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Fraction Mapping and Fraction Comparison Skills among Grade 4 Chinese Students: An Error Analysis

Abstract

Background. Mapping fraction symbols to magnitudes is easier for students to master than comparing fraction magnitudes. Fraction mapping assesses students' understanding of part-whole interpretations of fractions; fractions represent the parts of an object or set of objects. Fraction comparison assesses students' understanding of measurement interpretations of fractions; a fraction is a single numerical quantity, not a combination of two whole numbers.
Aim. To examine and compare the types of errors made by emergent fraction learners on fraction mapping and comparison tasks.
Sample. Grade 4 Chinese students (N = 1,036; 577 boys; M_{age} = 9.9 years).
Method. We examined performance and identified errors on fraction mapping and comparison tasks. For mapping, students converted pictorial representations into fraction notation. For comparison, they chose the larger of two symbolic fractions.
Results. Consistent with curriculum expectations, most students successfully mapped pictorial representations to fraction notation. In contrast, few students were able to accurately compare fraction magnitudes. Within each task, students' errors were consistent across trials, suggesting that they applied systematic but incorrect procedures. However, errors were not consistent between tasks and the correlation between mapping and comparison performance was weak.
Conclusion. Emergent fraction learners can acquire part-whole knowledge of fractions without acquiring measurement interpretations of fractions. Moreover, misconceptions about different interpretations of fractions need not overlap. Awareness of the types of errors that students make can assist educators in identifying misconceptions early so that students do not build their fraction knowledge on erroneous beliefs.

Word count: 245

Keywords: Fraction mapping, fraction comparison, Chinese students, fraction errors

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Fraction Mapping and Fraction Comparison Skills among Grade 4 Chinese Students: An Error Analysis

Many students have difficulty with fractions. These difficulties are global, occurring in North America (Siegler & Lortie-Forgues, 2017), Europe (Gabriel et al., 2013; Meert et al., 2010), and Asia (Chan et al., 2007). In the present study, we focused on emergent fraction learners from China. As a country, China scores at the top in overall performance on international mathematics assessments such as the Programme for International Student Assessment (PISA; OECD, 2018) and the Trends in International Mathematics and Science Study (TIMSS; Mullis et al., 2020). Nonetheless, like students in other countries, Chinese students have difficulty when they are first introduced to formal fraction notation in grades 3 and 4 (Xin & Liu, 2014). In the present study, we examined the errors that Chinese students made on fraction tasks.

Knowledge of fractions includes both part-whole and measurement interpretations (Cramer et al., 2008; English & Halford, 1995; Kilpatrick et al., 2001; Xin & Liu, 2014). Part-whole interpretations require students to recognize that fractions represent the parts of an object, or parts of a set of objects indicated by the fraction notation. For example, $\frac{1}{4}$ might refer to a pizza with one of four pieces eaten, or it might refer to four pizzas with one of them eaten (Hecht & Vagi, 2010). In both scenarios, the denominator represents the whole and the numerator represents the part. In contrast, measurement interpretations require students to think about a fraction as a single numerical quantity, not as a combination of two whole numbers. Measurement interpretations include the knowledge that the magnitude of two fractions can be compared, that fractions can be ordered from smallest to largest, and that fractions can be located on a continuous number line (Mazzocco & Devlin, 2008; Smith et al., 2005; Xin & Liu, 2014).

The measurement interpretation is also closely related to proportional reasoning, such that students must have a flexible understanding of how the quantity of a ratio is invariant across changes in measurement units (e.g., cutting a pizza into four equal slices and eating one slice is equivalent to cutting the same pizza into eight equal slices and eating two slices) (Lamon, 2005; Thomas & Saldanha, 2003).

The part-whole and measurement interpretations reflect two different aspects of fraction understanding, with the latter harder to master than the former (Thompson & Saldanha, 2003; Xin & Liu, 2014). Students who maintain beliefs that are grounded in their pre-existing whole-number knowledge can still understand part-whole interpretations of fractions. To understand measurement interpretations of fractions, however, students must revise their beliefs to recognize that a number can be infinitely divided (Stafylidou & Vosniadou, 2004; Xin & Liu, 2014).

Fraction Mapping

When fraction notation is introduced, students first learn to convert pictorial representations of magnitude to fraction symbols, and vice versa, as shown in Figure 1. To accurately map from external representations to fraction symbols, students must understand that fractions consist of two components: the numerator, which reflects the "part", and the denominator, which reflects the "whole" (Čadež & Kolar, 2018; Jiang et al., 2020).

(Insert Figure 1 approximately here)

One view of why fraction learning is difficult is that students' whole number knowledge interferes with learning about rational numbers (see reviews by Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017). To fit fractions into their existing whole number framework, students tend to process the whole number components of fractions discretely rather than processing fractions as a single magnitude (Ni & Zhou, 2005). Interference from students' whole number knowledge (i.e., whole number bias) can occur when they are learning fraction notation, that is, even before they are asked to compare and order fractions. For example, evidence of the whole number bias occurs when students are asked to identify the fraction based on the number of partitioned areas (e.g., Corwin et al., 1991). If 5 out of 8 equally partitioned sections are shaded, students may report, "Well, I can write five-eighths as five < 5 > or I can write it as five-eighths < $\frac{5}{8}$ >. It doesn't matter. It's the same thing." (Mack, 1995, p. 437). In this case, the student is focusing on the "part" as discrete pieces without considering the "whole". These students might be trying to add new knowledge about fractions to their pre-existing knowledge of whole numbers without updating their mental representations (Stafylidou & Vosniadou, 2004; Vosniadou, 1994).

Even if students understand that fraction notation represents a relation between a part and a whole, they can still have misconceptions about part-whole relations. For example, if a circle is divided into four equal parts with one shaded region, students might erroneously label the fraction as " $\frac{1}{3}$ " . This response suggests that students understand that fractions consist of a part and a whole, and can apply conventional fraction notation, but that they think fractions refer to discrete countable pieces (Saxe et al., 2005). Alternatively, students may apply nonconventional fraction notation, such as " $\frac{4}{1}$ " but still show some understanding that fractions represent part of a unit.

Overall, fractions can be difficult for students, even at most basic representational level. Outside of the classroom, students who do not understand fraction symbols may have difficulty following a recipe, reading time on an analog clock, or sharing equal portions of food among friends. To develop an understanding of more advanced fraction concepts, such as fraction magnitude, students must understand the notation, how the notation represents the magnitude of the fraction, and how fractions are related to natural numbers (Hecht et al., 2003; Hecht & Vagi, 2010; Mazzocco et al., 2013; Stafylidou & Vosniadou, 2004).

Fraction Magnitude Comparison

Difficulties with fraction learning may be explained by a conflict between new information about fraction symbols and prior knowledge of whole number symbols (Chi et al., 1994; Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017). For example, even if students can map fractions to their symbolic notations, they may still process the whole number components of fractions discretely, rather than holistically processing the numerator and denominator as a single magnitude. One way to assess students' understanding of fraction magnitude is through a fraction magnitude comparison task (e.g., Which fraction is larger, $\frac{1}{3}$ or $\frac{2}{7}$?).

Whole number bias may also be evident in fraction comparison tasks. For example, students might directly compare either the numerators or denominators of two fractions to determine which one is bigger, suggesting they are focusing on the whole number magnitudes (Bonato et al., 2007; DeWolf & Vosniadou, 2015; Meert et al., 2010). In a fraction comparison task, the relations among the components of fractions can influence whether the whole number bias leads to correct or incorrect responses. For example, congruent fraction pairs are those in which the larger fraction also has the larger components (e.g., $\frac{2}{5}$ vs. $\frac{4}{7}$; Ischebeck et al., 2009). If students have a whole number bias, they may correctly select $\frac{4}{7}$ as the larger of the two fractions because both its numerator and denominator are larger than the numerator and denominator for $\frac{2}{5}$. In contrast, incongruent fraction pairs are pairs in which the larger fraction has the smaller components (e.g., $\frac{2}{5}$ vs. $\frac{3}{8}$). If students have a whole number bias, they may incorrectly select $\frac{3}{8}$ because both the numerator and denominator are larger than the numerator and denominator for

$\frac{2}{5}$. Thus, when designing and scoring fraction comparison tasks, the types of fraction pairs (i.e., congruent and incongruent) need to be considered.

Similar to fraction mapping, students can have a partial understanding of fraction magnitude. Stafylidou and Vosniadou (2004) found that some students erroneously believed that the value of a fraction increased when the numbers that comprised the fraction decreased. For example, they knew that $\frac{1}{3}$ is greater than $\frac{1}{4}$ (i.e., smaller components = larger fraction) so they erroneously concluded that $\frac{2}{3}$ must also be greater than $\frac{4}{5}$. These students had some understanding that fractions do not operate in the same way as whole numbers, but still did not understand the relation between the numerator and denominator in a fraction. Thus, to comprehend measurement interpretations students need to expand their knowledge of whole numbers as a discrete system to rational numbers as a continuous system. This conceptual leap may lead students to make errors when comparing fraction magnitudes (Liu et al., 2012).

Fraction Learning for Chinese Students

An early part of fraction learning in China involves mapping pictorial representations of fractions, such as those shown in Figure 1, to symbolic fraction notations. These notations can be in written format, such as "$\frac{2}{3}$", or in oral format, such as "two-thirds". In East Asian languages, the notion of fraction parts is embedded in fraction names. For example, translating from English to Chinese or Korean, "two-thirds" becomes "of three parts, two". Because transparent part-whole relations are reflected in fraction names, the part-whole meaning may be easier to understand for East Asian students (Miura et al., 1999). However, this advantage for East Asian students may be limited to mapping fraction names to their conceptual referents (Paik & Mix, 2003; Mix & Paik, 2008). The support for part-whole interpretations that is provided by the language structure may be insufficient to overcome the complexity of measurement interpretations.

In China, fraction instruction begins in grade 3, focusing on pictorial representations to help students understand both part-whole and measurement interpretations (Ministry of Education, 2011). For example, beyond learning fraction notation, students may be introduced to fraction magnitude using pictorial representations, as shown in Figure 2. Alibali and Sidney (2015) argued that the amount of experience that students have with fractions influences whether they process fractions componentially or holistically. In grade 3, Chinese students work primarily with pictorial representations of fractions. Moreover, magnitude exercises focus on unit fractions and fractions with common denominators (i.e., congruent fractions). It is not until the latter half of grade 4 that students are exposed to more complicated measurement interpretations, including incongruent fraction pairs and reducible fractions. Thus, because students in the present study were tested at the end of the first semester of grade 4, they were still in the early stages of fraction learning. We therefore investigated performance and errors on two tasks: fraction mapping, a task that requires part-whole interpretations and uses pictorial representations, and fraction comparison, a task that requires measurement interpretations and uses only symbolic representations.

(Insert Figure 2 approximately here)

The Present Study

The goal of the present research was to study fraction knowledge in emergent learners by examining the errors Chinese students made on fraction mapping and fraction comparison tasks. Chinese students in grade 4 have received formal instruction on fraction notation and should therefore have a part-whole understanding of fractions. Thus, we expected their performance would be good on the fraction mapping assessment. However, we anticipated that some students would make errors that show evidence for fraction misconceptions and that students' errors would be consistent with the whole number bias and/or reflect an incomplete understanding of part-whole interpretations of fractions (Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017; Stafylidou & Vosniadou, 2004; Vosniadou, 1994). For example, when asked to write down the fraction represented by the shaded area of a figure (e.g., $\frac{1}{4}$), students may respond with whole numbers (e.g., "1" or "4"), reflecting whole number bias, or students may respond "$\frac{1}{3}$" (i.e., shaded over unshaded) or "$\frac{4}{1}$" (i.e., denominator over numerator), reflecting partial understanding of part-whole relations. Identifying the types of errors that students make is important so that their misconceptions about part-whole interpretations of fractions can be addressed before they learn more complex measurement interpretations.

Although Chinese students in grade 4 have been introduced to measurement interpretations of fractions, their exposure to magnitude representations is limited to unit fractions and fractions with common denominators. Moreover, exercises on fraction notation and comparison are focused on fractions with common components and include pictorial representations (Ministry of Education, 2011). Thus, students may find it difficult to compare symbolic digits in the absence of non-symbolic representations. Accordingly, we expected students would perform poorly on the fraction comparison task, particularly for incongruent fraction pairs. When fractions are presented only in symbolic format, students may rely on their knowledge of the whole number system to try and find relations among numbers. Their errors on the fraction comparison task, therefore, would reflect misconceptions about measurement interpretations of fractions. One possibility, consistent with a whole-number bias, is that some students may select the fraction with larger component(s) without considering the fraction as a whole. Moreover, we anticipated that regardless of whether students could accurately map fractions, students would make errors when comparing fractions because part-whole interpretations are not sufficient to master measurement interpretations.

Method

Participants

Grade 4 students (*N* = 1,036; 577 boys; M_{age} = 9.9 years; *SD* = .59) were recruited from 24 classrooms in two public elementary schools at the end of the first semester (December 2020). Ethics approval was obtained from Shandong Normal University. Students were invited to participate through letters sent home by the school, after approval from the principal. The schools were in a town with an economic level at the national average (National Bureau of Statistics, 2019).

Measures

This study is part of a larger longitudinal study investigating the development of math anxiety and control mechanisms. Thus, in addition to the measures described below, students completed a battery of questionnaires (e.g., perceived teacher support), cognitive measures (e.g., reasoning tasks), and mathematical measures (e.g., arithmetic); only measures relevant to the present hypotheses are described and analyzed. Two paper-and-pencil measures of fraction skills, fraction mapping and fraction comparison, were created. Data from these two tasks were also analyzed in a study investigating the relations between division and fractions (Xu et al., 2022).

Fraction Mapping

In this task (see Appendix A), students are presented with 20 items in two columns and have one minute to complete as many items as possible, in order. For each item, students are presented with a picture and asked to write down the fraction that corresponded to the shaded portion. Scoring is the total number of items correctly answered in one minute. Internal reliability based on accuracy on individual items was excellent, Cronbach's alpha = .98.

Fraction Comparison

In this task (see Appendix B), students are presented with 20 items in two columns and have one minute to complete as many items as possible, in order. For each item, students are asked to circle the larger of two fractions. Eleven items are congruent (5 common denominators), such that the relative magnitude of the numerator and/or denominator is consistent with the relative magnitude of the whole fraction (e.g., $\frac{1}{6} < \frac{5}{6}$; $\frac{5}{7} > \frac{2}{3}$). The other 9 items are incongruent (4 common numerators), such that the relative magnitude of the numerator and/or denominator is inconsistent with the relative magnitude of the whole fraction (e.g., $\frac{5}{8} < \frac{5}{6}$; $\frac{2}{3} > \frac{4}{7}$). Scoring is the total number of items correctly answered in one minute. Internal reliabilities based on accuracy on individual items for congruent and incongruent trials were excellent (Cronbach's alpha = .92 and .96, respectively).

Procedure

Testing took place during school hours in classrooms. All tasks were administered in a group during a 45-minute session. Two experimenters administered the assessments, with one focusing on administration (e.g., reading directions, keeping time) and one circulating the classroom to ensure students were following the instructions. Prior to the testing session, experimenters were provided with a detailed testing manual which carefully outlined testing and scoring procedures.

Results

Fraction Mapping Errors

We examined students' responses to the 20 items from the fraction mapping task. Four classifiable types of errors were made (see Table 1). First, students sometimes provided a whole number response that was equivalent to either the numerator (i.e., number of shaded regions) or denominator (i.e., total number of pieces). Second, students sometimes inverted the fraction, writing the "whole" as the numerator and the "part" as the denominator. Third, students sometimes correctly identified the numerator (i.e., shaded portion), but instead of the denominator representing the "whole", their denominator represented the remainder (i.e., unshaded portion). Fourth, students sometimes provided decimal responses instead of fraction responses.

Some errors could not be classified (i.e., miscellaneous) or they reflected careless errors, such as counting mistakes (e.g., responding with $\frac{5}{8}$ instead of $\frac{5}{9}$). Errors were independently coded by the two first authors. For reliability, both authors coded errors for the same 200 children (19.26% of the data). Inter-rater reliability was extremely high (Cohen's Kappa = .93). For those 200 trials, any inconsistent codes were discussed until the authors agreed with each other.

(Insert Table 1 approximately here)

Table 2 shows the frequency of responses to each of the fraction mapping items. Across all items the most common response was a correct response (50.5%) followed by no response (31.4%). Because the task was timed, most students did not have the opportunity to attempt all the items. The most common error across all items was inverting (7.6%), followed by miscellaneous errors (3.9%), whole number (2.8%), careless mistake (3.9%), shaded-unshaded (1.6%), and decimal (0.1%). Consistent with this pattern of overall errors, as shown in Table 1, more students made at least one inverting error, followed by shaded-unshaded errors, whole number, and decimal errors.

(Insert Table 2 approximately here)

In summary, most students were able to accurately map fractions to symbolic representations. On average, for trials in which they provided a response (i.e., excluding blank responses), students made errors on 24% of the total trials. Notably there are two anomalies in Table 2. First, students made fewer inverting errors on the fraction $\frac{4}{4}$ than on the other 19 items. For the fraction $\frac{4}{4}$, the numerator and denominator are the same and thus by default even if students inverted the fraction, they would still obtain the correct response. However, 13 students who made inverting errors on all other attempted trials responded with $\frac{4}{0}$ for the item $\frac{4}{4}$. Moreover, students made more careless errors on the fraction $\frac{2}{7}$ than on other items. For this item, the two shaded pieces were separated by one non-shaded piece and many students miscounted the number of non-shaded pieces, responding with either $\frac{2}{6}$ or $\frac{2}{8}$. These careless errors presumably reflect the requirement to do the task quickly, rather than students' misconceptions about fraction notation.

Because students were very consistent in the strategy that they chose, they could be categorized by their most frequent response (i.e., their modal response; see Figure 3). Across the trials, of the 1,036 students, 815 (78.5%) most often made accurate responses, 143 (13.8%) most often inverted, 31 (2.9%) most often responded with whole numbers, 26 (2.5%) most often had part-whole bias responses, and 16 (1.5%) most often responded with decimals. Lastly, only five students (0.5%) responded most often with miscellaneous responses that were unclassifiable and inconsistent, and thus they were excluded from further analyses.

(Insert Figure 3 approximately here)

Fraction Comparison Errors

We examined students' responses to the 20 items from the fraction comparison task. Unlike fraction mapping, this task did not allow for open-ended responses. Students had to circle the larger fraction which meant there were three possible responses: correct, incorrect, and blank. The frequency of each of these three possible responses is reported in Table 3. Only a small minority of students accurately compared fractions on both congruent and incongruent trials (i.e., obtained scores ≥ 80% on attempted trials; n = 55). The most noticeable pattern was the difference in correct versus incorrect responses for congruent and incongruent trials. For congruent trials, when students provided a response (i.e., excluding blank responses), most students responded correctly. For incongruent trials, when students provided a response, most students responded incorrectly. This pattern of high accuracy on congruent trials and low accuracy on incongruent trials is consistent with a strategy of selecting the fraction with the larger component(s) as the larger fraction, and thus suggests that many students showed a whole-number bias in this task.

(Insert Table 3 approximately here)

In further examining the pattern of errors, for congruent trials, a higher percentage of students responded accurately for fractions with common elements (i.e., common denominator; 85-89%), than for fractions without common elements (e.g., $\frac{4}{5}$ vs. $\frac{2}{3}$; 74-79%). Similarly, for incongruent trials, a higher percentage of students responded accurately for fractions with a common element (i.e., common numerator; 30-31%), than for fractions without common elements (e.g., $\frac{2}{3}$ vs. $\frac{4}{7}$; 18-19%). Thus, in general, most students selected the fraction with a larger component for all congruent trials, but based on the varying percentages, a small group of students appeared to adopt a different strategy when fractions did not have common denominators. Notably, accuracy was slightly higher (41%) for one trial: $\frac{1}{4}$ vs. $\frac{2}{3}$. It is possible that familiarity with these two fractions helped some students correctly select the larger fraction.

Fraction Mapping and Fraction Comparison

We examined the relations between fraction mapping and fraction comparison performance. Although fraction mapping was significantly correlated with fraction comparison, $r(1{,}035) = .14$, $p < .001$, the correlation was small and the significance reflects the large sample size. The R^2 value of .02 highlights the weak relation between these two measures. Similarly, as shown in Figure 4, the correlations between fraction mapping and congruent trials, $r(1{,}035) = -.06$, $p = .04$, and fraction mapping and incongruent trials, $r(1{,}035) = .17$, $p < .001$ were also weak.

(Insert Figure 4 approximately here)

Figure 5 shows the relation between the different fraction mapping errors and the four types of fraction comparison trials. Regardless of the types of errors students made on the mapping task, the pattern of performance on the fraction comparison task was similar: For all fraction mapping groups, performance was highest for congruent trials and lowest on incongruent trials. All groups had average scores of less than 50% on the incongruent trials, suggesting below chance performance. In summary, by the end of the first semester of grade 4, most students had learned to map between visual and symbolic representations, indicating part-whole interpretations of fractions, but had not yet grasped measurement interpretations.

(Insert Figure 5 approximately here)

Discussion

Fractions are a source of difficulty for many learners. In the present study, we examined fraction performance for Chinese emergent fraction learners using two tasks: a fraction mapping task to examine part-whole interpretations of fractions, and a fraction comparison task to examine measurement interpretations of fractions. Although previous research has established that part-whole interpretations are easier to master than measurement interpretations (Thompson & Saldanha, 2003; Xin & Liu, 2014), the relation between these two interpretations and the errors students make when completing tasks that require knowledge of these interpretations has not been established.

We chose to examine fraction mapping and fraction comparison performance for Chinese students in grade 4 for three reasons. First, we wanted to thoroughly examine the types of fraction errors that students make when they are being educated in a country that excels in mathematics achievement. Second, we aimed to compare performance on a task that students should have mastered by grade 4 (e.g., fraction mapping) and a task that students had not yet received formal training (e.g., symbolic fraction comparison) to determine whether students would make common errors across two different types of tasks. Third, we wanted to investigate the different types of responses (i.e., strategies) students would make when faced with a task beyond their current level of understanding.

Fraction Mapping Performance

Generally, as expected, students were quite accurate when mapping fractions. However, even in China, a country known for its elite mathematics performance, approximately 20% of students consistently made errors when mapping fractions, suggesting that these students had not mastered part-whole interpretations of fractions (Xin & Liu, 2014). The most common error was fraction inverting, where students wrote the denominator, or the “whole”, as the numerator, and the numerator, or the “part”, as the denominator. Prior research has suggested that part-whole interpretations of fractions may be easier to understand for East Asian students because the transparent part-whole relations are reflected in fraction names (Miura et al., 1999; Miura &

Okamoto, 2003; cf. Mix & Paik, 2008). More generally, a proportion of the superior mathematics performance of Chinese and other East Asian students can be explained by the Chinese-based system of number words and the simplicity of Chinese mathematical terms (Chan, 2014; Ngan Ng & Rao, 2010).

In contrast, the present study is an example of how number naming conventions may sometimes interfere with numerical processing. Seventeen percent of the Chinese students made one or more inverting errors when mapping fractions, indicating that they might have some understanding of the part-whole relation, but do not fully grasp the role of the numerator and denominator. The inverting error might be more common in East Asian languages that use the naming system in which the denominator, or whole, is stated before the numerator, or part (e.g., of three parts, two) than in languages, such as English, where the numerator is named first (e.g., two-thirds). Consistent with this possibility, for the few students who made decimal errors, their response also consisted of the "whole" followed by the "part" (i.e., 5.2 for $\frac{2}{5}$). Although use of the decimal notation itself may reflect students' confusion about rational number representations (i.e., students typically learn decimals and fractions simultaneously), the format of their response (denominator-decimal-numerator) suggests that their confusion is also related to the number naming system. Future research should explore whether students who speak other languages with different fraction structures also make inverting errors when they are learning fraction notation.

When students are first introduced to fractions, they need to revise their pre-existing knowledge about numbers (Chi et al., 1994; Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017) and thus errors on fraction tasks often reflect whole number biases or a partial understanding of part-whole interpretations of fractions. In the current study, only 3% of students showed evidence of the whole number bias, suggesting that most students knew that fractions should consist of both a numerator and denominator. However, an additional 3% of students made errors that suggested they only had a partial understanding of part-whole interpretations, believing that fractions refer to discrete countable pieces. Consistent with the findings of Saxe et al. (2005), these students recognized that fractions consist of two parts, and applied conventional fraction notation, however, they erroneously labelled fractions based on the shaded versus unshaded proportions (e.g., labelling $\frac{1}{4}$ as $\frac{1}{3}$).

Overall, even in the early stages of fraction learning, when instruction is focused on part-whole interpretations, we saw evidence for fraction misconceptions. One-fifth of students were unable to successfully map pictorial representations to fraction notations after a year of formal fraction instruction. Interestingly, when students made errors, they did so consistently. These consistent errors may be indicative of fraction misconceptions that need to be addressed before students learn measurement interpretations of fractions.

Fraction Comparison Performance

As expected for the fraction comparison task, students were quite accurate on congruent trials but had quite poor performance on incongruent trials. This study is the first to investigate errors on these two types of trials for Chinese students. The overall pattern of performance on the fraction comparison task is consistent with the findings from other countries: Students were more accurate for trials in which the larger components belonged to the fraction with the greater magnitude (Meert et al., 2010; Stafylidou & Vosniadou, 2004; Van Hoof et al., 2013). In line with the whole number bias, students in the present study consistently focused on whole number magnitudes, directly comparing either the numerators or denominators of two fractions to determine which fraction was bigger (Bonato et al., 2007; DeWolf & Vosniadou, 2015; Meert et al., 2010; Stafylidou & Vosniadou, 2004; Rinne et al., 2017).

In summary, although students were able to correctly answer some items on the fraction comparison task, most were unable to provide correct responses for both congruent and incongruent trials. These results were consistent with the Chinese curriculum expectations in grade 4, where exposure to magnitude representations is limited to unit fractions and fractions with common denominators. Most students had not developed adequate measurement interpretations of fractions.

Fraction Mapping and Fraction Comparison

Previous studies have found that fraction mapping is predictive of performance on more complex fraction tasks, such as fraction comparison (Douglas, 2020; Hecht et al., 2003; Hecht & Vagi, 2010; Lewis, 2016; Mazzocco et al., 2013). In contrast, we found a weak relation between fraction mapping and fraction comparison performance. Regardless of the types of errors students made on the fraction mapping task, performance patterns were similar for the fraction comparison task, with most students selecting the fraction with the larger component(s). Although few students demonstrated whole-number bias when mapping fractions, most students demonstrated this bias when comparing fractions. These results support the view that knowledge of fraction notation that allows students' to successfully map pictorial representations to symbols is necessary, but not sufficient for successful fraction comparison. Moreover, misconceptions about fractions can be present when students are learning about part-whole interpretations and measurement interpretations and these misconceptions need not overlap.

Educational Implications

Students from around the world struggle to learn fractions. In the present study, we focused on students in China because of their educational experiences and excellent performance on international mathematics assessments. Furthermore, we examined students who were in the early stages of their fraction learning, to pinpoint early misconceptions that students might have about part-whole and measurement interpretations of fractions.

We found that fractions were challenging both with respect to understanding part-whole interpretations and measurement interpretations. Even though most students could accurately map fractions, approximately one-fifth of students consistently made errors. These errors reflected various misconceptions about part-whole interpretations of fractions, with most errors indicating that students did not fully grasp that a fraction does not consist of two discrete pieces, but rather represents a single number. When we investigated how part-whole interpretations related to measurement interpretations, we found that knowledge of these two interpretations were essentially unrelated. Thus, educators need to be aware that misconceptions can be unique for each interpretation and can arise at any stage of fraction learning. That is, students may have one misconception about part-whole interpretations and a different misconception about measurement interpretations. Even in China, a country whose mathematics curriculum is organized such that difficult topics, such as fractions, are presented in small distinct subtopics so that students can master foundational concepts prior to moving to more advanced concepts (Li & Huang, 2013), we see evidence of early fraction misconceptions. The findings from the present study show that approximately 20% of students have misconceptions about fractions and thus mathematics educators should invest attention in how to assist students in overcoming the challenges of learning fractions.

Concepts can be challenging to learn when new information competes or interferes with previously acquired knowledge (Vosniadou, 1994, 2001, 2002). When the acquisition of new knowledge requires revisions to a well-established theoretical framework, learning failures, such as misconceptions, are more likely to occur. In the present study, we examined open-ended responses to one of the most basic and earliest-taught fraction concepts: converting pictorial representations into fraction notation. With our large sample size, we were able to identify several types of errors that students made consistently, demonstrating how prior knowledge interfered with early fraction interpretations. By being aware of the types of errors students make, educators can identify and refute misconceptions early so that students do not build their fraction foundation on erroneous beliefs.

Fraction knowledge is also important outside of school. For example, people rely on part-whole and measurement interpretations of fractions in their daily lives, such as when following a recipe, evaluating statistics used by media, making financial decisions, and evaluating prices while shopping. A classic example of poor fraction understanding in the real world comes from competing fast-food chains: One company lost money when they introduced their “third-pound burger” because consumers believed that the competitor’s “quarter pounder” was larger (Taubman, 2007). Thus, people’s ability to identify and correct fraction misconceptions has important implications, both in school and in everyday life.

Limitations and Future Research

In the present study we identified some of the errors that students make when mapping and comparing fractions. However, unlike the fraction mapping task, the fraction comparison task consisted of close-ended items. Thus, we could only speculate about the strategies students used to select the larger fraction. In the future, a fuller understanding of students’ errors and misconceptions about measurement interpretations of fractions could be captured by having students report their strategies as they compare and order fractions.

We examined the concurrent relation between performance on fraction mapping and fraction comparison tasks, however, with only a single timepoint, we could not explore the growth in knowledge for either part-whole or measurement interpretations of fractions. Future studies that follow students from when they are first introduced to fraction notation (i.e., grade 3) to the end of elementary school (i.e., grade 6) would provide further insights into the development of fraction knowledge as well as the development of fraction misconceptions.

In the present study we examined fraction performance among Chinese-educated students because of the strong mathematics curriculum in China and the country's excellent performance on international mathematics assessments. To see if these findings are generalizable, additional research is needed that explores the frequency and onset of different types of errors. Moreover, an in-depth comparison of different mathematics curricula from around the world would provide more insights into if and how cultural differences in educational experiences may influence fraction understanding.

Conclusion

We closely examined the types of errors students made on a fraction mapping task, which requires knowledge of part-whole interpretations of fractions, and a fraction comparison task, which requires knowledge of measurement interpretations of fractions. We found that neither performance nor the types of errors students made were related across the two tasks. In other words, students could hold independent misconceptions about the two interpretations of fractions. Early identification and refutation of misconceptions is critical so that students develop a strong understanding of fraction concepts which will provide the foundation for later, more advanced mathematical concepts.

References

Alibali, M. W., & Sidney, P. G. (2015). Variability in the natural number bias: Who, when, how, and why. *Learning and Instruction*, *37*, 56–61. https://doi.org/10.1016/j.learninstruc.2015.01.003

Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance*, *33*(6), 1410–1419. https://doi.org/10.1037/0096-1523.33.6.1410

Čadež, T. H., & Kolar, V. M. (2018). How fifth-grade pupils reason about fractions: A reliance on part-whole subconstructs. *Educational Studies in Mathematics*, *99*(3), 335–357. https://doi.org/10.1007/s10649-018-9838-z

Chan, W.-H., Leu, Y.-C., & Chen, C.-M. (2007). Exploring Group-Wise Conceptual Deficiencies of Fractions for Fifth and Sixth Graders in Taiwan. *The Journal of Experimental Education*, *76*(1), 26–57. https://doi.org/10.3200/JEXE.76.1.26-58

Chi, M. T. H., Slotta, J. D., & De Leeuw, N. (1994). From things to processes: A theory of conceptual change for learning science concepts. *Learning and Instruction*, *4*(1), 27–43. https://doi.org/10.1016/0959-4752(94)90017-5

Corwin, R. B., Russell, S. J., & Tierney, C. (1991). *Seeing fractions: A unit for the upper elementary grades*. California Department of Education.

Cramer, K., Wyberg, T., & Leavitt, S. (2008). The role of representations in fraction addition and subtraction. *National Council of Teachers of Mathematics*, *13*(8), 490–496. https://doi.org/10.5951/MTMS.13.8.0490

DeWolf, M., & Vosniadou, S. (2015). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction*, *37*, 39–49. https://doi.org/10.1016/j.learninstruc.2014.07.002

Douglas, H. (2020). *Fraction symbols and their relation to conceptual fraction knowledge for students in grades 4 and 6*. Unpublished doctoral dissertation.

English, L. D., & Halford, G. S. (1995). *Mathematics educations: Models and processes*. Lawrence Erlbaum.

Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology*, *4*. https://doi.org/10.3389/fpsyg.2013.00715

Hecht, S. A., Close, L., & Santisi, M. (2003). Sources of individual differences in fraction skills. *Journal of Experimental Child Psychology*, *86*(4), 277–302. https://doi.org/10.1016/j.jecp.2003.08.003

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, *102*(4), 843–859. https://doi.org/10.1037/a0019824

Ischebeck, A., Schocke, M., & Delazer, M. (2009). The processing and representation of fractions within the brain. *NeuroImage*, *47*(1), 403–413. https://doi.org/10.1016/j.neuroimage.2009.03.041

Jiang, Z., Mok, I. A. C., & Li, J. (2020). Chinese Students' Hierarchical Understanding of Part-whole and Measure Subconstructs. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-020-10118-1

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Lamon, S. J. (2005). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Lawrence Erlbaum.

Lewis, K. E. (2016). Beyond Error Patterns: A Sociocultural View of Fraction Comparison Errors in Students With Mathematical Learning Disabilities. *Learning Disability Quarterly*, *39*(4), 199–212. https://doi.org/10.1177/0731948716658063

Li, Y., & Huang, R. (2013). *How Chinese teach mathematics and improve teaching*. Routledge.

Liu, C., Xin, Z., & Li, X. (2012). The development of Chinese students' understanding of the concept of fractions from fifth to eighth grade. *Journal of Mathematics Education*, *5*(1), 45–62.

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education*, *26*(5), 422. https://doi.org/10.2307/749431

Mazzocco, M. M. M., & Devlin, K. T. (2008). Parts and 'holes': Gaps in rational number sense among children with vs. Without mathematical learning disabilities. *Developmental Science*, *11*(5), 681–691. https://doi.org/10.1111/j.1467-7687.2008.00717.x

Mazzocco, M. M. M., Myers, G. F., Lewis, K. E., Hanich, L. B., & Murphy, M. M. (2013). Limited knowledge of fraction representations differentiates middle school students with mathematics learning disability (dyscalculia) versus low mathematics achievement. *Journal of Experimental Child Psychology*, *115*(2), 371–387. https://doi.org/10.1016/j.jecp.2013.01.005

Meert, G., Grégoire, J., & Noël, M.-P. (2010). Comparing the magnitude of two fractions with common components: Which representations are used by 10- and 12-year-olds? *Journal of Experimental Child Psychology*, *107*(3), 244–259. https://doi.org/10.1016/j.jecp.2010.04.008

Ministry of Education. (2011). *Curriculum standards for school mathematics of nine-year compulsory education*. Beijing Normal University Press.

Miura, I. T., & Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 229–242). Lawrence Erlbaum Associates Publishers.

Miura, I. T., Okamoto, Y., Vlahovic-Stetic, V., Kim, C. C., & Han, J. H. (1999). Language supports for children's understanding of numerical fractions: Cross-national comparisons. *Journal of Experimental Child Psychology*, *74*(4), 356–365. https://doi.org/10.1006/jecp.1999.2519

Mix, K. S., & Paik, J. H. (2008). Do Korean fraction names promote part-whole reasoning? *Journal of Cognition and Development*, *9*(2), 145–170. https://doi.org/10.1080/15248370802022605

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science.* https://timssandpirls.bc.edu/timss2019/international-results/

Ngan Ng, S. S., & Rao, N. (2010). Chinese number words, culture, and mathematics learning. *Review of Educational Research*, *80*(2), 180–206. https://doi.org/10.3102/0034654310364764

Ni, Y., & Zhou, Y.-D. (2005). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. *Educational Psychologist*, *40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

OECD. (2018). *Programme for International Student Assessment.* http://www.oecd.org/pisa/publications/pisa-2018-results.htm

Paik, J. H., & Mix, K. S. (2003). U.S. and Korean Children's Comprehension of Fraction Names: A Reexamination of Cross-National Differences. *Child Development*, *74*(1), 144–154. https://doi.org/10.1111/1467-8624.t01-1-00526

Rinne, L. F., Ye, A., & Jordan, N. C. (2017). Development of fraction comparison strategies: A latent transition analysis. *Developmental Psychology*, *53*(4), 713–730. https://doi.org/10.1037/dev0000275

Saxe, G. B., Taylor, E. V., McIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis. *Journal for Research in Mathematics Education*, *36*(2), 137–157. https://www.jstor.org/stable/30034828

Siegler, R. S., & Lortie-Forgues, H. (2017). Hard Lessons: Why Rational Number Arithmetic Is So Difficult for So Many People. *Current Directions in Psychological Science*, *26*(4), 346–351. https://doi.org/10.1177/0963721417700129

Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology*, *51*(2), 101–140. https://doi.org/10.1016/j.cogpsych.2005.03.001

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, *14*(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015

Taubman, A. A. (2007). *Threshold resistance: The extraordinary career of a luxury retailing pioneer*. Harper Collins.

Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning. In *Research companion to the principles and standards for school mathematics* (J. Kilpatrick, G. Martin, D. Schifter (Eds.), pp. 95–114). National Council of Teachers of Mathematics.

Van Hoof, J., Lijnen, T., Verschaffel, L., & Van Dooren, W. (2013). Are secondary school students still hampered by the natural number bias? A reaction time study on fraction comparison tasks. *Research in Mathematics Education*, *15*(2), 154–164. https://doi.org/10.1080/14794802.2013.797747

Vosniadou, S. (1994). Capturing and modeling the process of conceptual change. *Learning and Instruction*, *4*(1), 45–69. https://doi.org/10.1016/0959-4752(94)90018-3

Xin, Z., & Liu, C. (2014). Chinese children's understanding of the fraction concept. In B. Sriraman, J. Cai, K. Lee, L. Fan, Y. Shimizu, C. Sam Lim, & K. Subramaniam (Eds.), *The first sourcebook on Asian research in mathematics education: China, Korea, Singapore, Japan, Malaysia, and India* (Vol. 1, pp. 515–540). Information Age.

Xu, C., Li, H., Di Lonardo Burr, S. M., Huang, B., LeFevre J-A., Si, J. (2022). Divide and conquer: Relations among arithmetic operations and emerging knowledge of fraction notation for Chinese students in grade 4. *Journal of Experimental Child Psychology, 217*, 105371. https://doi.org/10.1016/j.jecp.2021.105371

Table 1

Classifiable Errors on Fraction Mapping Task

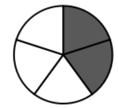
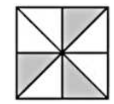
Error Name	**Description of Error**	**Example Responses to $\frac{2}{5}$** 	**Example Responses to $\frac{3}{8}$** 	**Percentage of Students who made Error at least Once**
Whole Number	Providing a whole number response equal to either the numerator or denominator	2	3	5%
Inverting	Inverting the numerator and denominator (i.e., whole/shaded)	$\frac{5}{2}$	$\frac{8}{3}$	17%
Shaded-Unshaded	Providing a response in which the numerator was the number of shaded pieces and the denominator was the number of unshaded pieces	$\frac{2}{3}$	$\frac{3}{5}$	10%
Decimal	Writing a fraction as a decimal	5.2	8.3	2%

Table 2

Frequency of Fraction Mapping Responses

Response	Item																				**Total**
	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{2}{7}$	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{3}{7}$	$\frac{7}{10}$	$\frac{5}{8}$	
Correct	825	777	759	895	781	783	767	601	669	683	665	564	513	428	345	242	114	87	53	46	**10,597**
Inverting	147	141	142	13	143	135	133	123	112	109	108	84	70	54	45	23	7	7	6	5	**1,607**
Whole-Number Bias	25	26	29	51	32	33	32	32	31	30	30	30	29	28	28	27	26	26	26	26	**597**
Part-Whole Bias	22	28	25	51	22	25	24	18	20	17	22	15	18	10	9	9	7	2	3	2	**349**
Decimal	16	15	15	12	16	16	14	14	13	10	10	9	7	6	6	5	3	2	2	2	**193**
Careless	1	1	6	3	11	1	1	154	51	17	13	54	21	3	2	9	16	6	5	5	**380**
Miscellaneous	0	44	54	3	14	14	16	6	15	12	29	10	0	2	0	0	1	0	0	0	**220**
Blank	0	4	6	8	17	29	49	88	125	158	159	270	378	505	601	721	862	906	941	950	**6,777**

Table 3

Frequency of Fraction Comparison Responses

Response	Item																				**Total**
	$\frac{1}{8}$ vs. $\frac{7}{8}$	$\frac{3}{4}$ vs. $\frac{1}{4}$	$\frac{3}{5}$ vs. $\frac{3}{7}$	$\frac{5}{9}$ vs. $\frac{5}{6}$	$\frac{1}{6}$ vs. $\frac{5}{6}$	$\frac{1}{9}$ vs. $\frac{1}{8}$	$\frac{3}{4}$ vs. $\frac{3}{5}$	$\frac{2}{5}$ vs. $\frac{2}{3}$	$\frac{5}{7}$ vs. $\frac{4}{7}$	$\frac{7}{9}$ vs. $\frac{8}{9}$	$\frac{2}{3}$ vs. $\frac{1}{4}$	$\frac{4}{5}$ vs. $\frac{2}{3}$	$\frac{1}{3}$ vs. $\frac{3}{4}$	$\frac{7}{8}$ vs. $\frac{3}{5}$	$\frac{3}{4}$ vs. $\frac{5}{6}$	$\frac{1}{4}$ vs. $\frac{2}{7}$	$\frac{3}{8}$ vs. $\frac{1}{3}$	$\frac{2}{3}$ vs. $\frac{4}{7}$	$\frac{5}{8}$ vs. $\frac{2}{3}$	$\frac{3}{4}$ vs. $\frac{5}{7}$	
Correct	917	903	315	312	874	324	316	316	876	873	407	709	734	709	678	654	646	151	142	139	**10,995**
Incorrect	118	132	719	721	159	707	712	703	142	134	576	247	191	183	182	181	170	639	634	633	**7,883**
Blank	0	0	1	2	2	4	7	16	17	28	52	79	110	143	175	200	219	245	259	263	**1,822**
% Correct[a]	89	87	30	30	85	31	31	31	86	87	41	74	79	79	79	78	79	19	18	18	**58**

Note. [a]Percent correct was calculated excluding blank responses (i.e., [correct/(correct + incorrect)]); Congruent trials are shaded.

Figure 1

Examples of Fraction Mapping Exercises Using Pictorial Representations

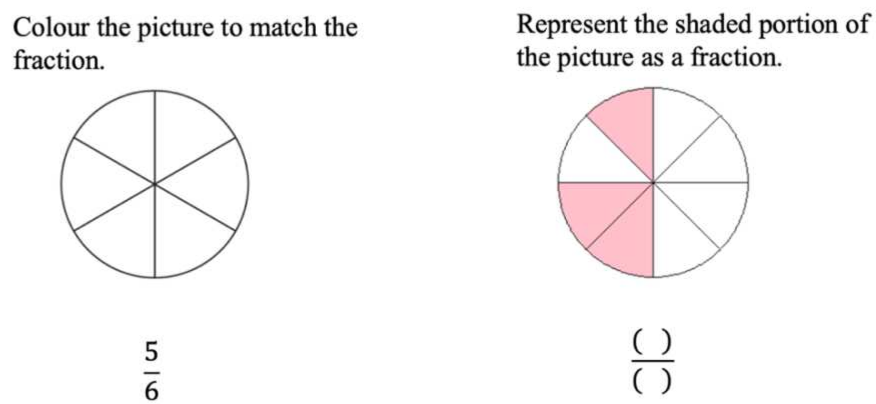


Figure 2

Examples of Fraction Magnitude Exercises Using Pictorial Representations

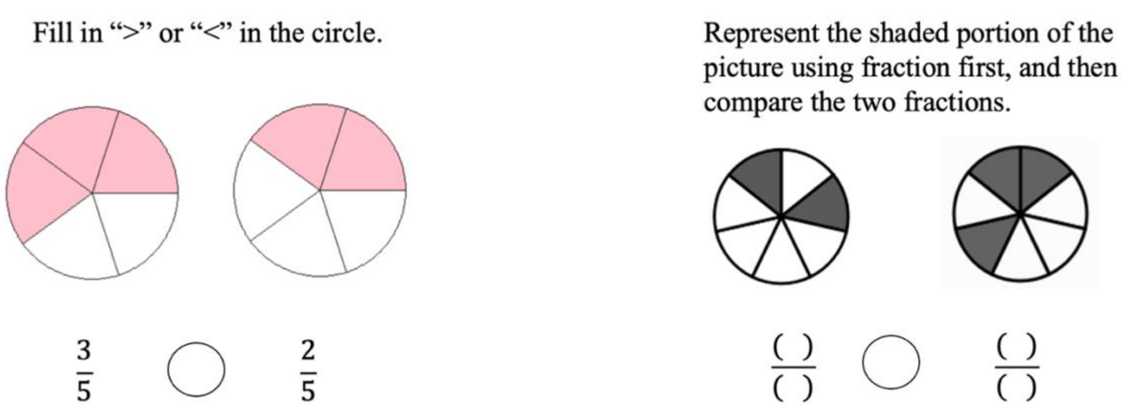


Figure 3

Percentage of Each Response Type by Item

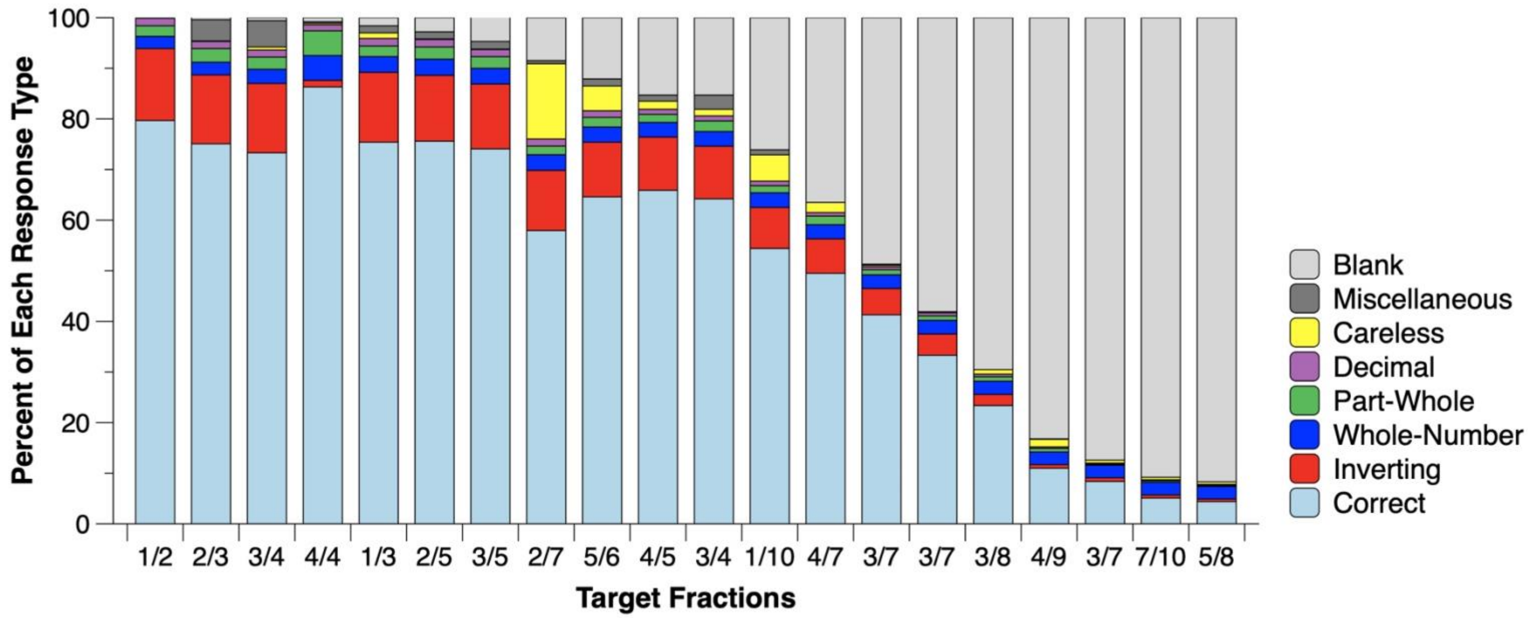
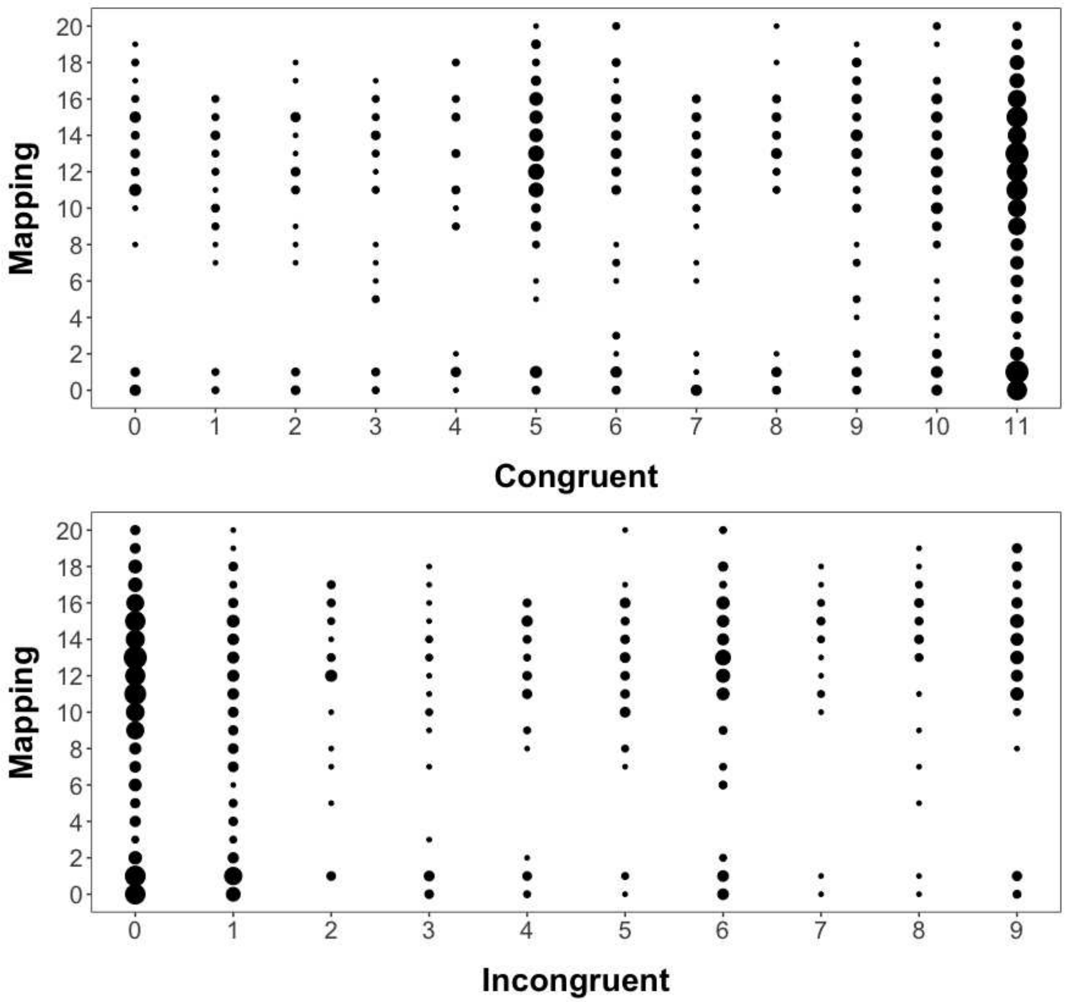


Figure 4

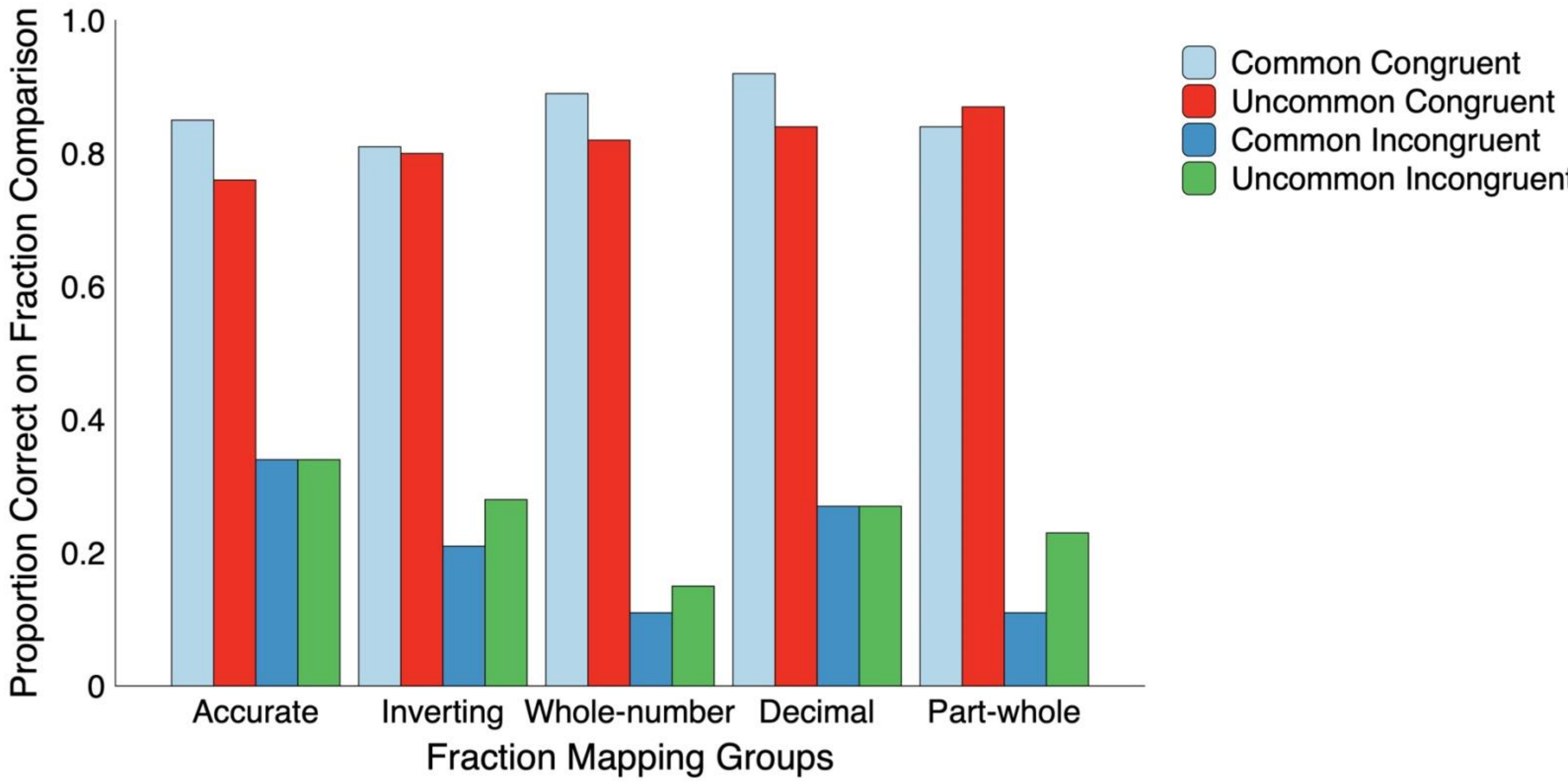
Scatterplots for Sum Scores on Fraction Mapping and Fraction Comparison Tasks



Note. The size of each point is proportionate to the frequency of that score

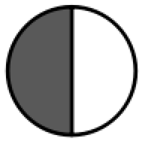
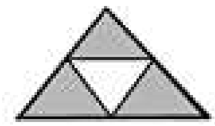
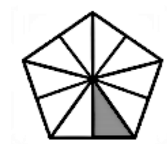
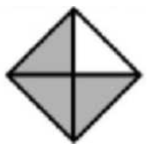
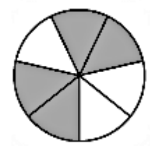
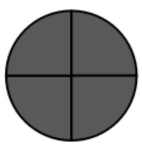
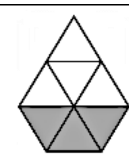
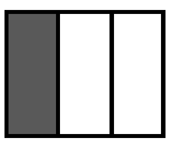
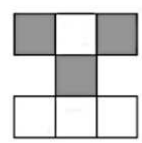
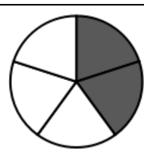
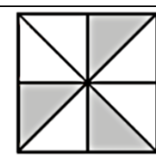
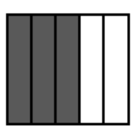
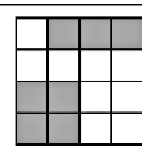
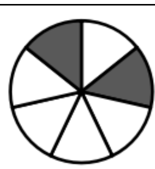
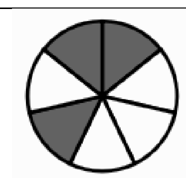
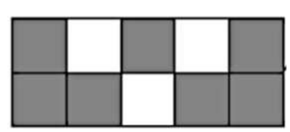
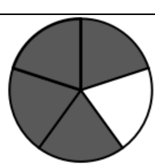
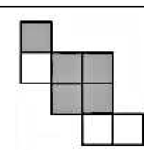
Figure 5

Proportion Correct of Each Type of Fraction Comparison Trials by Fraction Mapping Groups



Appendix A: Fraction Mapping Stimuli

Translated Instructions: Write down the fraction of the shaded area in order.

A1			A11		
A2			A12		
A3			A13		
A4			A14		
A5			A15		
A6			A16		
A7			A17		
A8			A18		
A9			A19		
A10			A20		

Appendix B: Fraction Comparison Stimuli

Translated Instructions: Circle the larger fraction in each pair in order.

A1	$\frac{1}{8}$	$\frac{7}{8}$	**A11**	$\frac{2}{3}$	$\frac{1}{4}$
A2	$\frac{3}{4}$	$\frac{1}{4}$	**A12**	$\frac{4}{5}$	$\frac{2}{3}$
A3	$\frac{3}{5}$	$\frac{3}{7}$	**A13**	$\frac{1}{3}$	$\frac{3}{4}$
A4	$\frac{5}{9}$	$\frac{5}{6}$	**A14**	$\frac{7}{8}$	$\frac{3}{5}$
A5	$\frac{1}{6}$	$\frac{5}{6}$	**A15**	$\frac{3}{4}$	$\frac{5}{6}$
A6	$\frac{1}{9}$	$\frac{1}{8}$	**A16**	$\frac{1}{4}$	$\frac{2}{7}$
A7	$\frac{3}{4}$	$\frac{3}{5}$	**A17**	$\frac{3}{8}$	$\frac{1}{3}$
A8	$\frac{2}{5}$	$\frac{2}{3}$	**A18**	$\frac{2}{3}$	$\frac{4}{7}$
A9	$\frac{5}{7}$	$\frac{4}{7}$	**A19**	$\frac{5}{8}$	$\frac{2}{3}$
A10	$\frac{7}{9}$	$\frac{8}{9}$	**A20**	$\frac{3}{4}$	$\frac{5}{7}$

Note: Congruent trials are A1, A2, A5, A9, A10, A12, A13, A14, A15, A16, A17.

Incongruent trials are A3, A4, A6, A7, A8, A11, A18, A19, A20.