Improvements to the CIELUV color space

Ling Wang,[1,2] Changjun Li,[1] Yang Xu,[1] Manuel Melgosa,[3] Kaida Xiao,[1,4] and Cheng Gao [1*]

[1] School of Computer and Software Engineering, University of Science and Technology Liaoning, Anshan, China
[2] Beijing Machinery and Equipment Research Institute, Beijing, China
[3] Department of Optics, University of Granada, Granada, Spain
[4] School of Design, University of Leeds, Leeds, UK

Correspondence
Cheng Gao, School of Computer and Software Engineering, University of Science and Technology Liaoning, Anshan, 114051, China. Email: 794962485@qq.com

ABSTRACT
Three new approximately uniform color spaces named MLUV, MLUV1, and MLUV2 were developed by modifying CIELUV lightness, $u'v'$ chromaticity coordinates, and u^*v^* color coordinates, respectively. Performance tests using the combined and four individual datasets employed at CIEDE2000 development showed that MLUV, MLUV1 and MLUV2 were significantly better than CIELUV. Using values of Standardized Residual Sum of Squares (CIE 217:2016) for predictions of the four individual datasets the ranking (from best to worst) was MLUV, MLUV2 and MLUV1, but for predictions of two ellipses datasets the ranking (from best to worst) was MLUV2, MLUV, and MLUV1. Overall, the MLUV2 space was found to be the best. Hence, it is expected that MLUV2 could be used for color specification and color difference evaluations, especially in industrial applications that depend on additive light mixing, such as color TV sets, video monitors, and lighting.

KEYWORDS
Uniform color space, uniform chromaticity scale, CIELUV, CIELAB, STRESS.

INTRODUCTION
Color difference refers mainly to the perceptual difference between two color samples. Color difference evaluation can be divided into subjective and objective. Subjective color difference evaluation is, for example, placing two samples side by side under specified lighting and observation conditions and asking real observers to complete a visual color difference task. These subjective assessments may be affected by different factors [1] and may also show poor repeatability. Objective color difference evaluation involves calculating a value from instrumental color measurements of the two samples in the pair, using a specific color difference formula. Color difference formulas and uniform color spaces are important tools in colorimetry. The current colorimetric system [2] is based on the XYZ tristimulus space proposed by the International Commission on Illumination (CIE). Following this proposal color scientists have devoted themselves to the study of color difference formulas. The simplest color difference formula should be the Euclidian distance between two points in the XYZ tristimulus space. However, it was soon

found that the XYZ tristimulus space was not uniform, which implies that color differences measured by such Euclidian distances were not at all proportional to the visual differences reported by observers with normal color vision. Therefore, more uniform color spaces began to be explored, using visual model theory [3,4] to replace XYZ distance with lightness and chromaticity (red-green and yellow-blue) differences in opponent color spaces. The Adams-Nickerson (ANLAB) color difference formula [5] is one of the earliest color difference formulas based on the following approximately uniform color space:

$$\begin{cases} L_{AN} = 0.23SV_Y \\ A_{AN} = S(V_X - V_Y) \\ B_{AN} = 0.4S(V_Y - V_Z) \end{cases} \tag{1}$$

where L_{AN}, A_{AN} and B_{AN} are three orthogonal axes, related to lightness, red-green and yellow-blue responses, respectively. V_X, V_Y, V_Z in Eq. (1) are the Munsell values based on Judd polynomial [6], and S is a parameter often designated in the literature as 42 or 40 [7]. For example, V_Y was determined from Y and Y_W values for sample and illuminant, respectively, using Eq. (2):

$$100(Y/Y_W) = 1.2219V_Y - 0.23111(V_Y)^2 + 0.23951(V_Y)^3 - 0.021009(V_Y)^4 + 0.0008404(V_Y)^5, \tag{2}$$

while V_X and V_Z were similarly determined from X and X_W, and Z and Z_W, respectively. The determination of Munsell values V_X, V_Y, V_Z using Eq. (2) is complicated. In 1958, Glasser et al. [8] reported that lightness L and tristimulus value Y roughly satisfied

$$L = c_1(100Y/Y_W)^{1/3} - c_2 \ , \tag{3}$$

where c_1 and c_2 were parameters dependent on experimental conditions, and Munsell value V_Y was roughly equal to $L/10$. Therefore, V_Y can be replaced by $\left[c_1\left(\frac{100Y}{Y_W}\right)^{\frac{1}{3}} - c_2\right]/10$, and similarly, V_X and V_Z can be replaced by $\left[c_1\left(\frac{100X}{X_W}\right)^{\frac{1}{3}} - c_2\right]/10$ and $\left[c_1\left(\frac{100Z}{Z_W}\right)^{\frac{1}{3}} - c_2\right]/10$, respectively. From these replacements and some empirical considerations, in 1976 CIE proposed an approximately uniform color space (UCS), named CIE 1976 $L^*a^*b^*$ (or CIELAB) [2, 9], defined as:

$$\begin{cases} L^* = 116\Phi(Y/Y_W) - 16 \\ a^* = 500\big(\Phi(X/X_W) - \Phi(Y/Y_W)\big) \\ b^* = 200\big(\Phi(Y/Y_W) - \Phi(Z/Z_W)\big) \end{cases} \tag{4}$$

where

$$\Phi(t) = \begin{cases} \left(\frac{841}{108}\right)t + \frac{16}{116} & ,t \le t^* \\ t^{\frac{1}{3}} & ,t > t^* \end{cases} , \quad \text{with} \quad t^* = \left(\frac{24}{116}\right)^3 . \tag{5}$$

CIELAB has been widely used in the surface color industry. Together with CIELAB, in 1976 CIE proposed another approximately UCS, named CIE 1976 $L^*u^*v^*$ (or CIELUV) [10], which was based on the CIE 1964 $U^*V^*W^*$ colour space [11] and work by Eastwood [12]. The uniform chromaticity scale associated to CIELUV is the CIE 1976 $u'v'$ diagram [2, 13],

$$\begin{cases} u' = \dfrac{4X}{X + 15Y + 3Z} \\ v' = \dfrac{9X}{X + 15Y + 3Z} \end{cases} , \quad (6)$$

and CIELUV has the following defining equations:

$$\begin{cases} L^* = 116\Phi(Y/Y_W) - 16 \\ u^* = 13L^*(u' - u'_W) \\ v^* = 13L^*(v' - v'_W) \end{cases} . \quad (7)$$

Here, u'_W and v'_W are determined using Eq. (6) from tristimulus values X_W, Y_W, Z_W of the reference white. Since the CIELUV color space incorporated the CIE 1976 $u'v'$ uniform chromaticity scale diagram, and straight lines in the $u'v'$ diagram readily express additive color mixture, the CIELUV color space is currently used in some industrial applications that depend on additive light mixing, such as color TV sets, video monitors, and lighting [14-20].

Since 1976, CIELAB-based color differences formulas such as CMC [21], CIE94 [22,23], and CIEDE2000 [24,25] have been developed. Currently, the most accurate color difference formula available is CIEDE2000, which has been jointly recommended as a standard by ISO and CIE [26]. Unfortunately, however, there are no color spaces associated to any of these three color difference formulas.

Recently, Du et al. [27] have proposed a new UCS named MLAB, which is based on CIELAB. Performance tests using the combined corrected (COM-corrected) dataset used for CIEDE2000 development [28], currently known as the "CIE combined corrected dataset" [29], showed that MLAB was better than CIELAB. However, since 1976 there has been no new color difference formula or UCS reported in the literature along the lines of CIELUV. For the current paper, new UCSs based on CIELUV have been developed and tested, as discussed in the sections that follow.

PROPOSED UNIFORM COLOR SPACES BASED ON CIELUV

The new approximately uniform color spaces (UCSs) introduced in this section have the same structure as CIELUV, with the values of their 9 parameters p_i (i=1,…,9) optimized using the CIE combined corrected dataset (COM-corrected).

The lightness of the new color spaces, denoted by L^*_m, is a modification of the lightness L^* proposed by CIELUV, and is defined as

$$L^*_m = (100 + p_1)\Phi_m(Y/Y_W) - p_1 \quad , \quad (8)$$

where the function $\Phi_m(t)$ is

$$\Phi_m(t) = \begin{cases} p_2 {t^*_m}^{(p_2-1)} t + \frac{p_1}{p_1+100} & ,0 \le t \le t^*_m \\ t^{p_2} & , \quad 1 \ge t > t^*_m \end{cases} , \quad (9)$$

and t^*_m is a function of parameters p_1 and p_2, defined as

$$t_m^* = \left(\frac{p_1}{(p_1+100)(1-p_2)}\right)^{\frac{1}{p_2}} \quad . \tag{10}$$

If $p_1 = 16$ and $p_2 = 1/3$, $\Phi_m(t)$ and t_m^* defined in Eqs. (9) and (10) become $\Phi(\mathrm{t})$ and t^* defined in Eq. (5). It was also verified that the lightness L_m^* defined by Eqs. (8)-(10) is always in the range 0-100, and its derivative is continuous.

The uniform chromaticity scales u_m', v_m' of the new proposed color spaces have the same structure as CIE 1976 u', v' coordinates, with following general expressions involving parameters p_3-p_7:

$$\begin{aligned} u_m' &= \frac{p_3 X}{p_4 X + p_5 Y + p_6 Z} \\ v_m' &= \frac{p_7 Y}{p_4 X + p_5 Y + p_6 Z} \end{aligned} \tag{11}$$

Finally, the red-green (u_m^*) and yellow-blue (v_m^*) color coordinates of the new color spaces are defined by the following expressions, with two additional parameters, p_8 and p_9:

$$\begin{aligned} u_m^* &= p_8 L_m^* (u_m' - u_{m,W}') \\ v_m^* &= p_9 L_m^* (v_m' - v_{m,W}') \end{aligned} \tag{12}$$

where, $u_{m,W}'$, and $v_{m,W}'$ are the uniform chromaticity coordinates of the reference white, determined using Eq. (11) from the tristimulus values X_W, Y_W, Z_W of the reference white.

The values of the parameters p_i (i=1,…,9) in Eqs. (8)-(12) were computed to achieve optimal predictions of visual color differences in the COM-corrected dataset [28], using the STRESS [28] function. Let ΔE_i be the color difference for a pair of samples in this dataset, computed using the Euclidian distance in the new color space L_m^*, u_m^*, v_m^*, and let ΔV_i be the visual difference for this pair of samples. The STRESS function, $f_{STRESS}(\{\Delta E_i\},\{\Delta V_i\})$ [28], is defined as

$$\mathrm{STRESS} = f_{STRESS}(\{\Delta E_i\},\{\Delta V_i\}) = 100\left(\frac{\sum_{i=1}^{N}(\Delta E_i - \gamma \Delta V_i)^2}{\sum_{i=1}^{N}\gamma^2 \Delta V_i^{\,2}}\right)^{1/2} \quad \text{with} \quad \gamma = \frac{\sum_{i=1}^{N}\Delta \mathrm{E}_i^2}{\sum_{i=1}^{N}\Delta \mathrm{E}_i \Delta V_i} \quad , \tag{13}$$

and measures the disagreement between the predictions made by the color difference formula and the visual differences. For example, STRESS =20 indicates a 20 percent disagreement between the predictions and the visual differences. Therefore, the smaller the STRESS value, the better the color difference formula or color space concerned performs. The function $f_{STRESS}(\{\Delta E_i\},\{\Delta V_i\})$ may be considered a function of parameters p_1-p_9, denoted as $F(p_1, p_2, \cdots, p_9)$. Furthermore, from comparison with the current CIELUV space, and also to minimize the objective function $F(p_1, p_2, \cdots, p_9)$, we decided that the parameters p_i (i=1,…,9) of the new color spaces should also satisfy

$$\begin{gathered} p_i > 0 \ \ for\ \ i = 1, 2 \cdots, 9 \\ 1 > p_2 > 0, \quad p_4 = 1 \end{gathered} \quad . \tag{14}$$

Thus, the determination of the 9 parameters p_1-p_9 of the new color spaces is modeled as the following nonlinear constraint optimization problem:

$$\begin{matrix} minimize\ F(p_1, p_2, \cdots, p_9) \\ \text{with constraint (14)} \end{matrix} . \quad (15)$$

Recently, Wang et al [30] have obtained values for the 9 parameters p_1-p_9 by numerically solving the constrained nonlinear optimization problem in Eq. (15), and the new color space obtained was named MLUV. The values of the 9 parameters for CIELUV and MLUV are listed in rows 2 and 3 of Table 1, respectively. It was found in [30] that MLUV was better than CIELUV, CIELAB, and MLAB [27].

Table 1. Values of parameters p_1-p_9 for CIELUV, MLUV [22], MLUV1 and MLUV2 color spaces.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
CIELUV	16	1/3	4	1	15	3	9	13	13
MLUV	29.2330	0.2311	1.2585	1.0000	1.5901	0.6963	1.0000	3.1168	5.0235
MLUV1	37.8517	0.2464	4.0000	1.0000	15.0000	3.0000	9.0000	5.0370	6.5817
MLUV2	30.9296	0.2372	1.0000	1.0000	2.3902	0.8180	1.0000	4.9343	7.9554

In addition, it was also found in [30] that for MacAdam's ellipses [31] the new uniform chromaticity scales, u'_m, v'_m, (see Eq. (11)) performed much worse than the original uniformity chromaticity scales, u', v', in terms of the global uniformity (GU) measure, although u'_m, v'_m outperformed u', v' in terms of the local uniformity (LU) measure [32]. Let α_i and β_i, $i = 1, 2, \cdots, n$, be the major and minor axes of the i^th^ ellipse in a given color space. Thus, $\pi\alpha_i\beta_i$ is the area of the i^th^ ellipse. Let A be the average area of all ellipses. The global uniformity (GU) measure [32] is defined as the STRESS value between $\pi\alpha_i\beta_i$ and $A_i = A$, with $i = 1,2, \cdots, n$, i.e., $\text{GU} = f_{STRESS}(\{\pi\alpha_i\beta_i\}, \{A_i\})$. In a uniform color space, all the ellipses should be circles with the same area, and therefore the GU measure for such a space should be zero. The local uniformity (LU) measure [32] is also defined in terms of the STRESS value, considering α_i/β_i and $r_i = 1$ with $i = 1,2, \cdots, n$, i.e., $\text{LU} = f_{STRESS}(\{\alpha_i/\beta_i\}, \{r_i\})$. In a uniform color space, the ellipses should be circles, all ratios α_i/β_i should be equal to 1, and therefore the LU measure should be zero. It can be concluded that the smaller the GU (or LU) measure, the more uniform the color space.

The uniform chromaticity scales u'_m and v'_m (Eq. (11)) are an important part of the new color spaces proposed. It is therefore desirable that the new color spaces not only outperform CIELUV in terms of STRESS value when predicting visual color difference datasets, but also that the associated u'_m, v'_m chromaticity coordinates at the same time perform as well as or better than the u', v' coordinates in terms of both the LU and GU measures from MacAdam's ellipse dataset. With this aim, we have in this paper adopted two sets of constraints for parameters p_1-p_9. The first set of constraints is to fix

$$p_3 = 4,\ p_5 = 15,\ p_6 = 3, \text{and } p_7 = 9, \quad (16)$$

resulting in the new uniform chromaticity coordinates u'_m, v'_m being identical to the original chromaticity coordinates u', v', respectively. In this case, the new color space has only 4 parameters to be optimized, and is named MLUV1. That is to say, MLUV1 is defined by numerically solving the following optimization problem:

$$\begin{array}{c} minimize\ F(p_1, p_2, \cdots, p_9) \\ \text{with constraints (14) and (16)} \end{array} \quad . \tag{17}$$

The values of parameters p_1-p_9 (i.e. the MLUV1 color space) are listed in the fourth row of Table 1.

Let $\mathrm{GU}(u'_m, v'_m)$, $(\mathrm{LU}(u'_m, v'_m))$, and $\mathrm{GU}(u', v')$ $(\mathrm{LU}(u', v'))$ be the GU (LU) measure, calculated from chromaticity coordinates u'_m, v'_m and u', v', respectively. It is clear that $\mathrm{GU}(u'_m, v'_m)$ is a function of the parameters p_3-p_7. We then adopt the following constraint condition:

$$g(p_3, p_4, \cdots, p_7) = \mathrm{GU}(u'_m, v'_m) - \mathrm{GU}(u', v') \le 0 \ . \tag{18}$$

Therefore, the optimization problem in (15) plus the new constraint (18) leads to the following new nonlinear optimization problem:

$$\begin{array}{c} minimize\ F(p_1, p_2, \cdots, p_9) \\ \text{with constraints (14) and (18)} \end{array} \tag{19}$$

The values of parameters p_1-p_9 obtained by the numerical solution of the optimization problem in (19), are listed in the last row of Table 1 and constitute a new color space named MLUV2. Note that Wang et al. [30] found that the chromaticity coordinates u'_m, v'_m, associated to the color space obtained by solving the optimization problem in (15), predicted MacAdam's ellipses better than the u', v' coordinates in terms of the LU measure, but not in terms of the GU measure. For this reason, only the GU measure has been considered in Eq. (18), along with the optimization problem posed in (19).

From Table 1 we can note that the values of parameter p_1 for MLUV, MLUV1, and MLUV2 are larger than for CIELUV, while the values of parameter p_2 for MLUV, MLUV1, and MLUV2 are smaller than for CIELUV. It can also be noted that $p_8 = p_9 = 13$ for CIELUV, but this is not true for MLUV, MLUV1, and MLUV2. The values of parameters p_3, p_5, p_6, and p_7 for MLUV and MLUV2 are much smaller than for CIELUV (or MLUV). This means that the chromaticity coordinates u'_m, v'_m associated to MLUV and MLUV2 are considerably different than the chromaticity coordinates u', v'. Furthermore, we can also note from Table 1 that for MLUV2 $p_3 = p_7 = 1$, which makes the chromaticity coordinates u'_m, v'_m associated to MLUV2 simpler than the u', v' coordinates associated to CIELUV.

PERFORMANCES OF THE NEW COLOR SPACES MLUV, MLUV1, AND MLUV2

First, we evaluated the performance of the new color spaces MLUV, MLUV1, and MLUV2 with respect to the four visual color difference datasets that constitute the COM-corrected dataset [28]. Next, the associated red-green and yellow-blue coordinates in the new proposed color spaces (u^*_m, v^*_m) and their uniform chromaticity coordinates (u'_m, v'_m) were also evaluated, from their predictions of two experimental ellipse datasets: the combined visual dataset (COMBVD) [24]; and MacAdam's dataset [31].

Predicting the COM-Corrected Visual Dataset

Using the STRESS index [28], we tested the performance of the new color spaces MLUV, MLUV1, and MLUV2, together with CIELUV, for the COM-corrected dataset and its four individual subsets (i.e. BFD-

P, Leeds, RIT-DuPont, and Witt). The results are listed in Table 2, including for comparison purposes the predictions made by CIELAB, CAM16-UCS [33] reported in [34] as current best available UCS, and CIEDE2000 (last row of Table 2), the current CIE/ISO-recommended color difference formula [26]. Note that using CAM16-UCS, in addition to the tristimulus values of the samples and illuminants, we assumed average surround, $Y_b = 20$, and $L_A = 40$ cd/m^2.

Table 2. STRESS values of CIELUV, MLUV, MLUV1, MLUV2, CIELAB, CAM16-UCS and CIEDE2000 color difference formulas for the BED-P, Leeds, RIT-DuPont, Witt, and COM-corrected visual datasets [28].

Space/Formula	BFD-P	Leeds	RIT-DuPont	Witt	COM-corrected
CIELUV	43.4	48.3	36.5	53.2	46.1
MLUV	38.7	36.3	25.3	45.6	38.8
MLUV1	40.1	38.2	27.8	47.0	40.4
MLUV2	38.5	36.8	25.8	45.9	38.9
CIELAB	42.5	40.1	33.4	51.7	43.9
CAM16-UCS	31.7	25.0	19.6	31.1	29.6
CIEDE2000	29.6	19.2	19.5	30.2	27.5

First, Table 2 shows that the three new color spaces (i.e. MLUV [30], MLUV1, and MLUV2) are better (i.e. have smaller STRESS values) than CIELUV and CIELAB for any of the visual datasets tested, but still far worse than CAM16-UCS. However, the three new color spaces are much simpler than CAM16-UCS. Second, CIELAB is better than CIELUV for any of the visual datasets tested, as reported before [28]. Third, of these three new color spaces and for the COM-corrected dataset, MLUV is slightly better than MLUV2 and both are better than MLUV1. Remember that the chromaticity coordinates u'_m, v'_m for MLUV1 are identical to the CIE 1976 chromaticity coordinates u', v' (see Table 1). Fourth, results achieved by CIEDE2000 are considerably better than those obtained by CIELUV, the three currently proposed new color spaces, CIELAB, and CAM16-UCS. However, remember that, unfortunately, CIEDE2000 is only a color difference formula, while other included in Table 2 are color spaces with their associated color difference formulas, as desirable.

To investigate the cases in which the STRESS values shown in Table 2 are statistically significant, we used the F-test test recommended by CIE [28]. To this aim, let

$$\mathrm{R} = \left(\frac{STRESS_A}{STRESS_B}\right)^2 \quad , \tag{21}$$

where subscripts A and B identify two different color spaces. The variable R in Eq. (21) is the ratio of two chi-squared variables and follows a Snedecor F-distribution with (N-1, N-1) degrees of freedom [28]. Here, N is the number of sample pairs in the visual dataset. Let F_C be the critical value of the F-distribution. If the value of R for a particular pair of spaces A and B is in the interval $[F_C, 1/F_C]$, the predictions of visual color differences made by the color spaces A and B are not statistically significantly different. Otherwise, the color space A is statistically significantly better or statistically significantly worse than the color space B when $\mathrm{R} < F_C$ or $R > 1/F_C$, respectively.

Table 3: R values (Eq. (21)) for different pairs of color spaces (CIELUV, MLUV, MLUV1, MLUV2, CAM16-UCS and CIELAB) and each of the visual datasets considered in Table 2. Values in

bold indicate that color spaces A and B are statistically significantly different, from values of critical intervals at 95% confidence level shown in the last row.

Space A in Eq. (21)	Space B in Eq. (21)	BFD-P	Leeds	RIT-DuPont	Witt	COM-corrected
MLUV	CIELUV	**0.795**	**0.565**	**0.481**	**0.735**	**0.708**
MLUV1	CIELUV	**0.854**	**0.626**	**0.580**	**0.781**	**0.768**
MLUV2	CIELUV	**0.787**	**0.581**	**0.500**	**0.744**	**0.712**
MLUV1	MLUV	1.074	1.107	1.207	1.062	**1.084**
MLUV2	MLUV	0.990	1.028	1.040	1.013	1.005
MLUV2	MLUV1	**0.922**	0.928	0.861	0.954	**0.927**
MLUV	CIELAB	**0.829**	0.819	**0.574**	**0.778**	**0.781**
MLUV1	CIELAB	**0.890**	0.907	**0.693**	0.826	**0.847**
MLUV2	CIELAB	**0.821**	0.842	**0.597**	**0.788**	**0.785**
MLUV	CAM16-UCS	**1.490**	**2.108**	**1.666**	**2.150**	**1.718**
MLUV1	CAM16-UCS	**1.600**	**2.335**	**2.012**	**2.284**	**1.863**
MLUV2	CAM16-UCS	**1.475**	**2.167**	**1.733**	**2.178**	**1.727**
CIELUV	CIELAB	1.043	**1.451**	1.194	1.059	1.103
$[F_C, 1/F_C]$		[0.928,1.077]	[0.799,1.252]	[0.800,1.249]	[0.825,1.212]	[0.964,1.038]

Table 3 shows R values for different color spaces A (column 1) and B (column 2), using several color difference datasets with confidence interval values $[F_C, 1/F_C]$ at 95% confidence level shown in the last row. Values in bold in Table 3 indicate that the corresponding color spaces A and B are statistically significantly different. First, from the values in rows 2-4 of Table 3 we can conclude that the three new color spaces proposed (MLUV, MLUV1, and MLUV2) are statistically significantly better than CIELUV for each of the four individual datasets and for the COM-corrected dataset. Second, from values in rows 5-7 of Table 3 comparing the three new colour spaces, we note that MLUV is statistically significantly better than MLUV1 for the COM-corrected dataset, but these two color spaces are not statistically significantly different for any of the four individual datasets (i.e. BFD-P, Leeds, RIT-DuPont, and Witt). Furthermore, MLUV2 and MLUV are not statistically significantly different for any of the four individual datasets nor for the COM-corrected dataset. More specifically, MLUV2 is better than MLUV for the BFD-P dataset, the opposite being true for the Leeds, RIT-DuPont, Witt, and COM-corrected datasets. Besides, MLUV2 is statistically significantly better than MLUV1 for the BFD-P and COM-corrected datasets, and MLUV2 is also better than MLUV1 (without statistical significance) for the Leeds, RIT-DuPont, and Witt datasets. Third, from values in rows 8-10 of Table 3 comparing the three new colour spaces (MLUV, MLUV1, and MLUV2) with CIELAB, we can note that MLUV and MLUV2 are significantly better than CIELAB for the BFD-P, RIT-DuPont, Witt, and COM-corrected datasets, and better than CIELAB (without statistical significance) for the Leeds dataset, while MLUV1 is significantly better than CIELAB for the BFD-P, RIT-DuPont, and COM-corrected datasets, and better than CIELAB (without statistical significance) for the Leeds and Witt datasets. Fourth, from values in rows 11-13 of Table 3 comparing the three new colour spaces (MLUV, MLUV1, and MLUV2) with CAM16-UCS, we can note that CAM16-UCS is significantly better than MLUV, MLUV1 and MLUV2 for the BFD-P, Leeds, RIT-DuPont, Witt, and COM-corrected datasets, as we expected. Finally, from values in last row of Table 3 comparing CIELUV and CIELAB, we can see that CIELAB is significantly better than CIELUV only for the Leeds dataset, and better than CIELUV (without statistical significance) for the

BFD-P, RIT-DuPont, Witt, and COM-corrected datasets.

As mentioned before, MLUV, MLUV1, and MLUV2 are statistically significantly better than CIELUV. This improvement comes in part from the modification of the lightness formula proposed by CIELUV (identical to the one proposed by CIELAB). Figure 1 shows a comparison of different proposed lightness formulas, plotting $(\text{Lightness}/100)^{0.4}$ against $(Y/Yw)^{0.4}$ for easier viewing of the differences between the color spaces considered. It can be noted that lightness for MLUV (red curve) and MLUV2 (blue curve) is greater than lightness for CIELUV (black curve). Remember that the chromaticity coordinates u'_m, v'_m for MLUV and MLUV2 are different from the CIE 1976 chromaticity coordinates u', v' used in CIELUV, which are identical to those in MLUV1 (see Table 1). As for lightness, Figure 1 shows that the proposals made by CIELUV (black curve) and MLUV1 (green curve) are similar, the lightness for MLUV1 being slightly smaller/greater than lightness for CIELUV for low/high Y/Yw values.

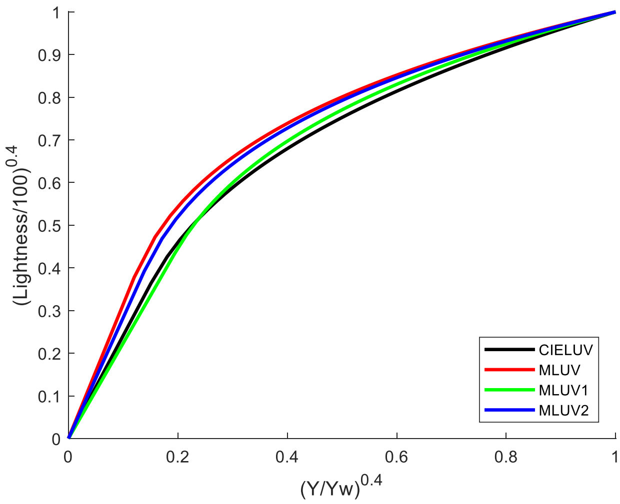

Figure 1: Comparison of lightness from CIELUV, MLUV, MLUV1, and MLUV2 color spaces.

Predicting the Combined Visual (COMBVD) and MacAdam's Ellipse Datasets

The experimental combined visual ellipse dataset (COMBVD) [24] was defined in CIELAB red-green (a^*) and yellow-blue (b^*) opponent color space, under CIE D65 illuminant and CIE 1931 standard observer, and includes 116 ellipses, centered at (a_i^*, b_i^*), or more exactly at (L_i^*, a_i^*, b_i^*), with major (α_i) and minor (β_i) axes, and rotation angles (θ_i), i=1,…,116. Figure 2 shows the COMBVD ellipses in the red-green and yellow-blue opponent color spaces defined in CIELUV (Fig. 2a), MLUV (Fig. 2b), MLUV1 (Fig. 2c), MLUV2 (Fig. 2d), CIELAB (Fig. 2e), and CAM16-UCS (Fig. 2f). MacAdam's experimental visual ellipse dataset [29] has 25 ellipses centered at (x_j, y_j) with major (α_j) and minor (β_j) axes, and rotation angles (θ_j), j=1,…,25, under CIE illuminant C and CIE 1931 observer. Figure 3 shows MacAdam's ellipses in the red-green and yellow-blue chromaticity coordinates associated to CIELUV (Fig. 3a), MLUV (Fig. 3b), MLUV1 (Fig. 3c), and MLUV2 (Fig. 3d), CIELAB (Fig. 3e), and CAM16-UCS (Fig. 3f), respectively, assuming illuminant C, CIE 1931 standard observer and $L^* = 50$. In a perfect uniform color space, all discrimination ellipses should be circles with the same area. Unfortunately, plots in Figures 2 and 3 indicate that all tested color spaces are far from perfectly uniform, as most of the ellipses plotted are not circles and have very different areas.

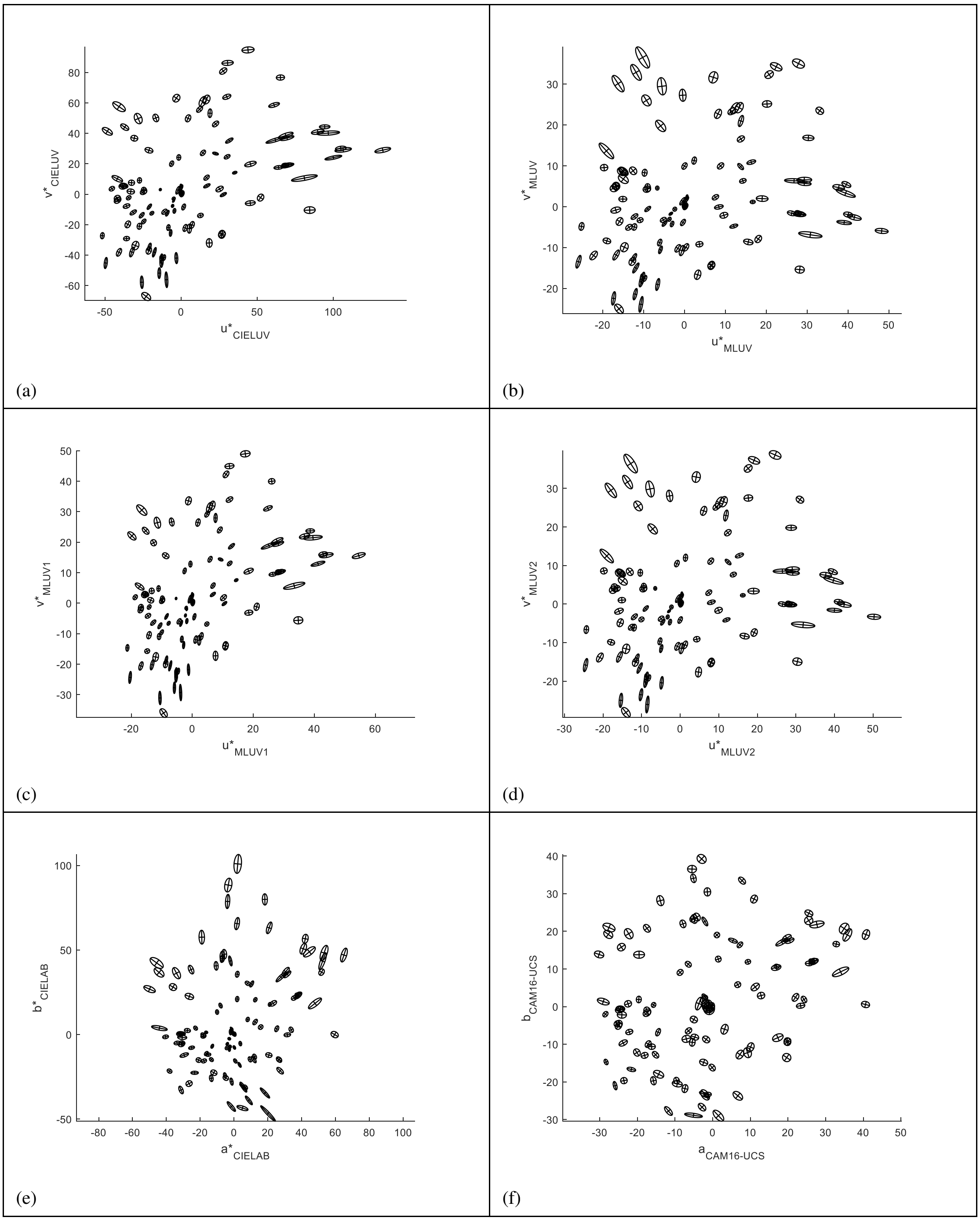

Figure 2: COMBVD ellipses [24] in red-green and yellow-blue spaces defined in CIELUV (a), MLUV (b), MLUV1 (c), MLUV2 (d), CIELAB (e), and CAM16-UCS (f).

While CAM16-UCS seems to show a nice uniformity, it is not easy to draw accurate conclusions from only a visual comparison of the six plots in Figures 2 or 3. Objective measures to quantify the uniformity of the color spaces mentioned are needed, and in this paper we used the local uniformity measure (LU) and the global uniformity measure (GU) [30,32]. The lower the values of LU or GU, the higher the uniformity. Table 4 shows values of the LU and GU measures for all color spaces tested (i.e. CIELUV, MLUV, MLUV1, MLUV2, CIELAB, and CAM16-UCS), considering the COMBVD and MacAdam's ellipse datasets, as well as the arithmetical mean of both datasets (last column in Table 4).

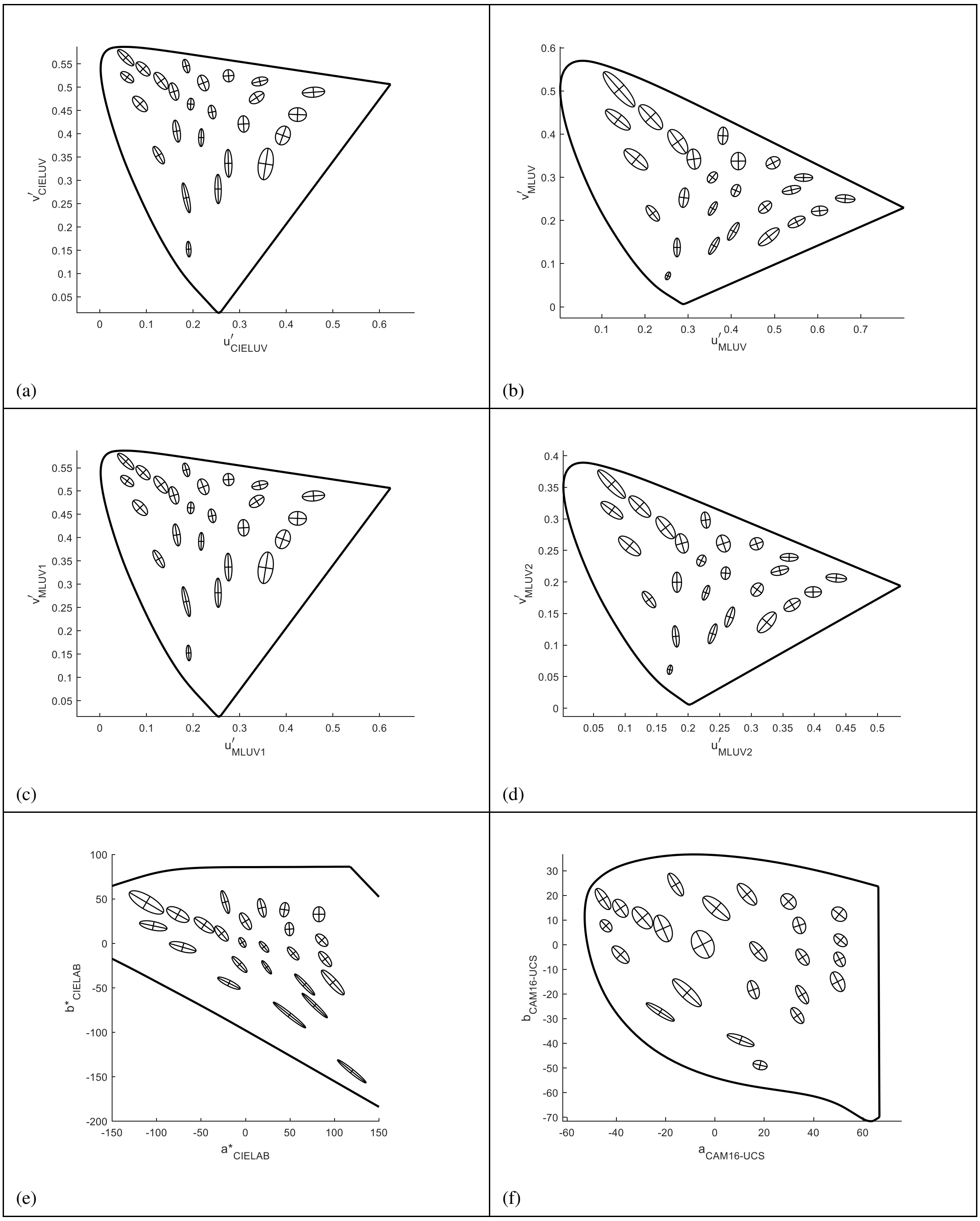

Figure 3: MacAdam's ellipses [31] in the uniform chromaticity diagrams u', v' proposed by CIELUV (a), MLUV (b), MLUV1 (c), and MLUV2 (d), together with CIELAB (e) and CAM16-UCS (f), using illuminant C, CIE 1931 standard observer and $L^* = 50$.

From Table 4, we can first note that MLUV achieved the lowest LU value for MacAdam's ellipse dataset, which means that MLUV ellipses are the closest to circles, and also that MLUV has the highest GU values, which means that MLUV ellipses have the greatest differences in area. For COMBVD ellipses, CAM16-UCS has the lowest LU value and CIELAB the largest GU value. Furthermore, we can see that CAM16-UCS was the best of all tested spaces in terms of LU and GU measures for the COMBVD ellipses. Second, Table 4 suggests that, considering the LU and GU measures, MLUV1 and CIELUV perform the same for MacAdam's ellipses, while MLUV1 is slightly worse than CIELUV for the COMBVD ellipses. Finally, from the LU and GU values shown in Table 4 we can also note that MLUV2 performed much better than CIELUV for MacAdam's ellipses. However, for the COMBVD ellipses, MLUV2 performed better than CIELUV in terms of LU measure, the opposite being true for the GU measure. Overall, from values shown in the last column of Table 4, we can state that for these ellipse datasets the ranking from best to worst color spaces is CAM16-UCS, MLUV2, MLUV, CIELUV, MLUV1, and CIELAB. Note that Tables 2 and 4 are all in terms of STRESS values. Therefore, if we average STRESS values in last columns of Tables 2 and 4 for each of the tested color spaces, the ranking from best to worst tested color spaces is CAM16-UCS (32.5), MLUV2 (39.3), MLUV (40.5), MLUV1 (41.9), CIELUV (44.7) and CIELAB (47.1).

Table 4: LU and GU measures for tested color spaces CIELUV, MLUV, MLUV1, MLUV2, CIELAB and CAM16-UCS using the COMBVD [24] and MacAdam's [29] ellipse datasets.

	COMBVD Ellipses		MacAdam's Ellipses		Mean of All
	LU	GU	LU	GU	
CIELUV	42.8	49.4	39.9	41.1	43.3
MLUV	36.9	53.0	27.2	51.4	42.1
MLUV1	42.9	49.8	39.9	41.1	43.4
MLUV2	38.8	50.2	28.8	40.9	39.7
CIELAB	39.5	60.7	52.9	48.2	50.3
CAM16-UCS	30.1	35.3	35.7	40.5	35.4

Finally, we note that Eastwood [12] showed that the CIE 1964 $U^*V^*W^*$ color space can be improved if the V^* axis is multiplied by 1.5 based on the Munsell renovation data, which resulted in the CIE 1976 $u'v'$ diagram. Hence Munsell renotation data can be also used in future research to evaluate the performance of new uniform chromaticity scale diagrams.

CONCLUSIONS

Three new color spaces (MLUV, MLUV1, and MLUV2) with the same structure as CIELUV were developed. It was found that the three color spaces were statistically significantly better than CIELUV in their predictions for the COM-corrected dataset and its four individual datasets (Tables 2 and 3), the range (from best to worst predictions) being: MLUV, MLUV2, and MLUV1. However, MLUV and MLUV2 were not statistically significantly different in predicting the visual color-difference datasets (Table 3). Furthermore, in predicting the COMBVD and MacAdam's ellipse datasets it was found that MLUV2 performed best and MLUV second best (see last column of Table 4). Overall, from Tables 2 and 4, we can state that MLUV2 performed best, followed by MLUV and MLUV1. CAM16-UCS is better than MLUV2 and other tested spaces, but MLUV2 is much simpler than CAM16-UCS. Therefore it may be expected that MLUV2 can be useful in color specification and color difference evaluation, especially in industrial fields that depend on additive light mixing, such as color TV sets, video monitors, and lighting applications. Full equations for the MLUV2 and worked examples are given in the next Appendix.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (grant numbers 61575090 and 61775169) and the Ministry of Science and Innovation of the National Government of Spain (grant number PID2019-107816GB-I00/SRA/10.13039/501100011033). Support from the Foundation of Liaoning Province Education Administration, and the Graduate Education Reform, Scientific & Technological Innovation and Entrepreneurship Project of the University of Science and Technology Liaoning is also acknowledged. Thanks also to Mr. Donald J. Murphy McVeigh for English revision.

ORCIDs
Changjun Li, https://orcid.org/0000-0002-9942-7690
Manuel Melgosa, https://orcid.org/0000-0002-7226-4190

Data Availability Statement
The data used to support the findings of this study are available from the corresponding author upon request. The data are not publicly available due to privacy restrictions.

Appendix: Full equations for MLUV2 with worked examples

Inputs: Tristimulus values X, Y, Z for the samples and illuminant (Xw, Yw, Zw).
Outputs: L^*_{MLUV2} for the lightness, u^*_{MLUV2} for the red-green coordinate, and v^*_{MLUV2} for the yellow-blue coordinate.

$$t^*_{MLUV2} = \left(\frac{30.9296}{130.9296 * 0.7628}\right)^{\frac{1}{0.2372}}. \quad \text{(A1)}$$

$$u'_{MLUV2,W} = \frac{X_W}{X_W + 2.3902Y_W + 0.8180Z_W}, \quad v'_{MLUV2,W} = \frac{Y_W}{X_W + 2.3902Y_W + 0.8180Z_W}. \quad \text{(A2)}$$

$$t = Y/Y_W \ . \quad \text{(A3)}$$

$$\Phi_{MLUV2}(t) = \begin{cases} \frac{0.2372 \cdot t}{(t^*_{MLUV2})^{0.7628}} + \frac{30.9296}{130.9296} & , \text{ if } 0 \le t \le t^*_{MLUV2} \\ t^{0.2372} & , \text{ if } 1 \ge t > t^*_{MLUV2} \end{cases}. \quad \text{(A4)}$$

$$u'_{MLUV2} = \frac{X}{X + 2.3902Y + 0.8180Z}, \quad v'_{MLUV2} = \frac{Y}{X + 2.3902Y + 0.8180Z}. \quad \text{(A5)}$$

$$L^*_{MLUV2} = 130.9296\Phi_{MLUV2}(\mathrm{t}) - 30.9296 \ . \quad \text{(A6)}$$

$$u^*_{MLUV2} = 4.9343L^*_{MLUV2}(u'_{MLUV2} - u'_{MLUV2,W}), \quad v^*_{MLUV2} = 7.9554L^*_{MLUV2}(v'_{MLUV2} - v'_{MLUV2,W}). \quad \text{(A7)}$$

Worked Examples

Tristimulus values X_W, Y_W and Z_W for the selected illuminant and related results using Eqs. (A1) and (A2) are listed in Table A1. Input tristimulus values $X\ Y\ Z$ for 6 samples and results using Eqs. (A3)-(A7) are listed in Table A2. Note that values in Table A1 are independent of selected samples.

X_W	Y_W	Z_W	t^*_{MLUV2}	$u'_{MLUV2,W}$	$v'_{MLUV2,W}$
96.91	100	108.6	0.0071	0.2281	0.2354

Table A1: The tristimulus values X_W, Y_W and Z_W for the illuminant and results using Eqs. (A1) and (A2).

	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
X	11.24	37.22	17.82	10.31	25.13	31.02
Y	10.05	34.63	19	13.37	23.84	42.7
Z	6.48	25.35	33.85	7.18	44.81	44.96
t	0.1005	0.3463	0.19	0.1337	0.2384	0.427
$\Phi_{MLUV2}(t)$	0.5798	0.7776	0.6744	0.6205	0.7117	0.8172
u'_{MLUV2}	0.2771	0.2645	0.196	0.2142	0.2116	0.1826
v'_{MLUV2}	0.2478	0.2461	0.209	0.2777	0.2007	0.2514
L^*_{MLUV2}	44.9896	70.8817	57.3699	50.3078	62.253	76.0683
u^*_{MLUV2}	10.8678	12.7066	-9.1039	-3.4712	-5.0865	-17.0885
v^*_{MLUV2}	4.4179	6.0063	-12.0748	16.9317	-17.1827	9.6588

Table A2: Tristimulus values $X\ Y\ Z$ for 6 samples and results using Eqs. (A3)-(A7).

REFERENCES

[1] CIE 101:1993. Parametric Effects in Colour-Difference Evaluation. CIE Central Bureau, Vienna, 1993.

[2] CIE 015:2018. Colorimetry, 4th Edition. CIE Central Bureau, Vienna, 2018.

[3] Adams EQ. A theory of color vision, Psychological Review 30(1), 56-76 (1923).

[4] Adams EQ. X-Z Planes in the 1931 I.C.I. System of Colorimetry, J. Opt. Soc. Am. 32(3), 168-173 (1942).

[5] McLaren K. The Adams—Nickerson Colour-difference Formula. J. Soc. Dyers Col. 86(8), 354-356 (1970).

[6] Newhall SM, Nickerson D, Judd DB. Final report of the OSA subcommittee on the spacing of the Munsell colors. J. Opt. Soc. Am. A 33(7), 385-418 (1943).

[7] McDonald R (editor), Colour Physics for Industry, Second Edition. Society of Dyers and Colourists, England, 1997.

[8] Glasser L G, McKinney A H, Reilley CD, and Schnelle PQ. Cube-root color coordinate system, J. Opt. Soc. Am. 48, 736 (1958).

[9] ISO CIE 11664-4:2019(E). Colorimetry — Part 4: CIE 1976 L*a*b* colour space. CIE Central Bureau, Vienna, 2019.
[10] Robertson A. Historical development of CIE recommended color difference equations. Color Res. Appl. 15(3), 167-170 (1990).
[11] CIE Proceedings 1963 (Vienna Session), Vol. B., pp. 209-220. Committee Report E-1.4.1: CIE Central Bureau, Paris, 1964.
[12] Eastwood D. A simple modification to improve the visual uniformity of the C.I.E. I964 U*V*W* colour space. In proceedings of the Second Congress of the International Colour Association, pages 293-296, published by Adam Hilger, London, 1973.
[13] ISO/CIE 11664-5:2016(E). Colorimetry — Part 5: CIE 1976 L*u*v* Color Space and u', v' Uniform Chromaticity Scale Diagram. CIE Central Bureau, Vienna, 2016.
[14] CIE TN 001:2014. Chromaticity difference specification for light sources. CIE Central Bureau, Vienna, 2014.
[15] Ohta N, Robertson AR. Colorimetry: Fundamentals and Applications. Wiley–IS&T Series in Imaging Science and Technology, John Wiley & Sons Ltd, West Sussex, England, 2005.
[16] Berns RS. Billmeyer and Saltzman's Principles of Color Technology, Third Edition. Wiley-Interscience, 2000.
[17] Nayatani Y , Sakai H. An integrated color-appearance model using CIELUV and its applications, Color Res. Appl. 33(2), 125-134 (2008).
[18] Kataoka T, Hinata H, Kako S. A new technique for detecting colored macro plastic debris on beaches using webcam images and CIELUV. Mar. Pollut. Bull. 64(9), 1829-1836 (2012).
[19] Pastilha R, Gupta G, Gross N, Hurlbert A. Temporal dynamics of daylight perception: Detection thresholds. J. Vision 20(13): 18 (2020).
[20] Clarke FJJ, McDonald R, Rigg B, Modification to the JPC79 color difference formula. J. Soc. Dyers Col. 100, 128-132 and 281-282 (1984).
[21] CIE 116:1995. Industrial color difference evaluation, CIE Central Bureau, Vienna, 1995.
[22] McDonald R, Smith KJ. CIE94—a new color difference formula. J. Soc. Dyers Col. 111, 376-379 (1995).
[23] Luo MR, Cui G, Rigg B. The development of the CIE 2000 colour-difference formula: CIEDE2000. Color Res. Appl. 26(5), 340-350 (2001).
[24] CIE 142:2001. Improvement to Industrial Colour-Difference Evaluation. CIE Central Bureau, Vienna, 2001.
[25] ISO/CIE 11664-6:2022(E). Colorimetry-Part 6: CIEDE2000 Color-Difference Formula. CIE Central Bureau, Vienna, 2022.
[26] Du F, Xu Y, Li CJ. Modified CIELAB Uniform Color Space. Acta Optica Sinica, 42(1): 0133001(2022, in chinese)
[27] CIE 217:2016. Recommended Method for Evaluating the Performance of Colour-Difference Formulae. CIE Central Bureau, Vienna, 2016.
[28] Carter RC, García PA, Melgosa M, Brill M. Metrics of color-difference formula improvement. J. Opt. Soc. Am. A 39, 1360-1370 (2022).
[29] Wang L, Li CJ, Xu Y and Gao C. A New Uniform Colour Space MLUV Based on the CIELUV. The 13th China Academic Conference on Printing and Packaging & Forum of Collaborative Innovation of Industry-University-Research, (online meeting), November 11-12, 2022, China; Digital Printing 221(6) (2022).

[30] MacAdam DL. Visual sensitivities to color differences in daylight. J. Opt. Soc. Am. 32(5), 247-274 (1942).

[31] Safdar M. Study on the Development of a Uniform Color Space for Perception-based Image Processing. PhD Thesis, Zhejiang University, 2017.

[32] Li CJ, Li ZQ, Wang ZF, Xu Y, Luo MR, Cui GH, Melgosa M, Brill MH, and Pointer MR. Comprehensive color solutions: CAM16, CAT16 and CAM16-UCS. Color Res. Appl. 42(6), 703-718 (2017).

[33] Luo MR, Xu Q, Pointer M, Melgosa M, Cui GH, Li CJ, Xiao KD, Huang M. A comprehensive test of colour-difference formulae and uniform colour spaces using available visual datasets. Color Res. Appl. 48(3), 267-282 (2023).

AUTHOR BIOGRAPHIES

Ling Wang is a graduate student in the Department of Computer Science, University of Science and Technology Liaoning, Anshan, China. She received her BS in computer science in 2020 from University of Changsha. Her current research interests are color difference formulas and uniform color spaces.

Changjun Li is a re-employed Professor at the Department of Computer Science, University of Science and Technology Liaoning, Anshan, China. He received his BS (1979), MS (1982), and PhD (1989) degrees in computational mathematics from Peking University (China), the Chinese Academy of Science, and Loughborough University (UK), respectively. His current research interests are chromatic adaptation transforms, color appearance modeling and uniform color spaces.

Yang Xu is a professor at the School of Computer Science and Software Engineering, University of Science and Technology Liaoning, Anshan, China. She received a BS in computer science and technology from Shenyang Jianzhu University, China, 2003, an MS in computer application from University of Science and Technology Anshan in 2006, and a PhD from Northeastern University in 2012. Her research focuses on image processing, computer vision and color appearance models.

Manuel Melgosa is a full professor in optics and a senior editor of this journal. He is currently Vice-president of International Affairs of the Lighting Committee of Spain, representative of Spain at CIE Divisions 1 and 8, and a member of CIE TCs 1-98, 8-14, 8-17, 8-18 and R1-60. His main research interests are color difference evaluation and applied colorimetry.

Kaida Xiao is an Associate Professor in Color and Imaging Science in the School of Design, University of Leeds, UK. His research interests are 3D color image reproduction, 3D color printing, 3D facial prosthesis printing, medical image capture and analysis, color appearance modeling, and image quality enhancement.

Cheng Gao is a research assistant in the Department of Computer Science, University of Science and Technology Liaoning. He received BS (2015) and MS (2018) degrees in software engineering from this university. His current research interests are the spectrum optimization of LEDs, color appearance modeling, and mesopic luminance computation.