is an arbitrary number. As the value of c continues to be increase $(c \rightarrow \infty)$ the shape of the sigmoid converges to a step function at the origin of the plane. It is very simple to derive all the expressions for $c = 1$ (Rasamoelina et al., 2020). By using the gradient descent method, this algorithm leads us towards required solution of the problem by stabilizing the weights assigned to the input layer which helps to minimize the error function defined as: $E(\nu_j, \omega_j, \alpha_j) = \sum_{i=1}^{n} (y_i - \sigma(x_i))^2$, where $j = 1,2,3$. That error function presents the mean squared error in between experimental data curve and neural network fitted curve. There are nine parameters used in back propagation algorithm. The error function used in the BP-Algorithm must be continuous and differentiable as well, because this method requires calculations of the gradient for the error function at each iteration (Yang et al., 2020).

Trapezium rule is also one of the methods to solve a definite integral. More precisely, when it is difficult or impossible to find the exact value of a given definite integral, we use a method known as "The trapezium Rule" to find its approximate value (Al Azah, 2017). The formula given is:

$$\int_a^b f(x)dx \approx \frac{1}{2} h[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n] \tag{7}$$

Since we find out analytical formula for $\sigma_f\ (\varepsilon_f)$ in the form of ANN model. Now we can solve two general mathematical models of the coefficient of fibre stress utilization in yarn by using two familiar methodologies called the Trapezium method and the 3-point Gaussian quadrature method (Brass et al., 1997). First, we have to calculate the integral part of that equation (1). Let's say,

$$I_g = \int_0^{\gamma_D} \sigma_f(\epsilon_f) \tan(\gamma) d\gamma \tag{8}$$

By Gaussian integral transformation:

$$I_g = \frac{b-a}{2} \int_{-1}^{1} \sigma_f\Big(\epsilon_f\big(g(t)\big)\Big) \tan\big(g(t)\big) dt, g(t_i) = \frac{b-a}{2} t_i + \frac{b+a}{2} \tag{9}$$

where $i = 1,2,3$.

From Equation (8)

$$I_g = \frac{\gamma_D}{2} \sum_{i=1}^{3} w_i \sigma_f\big(\epsilon_f(g(t_i))\big) \tan\big(g(t_i)\big) \tag{10}$$

$$I_g = \frac{\gamma_D}{2} \Big(w_1 \sigma_f\big(\epsilon_f(g(t_1))\big) \tan\big(g(t_1)\big) + w_2 \sigma_f\big(\epsilon_f(g(t_2))\big) \tan\big(g(t_2)\big) + w_3 \sigma_f\big(\epsilon_f(g(t_3))\big) \tan\big(g(t_3)\big) \Big) \tag{11}$$

Substituting $\gamma = g(t_i)$ in Equation (2) in respect to Gaussian Quadrature method, we can re-write as follows

$$\epsilon_f(g(t_i)) = \epsilon_y\big(\cos^2(g(t_i)) - \lambda_r \sin^2(g(t_i))\big).$$

According to the Gaussian Quadrature weights table, the following weights and points can be used:

$$w_1 = 5/9, \quad w_2 = 8/9, \quad w_3 = 5/9$$

$$t_1 = -\sqrt{\frac{3}{5}}, \quad t_2 = 0, \quad t_3 = \sqrt{\frac{3}{5}}.$$

Now from Equation (3), let's say,

$$I_g = \int_0^{\theta_v} \sigma_f\big(\epsilon_f(\theta)\big) \cos^2(\theta) u(\theta, \gamma) = \frac{b-a}{2} \int_{-1}^{1} \sigma_f\Big(\epsilon_f\big(g(t)\big)\Big) \cos^2\big(g(t)\big) u\big(g(t), \gamma, \beta\big) dt, \tag{12}$$

where $a = 0$ and $b = \theta_\nu$. From 3-point Gaussian Quadrature rule,

$$g(t_i) = \frac{\theta_v}{2}(1 + t_i) \tag{13}$$

where $i = 1,2,3$. From equation (12),

$$I_g = \frac{\theta_v}{2} \sum_{i=1}^{3} w_i \sigma_f\big(\epsilon_f(g(t_i))\big) \cos^2(g(t_i)) u(g(t_i), \gamma, \mathrm{B}) \tag{14}$$

Here we have to compute the values of $\epsilon(g(t_1))$, $\epsilon(g(t_2))$ and $\epsilon(g(t_3))$ from Equation (13). For 3-point Gaussian Quadrature method,

$$\epsilon_{f(g(t_i))} = \epsilon_y\big(\cos^2(g(t_i)) - \lambda \sin^2(g(t_i))\big) \tag{15}$$

From Equation (1), we can write:

$$\psi_2(\epsilon_y) = \frac{\sigma_y(\epsilon_y)}{\sigma_f(\epsilon_y)} = \frac{2}{\sigma_f(\epsilon_y) \tan^2(\gamma_D)} \int_0^{\gamma_D} I_g \frac{\sin(\gamma)}{\cos^3(\gamma)} d\gamma \tag{16}$$

Now again applying Gaussian Quadrature method,

$$\psi_2(\epsilon_y) = \left[\frac{2}{\sigma_f(\epsilon_y)\tan^2(\gamma_D)}\right]\frac{\gamma_D}{2}\int_{-1}^{1} I_g\left(g(t)\right)\frac{\sin\left(g(t)\right)}{\cos^3\left(g(t)\right)}dt \tag{17}$$

where $g(t) = \frac{\gamma_D}{2}(1+t),\ \ i = 1,2,3.$

From Equation (17), we can write as follows;

$$\psi_2(\epsilon_y) = \left[\frac{2}{\sigma_f(\epsilon_y)\tan^2(\gamma_D)}\right]\frac{\gamma_D}{2}\sum_{i=1}^{3} w_i I_g\left(g(t_i)\right)\frac{\sin\left(g(t_i)\right)}{\cos^3\left(g(t_i)\right)} \tag{18}$$

That expression was simplified through a MATLAB simulation. In previous studies (Zubair, Eldeeb, et al., 2017; Zubair, Neckář, et al., 2017; Zubair, Neckar, et al., 2017; Zubair et al., 2020), the following distribution function was used for both models. The distribution function $u(\theta,\gamma,\beta)$ which depends upon β angular preference of fibres in yarn, twist angle γ and fibre inclination angles varying from 0^o to 90^o, is given as follows:

$$u\ (\theta,\gamma,\beta) = \frac{1}{\pi}\frac{\beta}{\beta^2-\left(\beta^2-1\right)\cos^2(\theta+\gamma)} + \frac{1}{\pi}\frac{\beta}{\beta^2-\left(\beta^2-1\right)\cos^2(\theta-\gamma)} \tag{19}$$

The above distribution function was replaced by a Gaussian distribution function to recalculate the fibre stress utilization which is given as follows:

$$u\ (\theta,\gamma,\beta) = \frac{1}{\beta\sqrt{2\pi}}\ e^{-1/2\left(\frac{\theta-\gamma}{\beta}\right)^2} \tag{20}$$

and another proposed distribution function has been introduced for modification of model of fibre stress utilization and is given below.

$$u\ (\theta,\gamma,\beta) = \frac{1}{\beta-(\theta-\gamma)} \tag{21}$$

3.2. Experimental coefficient of fibre stress utilization

Coefficient of fibre stress utilization in yarn helical structure describes the tensile behavior of yarn due to strain through the yarn axis and it can be defined as the ratio between specific stress in yarn $\sigma_y\ (\varepsilon_y)$ due to yarn axial strain and specific stress in fibre $\sigma_f\ (\epsilon_y)$ due to fibre axial strain. It can be written as:

$$\psi(\epsilon_y) = \frac{\sigma_y(\epsilon_y)}{\sigma_f(\epsilon_y)} \tag{22}$$

The coefficient of fiber stress utilization up to the breaking point which is the ratio between the specific stress of yarn up to breaking point, σ_{yb} and specific stress of fiber up to the point of break σ_{fb}. The experiment fiber stress utilization, $\psi\ (\varepsilon_y)$ can be evaluated from the simple relationship, $\psi\ (\varepsilon_y) = \frac{\sigma_{yb}}{\sigma_{fb}}$ demonstrated as in Figure 5.

The prediction of fiber stress utilization up to the breaking point of fiber in yarn was necessary because yarn strength is the strength of yarn at the breaking point, which is the important parameter for spinner to evaluate the quality of the staple spun yarns and for the process optimization. Therefore, it is necessary to predict the fiber stress utilization up to the breaking point which can further help to predict the yarn specific stress knowing fiber specific stress using the equation, $\sigma_{yb} = \sigma_{fb} \times \psi\ (\varepsilon_y)$.

We need to predict the fiber stress utilization at the point of break because before the process of break all the fiber are in working position. No fiber is broken and all contribute in fiber stress utilization but at the point of break majority of the fibers are broken when yarn is broken during the application of tensile force. Staple spun yarns have many applications in weaving and knitting where yarn bears many stresses during the conversion of yarn into fabric manufacturing process, so it is important to know the breaking strength of the yarn before weaving and knitting process for the weavers. Therefore, it is necessary to know the value of fiber stress utilization at the point of break to determine the breaking strength of yarn.

All the mean curves as shown in Figure 2 for fibre and yarn were used to evaluate the experimental coefficient of fibre stress utilization in yarns under study using equation (22) and are shown in the Figure 6. The shape of each type of fibre stress utilization curves for three different technologies—SIRO, Compact and SIRO-Compact—have similarities. The experimental fibre stress utilization for the yarn produced from SIRO-Compact is higher as compared with the yarns produced from the SIRO and Compact technologies which might be the effect of hybrid technologies for SIRO-Compact staple spun yarn production.

3.3. Predicted and experimental coefficient of fibre stress utilization

The theoretical fibre stress utilization was evaluated using three different distributions Dirac delta, Gamma and Gaussian distribution, as given in equations (19), (20) and (21), respectively. In previous studies, the fibre stress

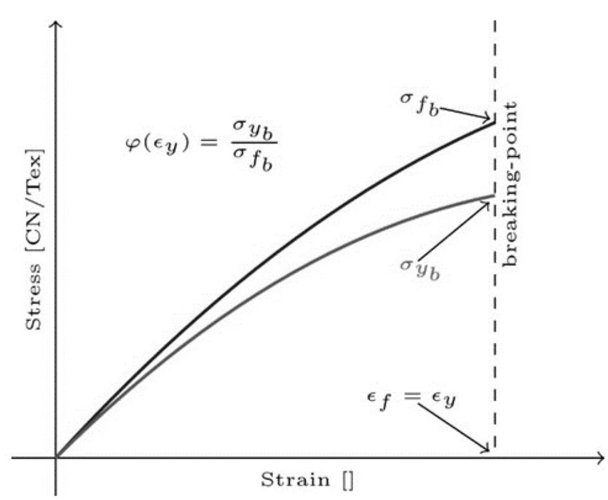

Figure 5. Fiber stress utilization in yarn.

utilization was predicted before the break point of ring, rotor and air-jet staple spun yarn (Zubair, Neckář, et al., 2017; Zubair et al., 2020). In this study, the modified model applying different distribution functions was used to predict the fibre stress utilization up to break point. The experimental and predicted coefficient of fibre stress utilization curves were obtained using equation (18) for three different types of yarns 20/1, 30/1 and 40/1 produced from three technologies: SIRO, Compact and SIRO-Compact. The values of the constant β for angular distribution of fibre in yarn used to evaluate the coefficient of fibre stress utilization are recorded in Table 2. The higher values of β for Equation (19) represent that the fibre have improved distribution in the yarn, while the lower values for Equations (20) and (21) exhibit better directional distribution of fibre in the yarn. The constants used for the distribution functions are given in Table 2.

The experimental and predicted fibre stress utilization from equations (1), (3), (17) and (18) and using distribution function from equations (19), (20) and (21) for compact yarn are shown in Figures 7, 8 and 9 respectively. The predicted fibre stress utilization curves from the single integral equations, both from trapezium and Gaussian method, are far from the experimental fibre stress utilization curves and lies above the experimental curves. It is evident that the predicted fibre stress utilization using double integral equation and three distributions for three types of yarn count (20/1, 30/1 and 40/1) produced from compact technology captured well the experimental fibre stress utilization. The constants used for directional distribution of fibre were 3, 0.3 and 1.25, respectively, for all three types of compact yarn.

The results of fibre stress utilization in yarn are in good agreement with three different types of distributions for three types of yarn counts. The coefficient of fibre stress utilization predicted from single integral with Trapezium and Gaussian methods lies above the experimental fibre stress utilization curves due to consideration of same angular position of fibre helix in the yarn. When the normal distributions from equations (19), (20) and (21) were applied to incorporate the effect of different angular distribution of fibre in yarn, the predicted fibre stress utilization curves approached near the experiment fibre stress utilization curves for all three types of technologies.

The experimental and predicted fibre stress utilization from Equations (1), (2), (3), (11) and (18) and using distribution function from equations (19), (20) and (21) for SIRO yarn are shown in Figures 10, 11 and 12, respectively. It is evident that the predicted fibre stress utilization for three types of yarn count (20/1, 30/1 and 40/1) produced from SIRO technology captured well the experimental fibre stress utilization. The predicted fibre stress utilization curves from the single integral equations both from trapezium and Gaussian method are far from the experimental fibre stress utilization curves and lies above the experimental curves. The constants for directional distribution of fibre used as 4.5, 0.25 and 0.20, respectively, for all three SIRO yarns.

The experimental and predicted fibre stress utilization from equations (1), (3), (17) and (18) and using distribution function from equations (19), (20) and (21) for SIRO-compact yarn are shown in Figures 13, 14 and 15, respectively. It is evident that the predicted fibre stress utilization for three type of studied yarn count (20/1, 30/1 and 40/1) produced from SIRO-compact technology captured well the experimental fibre stress utilization. The predicted fibre stress utilization curves from the single integral equations,

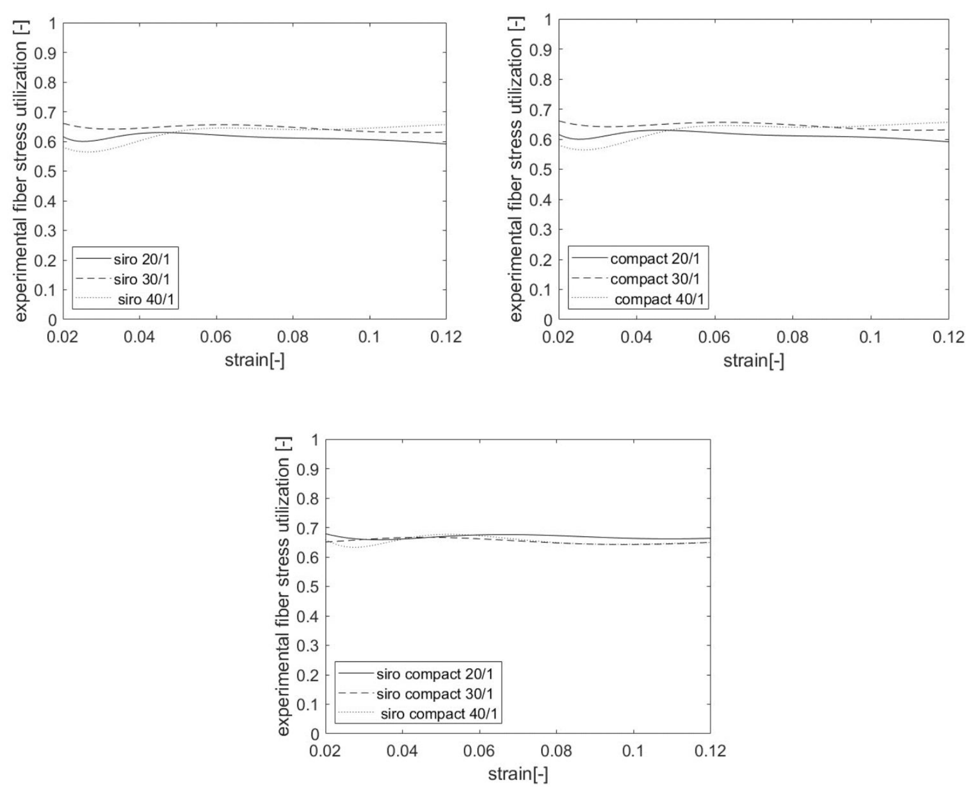

Figure 6. Experimental fibre stress utilization of SIRO, compact & SIRO-compact yarns.

Table 2. Constant for angular preference of fibre in yarn.

Yarn ring spinning technologies	Value of β for Equation (19)	Value of β for Equation (20)	Value of β for Equation (21)
Compact	3	0.3	1.25
SIRO	4.5	0.25	1.2
SIRO compact	6	0.2	1.15

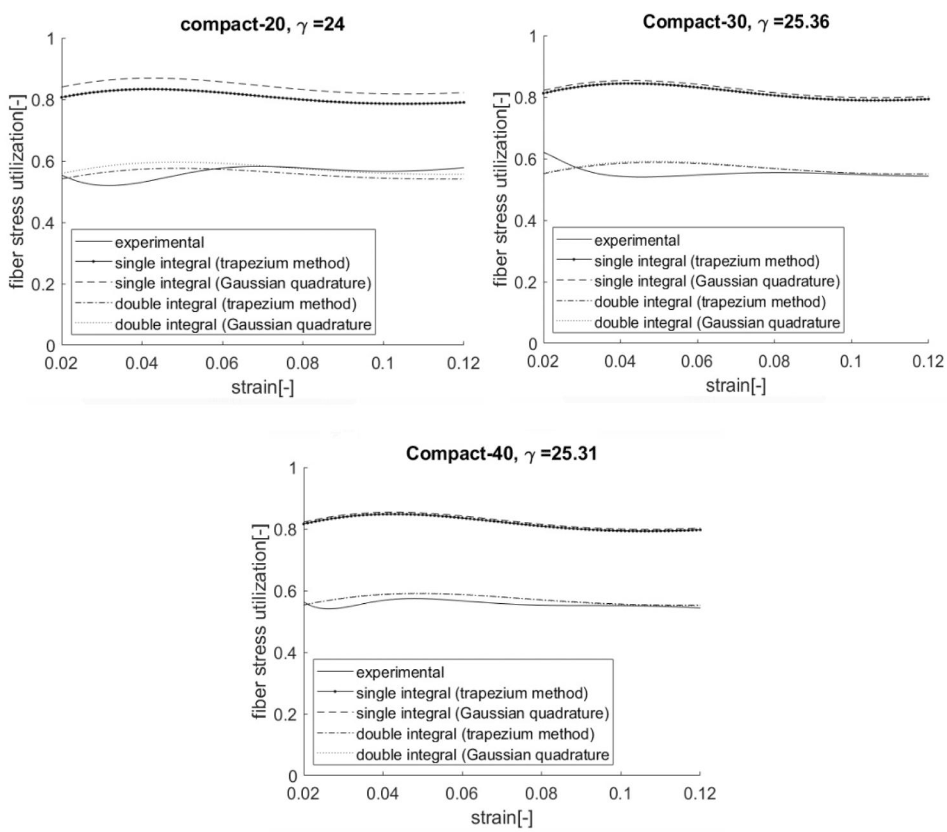

Figure 7. Experimental and predicted fibre stress utilization from Equation (19) and $\beta = 3$.

both from trapezium and Gaussian method, are far from the experimental fibre stress utilization curves and lies above the experimental curves. The constants used for directional distribution of fibre were 6, 0.20 and 1.15, respectively, for all three types for SIRO-compact yarn.

It is evident that the fibre stress utilization curves obtained from different distributions are in good agreement with the experimental fibre stress utilization curves for three different yarn linear densities and technologies. This shows that the helical structure of SIRO, Compact and SIRO-compact yarn consists of fibre elements having different inclination 'θ' with yarn strip is more accurate as compared to the helical structure of yarn having general angle of fibre elements along 'γ' with yarn strip. The modified model for prediction is valid for the SIRO, Compact and SIRO-compact yarn production technologies. The extrapolation technique improved the model by predicting the fibre stress utilization up to the breaking point of yarn which is useful for spinners.

4. Conclusion

The main theme of the work is to model the coefficient of fibre stress utilization in modified ring spun yarn (SIRO, Compact and SIRO-compact) from different models, distribution functions. The experimental data of fibre and yarn stress-strain have multiple curves for different yarns. The extrapolated mean tensile stress-strain curves from experimental data of fibre and yarns were approximated by an analytical method by means of artificial neural network as input for the model. The extrapolated analytical stress-strain curves help us to approximate analytical solving of single and double integral models up to the breaking point of fibre and yarn. The new distribution functions Gamma and Gaussian distribution were introduced in parallel with Dirac delta function for prediction of fibre stress utilization in SIRO, Compact and SIRO-compact yarns. The different fibre directional distribution parameters B were introduced in distribution functions to predict the fibre stress utilization. The results of fibre stress utilization numerically predicted from the different models with varying distributions functions are in good agreement with the experimental fibre stress utilization for all types of SIRO, Compact and SIRO-compact yarns. It can be concluded that SIRO-compact yarn exhibited better fibre stress utilization as compared to SIRO and SIRO-compact yarn which might be due to the improved fibre orientation in SIRO-compact yarn. The new methodology is efficient from computational prospectus as well as prediction up to the breaking point of fibre and yarns.

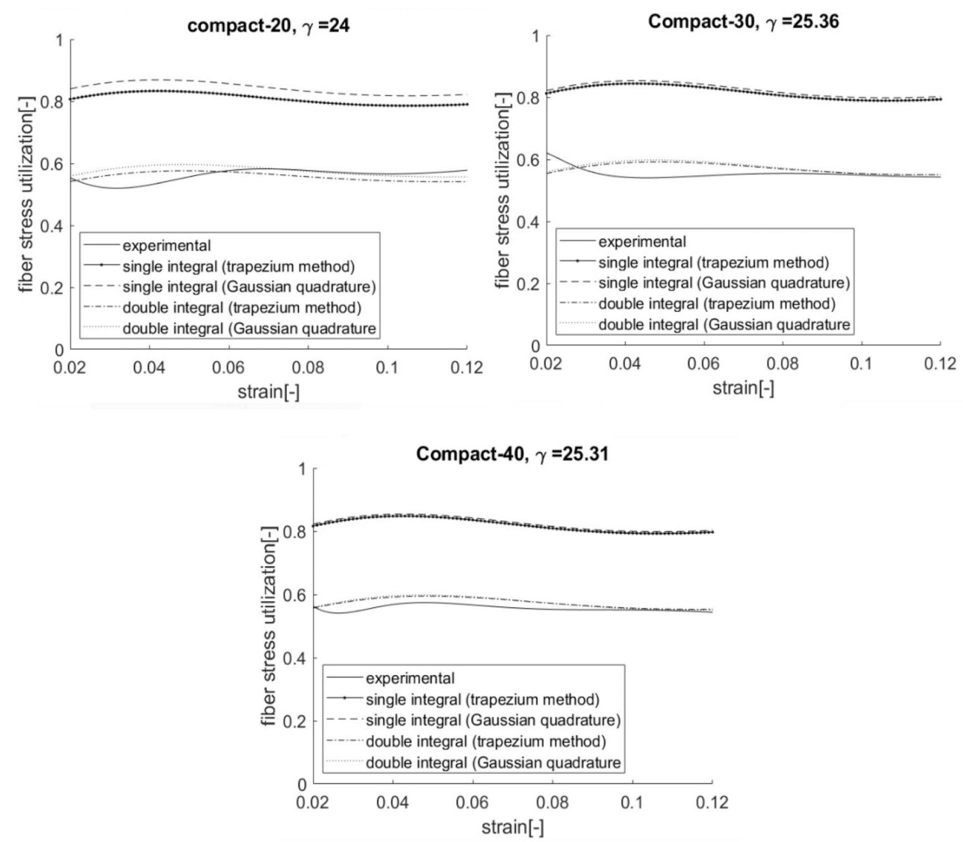

Figure 8. Experimental and predicted fibre stress utilization from Equation (20) and $\beta = 0.3$.

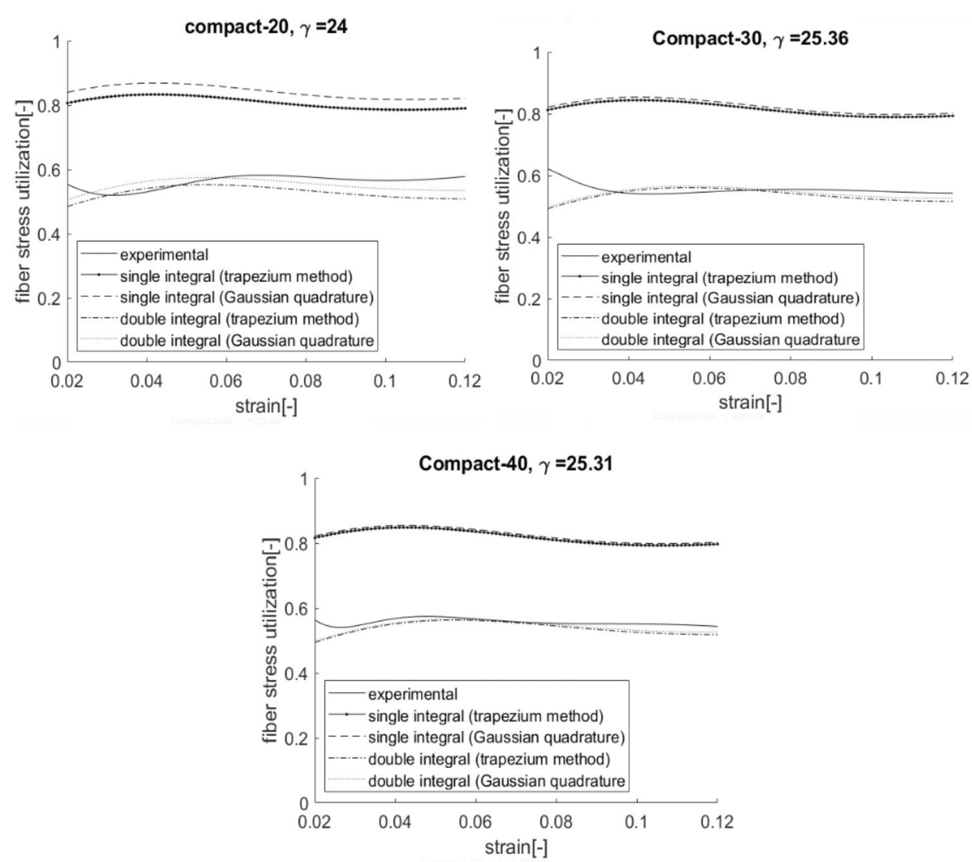

Figure 9. Experimental and predicted fibre stress utilization from Equation (21) and $\beta = 1.25$.

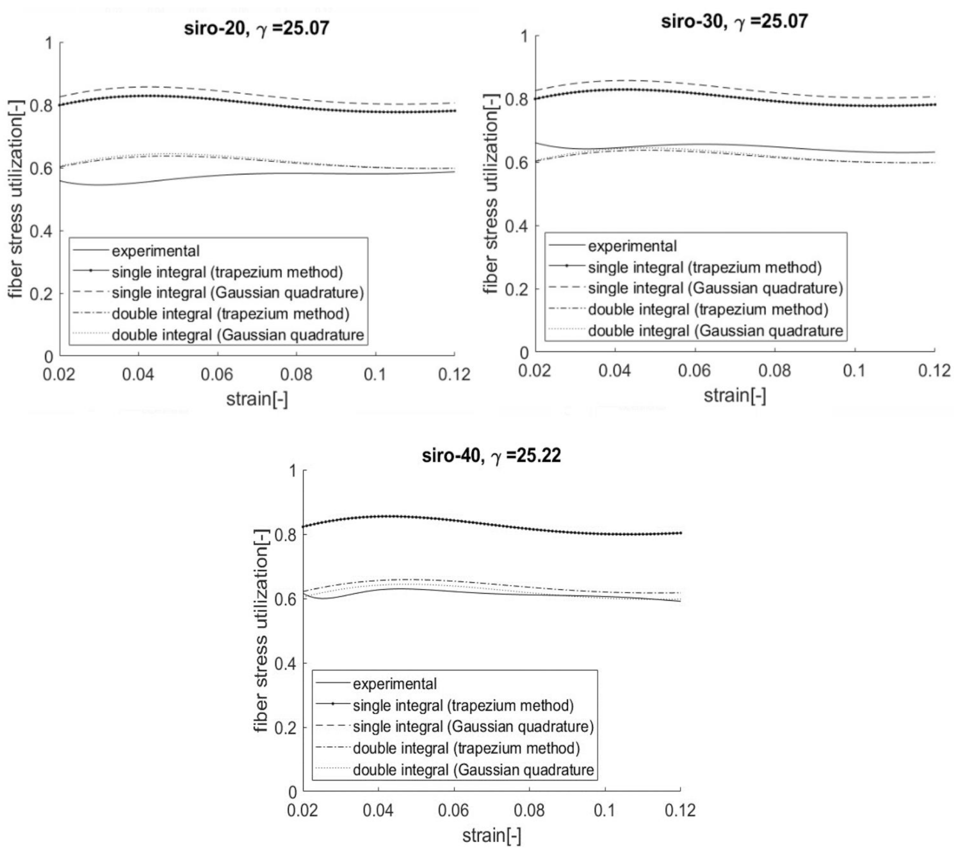

Figure 10. Experimental and predicted fibre stress utilization from Equation (19) and $\beta = 4.5$.

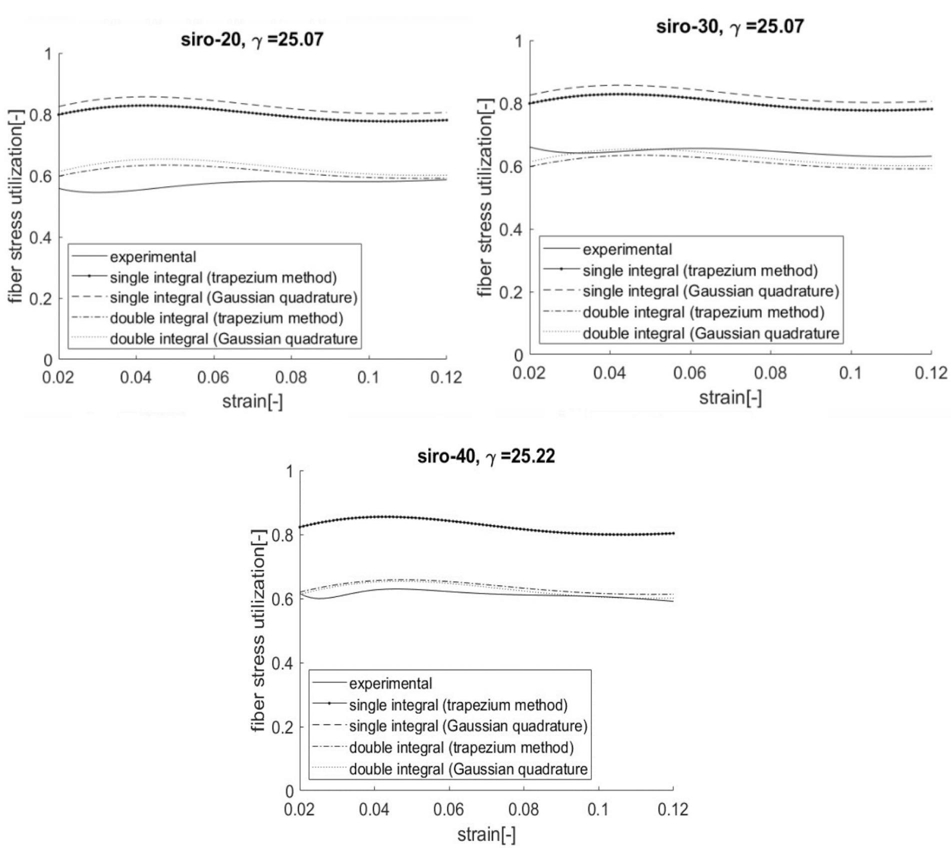

Figure 11. Experimental and predicted fibre stress utilization from (20) and $\beta = 0.25$.

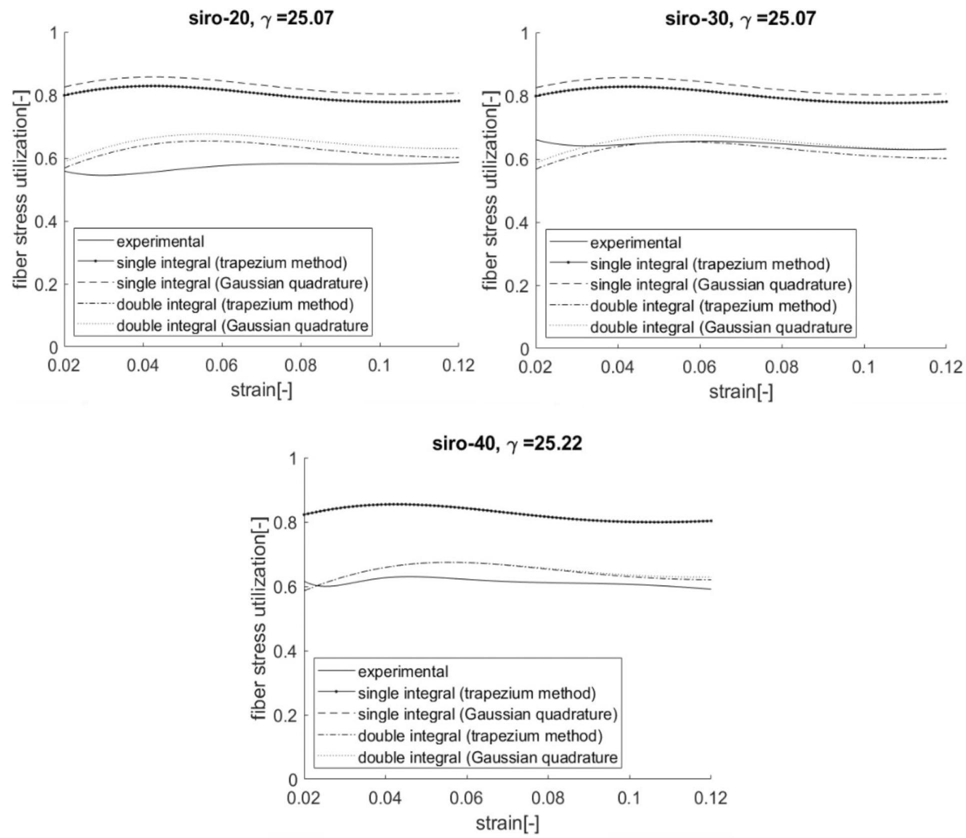

Figure 12. Experimental and predicted fibre stress utilization from distribution function Equation (21) ($\beta = 1.20$).

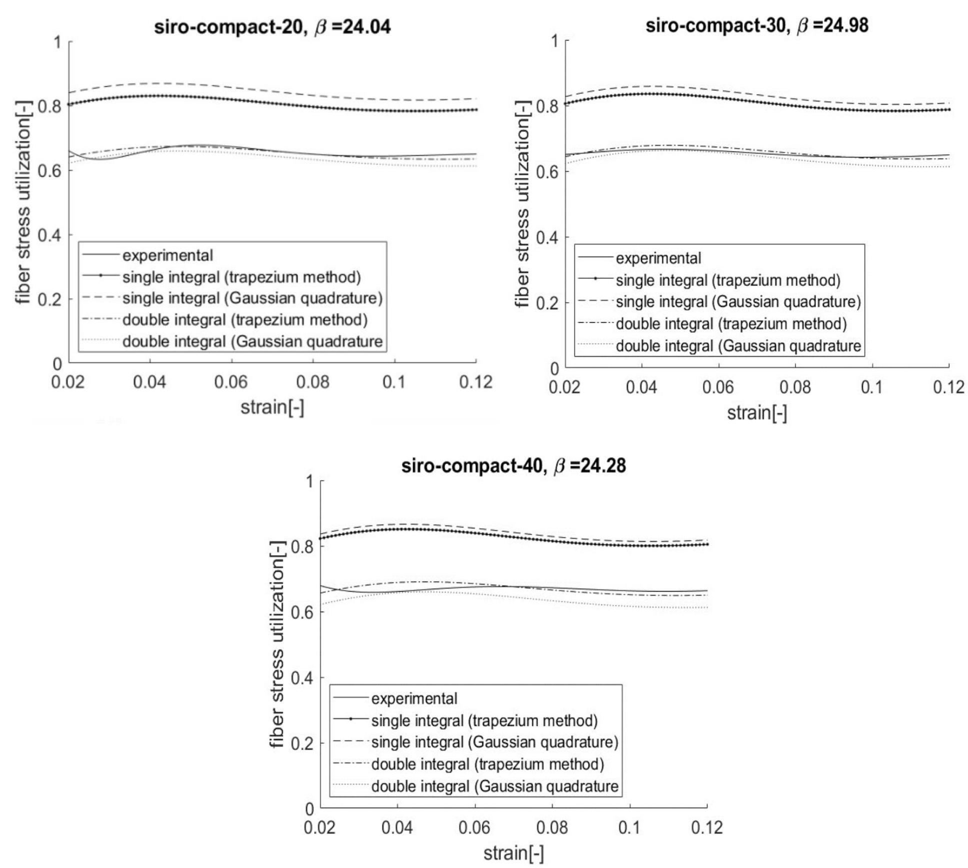

Figure 13. Experimental and predicted fibre stress utilization from distribution function Equation (19) ($\beta = 6$).

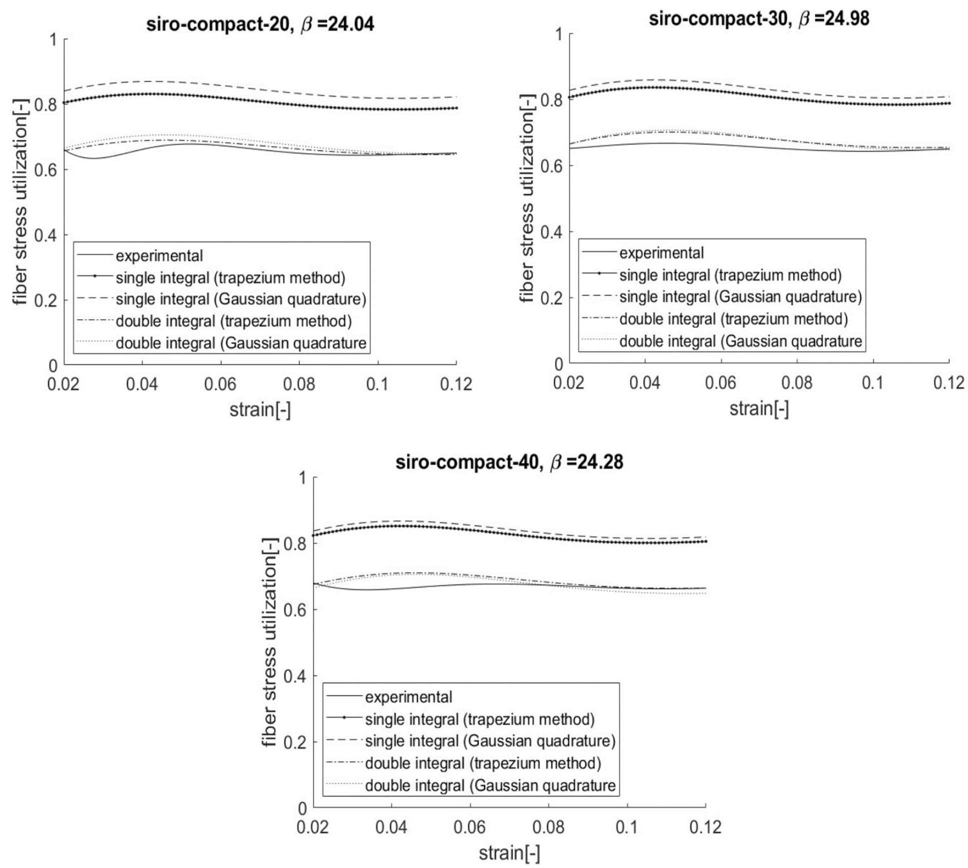

Figure 14. Experimental and predicted fibre stress utilization from distribution function Equation (20) ($\beta = 0.20$).

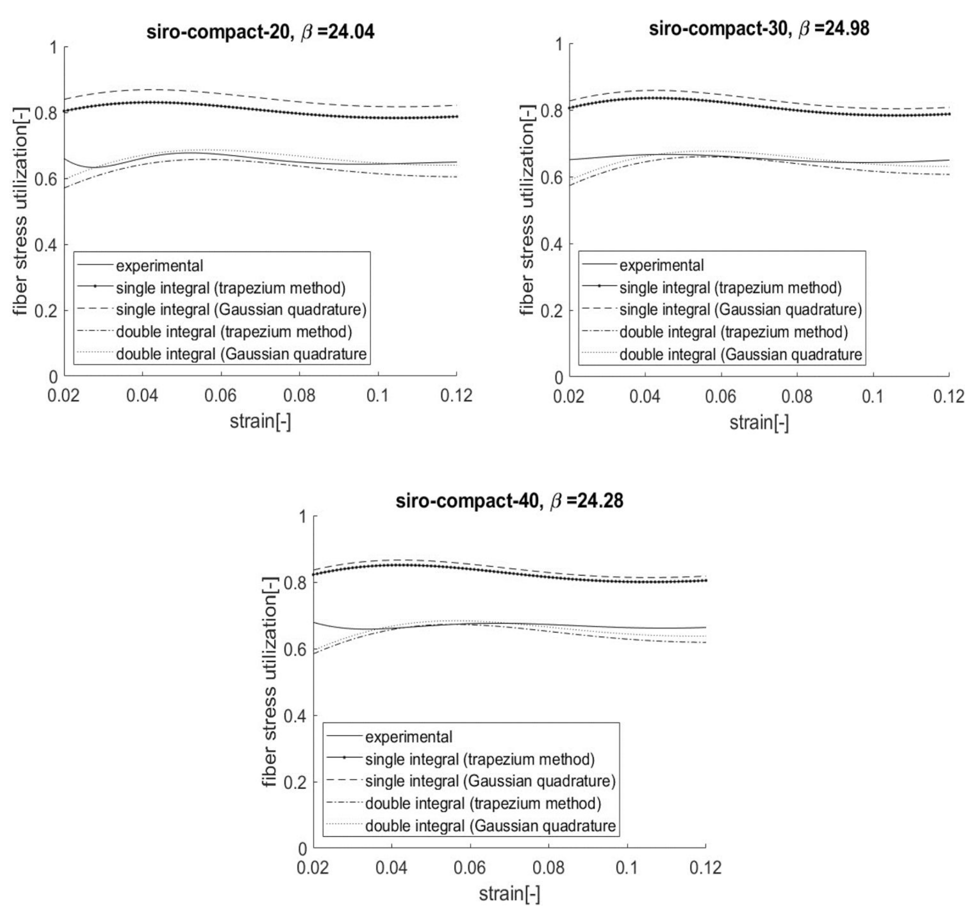

Figure 15. Experimental and predicted fibre stress utilization from distribution function Equation (21) ($\beta = 1.15$).

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Abdul Jabbar http://orcid.org/0000-0002-7751-5489

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