Article:

Chowdhury, S.M. and Kim, S.-H. orcid.org/0000-0001-6171-5382 (2017) “Small, yet beautiful”: Reconsidering the optimal design of multi-winner contests. Games and Economic Behavior, 104. pp. 486-493. ISSN 0899-8256

https://doi.org/10.1016/j.geb.2017.05.012

Published as: Chowdhury, S.M., & Kim, S.H. (2017). "Small, yet Beautiful": Reconsidering the Optimal Design of Multi-winner Contests.
Games and Economic Behavior, 104, 486-493.

"Small, yet Beautiful": Reconsidering the Optimal Design of Multi-winner Contests *

Subhasish M. Chowdhury[†] and Sang-Hyun Kim[‡]

This version: September 27, 2015

Abstract

We reconsider whether a grand multi-winner contest elicits more equilibrium effort than a collection of sub-contests. Fu and Lu (2009) employ a sequential winner-selection mechanism and find support for running a grand contest. We show that this result is completely reversed if a simultaneous winner-selection mechanism or a sequential loser-elimination mechanism is implemented. We then discuss the optimal allocation of players and prizes among sub-contests, and the case in which there is restriction in the number of sub-contests.

.

JEL Classifications: C72; D72; D74

Keywords: Contest design; Multiple winner; Group-size; Selection mechanism

*We appreciate the useful comments of Qiang Fu, Jingfeng Lu and the seminar participants at the University of East Anglia. Any remaining errors are our own.

†Corresponding author. School of Economics, Centre for Behavioural and Experimental Social Science, and Centre for Competition Policy, University of East Anglia, Norwich Research Park, Norwich NR4 7TJ, UK. Phone: +44 160-359-2099. E-mail: s.modak-chowdhury@uea.ac.uk

‡School of Economics, and Centre for Competition Policy, University of East Anglia, Norwich Research Park, Norwich NR4 7TJ, UK. Phone: +44 160-359-3422. E-mail: sang-hyun.kim@uea.ac.uk

1 Introduction

Contests, in which players exert costly and irreversible efforts to win a prize, are ubiquitous in day-to-day life. In cases such as war, terrorism or territorial conflicts, contests are not designed by an organizer. However, there are very many situations including sports, patent race, promotion tournament, crowd sourcing, legal battle etc. in which an organizer organizes the contest and contest design issues become highly important. The topic of optimal design of contests, hence, has been an active area of research. In the literature one of the most frequently attempted questions is how to maximize the total effort exerted in a contest. For a contest with noisy outcome it is a further important question whether arranging a grand contest elicits higher equilibrium effort than what arranging several sub-contests does.

For a single-winner setting, Moldovanu and Sela (2001, 2006) show that under certain conditions a grand contest indeed elicits higher effort. Adding an important contribution to this area, Fu and Lu (2009) characterize the optimal structure for multi-winner contests.[1] They employ a nested winner-selection procedure as in Clark and Riis (1996) and show that a grand contest elicits greater equilibrium efforts than what a collection of mutually exclusive and exhaustive sub-contests does. This is an important finding since this extends the single-winner contest results of Moldovanu and Sela (2001, 2006) into multi-winner settings, and provides with clear policy design prescriptions.

In the specific mechanism employed above, the winners are selected sequentially. Players simultaneously exert their effort, and K winners are selected by K consecutive draws. Once a winner is selected through a Tullock (1980) contest success function, he/she is immediately removed from the pool of candidates up for the next draw. This procedure is repeated until all the prizes are exhausted.

In the field, however, the winner-selection procedure in a multi-winner contest is not always the one suggested above. Clark and Riis (1996) mention that when "*the imperfectly discriminating rent-seeking contest [...] ha(s) several winners, there is no unique method for selecting those winners*". Indeed, the very first winner-selection mechanism suggested in the multi-winner contest literature is by Berry (1993), who considers a one-shot winner-selection mechanism. Under this, the players exert effort and the set of winners are taken

[1] In these contests there are multiple prizes, but a contestant can win at most one prize.

out simultaneously. The probability of a player to win one of the K number of prizes is the sum of efforts exerted by any combination of a K-player group that includes that specific player, divided by the sum of efforts exerted by any combination of a K-player group. There are different instances in which either a simultaneous (Berry, 1993) or a sequential (Clark and Riis, 1996) winner-selection mechanism is employed in the field.

There are both pros and cons of employing the simultaneous mechanism. Clark and Riis (1996) show that with this mechanism the very first prize is allocated according to the effort outlays whereas all the other prizes are implicitly allocated randomly - allowing for an incentive to free-ride. Chowdhury and Kim (2014), on the other hand, find an equivalence of the simultaneous mechanism to a mechanism in which the losers are sequentially taken out - essentially providing a microfoundation for the contest success function arising out of the simultaneous mechanism.[2] Since the loser-elimination mechanism is well implemented in real life, this helps one to reformulate those real life situations as well. Hence, it is important to understand whether the answer to the original question (of comparing grand contest with sub-contests) depends on the particular winner-selection mechanism implemented.

In this study we reconsider such comparison of a grand contest with a collection of mutually exclusive and exhaustive sub-contests from a design point of view. We employ the simultaneous winner-selection mechanism (Berry, 1993) and find that the result of Fu and Lu (2009) gets reversed, i.e., a collection of sub-contests elicit a higher level of equilibrium effort than what a grand multi-winner contest does. In such a situation we characterize the optimal allocation of players and prizes for the symmetric case. We further show that under the sequential loser-elimination mechanism (Chowdhury and Kim, 2014) the Fu and Lu (2009) result may again be reversed. We then characterize the optimal contest structure when the number of sub-contests is limited to two.

[2]Moreover, Chowdhury and Kovenock (2012) find effort equivalence between the simultaneous mechanism and a situation in which the players in a multi-winner contest are connected by a ring network. de Palma and Munshi (2013) find that the simultaneous mechanism has a probabilistic foundation.

2 Model and main result

2.1 Contest design under simultaneous winner-selection

Consider N identical players competing for K indivisible prizes with $N > K \geq 2$. The common values of the prizes are v_1, v_2, ..., v_K, and a player can win at most one prize. The contest designer can run the grand contest by putting all the players and the prizes together or run M small contests by dividing the contestants and the prizes into mutually exclusive groups. Let n_g be the number of contestants in group g and $\mathbf{v}_g = (v_1^g, v_2^g, ..., v_{k_g}^g)$ be the vector of values of prizes allocated to group g where k_g be the number of prizes in group g. We also define a collection of contests $\mathbf{C} = \{c_g\}_{g=1}^M$ of which entry is $c_g = \{n_g, \mathbf{v}_g\}$. Since the groups are mutually exclusive and exhaustive, given the sets of contestants and of prizes, collection $\mathbf{C}$ defines a contest structure. Therefore, $\sum_{g=1}^M n_g = N$ and $\sum_{g=1}^M |\mathbf{v}_g| = \sum_{g=1}^M k_g = K$ where $|\mathbf{v}_g|$ is the number of elements in vector $\mathbf{v}_g$. We allow both a contest without a prize ($k_g = 0$ but $n_g > 0$) and that without a player ($k_g > 0$ but $n_g = 0$). In other words, the designer can throw prizes or players away, but without any loss, it is assumed that there is no contest without a prize nor a player ($k_g = 0$ and $n_g = 0$). Throughout the paper, we assume linear cost function with unit marginal cost.

Let us first consider the problem of player i who is allocated to group g with $k_g \geq 1$. If every player in the group other than player i expends the same amount of effort x_{-i}, the simultaneous winner-selection mechanism (Berry, 1993) generates the following contest success function:

$$P_i^{SM}(x_i, x_{-i}) = \min\left\{\frac{(k_g-1)x_{-i}+x_i}{(n_g-1)x_{-i}+x_i}, 1\right\}$$

where x_i is the effort of player i in g. In his original paper, Berry considers only the case with identical prizes. Here, we assume that when prizes are heterogeneous, the prizes allocated to contest c_g are randomly assigned to the winners in c_g.[3] Letting $\overline{v}^g$ denote the expected

[3] Chowdhury and Kim (2014) show that a mechanism that sequentially eliminates losers also generates the same contest success function if prizes are homogeneous. If prizes are heterogeneous, however, this does not have to be the case. In the next section we examine whether our main result remains valid with the Chowdhury-Kim mechanism with an example.

value of the prize, $\left(\sum_{j=1}^{k_g} v_j^g\right)/k_g$, we can write the objective function of player i as

$$\pi_i(x_i, x_{-i}|n_g, \mathbf{v}_g) = \overline{v}^g P_i^{SM}(x_i, x_{-i}) - x_i,$$

thus the symmetric equilibrium effort is

$$x_g^{SM} = \begin{cases} \overline{v}^g (n_g - k_g)/n_g^2 & \text{if } n_g \geq k_g \geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{1}$$

Let $T^{SM}(\mathbf{C})$ denote the total equilibrium effort with the simultaneous winner selection mechanism, i.e.,

$$T^{SM}(\mathbf{C}) = \sum_{g=1}^{M} n_g \times x_g^{SM} \tag{2}$$

The following proposition states that the grand contest never maximizes the total effort if the simultaneous mechanism is implemented.

Proposition 1 *Consider a contest structure $\mathbf{C}'$. If $n'_g \geq k'_g \geq 2$ for a contest $c'_g \in \mathbf{C}'$, there exists another structure $\mathbf{C}''$ such that $T^{SM}(\mathbf{C}') < T^{SM}(\mathbf{C}'')$.*

Proof. It is clear from (1) that x_g^{SM} increases as k_g decreases if $n_g \geq k_g$. Suppose that we throw away the least valued prize from a contest with $k_g \geq 2$, and let $\overline{v}_{new}^g$ denote the new expected value of the prize in the contest. Then $\overline{v}_{new}^g \geq \overline{v}^g$, and the effort elicited by the new contest is

$$\frac{\overline{v}_{new}^g (n_g - k_g + 1)}{n_g} > \frac{\overline{v}^g (n_g - k_g)}{n_g} = \text{the effort from the old contest } g.$$

■

This proposition simply states that in the optimal contest structure, there should not be a sub-contest with more than one prize. To characterize the optimal structure further, let us assume, following Berry (1993), that the prizes are identical (i.e., $v_1 = v_2 = ... = v_K = v$), and define $\overline{K} = \min\{K, \lfloor N/2 \rfloor\}$ where $\lfloor N/2 \rfloor$ is the largest integer not greater than $N/2$. Then we can show that with identical prizes, the total equilibrium effort is maximized by the symmetric structure.

Proposition 2 *Suppose that the prizes are identical. In the optimal structure, only $\overline{K}$ prizes are used, and N players are divided as symmetrically as possible into $\overline{K}$ groups. In each sub-contest, the players compete for a single prize.*

Proof. To show that fewer than $\overline{K}+1$ prizes should be used, suppose that $K > N/2$. If $K'(> N/2)$ prizes are used, that is, if K' prizes are allocated to contests with at least one player, then there must be a contest c_g such that $n_g \geq k_g \geq 2$ or $k_g \geq n_g \geq 1$. If $n_g \geq k_g \geq 2$, as shown in Proposition 1, the total effort can be increased further. If $k_g \geq n_g \geq 1$, on the other hand, because the players in contest c_g does not expend any effort, they can be moved to another contest in which no prize is given for free. Thus, fewer than $\overline{K}+1$ prizes should be used.

Proposition 1 shows that for any sub-contest with $n_g > k_g$, k_g must be 1. In such a sub-contest, the elicited equilibrium effort is

$$e(n_g) = \frac{(n_g - 1)}{n_g} v.$$

Notice that $e(n_g)$ is increasing and concave in n_g, from which we infer the following. First, throwing away a player reduces the total effort because $e(n_g)$ is increasing in n_g. Second, using fewer than $\overline{K}$ prizes is not optimal because $e(n_g)$ is concave, which means that having more smaller sub-contests is better than having fewer larger ones. Therefore, exactly $\overline{K}$ prizes are used in the optimal structure. If we ignore the integer problem, then due to the concavity of $e(n_g)$, the total effort $\sum_{g=1}^{\overline{K}} v(n_g - 1)/n_g$ will be maximized when $n_i = n_j$ for all i, j. ■

When the prizes are identical, it is optimal to run many symmetric sub-contests. If the prizes are heterogeneous, however, it may be optimal to make more players compete for a more valuable prize for which each player is willing to expend more effort. To see this in a clearer manner, let us consider four players ($N = 4$) competing for two different prizes, 1 and 2 ($K = 2$), with values $v_1 \geq v_2$. Because according to (1), $n_g x_g^{SM}$, the effort elicited by contest c_g, increases in n_g, allocating a player to a contest without a prize (i.e., excluding a player) is never optimal. And we know that the grand contest does not maximize the total effort. Therefore, we only need to consider how to allocate the four players to two contests each of which has a single prize.

If two players are allocated to each contest ("2-2" structure), each of them is the standard Tullock contest. Therefore, the equilibrium effort of a player who competes for prize $k(= 1, 2)$ is $v_k/4$, and the total effort is $T^{\text{2-2}} = (v_1 + v_2)/2$. If, on the other hand, all four of them compete for the more valuable prize ("4-0" structure), according to (1), the total effort is $T^{4\text{-}0} = 3v_1/4$.[4] Thus, if the prizes are sufficiently heterogeneous (more precisely if $v_1 > 2v_2$), the most asymmetric contest structure - in which the lower value prize is excluded - maximizes the total effort.

2.2 Comparison with sequential winner-selection

The results above contrast sharply with the result of Fu and Lu (2009) who employs the sequential winner-selection mechanism à la Clark and Riis (1996) for each contest. In this mechanism, the probability that player i is selected in the k^{th} draw is

$$P_{ik}^{SQ}(x_i, x_{-i}) = \sum_{\forall \Omega_k} \left[\Pr(\Omega_k)\, I(i \in \Omega_k)\, p_i(x_i, x_{-i}|\Omega_k)\right]$$

where Ω_k is a set of $N-(k-1)$ players, $\Pr(\Omega_k)$ is the probability that the set of the remaining contestants for the k^{th} draw are Ω_k, $I(i \in \Omega_k)$ is the indicator function that takes value 1 if $i \in \Omega_k$ and 0 otherwise, and $p_i(x_i, x_{-i}|\Omega_k) = x_i / \sum_{j \in \Omega_k} x_j$. It can be shown that in the symmetric equilibrium, this mechanism elicits total effort as much as

$$Nx_i^{SQ} = \sum_{k=1}^{K} \left[v_k \left(1 - \sum_{l=0}^{k-1} \frac{1}{N-l} \right) \right]. \tag{3}$$

Suppose, as before, there are four players ($N = 4$) competing for two different prizes, 1 and 2 ($K = 2$), with values $v_1 \geq v_2$. As shown above, the maximized total effort with two small contests is

$$T^{small} = \max\left\{ \frac{v_1 + v_2}{2}, \frac{3}{4}v_1 \right\}.$$

If all the prizes and the players are put in one grand contest and the simultaneous winner-selection mechanism is implemented, then according to (2) the total equilibrium effort is

$$T^{SM} = \frac{\overline{v}\,(N-K)}{N} = \frac{v_1 + v_2}{4}.$$

On the other hand, when the sequential winner-selection mechanism is implemented, then according to (3) the total effort is

$$T^{SQ} = v_1 \left(1 - \frac{1}{4}\right) + v_2 \left(1 - \frac{1}{4} - \frac{1}{3}\right) = \frac{3v_1}{4} + \frac{5v_2}{12}.$$

This example clearly shows that $T^{SQ} > T^{small} > T^{SM}$.

[4] Structure "3-1" (making only three players compete for prize 1) is dominated by structure "4-0" in terms of the total elicited effort because in structure "3-1" one player is wasted.

3 Further analyses

In relation with Fu and Lu (2009), it emerges that the 'Beauty of Bigness' result is not independent of the winner-selection mechanism employed in the exercise. If a mechanism suffers with free-riding problem as in the simultaneous mechanism (Berry, 1993) does, putting everything together may make things worse by allowing free-ridings in a greater scale. In such cases a collection of small contests would be more 'beautiful' to implement. Chowdhury and Kim (2014) show that a sequential loser-elimination mechanism also suffers from the challenge of free-riding. It is, hence, worth investigating whether implementing the loser-elimination mechanism can also reverse the result of Fu and Lu (2009). Further, even when a collection of sub-contests elicit higher effort, organizing a high number of contests might be costly. It will then be interesting to understand the optimal allocation of players in sub-contests, when such restrictions arise. We approach these questions below.

3.1 Sequential loser-elimination mechanism

In the previous section we examined whether the optimality of the grand contest remains valid if the simultaneous winner-selection mechanism is employed. Here, using a simple example, we repeat the analysis with the sequential loser-elimination mechanism (Chowdhury and Kim, 2014) that is prevalent in the field.

Suppose, as before, there are four contestants ($N = 4$) competing for two different prizes, 1 and 2 ($K = 2$), with values $v_1 \geq v_2$. As shown above, when two small contests are run, the total effort is

$$T^{small} = \max\left\{\frac{v_1 + v_2}{2}, \frac{3}{4}v_1\right\}.$$

When all the prizes and the players are put in one grand contest, according to the sequential loser-elimination mechanism, after two losers are eliminated, the third loser gets the second prize, and the survivor gets the first prize. Chowdhury and Kim (2014) show that if everyone but player i expends the same amount of effort x_{-i}, then the probability for i to win a prize under this mechanism is:

$$P_i^{LE}(x_i, x_{-i}) = \frac{(K-1)x_{-i} + x_i}{(N-1)x_{-i} + x_i} = \frac{x_{-i} + x_i}{3x_{-i} + x_i}$$

Thus, the expected payoff for player i is

$$\begin{aligned}\pi_i(x_i, x_{-i}) &= \frac{x_{-i} + x_i}{3x_{-i} + x_i}\left[v_2 + \frac{x_i}{x_{-i} + x_i}(v_1 - v_2)\right] - x_i \\ &= \frac{v_1 x_i + v_2 x_{-i}}{3x_{-i} + x_i} - x_i\end{aligned}$$

from which the symmetric equilibrium effort with the loser-elimination mechanism is derived as $(3v_1 - v_2)/16$, and the total effort as

$$T^{LE} = \frac{3v_1 - v_2}{4}$$

which is greater than T^{SM} and smaller than T^{small}.

3.2 Limit in the number of contests

Thus far, we have assumed that there is no additional cost for the designer to organize more contests, and showed that dividing a grand contest into smaller ones can increase the total effort. Let us now suppose that there exist operational costs for running these contests, which increases in the number of groups. Because of the costs, one cannot run more than two contests, i.e., $M \leq 2$.[5] An interesting question is what the optimal contest structure looks like when this constraint is imposed. In the following analysis, for the sake of simplicity we assume that the prizes are identical ($v_1 = v_2 = ... = v_K = v$), in which case the sequential loser-elimination mechanism is best-response equivalent to the simultaneous winner selection mechanism.

The following proposition, ignoring the integer problem, characterizes the optimal contest structure given the cost constraint.

Proposition 3 *If M cannot be greater than 2, then the total effort is maximized by* $\mathbf{C}$ *such that $k_1^* = 1$ and*

$$n_1^* = \begin{cases} \frac{N\left(\sqrt{K-1}-1\right)}{K-2} & \text{if } K > 2 \\ N/2 & \text{if } K = 2 \end{cases}.$$

[5]One may generalize this with a generic convex cost function that considers the number of contests. But, to provide with a simple and clear example, here we consider the case in which the cost is zero for up to two contests and then it becomes infinity.

Proof. Without loss of generality, suppose $n_1 \leq N/2$. Because the prizes are identical, the total effort can be written as

$$\begin{aligned} n_1 x_1 + n_2 x_2 &= v\left[\frac{n_1 - k_1}{n_1} + \frac{n_2 - k_2}{n_2}\right] \\ &= v\left[\frac{N-K}{N-n_1} + \frac{(n_1 - k_1)(N - 2n_1)}{n_1(N-n_1)}\right]. \end{aligned}$$

This shows that $n_1x_1+n_2x_2$ is maximized when k_1 is the minimized. Therefore $k_1^* = 1$. Note that if $k_1 = 0$, some players are thrown away, which is not optimal as shown in the previous section. Given $k_1^* = 1$, the expression is maximized at $n_1^* = N\left(\sqrt{K-1} - 1\right)/(K-2)$ which satisfies assumption $n_1 \leq N/2$ and converges to $N/2$ when K goes to 2. ■

This proposition shows that if the number of contests cannot be as big as the number of prizes, the total effort can be maximized by an asymmetric contest structure. Observe that $N\left(\sqrt{K-1} - 1\right)/(K-2)$ is larger than N/K, meaning that when $K > 2$, only a single prize is allocated in group 1, but there are disproportionately many contestants in the group. So, in the optimal structure, two small contests are organized. In one of these contests players face a fierce competition, while in the other, players compete in a more relaxed manner.

4 Discussion

In this study we reconsider design of multi-winner contests. Fu and Lu (2009) employ a sequential winner-selection mechanism and find that a grand contest always elicits higher equilibrium effort than a collection of sub-contests. We show that the result is completely reversed if a simultaneous winner-selection mechanism is implemented. This happens because the simultaneous winner-selection mechanism, unlike the sequential winner-selection mechanism suffers with the issue of free-riding. When a grand contest is implemented under such mechanism, the degree of free-riding increases resulting in lower effort level. Following similar logic, with a sequential loser-elimination mechanism, a grand contest elicits lower effort.

These results are of importance for several reasons. First, they show that the optimal design of a multi-winner contest depends crucially on the type of winner-selection mechanism.

Hence, depending on the objective of the designer, a combination of a winner-selection mechanism and a grand or sub contest should be employed.

Furthermore, given the number of players and the number of prizes, the above result together with Fu and Lu (2009) provides one with a clear ranking among the mechanisms as follows: *Grand contest with sequential winner selection* $>$ *Collection of sub-contests with sequential winner selection* $\geq$ *Collection of sub-contests with simultaneous winner selection* $>$ *Grand contest with simultaneous winner selection.*

If a contest designer faces an unconstrained choice of which mechanism to be employed, this ranking clearly shows that running a grand contest with sequential winner selection mechanism would be preferred - triumphing the 'Beauty of Bigness'. In the field, however, a simultaneous mechanism might already be in place and would be costly to replace. The current study prescribes that in such a case it is preferred to implement a collection of small sub-contests, as in terms of total equilibrium effort, they are 'Small, yet Beautiful'.

Moreover, this study also indicates that if it is possible to employ the loser-elimination mechanism, then it might elicit more effort than the simultaneous winner-selection mechanism, and can be a compromise if a sequential winner-selection mechanism cannot be employed.

Finally, it is well known that in the collective rent-seeking contests (*à la* Nitzan, 1991), a part of the prize is allocated according to the effort outlays and the rest is allocated randomly. Since that is also the case for both the simultaneous winner-selection and the sequential loser-elimination mechanisms multi-winner contests, the current result indicates that in such collective contests it might be possible to elicit higher effort by splitting the prize from a grand prize into several small prizes.

References

[1] Berry, S. K. (1993). Rent-seeking with multiple winners, *Public Choice*, 77, 437-443.

[2] Chowdhury, S. M., & Kim, S. H. (2014). A note on multi-winner contest mechanisms. *Economics Letters*, 125(3), 357-359.

[3] Chowdhury, S. M., & Kovenock, D. (2012). A combinatorial multiple winner contest with package designer preferences. Paper presented at the Coalition Theory Network Conference.

[4] Clark, D. & Riis, C. (1996). A multi-winner nested rent-seeking contest, *Public Choice*, 87, 177-184.

[5] Fu, Q., & Lu, J. (2009). The beauty of "bigness": On optimal design of multi-winner contests. *Games and Economic Behavior*, 66(1), 146-161.

[6] Moldovanu, B., & Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review*, 542-558.

[7] Moldovanu, B., & Sela, A. (2006). Contest architecture. *Journal of Economic Theory*, 126(1), 70-96.

[8] Nitzan, S. (1991). Collective rent dissipation. *Economic Journal*, 1522-1534.

[9] de Palma, A., & Munshi, S. (2013). A generalization of Berry's probability function. *Theoretical Economics Letters*, 3(05), 12.

[10] Tullock, G. (1980). Efficient Rent Seeking. In James M. Buchanan, Robert D. Tollison, Gordon Tullock, (Eds.), *Toward a theory of the rent-seeking society.* College Station, TX: Texas A&M University Press, pp. 97-112.