



UNIVERSITY OF LEEDS

This is a repository copy of *Domain and range of the CIECAM16 forward transformation*.

White Rose Research Online URL for this paper:

<https://eprints.whiterose.ac.uk/201442/>

Version: Accepted Version

---

**Article:**

Gao, C. [orcid.org/0000-0003-2233-2914](https://orcid.org/0000-0003-2233-2914), Melgosa, M. [orcid.org/0000-0002-7226-4190](https://orcid.org/0000-0002-7226-4190), Xiao, K. [orcid.org/0000-0001-7197-7159](https://orcid.org/0000-0001-7197-7159) et al. (2 more authors) (2023) Domain and range of the CIECAM16 forward transformation. *Optics Express*, 31 (14). pp. 23702-23713. ISSN 1094-4087

<https://doi.org/10.1364/oe.493856>

---

© 2023 Optica Publishing Group. One print or electronic copy may be made for personal use only. Systematic reproduction and distribution, duplication of any material in this paper for a fee or for commercial purposes, or modifications of the content of this paper are prohibited.

**Reuse**

Items deposited in White Rose Research Online are protected by copyright, with all rights reserved unless indicated otherwise. They may be downloaded and/or printed for private study, or other acts as permitted by national copyright laws. The publisher or other rights holders may allow further reproduction and re-use of the full text version. This is indicated by the licence information on the White Rose Research Online record for the item.

**Takedown**

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing [eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk) including the URL of the record and the reason for the withdrawal request.



[eprints@whiterose.ac.uk](mailto:eprints@whiterose.ac.uk)  
<https://eprints.whiterose.ac.uk/>

# The Domain and Range of the CIECAM16 Forward Transformation

CHENG GAO,<sup>1,2</sup> MANUEL MELGOSA,<sup>3</sup> KAIDA XIAO,<sup>4</sup> MICHAEL R. POINTER,<sup>4</sup>  
AND CHANGJUN LI,<sup>2,\*</sup>

<sup>1</sup> School of Electronics and Information Engineering, University of Science and Technology Liaoning, Anshan, China

<sup>2</sup> School of Computer and Software Engineering, University of Science and Technology Liaoning, Anshan, China

<sup>3</sup> Department of Optics, University of Granada, Granada, Spain

<sup>4</sup> School of Design, University of Leeds, Leeds, UK.

\* [cjliustl@sina.com](mailto:cjliustl@sina.com)

**Abstract:** The domain and range of the CIECAM16 forward transformation was numerically determined and visualized for CIE standard illuminants, using a linear programming approach that provides the gamuts and colour solids for optimum colours. The effect of the surround, adapting luminance, and luminance of the background on the range of the CIECAM16 forward transformation were individually analyzed, showing that their ranges increased when the surround changed from dark to dim or average, the adapting luminance increased, or the luminance of the background decreased. The proposed methodology for the determination and visualization of the domain and range of the CIECAM16 forward transformation can be used for any illuminant, as well as for CIECAM02, CAM16, CAM02-UCS and CAM16-UCS. The findings of this paper not only solve the long-term unresolved domain and range problems of the CIE colour appearance models, but also find applications in cross-media colour reproduction. Furthermore, it was also found that some non-CIE colours are inside the International Color Consortium Profile Connection Space (ICC PCS), and some CIE colours are not included in that space.

© 2023 Optica Publishing Group under the terms of the Optica Open Access Publishing Agreement

## 1. Introduction

In 2022, CIE recommended CIECAM16 [1], a new colour appearance model for related colours and colour management applications, which replaces CIECAM02 [2]. This new model is mainly based on the CAM16 colour appearance model described by Li et al. [3], with small changes based on the work by Wang et al. [4] and Gao et al. [5]. Like CIECAM02 [2] and CAM16 [3], CIECAM16 [1] also has a forward and an inverse transformation. The CIECAM16 forward transformation can predict colour appearance attributes such as lightness  $J$ , colourfulness  $M$ ,

and hue angle  $h$ , from the CIE tristimulus values of a sample under the CIE 1931 standard observer and a specified illuminant and surround (i.e. average, dim or dark surround). Unlike CIECAM02, in CIECAM16, both the chromatic and the luminance adaptations are completed in the same CAT16 cone-like space [3]. Hence, CIECAM16 is simpler than CIECAM02, and, in addition, the prediction of visual colour appearance experimental datasets gives performance that is as good or better than that of CIECAM02 [1-3].

Although CIECAM16 solved some problems associated with CIECAM02, such as the so-called “Yellow-Blue” and “Purple” problems [6-8, 3], some additional questions related to CIECAM16 are still unsolved. In the clause “Future research” of reference [1] four different items were mentioned, the first one being the range of the forward transformation of CIECAM16. In colour management [9, 10] both the forward and inverse transformations of CIECAM16 are needed. Furthermore, for efficient colour image reproduction, both transformations on regular grids are used as look up tables and, in a later stage, interpolations are also sometimes needed. In previous colour appearance models, such as CIECAM97s [11] and CIECAM02, the domains for the forward and inverse transformations were also unknown. Thus, regular grids were chosen that were large enough to cover these unknown domains. Specifically, the International Color Consortium (ICC) [9] chose regular grids in CIELAB colour space, with  $L_i^*$  between 0 and 100, and  $a_j^*$  and  $b_k^*$  between  $-128$  and  $127$ . When these grids  $L_i^*$ ,  $a_j^*$ ,  $b_k^*$  are transformed to tristimulus values  $X, Y, Z$ , some values were beyond the domain of the CIECAM02 forward transformation [9,10], and this resulted in a problem with computational failure. For CIECAM16, the domain of its forward transformation is all the  $X, Y, Z$  values with chromaticity coordinates inside the CIE  $x, y$  chromaticity diagram [3]. However, this is too general and further knowledge on the exact boundary or gamut of the domain and range of CIECAM16 would be useful. In fact, investigation into the domains of the forward and inverse transformations started when CIECAM97s was recommended. It is known that the domains and ranges for CIECAM97s, CIECAM02, and CIECAM16 are dependent on the illumination, luminance level and viewing surround. Therefore, there is no way to derive analytical formulae defining such domains and ranges. For the current CIECAM16, it is possible to derive the domain numerically, which is the main objective of this paper.

Note that domain and range are mathematical terms [1]. The domain is a set of one or more variables of a function. For example, if the function is the square of the variable  $x$ , the domain of this function is the set of all real numbers. The range of a function is the set of the output of a function for all variables in the domain. Hence, in the previous example, the range is the set of all non-negative real numbers. The range of a function is the domain of the inverse of this function.

## 2. The Boundary of the Domain of the Forward Transformation in CIECAM16

Let  $\Omega_F$  be the domain of the forward transform in CIECAM16 [1]. The exact definition of  $\Omega_F$  in a mathematical sense is not important to us. Instead, we search the domain  $\Omega_F$  which

contains all CIE colours, defined as all tristimulus values  $X, Y, Z$  under a given viewing condition, given by

$$\begin{aligned} X &= \kappa \int_a^b E(\lambda) r(\lambda) \bar{x}(\lambda) d\lambda \\ Y &= \kappa \int_a^b E(\lambda) r(\lambda) \bar{y}(\lambda) d\lambda . \\ Z &= \kappa \int_a^b E(\lambda) r(\lambda) \bar{z}(\lambda) d\lambda \end{aligned} \quad (1)$$

where  $E(\lambda)$  is the spectral power distribution of the illuminant or light source,  $r(\lambda)$  (with values between 0 and 1 for non-fluorescent materials) is the spectral reflectance of the considered sample,  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ , and  $\bar{z}(\lambda)$  are the CIE 1931 colour-matching functions, and  $\kappa$  is a scaling factor [11] defined by

$$\kappa = 100 / \int_a^b E(\lambda) \bar{y}(\lambda) d\lambda \quad . \quad (2)$$

From Eq. (14) of Li et al. [3] it follows that the corresponding values in cone space,  $R, G, B$ , are given by

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M_{16} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

where  $M_{16}$  is the CAT16 matrix [3]. Thus, all  $X, Y, Z$  values in  $\Omega_F$  defined by Eq. (1) are valid inputs to the forward transformation of CIECAM16.

In most cases all integrands involved in Eq. (1) have no analytical expressions, hence CIE [12] recommended to replace the integrations in Eq. (1) by 1 nm summations:

$$\begin{aligned} X &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{x}(\lambda_j) \\ Y &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{y}(\lambda_j) , \\ Z &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{z}(\lambda_j) \end{aligned} \quad (4)$$

where,

$$\kappa = 100 / \sum_{j=1}^n E(\lambda_j) \bar{y}(\lambda_j) . \quad (5)$$

and wavelengths  $\lambda_j$  are uniformly sampled in the visible wavelength range between 380 nm and 780 nm at 1 nm intervals.

Note that for most practical purposes, the summations in Eq. (4) may be approximated by using wavelengths intervals equal to 5 nm over the wavelength range 380 nm to 780 nm [12].

Note also that for  $r(\lambda_j) \equiv 1$  the  $X, Y, Z$  values computed using Eqs. (1) or (4) are the tristimulus values of the assumed illuminant (or light source), denoted as  $X_w, Y_w$  and  $Z_w$  respectively. Since

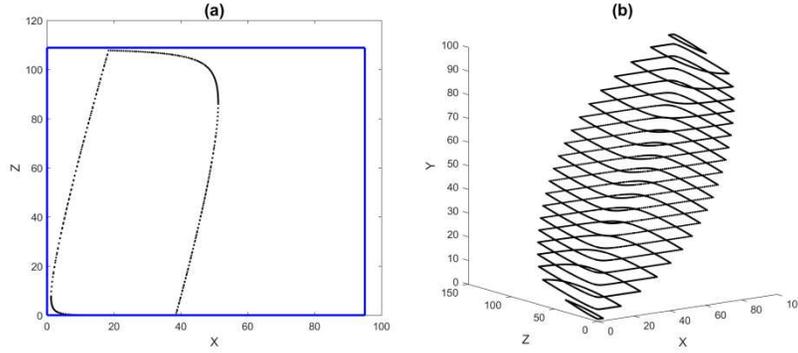
$$0 \leq r(\lambda_j) \leq 1 \quad (6)$$

for all non-fluorescent surface colour samples, all  $X, Y, Z$  tristimulus values are located inside a cube with  $X$  direction between 0 and  $X_w$ ,  $Y$  direction between 0 and  $Y_w (= 100)$ , and  $Z$

direction between 0 and  $Z_w$ . Can we say this cube, denoted as  $\Omega_{XYZ}$ , is equivalent to  $\Omega_F$ ? The answer has to be no. There are  $X, Y, Z$  values inside this cube that do not represent any CIE colour. In fact, if  $Y$  is fixed to be 18.4187, which corresponds to lightness  $L^* = 50$ , the tristimulus values  $X, Y, Z$  satisfy:

$$\begin{aligned} X &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{x}(\lambda_j) \\ Y &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{y}(\lambda_j) = 18.4187. \\ Z &= \kappa \sum_{j=1}^n E(\lambda_j) r(\lambda_j) \bar{z}(\lambda_j) \end{aligned} \quad (7)$$

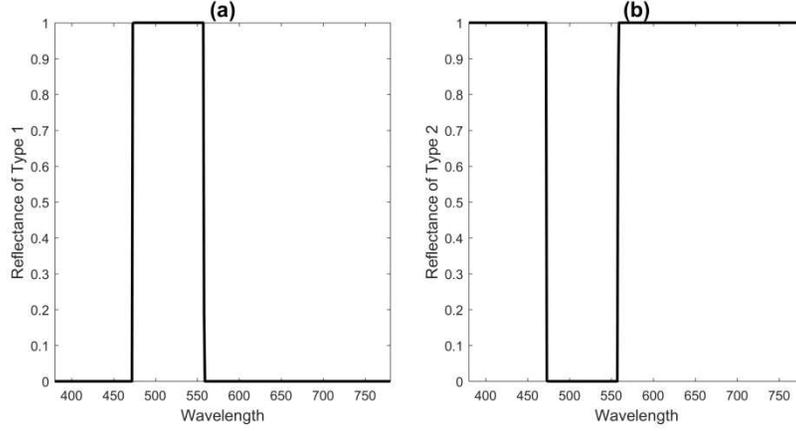
Based on Eq. (7) and inequality (6), Li et al. [13] proposed a linear programming approach to find all proper tristimulus values  $X$  and  $Z$  and their boundary. It was found by this linear programming approach that the valid tristimulus values  $X$  and  $Z$  should be enclosed by the dotted curve shown in Figure 1(a). Thus, this black dotted curve is the boundary or gamut for the tristimulus values  $X$  and  $Z$  when  $Y$  is fixed to a value of 18.4187. It can be considered that the domain enclosed by this dotted curve, together with the fixed  $Y$  value, includes all CIE colours. The blue square shown in Figure 1 (a) is the boundary found cutting the cube  $\Omega_{XYZ}$  by the plane  $Y = 18.4187$  assuming CIE illuminant D65 ( $X_w = 95.04$ ;  $Z_w = 108.88$ ). Figure 1(a) shows that many  $X, Z$  values enclosed by the blue square are outside the black dotted curve. Thus, in general, many  $X, Y, Z$  values inside the cube  $\Omega_{XYZ}$  do not represent CIE colours.



**Fig. 1.** (a) Black dotted curve: the boundary in  $X, Z$  plane when  $Y = 18.4187$  under CIE illuminant D65; blue square: the boundary when cutting the cube  $\Omega_{XYZ}$  by the plane  $Y = 18.4187$  under illuminant D65. (b) The gamut  $\Omega_F$  in  $X, Y, Z$  space for  $Y = 1, 5, 10, \dots, 90, 95$  and  $99$ , under illuminant D65.

Based on the above discussion, when  $Y = 18.4187$ , the dotted curve shown in Figure 1(a) can be considered the boundary of  $\Omega_F$  under CIE illuminant D65. Now, if we sample  $Y$  regularly, between 0 and 100, for each  $Y$  value, we can find a similar region enclosed by a dotted curve, like the one in Figure 1(a) in the  $X, Z$  plane, resulting in the gamut  $\Omega_F$  under D65. Figure 1(b) shows the gamut  $\Omega_F$  under D65 with  $Y$  set to a value of 1, 5, 10, ..., 90, 95 and 99, respectively. Hence, sampling in the  $Y$  direction is determined first, and next the colour

volume or gamut  $\Omega_F$  as shown in Figure 1 is obtained. Thus, any regular grid must be on or inside the boundary of this colour volume.

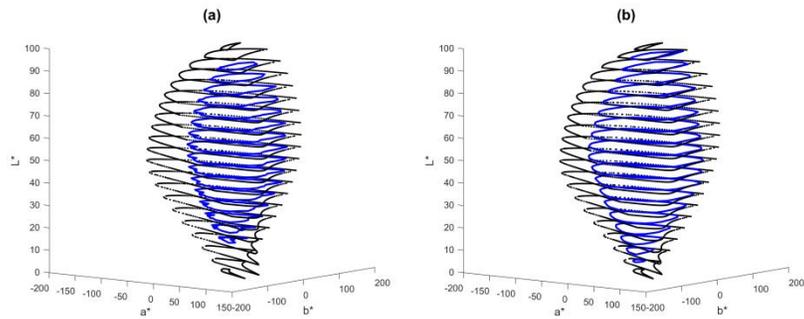


**Fig. 2.** Reflectance of Type 1 (left) and Type 2 (right) optimum colours.

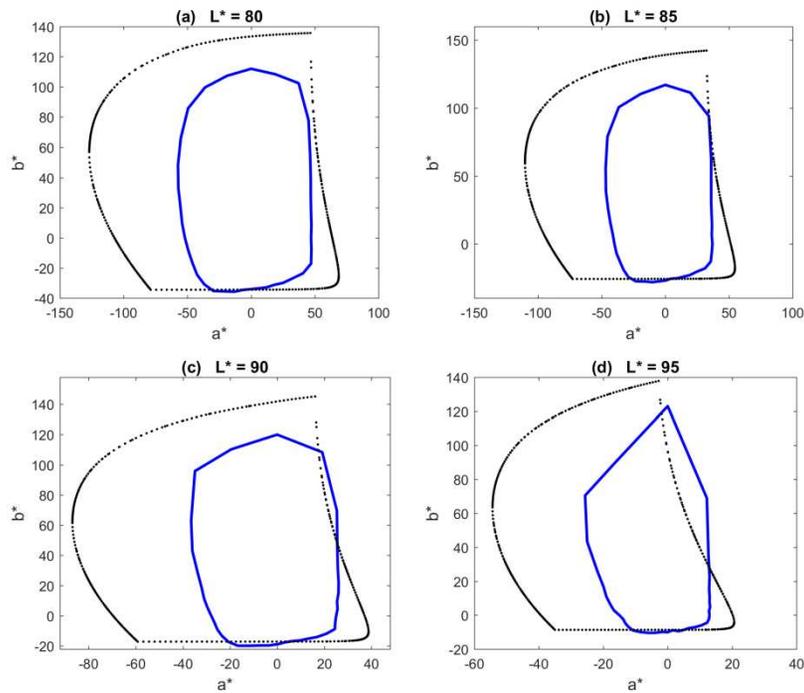
Note that reflectances obtained using the linear programming approach [13] have essentially a mountain-like or a valley-like shape, as shown in Figure 2. In the literature [14], the former is known as Type 1 reflectance and the latter as Type 2 reflectance. Both types of reflectances provide the so-called ‘optimum colours’ and are different from the reflectances of real surface colours. Hence, the colour volume or the gamut of  $\Omega_F$  determined by the proposed approach is called the ‘optimum colour solid’ [13, 14].

Since the CIELAB space is more uniform than the XYZ space, it is convenient if the regular grids for  $\Omega_F$  be defined in the  $L^*$ ,  $a^*$ ,  $b^*$  space. Figure 3 shows the gamuts (black dotted curves)  $\Omega_F$  under illuminants C (a) and D50 (b) in the  $L^*$ ,  $a^*$ ,  $b^*$  space with  $L^*$  being set to a value of 1, 5, 10, ..., 90, 95 and 99, respectively. The blue solid curves in (a) and (b) are the real surface colour gamuts from Pointer [15] (a) and ISO [16] (b), respectively. It can be seen that Pointer’s gamut is inside the gamut  $\Omega_F$ , but the ISO gamut is not completely inside the optimum colour solid. Specifically, Figure 4 shows parts of the blue curves outside the black dotted curve for  $L^*$  values of 80, 85, 90 and 95. Thus, the ISO gamut is too large in some regions, which is consistent with the earlier findings of Li et al. [17].

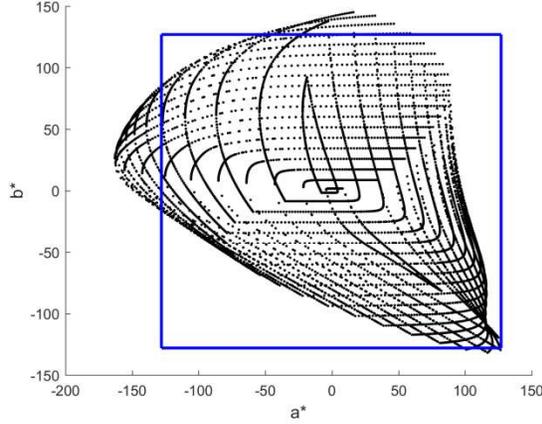
Note that the domain under any illuminant for the forward CIECAM16 can be obtained and visualized using the above approach. The reason to show the domain under illuminant C in Figure 3(a) is to compare with Pointer’s gamut of real surface colours which was defined in terms of  $L^*$ ,  $C_{ab}^*$ ,  $h_{ab}$  under illuminant C. Similarly, the reason to show the domain under illuminant D50 in Figure 3(b) is to compare with the ISO gamut of real surface colours and the ICC [9] profile connection space (PCS), which were defined in terms of  $L^*$ ,  $C_{ab}^*$ ,  $h_{ab}$  and  $L^*$ ,  $a^*$ ,  $b^*$ , respectively, under illuminant D50.



**Fig. 3.** The black dotted curves are the gamuts (optimum colour solids)  $\Omega_F$  in  $L^*$ ,  $a^*$ ,  $b^*$  space under illuminants C (a) and D50 (b) with  $L^*$  being 1, 5, 10, ..., 90, 95 and 99, respectively. The blue solid curves are the real surface colour gamuts from Pointer [15] (a) and ISO [16] (b), respectively.



**Fig. 4.** ISO gamut [15] (blue) and  $\Omega_F$  (black) in  $a^*$   $b^*$  under illuminant D50, for  $L^* = 80$ , 85, 90 and 95.



**Fig. 5.** Gamuts  $\Omega_F$  in the  $a^* b^*$  plane under illuminant D50 with  $L^*$  being 1, 5, 10, ..., 95 and 99 respectively. The blue square is the boundary when cutting the ICC cube  $\Omega_{PCS}$  by any fixed  $L^*$  plane in the  $L^*, a^*, b^*$  space.

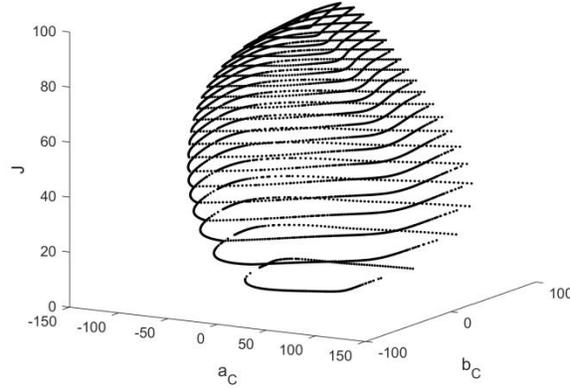
Note also that ICC [9] defined the PCS, denoted as  $\Omega_{PCS}$ , in  $L^*, a^*, b^*$  space under illuminant D50 with  $L^*$  being between 0 and 100, and  $a^*$  and  $b^*$  being between  $-128$  and  $127$ . Figure 5 shows  $\Omega_F$  gamuts in the  $a^*, b^*$  plane under illuminant D50 for  $L^*$  being 1, 5, 10, ..., 95 and 99 respectively. The blue square in Figure 5 is the boundary when cutting the ICC cube  $\Omega_{PCS}$  by a fixed  $L^*$  plane in the  $L^*, a^*, b^*$  space. It can also be seen that the blue square contains many  $a^*$  and  $b^*$  values which are not inside the domain enclosed by the black curves, which implies that the ICC cube  $\Omega_{PCS}$  contains many non-real-world colours. On the other hand, it is interesting to note that, parts of the areas enclosed by the dotted curves are outside the blue square, which may indicate that some real colours are not covered by the ICC cube  $\Omega_{PCS}$  as well.

In summary, the linear programming approach developed by Li et al. [13] is proposed for determining the colour solid  $\Omega_F$  or domain of the CIECAM16 forward transformation under any viewing condition.  $\Omega_F$  can be defined in  $X, Y, Z$  or  $L^*, a^*, b^*$  spaces. Inequality (3) is satisfied for all  $X, Y, Z$  in  $\Omega_F$ . Furthermore,  $\Omega_F$  covers all real surface colours. Hence  $\Omega_F$  can be used as the domain for the forward transformation of CIECAM16. The range of the forward transformation or, equivalently, the domain of the inverse transformation of CIECAM16, will be denoted by  $\Omega_R$  and will be discussed in the next section.

### 3. The Range of the CIECAM16 Forward Transformation

Determining the domain of the CIECAM16 inverse transformation is equivalent to find the range  $\Omega_R$  of the forward transformation. In the previous section, we proposed how to compute the domain  $\Omega_F$  of the forward transformation of CIECAM16 under any illuminant. Thus, the

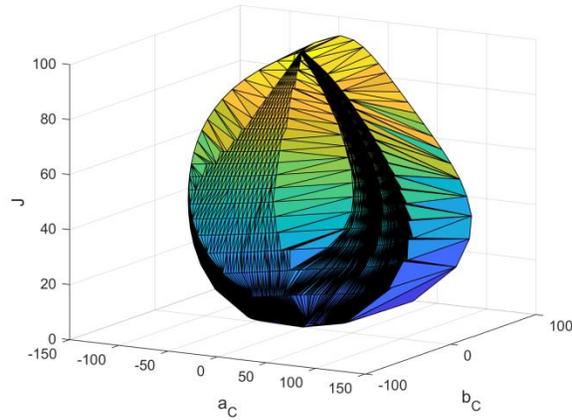
range  $\Omega_R$  can be determined using the forward transformation, with the appropriate values of surround, background, and adapting luminance, under the specified illuminant. Let the domain  $\Omega_F$  in  $X, Y, Z$  space be determined under CIE illuminant D65 with  $Y$  being 1, 5, 10, ..., 90, 95 and 99, as shown in Figure 1(b). Figure 6 shows the range  $\Omega_R$  in  $J, a_C, b_C$  space [18] with average surround,  $L_A = 40 \text{ cd m}^{-2}$ , and  $Y_b = 20$ .



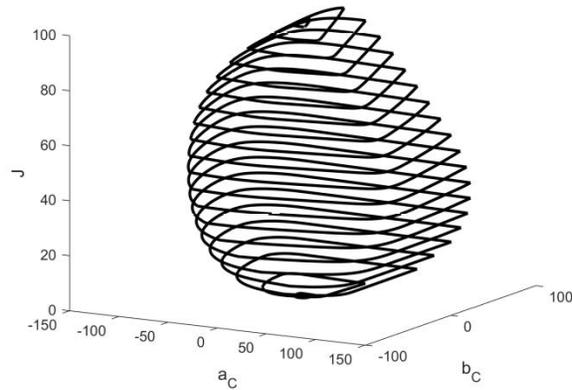
**Fig. 6.** The gamut or colour solid for the range  $\Omega_R$  of CIECAM16 in  $J, a_C, b_C$  space, using the  $\Omega_F$  shown in Figure 1(b) as input, with average surround,  $L_A = 40 \text{ cd m}^{-2}$  and  $Y_b = 20$ , under CIE illuminant D65.

First, note that any single layer or dotted curve in Figure 1(b) was transferred to a single dotted curve in  $J, a_C, b_C$  space. Thus, as expected, the boundary of the domain  $\Omega_F$  was transferred to the boundary of the range  $\Omega_R$ . Second, note that all points in each layer or dotted curve shown in Figure 1(b) have the same  $Y$  value. However, the points on the corresponding layer in the  $J, a_C, b_C$  space have different lightness values  $J$ , which makes the representation of regular grids in  $J, a_C, b_C$  space difficult. To overcome this problem, the convex hull [19] of  $\Omega_R$  is computed using all points on dotted curves shown in Figure 6, together with coordinates  $J, a_C, b_C$  of the black and white points, obtained using the forward transformation with tristimulus values  $(0, 0, 0)$  and  $(X_W, Y_W, Z_W)$  as inputs, respectively. The computed convex hull is shown in Figure 7. The surface of this convex hull comprises many triangle-shape facets. Note that the colours used in Figure 7 are only to aid viewing the different triangle-shape facets. It is clear that the intersection of the surface of this convex hull and a plane in  $J, a_C, b_C$  space with  $J$  being a constant is a closed curve in  $a_C, b_C$  space. Figure 8 shows the closed curves obtained using the fixed  $J$  plane, with  $J$  being 1, 5, 10, ..., 90, 95 and 99 in  $J, a_C, b_C$  space, which may be considered as the gamut of the range  $\Omega_R$ . The difference between the colour solids shown in Figures 6 and 8 is that values of lightness  $J$  for dotted curves in each layer are

different in Figure 6 and constant in Figure 8. Hence, it is more appropriate to consider the colour solid shown in Figure 8 as representative of the range  $\Omega_R$  of the forward transformation of CIECAM16 (or the domain of its inverse transformation).



**Fig. 7.** The convex hull of the range  $\Omega_R$  shown in Figure 6.



**Fig. 8.** Gamut and colour solid for the range of CIECAM16 forward transformation in  $J$ ,  $a_C$ ,  $b_C$  space [18] using the input  $X, Y, Z$  shown in Figure 1(b) with average surround,  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$  under CIE illuminant D65.

Note that  $a_C$ ,  $b_C$  [18] are defined in terms of perceptual attributes chroma  $C$  and hue angle  $h$  (in degrees) using the following expressions:

$$\begin{aligned} a_C &= C \cdot \cos(\pi h/180) \\ b_C &= C \cdot \sin(\pi h/180) \end{aligned} \quad (8)$$

Similarly,  $a_M$ ,  $b_M$  or  $a_s$ ,  $b_s$  can be defined in terms of perceptual attributes colourfulness  $M$  and saturation  $s$ , respectively. The colour space  $J$ ,  $a_C$ ,  $b_C$  was considered above. However, depending on the application, other spaces such as  $J$ ,  $a_M$ ,  $b_M$  or  $Q$ ,  $a_C$ ,  $b_C$ , can be used.

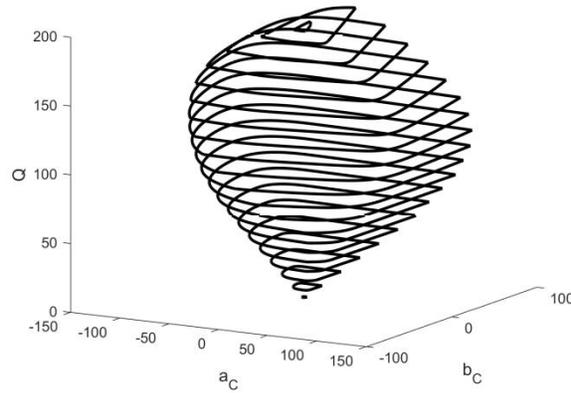
Now we can summarize the main steps to obtain the gamut and colour solid for the range  $\Omega_R$ :

Step 1) Determine the gamut of the domain  $\Omega_F$  under the specified illuminant as discussed in section 2;

Step 2) Obtain boundary points in a colour appearance space, for example  $J$ ,  $a_C$ ,  $b_C$ , using the forward transformation under specific viewing conditions, which include surround, background, and adapting luminance, using as inputs the points on the gamut of the domain  $\Omega_F$  together with the tristimulus values of the black and white samples;

Step 3) Determine the convex hull using the boundary points obtained in Step 2;

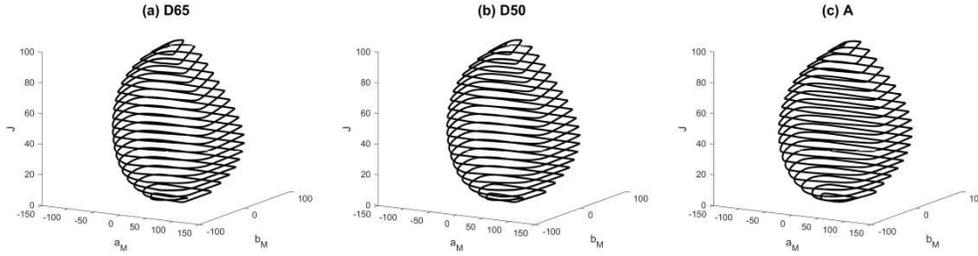
Step 4) Find the intersect closed curves of the convex hull obtained in Step 3 at a series of fixed values of lightness,  $J$  (or brightness,  $Q$ ) planes for the gamut (colour solid) of the range  $\Omega_R$ .



**Fig. 9.** Gamut and colour solid for the range of CIECAM16 forward transformation in  $Q$ ,  $a_C$ ,  $b_C$  space using the input  $X, Y, Z$  values shown in Figure 1(b) with average surround,  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$  under illuminant D65.

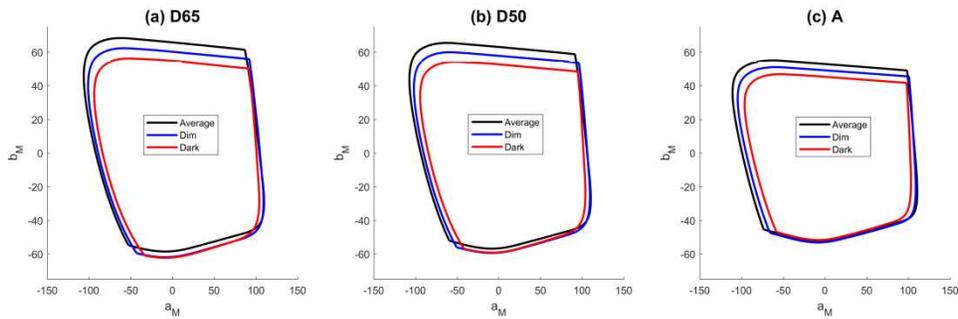
Based on the above 4 steps, the range  $\Omega_R$  can be obtained for any illuminant and viewing conditions. As an example, Figure 9 shows the gamut and colour solid for  $\Omega_R$  in  $Q$ ,  $a_C$ ,  $b_C$  space under illuminant D65, average surround,  $L_A = 40 \text{ cd m}^{-2}$  and  $Y_b = 20$ . Similarly, Figure 10 shows the gamuts and colour solids for the range of CIECAM16 forward transformation in  $J$ ,  $a_M$ ,  $b_M$  space with average surround,  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$  under

illuminants D65 (a), D50 (b) and A (c), which are the three standard illuminants currently adopted by the CIE.

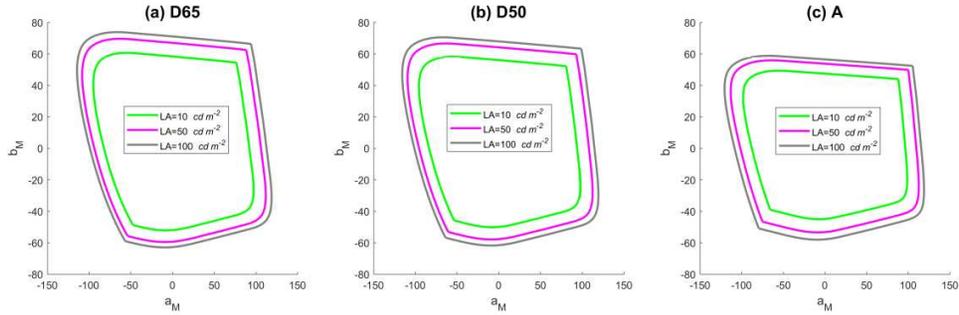


**Fig. 10.** Gamuts and colour solids for the range of CIECAM16 forward transformation in  $J$ ,  $a_M$ ,  $b_M$  space with average surround,  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$  under CIE illuminant D65(a), illuminant D50 (b) and illuminant A (c).

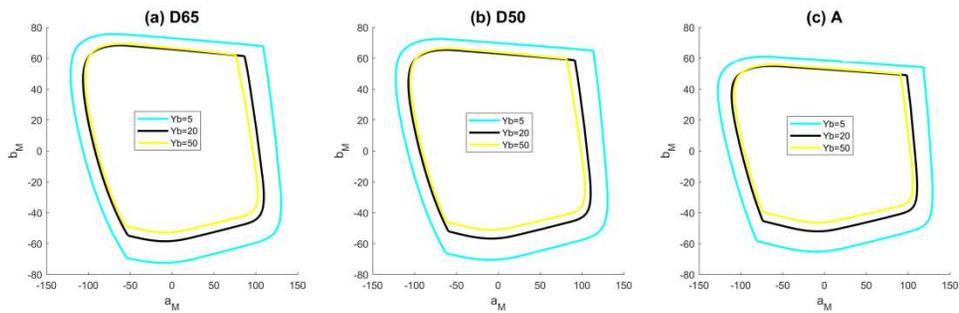
Based on the determined range  $\Omega_R$ , the influence of the surround, adapting luminance level and background can be further investigated. Figure 11 shows the gamut of  $\Omega_R$  in  $a_M$ ,  $b_M$  space when  $J$  is fixed to 50 for average (black curve), dim (blue curve) and dark (red curve) surrounds, under illuminants D65, D50 and A. In Figure 11 the values of the adapting luminance and background were  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$ , as used in Figure 10. It can be noted that the areas covered by the black/red curves are the largest/smallest, which confirms the expectation that the volumes of the gamuts [20-22] decrease when we change from average to dim and dark surround. Figure 12 shows the gamut of  $\Omega_R$  in  $a_M$ ,  $b_M$  space when  $J$  is fixed to 50 for  $L_A = 10 \text{ cd m}^{-2}$  (green curve),  $50 \text{ cd m}^{-2}$  (magenta curve),  $100 \text{ cd m}^{-2}$  (middle grey curve) with average surround,  $Y_b = 20$ , under illuminants D65 (a), D50 (b) and A (c). It can be noted that with larger  $L_A$  values the area enclosed by the curve increases, which is consistent with the Hunt effect [22]. Figure 13 shows the gamuts for the range of CIECAM16 forward transformation in  $a_M$ ,  $b_M$  space, when  $J$  is fixed to 50, with average surround,  $L_A = 40 \text{ cd m}^{-2}$ , and  $Y_b = 5$  (cyan curve), 20 (black curve) and 50 (yellow curve), under illuminants D65 (a), D50 (b) and A (c). It can be noted that when  $Y_b$  is smaller, the area enclosed by the curve is larger, which is consistent with the lightness contrast and Hunt effects [22, 23]. The lightness contrast effect tells us the perceived lightness increases when colours are viewed against a darker background. Then, from Hunt effect, the lighter colours appear more colorful. Hence, in Figure 13 the areas enclosed by the black/red curves are the largest/smallest.



**Fig. 11.** The gamuts for the range of CIECAM16 forward transformation in  $a_M$ ,  $b_M$  space, when  $J$  is fixed to 50, for average (black curve), dim (blue curve) and dark (red curve) surrounds,  $L_A = 40 \text{ cd m}^{-2}$ ,  $Y_b = 20$ , under illuminants D65 (a), D50 (b) and A (c).



**Fig. 12.** The gamuts for the range of CIECAM16 forward transformation in  $a_M$ ,  $b_M$  space, when  $J$  is fixed to 50, with average surround,  $Y_b = 20$ , and  $L_A = 10 \text{ cd m}^{-2}$  (green curve),  $50 \text{ cd m}^{-2}$  (magenta curve) and  $100 \text{ cd m}^{-2}$  (middle grey curve), under illuminants D65 (a), D50 (b) and A (c).



**Fig. 13.** The gamuts for the range of CIECAM16 forward transformation in  $a_M$ ,  $b_M$  space, when  $J$  is fixed to 50, with average surround,  $L_A = 40 \text{ cd m}^{-2}$ , and  $Y_b = 5$  (cyan curve), 20 (black curve) and 50 (yellow curve), under illuminants D65 (a), D50 (b) and A (c).

Finally, we can add that the domain  $\Omega_F$  for the CIECAM16 forward transformation is also the domain for CIECAM02 and CAM16. In fact, in reference [23] the robustness of CIECAM02 was tested using the domain  $\Omega_F$ . The range  $\Omega_R$  for CIECAM02 and CAM16 forward transformations can be determined in a similar way to that for CIECAM16. The proposed methodology can be also used to determine and visualize ranges in the CAM16-UCS [3] and CAM02-UCS [12, 25] uniform colour spaces.

#### 4. Conclusions

CIE has recommended a new colour appearance model, CIECAM16 [1], to replace CIECAM02 [2]. Unlike CIECAM02, in CIECAM16 the chromatic and luminance adaptations are completed in the same CAT16 cone-like space [3]. Hence, CIECAM16 is simpler than CIECAM02, and the performance of CIECAM16 in predicting visual colour appearance datasets is as good as or better than that of CIECAM02 [3]. However, in reference [1] four unsolved problems were identified, the first being the range of the forward transformation which has been analyzed in the current paper.

In this paper, first the domain  $\Omega_F$  of the forward transformation under any illuminant was numerically determined and visualized using the linear programming approach in reference [13] to find the gamut and colour solid of the optimum colours [12]. Second, the gamuts and colour solids of the ranges  $\Omega_R$  of the CIECAM16 forward transformation under different viewing conditions were determined and visualized using the identified domain  $\Omega_F$ . The ranges  $\Omega_R$  can be represented in different colour spaces such as  $J, a_M, b_M$  or  $J, a_C, b_C$ , or  $Q, a_C, b_C$ , depending on the application. Finally, the effects of the surround, adapting luminance level, and background were investigated separately, showing ranges  $\Omega_R$  for three standard CIE illuminants (D65, D50 and A). It was found that the ranges increase when the surround changes from dark to dim and average, and also that ranges increase with the increase of the adapting luminance or the decrease of the luminance of the background.

The proposed methodology for determining and visualizing the domain and range of the CIECAM16 forward transformation can be also applied to determine those for CIECAM02, CAM16, CAM02-UCS and CAM16-UCS. The analysis and results reported in this paper provide a solution to one of the long-term unresolved domain and range problems of the CIE colour appearance models, and will have application in cross-media colour reproduction.

Furthermore, it is shown that part of the ICC PCS  $\Omega_{PCS}$  under CIE illuminant D50 is outside of the domain  $\Omega_F$  (see Figure 5), which implies that the ICC PCS cube  $\Omega_{PCS}$  contains non-CIE (i.e. unreal) colours. It is also shown that some  $X, Y, Z$  values locate inside  $\Omega_F$ , but outside  $\Omega_{PCS}$  (see Figure 5), which implies that some CIE colours are not covered by the ICC PCS.

**Funding.** National Natural Science Foundation of China (61775169, 61575090); Department of Education of Liaoning Province (LJKQZ2021127, LJKZ0310, LJKZ0291); University of Science and Technology Liaoning (LKDYC202103, 2021YQ04). Ministry of Science and Innovation, and State Research Agency of the government of Spain, grant number PID2019-107816GB-I00/SRA/10.13039/501100011033.

**Disclosures.** The authors declare no conflicts of interest.

**Data Availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

## References

1. CIE 248:2022, “The CIE 2016 colour appearance model for colour management systems: CIECAM16,” CIE Central Bureau, Vienna, 2022.
2. CIE 159:2004, “A colour appearance model for colour management systems: CIECAM02,” CIE Central Bureau, Vienna, 2004.
3. C. J. Li, Z. Q. Li, Z. F. Wang, Y. Xu, M. R. Luo, G. H. Cui, M. Melgosa, M. H. Brill, and M. R. Pointer, “Comprehensive color solutions: CAM16, CAT16 and CAM16-UCS,” *Color Res. Appl.* **42**(6), 703–718 (2017).
4. Z. F. Wang, C. Gao, Y. Xu, M. Melgosa, M. H. Brill, M. R. Pointer, and C. J. Li, “Further investigation on the modified hyperbolic function in the CAM16 color appearance model,” *Color Res. Appl.* **44**(3), 359–366 (2019).
5. C. Gao, Z. F. Wang, Y. Xu, M. Melgosa, K. D. Xiao, M. H. Brill, and C. J. Li, “The von Kries chromatic adaptation transform and its generalization,” *Chin. Opt. Lett.* **18**(3), 030201-033601 (2020).
6. M. H. Brill, “Irregularity in CIECAM02 and its avoidance,” *Color Res. Appl.* **31**(2), 142–145 (2006).
7. M. H. Brill and S. Süsstrunk, “Repairing gamut problems in CIECAM02: A progress report,” *Color Res. Appl.* **33**(5), 424–426 (2008).
8. J. Jiang, Z. F. Wang, M. R. Luo, M. Melgosa, M. H. Brill, and C. J. Li, “Optimum solution of the CIECAM02 yellow-blue and purple problems,” *Color Res. Appl.* **40**(5), 491–503 (2015).
9. I. Tastl, M. Bhachech, N. Moroney, and J. Holm, “ICC Color Management and CIECAM02,” in *13th Color and Imaging Conference (CIC)* (2005), pp. 217–223.
10. R. Guay, and M. Shaw, “Dealing with Imaginary Color Encodings in CIECAM02 in an ICC Workflow,” in *13th Color and Imaging Conference (CIC)* (2005), pp. 2005–318.
11. M. R. Luo, and R. W. G. Hunt, “The structure of the CIE 1997 colour appearance model (CIECAM97s),” *Color Res. Appl.* **23**(3), 138–146 (1998).
12. CIE 015:2018, “Colorimetry, 4<sup>th</sup> edition,” CIE Central Bureau, Vienna, 2018.
13. C. J. Li, M. R. Luo, M-S Cho, and J-S Kim. “Linear programming method for computing the gamut of object color solid,” *J. Opt. Soc. Am. A* **27**(5), 985–991 (2010).
14. F. Martínez-Verdú, E. Perales, E. Chorro, D. de Fez, V. Viqueira, and E. Gilabert. “Computation and visualization of the MacAdam limits for any lightness, hue angle and light source,” *J. Opt. Soc. Am. A* **24**(6), 1501–1515 (2007).
15. M. R. Pointer. “The gamut of real surface colours,” *Color Res. Appl.* **5**(3), 145–155 (1980).

16. ISO 12640-3:2007, "Graphic technology - Prepress digital data exchange – Part 3: CIELAB standard colour image data (CIELAB/SCID)," ISO, 2007.
17. C. J. Li, M. R. Luo, M. R. Pointer, and P. Green. "Comparison of real colour gamuts using a new reflectance database," *Color Res. Appl.* **39**(5), 442-451 (2014).
18. N. Moroney, M. D. Fairchild, R. W. G. Hunt, C. J. Li, M. R. Luo, and T. Newman, "The CIECAM02 color appearance model," in *10th Colour Imaging Conference (CIC)* (2002), pp. 23-27.
19. C. B. Barber, D. P. Dobkin, and H. T. Huhdanpaa, "The Quickhull algorithm for convex hulls," *ACM Trans. Math. Softw.* **22**(4), 469-483 (1996).
20. N. Moroney, "A comparison of the CIELAB and CIECAM97s," in *6th Colour Imaging Conference (CIC)* (1998), pp. 17-21.
21. C. J. Li, M. R. Luo, and R. W. G. Hunt. "A revision of the CIECAM97s model," *Color Res. Appl.* **25**(4), 260-266 (2000).
22. M. R. Luo and C. J. Li, *CIECAM02 and its associated colour spaces. Colorimetry: Understanding the CIE system*, (John Wiley & Sons, Inc., 2007), Chapter 11.
23. M. R. Luo, *The CIE 1997 colour appearance model: CIECAM97s. Colour Engineering: Achieving Device Independent Color*, (John Wiley & Sons, Inc., 2002), Chapter 4.
24. C. J. Li, and M. R. Luo. "Testing the robustness of CIECAM02," *Color Res. Appl.* **30**(2), 99-106 (2005).
25. M. R. Luo, G. H. Cui, and C. J. Li. "Uniform colour spaces based on CIECAM02 colour appearance model," *Color Res. Appl.* **31**(4), 320-330 (2006).