Proceedings Paper:
Nassaj, A., Abiri Jahromi, A., Li, K. et al. (2 more authors) (2023) Linear Distribution System State Estimation by Hybrid Synchronized/Unsynchronized Measurements. In: 2023 IEEE 13th International Workshop on Applied Measurements for Power Systems (AMPS). 13th IEEE International Workshop on Applied Measurements for Power Systems (AMPS 2023), 27-29 Sep 2023, Bern, Switzerland. IEEE . ISBN 979-8-3503-1240-9

https://doi.org/10.1109/AMPS59207.2023.10297208

Linear Distribution System State Estimation by Hybrid Synchronized/Unsynchronized Measurements

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Abstract— **This paper proposes a linear state estimation method for unbalanced distribution systems using available synchronized/unsynchronized measurements. The paper demonstrates that circuit properties can readily be taken advantage of to linearly express power measurements in terms of bus voltages and line currents. This results in a linear distribution system state estimation (LDSSE) process, which involves two stages. In stage one, the three-phase voltage phase angles are estimated using available measurements and network parameters. By formulating voltage/current phasors based on these phase angles, linear equations are developed that enable the estimation of voltage phasors. The linear nature of the proposed method significantly reduces the computation time while maintaining the solution's accuracy. The superiority of the proposed LDSSE method over existing methods is verified using extensive simulation studies conducted on different test feeders.**

Keywords—Hybrid measurements, linear state estimation, unbalanced distribution system, weighted least squares.

I. INTRODUCTION

State estimation (SE) is a crucial process that involves filtering measurement errors to obtain an accurate representation of the power system's current state [1]. The abundance of measurements on transmission systems has made state estimation a normal practice at extra-high voltage levels, where SE is also a prerequisite for energy management applications such as contingency analysis and congestion management [1, 2]. SE does not hold much significance for passive distribution systems characterized by slow demand variations that can effectively be managed via load forecasting. Nevertheless, passive distribution systems are evolving into active systems that require real-time monitoring [3], owing to the increasing integration of distributed generations, battery energy storage systems, and electric vehicle charging stations. In this context, distribution system state estimation (DSSE) becomes crucial for real-time monitoring and control purposes.

Except for some pioneering research on DSSE in the 1990s [4-6], this field has gained most of its popularity in recent years. DSSE is associated with a greater number of challenges compared to transmission SE due to factors such as insufficient observability, unbalanced radial configuration, and a high ratio of r/x [7, 8]. Extensive work has been carried out to alleviate these challenges [9-13]. The observability problem is considered as a major concern due to limited metering instruments compared to the large size of distribution systems. Distribution systems are normally under-determined, which means they are monitored by a small number of real-time measurements (which are typically less than the number of state variables [7]). However, the integration of smart meters, advanced metering infrastructures (AMIs), and micro phasor measurement units (μPMUs) hold great promise for implementing DSSE [8]. The usage of pseudo-measurements and virtual measurements is primarily aimed at increasing measurement redundancy in distribution systems [9-11]. The unbalanced operation caused by single- or double-phase loads and untransposed lines is deemed to be another major issue. This is the main reason that DSSE is usually developed per phase based on the three-phase models of equipment [12]. Radial configuration and high r/x ratio also present complications that can lead to an ill-conditioned gain matrix and potential algorithm divergence in conventional DSSE (CDSSE) [13].

There are other research gaps that need to be addressed for DSSE. Dealing with nonlinear equations developed based on conventional measurements is a major concern. Conventional measurements can provide unsynchronized voltage/current magnitudes and active/reactive powers in distribution systems. CDSSE formulations based on these measurements are nonlinear, which require iterative solving processes such as the Gauss-Newton method [5], Forward-Backward sweep method [14], or their combination [15]. Iterative CDSSE methods require both initialization and iteration, suffering from the possibility of divergence and the multiplicity of solutions. Due to the three-phase modeling of a large number of buses in distribution systems, iterative solutions would be highly time-consuming. The radial configuration and high r/x ratios in distribution systems increase the possibility of an ill-conditioned gain matrix, thus, inaccurate results or even divergence.

Linear DSSE (LDSSE) methods are introduced to address some of the challenges associated with CDSSE. A primitive solution for LDSSE harnesses a fast decoupled algorithm, where after applying some simplifications, the linearized problem is solved iteratively [16]. To avoid iteration in LDSSE, a trivial approach is to make the equation linear by installing a large number of μPMUs to cover the distribution system with enough measurement redundancy [17, 18]. However, the realization of synchrophasor redundancy in distribution systems appears to be quite challenging. This is why some recent works focus on LDSSE using small voltage drops provided by conventional measurements [19]. Although this approach can be considered more practical for employing SCADA data rather than synchrophasors, simplifications made in modeling could easily pose unreliable results.

This paper presents a two-stage LDSSE method based on purely conventional or hybrid measurements. In contrast to the existing LDSSE methods found in the literature, the proposed method relies on inherently linear equations, eliminating the requirement for linearization. As a result, the estimation of three-phase voltage phasors using the proposed LDSSE method does not involve any iteration or simplification in the modeling process.

The rest of this paper is organized as follows. Section II discusses pressing challenges in the context of distribution systems state estimation. The proposed LDSSE is detailed in Section III. The performance evaluation is carried out in Section IV. Section V provides the conclusions.

II. Basic Concepts

A. Distribution System State Estimation (DSSE)

The state estimation runs in the control center to provide a picture of the power system's current condition. The principal duty of SE is filtering out measurement errors and providing operators with reliable data on system variables [1]. Unlike transmission systems, there is insufficient redundancy to implement SE in distribution systems. By exploiting historical load data as pseudo-measurements as well as zero-injection buses as virtual measurements, the observability of distribution systems can be improved [7]. The penetration of real-time meters, including SCADA, smart meters, μPMUs, and AMIs, assesses the feasibility of implementing DSSE [11]. Utilizing network parameters and real-time measurements, SE equations may be developed linearly [20] or nonlinearly [2]. It goes without saying that with the same level of accuracy, linear formulations would be favored, owing to the inherent challenges associated with the solution of nonlinear problems.

DSSE methods are developed based on unbalanced modeling of distribution systems, where a three-phase model of equipment is required [21]. Dealing with unbalanced systems and developing three-phase models increases the complexity of SE in distribution systems compared to transmission systems. Let us assume the reference bus's voltage phase angle is known in a n-bus system. The length of the state vector in transmission systems is $2n-1$, while in distribution systems is $6n-3$. Therefore, the size of the SE problem in distribution systems is three times greater than that of transmission systems. Considering the size of the problem and the ill-conditioned gain matrix in DSSE (due to the radial configuration with high r/x), the possibility of divergence is significant.

B. Hybrid Measurements

Measurements in power systems can be categorized into two main groups: Conventional and synchrophasor measurements. Conventional measurements provided by smart meters and SCADA are unsynchronized (i.e., voltage/current magnitudes and active/reactive powers). The refresh rate of conventional measurements is within a few seconds (typically every 2 sec) [1]. Conventional measurements are appropriate for static SE, where the rate of change of variables can be considered slow due to the steady-state condition of the power system [22]. On the other hand, synchrophasors can trace the dynamics of power systems by providing time-synchronized voltage and current phasors. The typical reporting rate of synchrophasors is 50/60 samples per second, which is adequate for dynamic SE purposes [23]. SE by only synchrophasors is not yet applicable in many existing distribution systems due to insufficient instrumentation or enough infrastructure. In this context, developing state estimators that can function with unsynchronized or hybrid measurements are considered quite advantageous [20].

C. Weighted Least Squared based Estimation

In SE, a regression model is developed to predict the values of target variables while minimizing the adverse impact of measurement errors. In the case of a nonlinear formulation, the first-order approximation of the Taylor series can be employed to linearize the equations. Then, the linearized formulation is solved iteratively. Solving linear equations does not involve initialization or iteration. To find the best-fit for the states, various regression methods can be applied, such as WLS, least absolute values (LAV), or generalized maximum-likelihood (GM) [7]. Within the class of linear unbiased estimators, the least squares estimator provides the lowest variance based on Gauss–Markov theorem [24]. If the measurement errors can be assumed to have Gaussian distribution with mean zero, then WLS is also the maximum likelihood estimator [24].

WLS is one of the straightforward methods used for DSSE [22, 25]. The DSSE problem can be modeled as a set of linear equations as follows:

$$\boldsymbol{Hx} + \boldsymbol{e} = \boldsymbol{z} \tag{1}$$

where $\boldsymbol{H}$ is the representative of the coefficient matrix and $\boldsymbol{x}$ denotes the vector of states. Also, $\boldsymbol{z}$ and $\boldsymbol{e}$ denote the vectors of measurements and measurement errors, respectively.

The closed-form solution of (1) by WLS would be [24]

$$\hat{\boldsymbol{x}} = \boldsymbol{PH}^{*}\boldsymbol{R}^{-1}\boldsymbol{z} \tag{2}$$

$$\boldsymbol{P} = (\boldsymbol{H}^{*}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1} \tag{3}$$

where $\hat{\boldsymbol{x}}$ denotes the vector of estimated states, and $\boldsymbol{P}$ and $\boldsymbol{R}$ denote the covariance matrices of estimated states and measurements, respectively.

III. The Proposed LDSSE

The proposed linear state estimation method for unbalanced distribution systems is described in this section. Linear SE methods in the literature are highly dependent on redundant synchrophasor data [17]. Other methods take advantage of slight voltage drops within distribution lines to develop LDSSE using simplifications [19]. The proposed LDSSE in this paper can be formulated with both purely conventional (unsynchronized) or hybrid (a mix of synchronized and unsynchronized) measurements with no simplification in modeling.

The proposed LDSSE contains two major stages. In the first stage, the voltage phase angles of the whole system are calculated based on the available measurements. Then, using the voltage phase angles from the first stage, the final voltage phasors are estimated in the second stage. The mentioned topics are explained in the following subsections.

A. Computing Unbalanced Voltage Phase Angles

To develop linear equations using conventional measurements, first, the local variables are defined as

$$v_k^\alpha = |v_k^\alpha| \quad (4)$$

$$i_{kl}^\alpha = |i_{kl}^\alpha| e^{j\varphi_{kl}^\alpha} = (p_{kl}^\alpha - jq_{kl}^\alpha)/|v_k^\alpha| \quad (5)$$

$$\varphi_{kl}^\alpha = -\tan^{-1}(q_{kl}^\alpha/p_{kl}^\alpha) \quad (6)$$

where $|v_k^\alpha|$ is the measured voltage magnitude of phase α at the bus k, $|i_{kl}^\alpha|$ and φ_{kl}^α denote the measured current magnitude and power factor phase angle of the same phase at the branch kl. Also, p_{kl}^α and q_{kl}^α are the representative of measured active and reactive powers of the same phase and branch. The local variables in (4)-(6) are built up by only conventional measurements from SCADA or smart meters.

The three-phase voltage phase angles of the reference bus are considered to be known based on the proposed solution in [26, 27]. Hence, for the reference bus or any bus equipped with µPMUs, the following equation is applied

$$\tilde{v}_k^\alpha = |v_k^\alpha| e^{j\delta_k^\alpha} = v_k^\alpha e^{j\delta_k^\alpha} \quad (7)$$

where δ_k^α denote voltage phase angle of phase α at the bus k. Further, (7) is used for only reference bus in the event of purely conventional measurements.

Now, the relationship between synchronized voltage and current phasors can be expressed as

$$\frac{\tilde{\imath}_{kl}^\alpha}{\tilde{v}_k^\alpha} = \frac{|i_{kl}^\alpha| e^{j\varphi_{kl}^\alpha + \delta_k^\alpha}}{|v_k^\alpha| e^{j\delta_k^\alpha}} = \frac{|i_{kl}^\alpha| e^{j\varphi_{kl}^\alpha}}{|v_k^\alpha|} = \frac{i_{kl}^\alpha}{v_k^\alpha} \quad (8)$$

where $\tilde{v}_k^\alpha$ is the synchronized voltage phasor of phase α at the bus k, and $\tilde{\imath}_{kl}^\alpha$ denotes the synchronized current phasor of the same phase at the branch kl. Consequently, (8) can be rewritten as the following equation

$$\tilde{\imath}_{kl}^\alpha = (i_{kl}^\alpha/v_k^\alpha)\tilde{v}_k^\alpha = A_{kl}^\alpha \tilde{v}_k^\alpha \quad (9)$$

The above equation dedicates a linear relationship between synchronized current and voltage phasors using local variables gathered from conventional measurements. Considering the π model of a three-phase line, Kirchhoff's current law yields

$$\boldsymbol{Y}_{kl}^{abc}(\tilde{\boldsymbol{v}}_k^{abc} - \tilde{\boldsymbol{v}}_l^{abc}) + \boldsymbol{B}_{kl}^{abc}\tilde{\boldsymbol{v}}_k^{abc} = \tilde{\boldsymbol{\imath}}_{kl}^{abc} \quad (10)$$

where $\tilde{\boldsymbol{v}}_k^{abc}$ and $\tilde{\boldsymbol{v}}_l^{abc}$ denote the vectors of three-phase voltage phasors at buses k and l, respectively, $\tilde{\boldsymbol{\imath}}_{kl}^{abc}$ is the vector of three-phase current phasors from bus k to bus l. Also, $\boldsymbol{Y}_{kl}^{abc}$ and $\boldsymbol{B}_{kl}^{abc}$ denote series admittance and shunt susceptance matrices of branch kl. In the case of having current phasors from µPMUs, they can be directly used in (10). Now, applying (9) to (10) yields

$$\left[\boldsymbol{A}_{kl}^{abc} - (\boldsymbol{Y}_{kl}^{abc} + \boldsymbol{B}_{kl}^{abc})\right]\tilde{\boldsymbol{v}}_k^{abc} + \boldsymbol{Y}_{kl}^{abc}\tilde{\boldsymbol{v}}_l^{abc} = 0 \quad (11)$$

By (11), a linear equation with respect to voltage phasors as state variables can be developed for those branches equipped with conventional measurements (at just one end of the line). $\boldsymbol{A}_{kl}^{abc}$ is a diagonal 3×3 complex matrix developed based on (9), which its elements are defined as

$$A_{kl}^\alpha = (|i_{kl}^\alpha| e^{j\varphi_{kl}^\alpha})/|v_k^\alpha| = (p_{kl}^\alpha - jq_{kl}^\alpha)/|v_k^\alpha|^2 \quad (12)$$

The above equation can be developed by voltage magnitude as well as either current magnitude or power measurements in distribution system lines. On the other hand, (11) can be extended for conventional current or power injection measurements as follows:

$$\left[\boldsymbol{A}_k^{abc} - \sum_{l\epsilon c}(\boldsymbol{Y}_{kl}^{abc} + \boldsymbol{B}_{kl}^{abc})\right]\tilde{\boldsymbol{v}}_k^{abc} + \sum_{l\epsilon c}\boldsymbol{Y}_{kl}^{abc}\tilde{\boldsymbol{v}}_l^{abc} = 0 \quad (13)$$

where c refers to those buses connected to bus k, and $\boldsymbol{A}_k^{abc}$ is computed based on (12) for injected currents/powers to the relevant bus. Thus, the linear set of equations for conventional current/power injection meters is developed.

To improve the observability problem in DSSE, zero-injection buses are considered here. The linear equation for zero-injection buses can be defined as

$$\sum_{l\epsilon c}(\boldsymbol{Y}_{kl}^{abc} + \boldsymbol{B}_{kl}^{abc})\tilde{\boldsymbol{v}}_k^{abc} + \sum_{l\epsilon c}\boldsymbol{Y}_{kl}^{abc}\tilde{\boldsymbol{v}}_l^{abc} = 0 \quad (14)$$

Using (14) can improve the observability of the system in the DSSE problem. As can be seen, the proposed LDSSE can cover all types of measurements straightforwardly.

Now, the LDSSE can be formulated using conventional measurements in (7), (11), (13), and (14) for unbalanced voltages of reference bus, current/power flows, current/power injections, and zero injections, respectively. Also, (7) and (10) are utilized for voltage and current phasors measured by µPMUs, respectively. Then, the initial three-phase voltage phasors are determined in the first stage by utilizing ordinary least-squares (i.e. (2) with identity measurements covariances). The presence of measurement data in the coefficient matrix degrades the effectiveness of least-squares estimation. The reason behind this issue is the sensitivity of the least-squares method to errors in the coefficient matrix. In addition, deriving the covariance of each equation for WLS is not straightforward. Therefore, only estimated voltage phase angles in the first stage are exploited to estimate the final states by the second stage.

B. Estimating Three-Phase Voltage Phasors

Having the three-phase voltage phase angles from the first stage, the current and voltage phasors can be rewritten based on these. This helps to move the measurements from the coefficient matrix to the measurement vector. This enables us to take advantage of WLS estimation, as will be described in Subsection *C*. The voltage and current phasors expressed in terms of the voltage phase angles derived in the first stage and the magnitude measurements are referred to as pseudo-synchronized phasors. This is to signify the difference in reporting times of conventional measurements and the fact that these measurements can be combined with synchrophasors since the power system is assumed to be in a steady-state condition.

To develop pseudo-synchronized voltage phasors similar to (7), the following equation is used

$$\tilde{v}_k^\alpha = \hat{v}_k^\alpha = v_k^\alpha e^{j\hat{\delta}_k^\alpha} \quad (15)$$

where $\hat{v}_k^\alpha$ denotes the estimated voltage phasor of phase α at bus k formed by v_k^α and $\hat{\delta}_k^\alpha$ as the relevant locally measured voltage magnitude and estimated phase angle. In the case of the availability of synchrophasors, the measured voltage phasor is directly used in (15). Combining local measurements (such as SCADA or smart meters) and computed phase angles gives pseudo-synchronized phasors.

Similar to voltage phasors, pseudo-synchronized current phasors are defined as

$$\hat{i}_{kl}^{\alpha} = i_{kl}^{\alpha} e^{j\hat{\delta}_k^{\alpha}} \tag{16}$$

Depending on the availability of either current magnitude or active/reactive power, i_{kl}^{α} is formed by (5). Taking synchrophasor data into account, the current phasor measured is directly applied to (16). Using (16) in (10)

$$\left(\boldsymbol{Y}_{kl}^{abc} + \boldsymbol{B}_{kl}^{abc}\right)\tilde{\boldsymbol{v}}_k^{abc} - \boldsymbol{Y}_{kl}^{abc}\tilde{\boldsymbol{v}}_l^{abc} = \hat{\boldsymbol{\imath}}_{kl}^{abc} \tag{17}$$

This equation can be extended for current injection measurement based on (13). Now, by employing (14) for virtual measurements, and (15) and (17) for built phasors, the linear set of equations is developed as follows:

$$\begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \boldsymbol{H}_3 \end{bmatrix} [\tilde{\boldsymbol{v}}] = \begin{bmatrix} \hat{\boldsymbol{v}} \\ \hat{\boldsymbol{\imath}} \\ \hat{\boldsymbol{o}} \end{bmatrix} \tag{18}$$

where $\tilde{\boldsymbol{v}}^{\alpha}$ denotes the vector of three-phase voltage phasors with $3n$ states for a n-bus distribution system. $\boldsymbol{H}_1$ is a $u \times 3n$ identity matrix corresponding to the vector $\hat{\boldsymbol{v}}$ with u voltage phasor measurements. $\boldsymbol{H}_2$ is a $w \times 3n$ matrix consisting of network parameters related to the vector $\hat{\boldsymbol{\imath}}$ with w current phasor measurements. The elements of vectors $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{\imath}}$ can contain either measured synchrophasors or computed pseudo-synchronized phasors. Also $\boldsymbol{H}_3$ is a $l \times 3n$ matrix containing network parameters respected to the null vector $\hat{\boldsymbol{o}}$ for zero injection buses. Here, contrary to the first stage, the coefficient matrix $[\boldsymbol{H}_1 \quad \boldsymbol{H}_2 \quad \boldsymbol{H}_3]^T$ is error-free and contains only network parameters.

C. *Defining Weights for WLS*

To solve (18) by WLS in (2), the proper covariance should be defined for equations in this stage. Taking the complex form of a phasor into account

$$z = z^t + e = r^t e^{j\theta^t} + e = re^{j\theta} \tag{19}$$

where z^t and e are the representative of the true value and error of phasor z . r^t and θ^t denote true values of magnitude and phase angle of the phasor z^t. r and θ denote the measured values of magnitude and phase angle of the phasor z. The errors in both magnitude and phase angle of e are assumed to be independent (provided using μPMUs), with each following a Gaussian distribution. Specifically, $r \sim \mathcal{N}(r^t, \sigma_r^2)$ and $\theta \sim \mathcal{N}(\theta^t, \sigma_\theta^2)$, with σ_r^2 and σ_θ^2 representing the variance of the magnitude and phase angle errors, respectively. For those phase angles gathered from the first stage, σ_θ^2 would be negligible and is assumed to be zero. The covariance of e associated with its magnitude and phase angle errors is then defined as [28]

$$\sigma_z^2 = r^2\left(1 - e^{-\sigma_\theta^2}\right) + \sigma_r^2\left(2 - e^{-\sigma_\theta^2}\right) \tag{20}$$

Using (20), a real-valued variance can be extracted for the error in a polar phasor. This equation applies to equations developed by (15) and (17). The variance of the virtual measurements is considered to be 10^{-6} here.

IV. Simulation Results

In order to evaluate the accuracy and effectiveness of the proposed LDSSE, extensive simulation studies are carried out. For each simulation, different measurement types and variances are investigated. Measurement errors are assumed to be independent and have normal distributions with standard deviation σ. The three-sigma criterion is used to report the error range [29]. All codes are developed in MATLAB R2021b and run on a Core i7 CPU with 32-GB RAM. To be able to draw solid conclusions from each case study, Monte Carlo simulations are run and the obtained results are compared with the true values. It is noteworthy that load flow results for each case study are acquired from OpenDSS [30]. For all test systems, the minimum number of measurements with their location is determined by a linear programming method, as described in [31].

In Subsection A, the efficiency of the proposed LDSSE method using various levels of measurement errors is analyzed. Afterward, the performances of the proposed LDSSE and CDSSE are compared on various test feeders in Subsection B. In different simulations, the ratio between synchronized and unsynchronized measurements is varied to study its impact.

A. *Evaluating the Accuracy of Proposed LDSSE*

The performance of the proposed LDSSE is evaluated here using Monte Carlo simulations considering different measurement error levels. The IEEE 13-bus test feeder [32], as a well-known unbalanced distribution system, is used for this purpose. Taking advantage of zero injection at 632, 633, 680, and 684 buses, 645, 650 (reference), 684, and 692 buses are considered to be equipped with SCADA measurements. In fact, only 4 buses provide real-time measurements out of 13 buses. Here, conventional measurements are considered to show the effectiveness of the proposed method. 10,000 simulations are run for the test system considering three different measurement error levels. Results for voltage magnitude and phase angle errors (i.e., mismatches between estimated and true values) of the system are shown in Fig. 1. The results in Fig. 1 (a) and (b) are linked to three different levels of measurement errors. The errors in real-time conventional measurements and pseudo-measurements are assumed to be 1% and 10%, 3% and 30%, and 5% and 50%, respectively. Also, about one-third of the measurements are assumed to be coming from pseudo-measurements. Both figures are representative of the accumulative mismatch between the estimated values and true values in all phases. As can be seen, the proposed LDSSE is unbiased regarding estimated voltage magnitudes and phase angles.

Figure 2 demonstrates the root mean square error (RMSE) of phase b voltage phasors. Using RMSE, a real-valued index for examining the accuracy of estimated voltage phasors is obtained by

$$RMSE = \sqrt{\sum\nolimits_{i=1}^{n} \left|\tilde{v}_i^{tru} - \tilde{v}_i^{est}\right|^2 / n} \tag{21}$$

where $\tilde{v}_i^{tru}$ and $\tilde{v}_i^{est}$ denote the true and estimated voltage phasors, and n is the number of states.

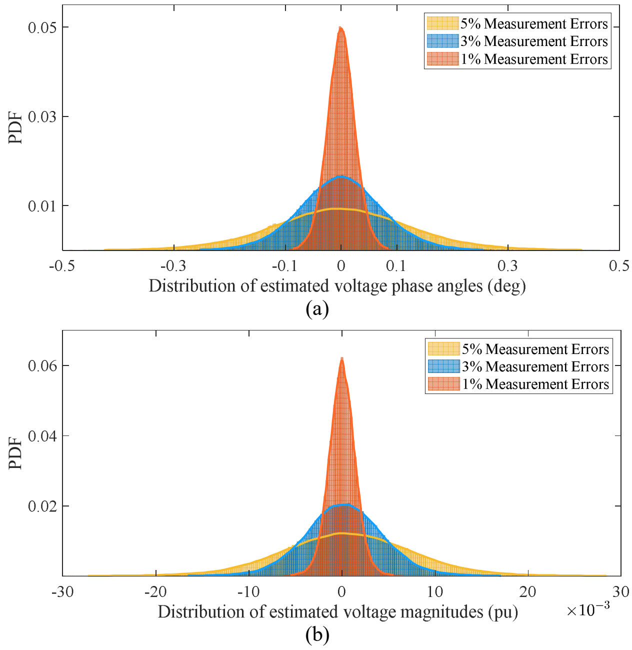

Fig. 1. Estimation errors of proposed LDSSE using various measurement errors for (a): Voltage phase angles and (b): Voltage magnitudes.

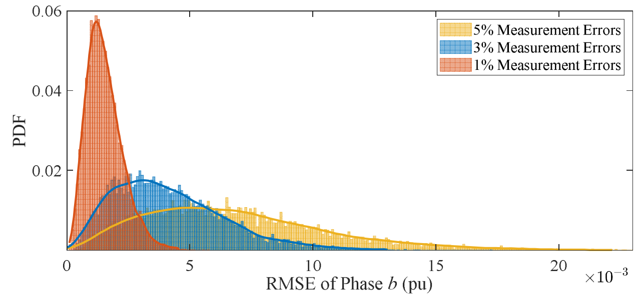

Fig. 2. RMSE of proposed LDSSE using various measurement errors.

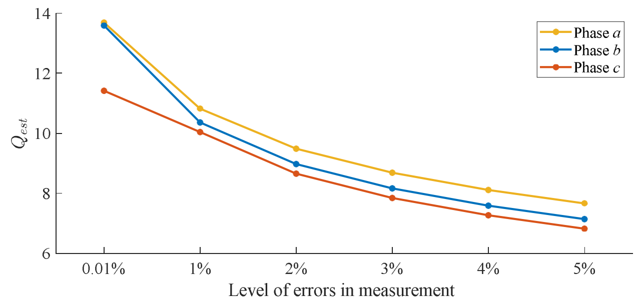

Fig. 3. Quality of estimates using the proposed LDSSE.

Due to the close behavior of three phases, only the results of one phase (phase b) is shown here. Utilizing the proposed LDSSE, accurate and reliable results may be achieved for each phase of unbalanced distribution systems.

The quality of estimates is also reported using [9]

$$Q_{est} = \ln(1/\mathrm{tr}(\boldsymbol{P})) \tag{22}$$

where Q_{est} is a statistical measure denoting the quality of the estimator and $\mathrm{tr}(\boldsymbol{P})$ is the trace of the states covariance matrix. This quality index expresses, in principle, the sensitivity of the estimator to the level of measurement errors [9]. Several error levels are studied and the average quality index for 10,000 simulations on the IEEE 13-bus test feeder is presented in Fig. 3. This figure represents the results for each phase separately. In this simulation, the variation range of measurement errors is altered from 0.01% to 5%. The first case runs LDSSE with hybrid measurements (those that are provided by µPMUs and SCADA along with pseudo-measurements). Other cases use unsynchronized data provided by SCADA and pseudo-measurements. In all cases, pseudo-measurements constitute 20% of all measurements. As shown in Fig. 3, increasing the level of errors degrades the quality of estimation by the proposed LDSSE as expected.

B. Comparison Under Various Test Feeders

The performance of the proposed LDSSE is compared with that of the CDSSE method in this subsection. To this end, several test feeders are chosen to conduct the simulations, namely, the IEEE 13-bus, 37-bus, 123-bus, 906-bus (European low voltage test feeder), and 8500-bus test feeders [32]. With conventional measurements, the CDSSE becomes nonlinear and is solved iteratively by the Gauss-Newton algorithm [5]. Regardless of the ratio between the numbers of synchronized and unsynchronized measurements, the proposed LDSSE remains fast and reliable. Both CDSSE and LDSSE methods take advantage of WLS to estimate the system state.

Table I summarizes obtained results (RMSE and computation time) on different test feeders using CDSSE and LDSSE. In this case study, the level of errors for µPMUs, SCADA, and pseudo-measurements are set to 0.01%, 5%, and 30%. It is assumed that 20% of measurements are provided by µPMUs. This is 30% and 50% for SCADA and pseudo-measurements, respectively. All zero-injection buses are considered virtual measurements with a variance of 10^{-6}. The results obtained from Monte Carlo simulations with 10,000 runs for small test feeders (i.e., 13-bus, 37-bus, and 123-bus systems) as well as 1,000 runs for large-scale test feeders (i.e., 906-bus and 8500-bus systems).

As can be seen in Table I, the RMSE by CDSSE is less than this by LDSSE for the 13-bus test feeder as the smallest case study. For rest test feeders, LDSSE presents more accuracy with less RMSE compared to the results of the conventional method. On the other hand, the LDSSE highly outperforms the CDSSE regarding computation time. As such, the computation burden of the proposed LDSSE is far less than the conventional method. Increasing the scale of the test system yields less efficiency in the performance of the CDSSE. In other words, the inaccuracy and computation burden of CDSSE regarding problem size (system scale) are increasing compared to the proposed LDSSE. The difference between the LDSSE and CDSSE is more tangible regarding computation burden. As such, for a large-scale system like the 8500-bus test feeder, LDSSE is approximately 20 times faster than CDSSE. Relying on inherently linear equations is the main attribute of the proposed LDSSE, by which accurate results can be achieved with no need for iteration. While, CDSSE methods are based on approximately linearized equations solved iteratively. Due to the considerable scale of real-world distribution systems, conventional iterative DSSE methods are prone to divergence possibility and large computation time. However, the proposed two-stage LDSSE reliably yields accurate results in a fast and straightforward way, applicable to unbalanced distribution systems of any realistic size.

TABLE I. COMPARING THE PERFORMANCE OF CDSSE AND LDSSE

Test Feeders	RMSE (pu) $\times 10^{-3}$		Computation Time (s) $\times 10^{-3}$	
	CDSSE	***LDSSE***	***CDSSE***	***LDSSE***
13-bus	7.6398	10.4577	2.4230	0.5509
37-bus	4.2776	3.6095	10.9941	1.8823
123-bus	5.7661	4.8005	165.6935	11.6004
906-bus	16.1939	10.8291	44683.96	2636.51
8500-bus	17.9107	10.9819	175789.82	8897.23

V. CONCLUSION

This paper puts forward a linear state estimation method for unbalanced distribution systems using hybrid synchronized/unsynchronized measurements. The proposed method estimates the system states through a two-stage process. It first computes the voltage phase angles and then estimates the three-phase voltage phasors. This linear distribution system state estimation (LDSSE) method outperforms conventional distribution system state estimation (CDSSE) in terms of both speed and accuracy. As per simulation results, the differences are more significant for large-scale distribution systems, where CDSSE requires a significant amount of time to converge (if successful at all). This is while the LDSSE method offers a much faster solution with higher accuracy (e.g., about 20 times faster on the 8500-bus test feeder). These improvements along with the guaranteed convergence emanate from formulating the problem linearly, even with purely SCADA measurements. This eliminates the need for iteration or simplification (in modeling). Extensive simulations conducted on various test feeders validate the accuracy and computational efficiency of the proposed LDSSE method.

REFERENCES

[1] F. F. Wu, K. Moslehi, and A. Bose, “Power system control centers: past, present, and future,” *Proceedings of the IEEE,* vol. 93, no. 11, pp. 1890-1908, 2005.

[2] A. Abur, and A. G. Exposito, *Power system state estimation: theory and implementation*: CRC press, 2004.

[3] F. Ahmad, A. Rasool, E. Ozsoy, R. Sekar, A. Sabanovic, and M. Elitaş, “Distribution system state estimation-A step towards smart grid,” *Renewable and Sustainable Energy Reviews,* vol. 81, pp. 2659-2671, 2018.

[4] M. E. Baran, and A. W. Kelley, “State estimation for real-time monitoring of distribution systems,” *IEEE Transactions on Power Systems,* vol. 9, no. 3, pp. 1601-1609, 1994.

[5] C. N. Lu, J. H. Teng, and W. E. Liu, “Distribution system state estimation,” *IEEE Transactions on Power Systems,* vol. 10, no. 1, pp. 229-240, 1995.

[6] M. E. Baran, and A. W. Kelley, “A branch-current-based state estimation method for distribution systems,” *IEEE Transactions on Power Systems,* vol. 10, no. 1, pp. 483-491, 1995.

[7] K. Dehghanpour, Z. Wang, J. Wang, Y. Yuan, and F. Bu, “A survey on state estimation techniques and challenges in smart distribution systems,” *IEEE Transactions on Smart Grid,* vol. 10, no. 2, pp. 2312-2322, 2019.

[8] A. Primadianto, and C. Lu, “A review on distribution system state estimation,” *IEEE Transactions on Power Systems,* vol. 32, no. 5, pp. 3875-3883, 2017.

[9] R. Singh, B. C. Pal, and R. A. Jabr, “Distribution system state estimation through Gaussian mixture model of the load as pseudo-measurement,” *IET generation, transmission & distribution,* vol. 4, no. 1, pp. 50-59, 2010.

[10] E. Manitsas, R. Singh, B. C. Pal, and G. Strbac, “Distribution system state estimation using an artificial neural network approach for pseudo measurement modeling,” *IEEE Transactions on power systems,* vol. 27, no. 4, pp. 1888-1896, 2012.

[11] M. Ajoudani, A. Shiekholeslami, and A. Zakariazadeh, “Improving state estimation accuracy in active distribution networks by coordinating real-time and pseudo-measurements considering load uncertainty,” *IET Generation, Transmission & Distribution,* vol. 16, no. 8, pp. 1620-1638, 2022.

[12] M. Bazrafshan, and N. Gatsis, “Comprehensive modeling of three-phase distribution systems via the bus admittance matrix,” *IEEE Transactions on Power Systems,* vol. 33, no. 2, pp. 2015-2029, 2018.

[13] L. Whei-Min, T. Jen-Hao, and C. Shi-Jaw, “A highly efficient algorithm in treating current measurements for the branch-current-based distribution state estimation,” *IEEE Transactions on Power Delivery,* vol. 16, no. 3, pp. 433-439, 2001.

[14] D. Youman, H. Ying, and Z. Boming, “A branch-estimation-based state estimation method for radial distribution systems,” *IEEE Transactions on Power Delivery,* vol. 17, no. 4, pp. 1057-1062, 2002.

[15] W. Haibin, and N. N. Schulz, “A revised branch current-based distribution system state estimation algorithm and meter placement impact,” *IEEE Transactions on Power Systems,* vol. 19, no. 1, pp. 207-213, 2004.

[16] W. M. Lin, and J. H. Teng, “Distribution fast decoupled state estimation by measurement pairing,” *IEE Proceedings-Generation, Transmission and Distribution,* vol. 143, no. 1, pp. 43-48, 1996.

[17] D. A. Haughton, and G. T. Heydt, “A linear state estimation formulation for smart distribution systems,” *IEEE Transactions on Power Systems,* vol. 28, no. 2, pp. 1187-1195, 2013.

[18] A. Salehi, M. Fotuhi-Firuzabad, S. Fattaheian-Dehkordi, M. Gholami, and M. Lehtonen, “Developing an optimal framework for PMU placement based on active distribution system state estimation considering cost-worth analysis,” *IEEE Access,* vol. 11, pp. 12088-12099, 2023.

[19] Y. Zhang, and J. Wang, “Towards highly efficient state estimation with nonlinear measurements in distribution systems,” *IEEE Transactions on Power Systems,* vol. 35, no. 3, pp. 2471-2474, 2020.

[20] A. S. Dobakhshari, S. Azizi, M. Paolone, and V. Terzija, “Ultra fast linear state estimation utilizing SCADA measurements,” *IEEE Transactions on Power Systems,* vol. 34, no. 4, pp. 2622-2631, 2019.

[21] W. H. Kersting, "Distribution system modeling and analysis," *Electric Power Generation, Transmission, and Distribution: The Electric Power Engineering Handbook*, pp. 26-1: CRC press, 2018.

[22] M. E. El-Hawary, *Advances in electric power and energy: static state estimation*: John Wiley & Sons, 2020.

[23] J. Zhao, A. Gómez-Expósito, M. Netto, L. Mili, A. Abur, V. Terzija, I. Kamwa, B. Pal, A. K. Singh, J. Qi, Z. Huang, and A. P. S. Meliopoulos, “Power system dynamic state estimation: motivations, definitions, methodologies, and future work,” *IEEE Transactions on Power Systems,* vol. 34, no. 4, pp. 3188-3198, 2019.

[24] K. S. Miller, “Complex linear least squares,” *SIAM Review,* vol. 15, no. 4, pp. 706-726, 1973.

[25] R. Singh, B. C. Pal, and R. A. Jabr, “Choice of estimator for distribution system state estimation,” *IET generation, transmission & distribution,* vol. 3, no. 7, pp. 666-678, 2009.

[26] R. Schincariol da Silva, T. R. Fernandes, and M. C. de Almeida, “Specifying angular reference for three-phase distribution system state estimators,” *IET Generation, Transmission & Distribution,* vol. 12, no. 7, pp. 1655-1663, 2018.

[27] A. L. Langner, and A. Abur, “Formulation of three-phase state estimation problem using a virtual reference,” *IEEE Transactions on Power Systems,* vol. 36, no. 1, pp. 214-223, 2021.

[28] A. S. Dobakhshari, M. Abdolmaleki, V. Terzija, and S. Azizi, “Online non-iterative estimation of transmission line and transformer parameters by SCADA data,” *IEEE Transactions on Power Systems,* vol. 36, no. 3, pp. 2632-2641, 2021.

[29] I. Hughes, and T. Hase, *Measurements and their uncertainties: a practical guide to modern error analysis*: OUP Oxford, 2010.

[30] "OpenDSS: distribution system simulator," Available: https://www.epri.com/pages/sa/opendss.

[31] S. Azizi, A. S. Dobakhshari, S. A. N. Sarmadi, and A. M. Ranjbar, “Optimal PMU placement by an equivalent linear formulation for exhaustive search,” *IEEE Transactions on Smart Grid,* vol. 3, no. 1, pp. 174-182, 2012.

[32] "IEEE PES test feeder," Available: https://cmte.ieee.org/pes-testfeeders/resources/.