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Audit Market Measures in Audit Pricing Studies: The Issue of Mechanical Correlation

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ABSTRACT Mechanical correlation bias is inherent in audit pricing studies when independent variables (X) are derived from firm level audit fees (Y). Such variables are endogenous by construction leading to biased estimates, since (mechanically) X determines Y and Y determines X. After reviewing the extant auditing/accounting literature where mechanical correlation obtains we employ mathematical derivations and simulations to quantify the bias associated with a range of mechanically correlated market competition and industry specialist variables. Since auditor market competition variables are important to regulators and antitrust authorities, we analyze the mechanical correlation issue with regard to an extant study which introduces a novel measure of audit market competition (derived from audit fees). The study provides evidence that smaller incumbent auditors are pressured into offering lower fees when competing against a large local audit firm. However, when the current client's audit fee is 'decoupled' from this new competition measure to mitigate bias, it is statistically insignificant in our multivariate regression analysis. Additionally, we employ auditee sales and total assets to construct proxies for competition variables (which are not mechanically correlated) and find them to be statistically insignificant. We conclude with suggestions of how to address the issue of mechanical correlation in future studies.

Keywords: Audit pricing; Competition measures; Endogeneity; Mechanical correlation; Mathematical coupling; Proxy variables; Simulations

1. Introduction

Empirical questions related to how audit markets function, the pricing of auditors' industry specialization, and the impact of market competition on audit fees have been widely addressed in the audit pricing literature. Empirical evidence reported on these issues is important for companies seeking higher quality audits by industry specialists (Craswell et al., 1995; Ferguson & Stokes, 2002), whilst evidence on the effects of auditor market competition variables on audit pricing is

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of key import to regulators and antitrust authorities that seek to understand the implications of a market dominated by a few large audit firms (e.g. DeFond & Zhang, 2014; McKinnon, 2015). Auditor competition variables have been constructed to capture the nature of competition within various market boundaries/industry sectors and with reference to industry specialism (Ferguson et al., 2003; Francis et al., 2005). In modeling audit pricing, extant studies derive their measures of industry specialization and market competition from aggregations of firm level audit fees. These measures are endogenous due to mechanical correlation[1] between the dependent and independent variables, leading to biased estimates.

Mechanical correlation occurs where an explanatory variable (X) is derived from the dependent variable (Y), leading to X being endogenous by construction. More specifically, Y determines X since Y and X are related mathematically. As highlighted in the next section, in the accounting literature the classic (and intuitive) example of mechanical correlation is where Y and X share a common denominator. Mechanical correlation is present in accounting studies which employ mean values of the dependent variable to construct an explanatory variable to proxy for peer group effects. However, prior audit pricing studies have not sufficiently considered this form of mechanical endogeneity.

As described in Section 2, a number of empirical approaches have been advocated to alleviate the bias associated with mechanically correlated variables. These include estimation with the instrumental variable (IV) two-stage least squares approach (2SLS), using the lag of X in place of X, developing a proxy explanatory variable for X (in audit pricing research based on client size) and 'decoupling' (see below) the direct empirical link between the mechanically correlated dependent and independent variables.

After employing mathematical derivations to quantify mechanical correlation bias, we employ simulation analysis in an audit pricing setting, using a range of market share variables constructed from firm level audit fees. Inter alia, we find that (concordant with our mathematical derivations) the mean bias attributable to mechanical correlation is substantial and persists across all variables; but that bias reduces when employing company size-based proxy variables or decoupled ones. In accord with expectations, generally, the bias reduces inversely with market size (in terms of the number of observations in each market).

Employing contemporary data, we then examine mechanical correlation with reference to the recent empirical study of Chu et al. (2018). These authors develop a new market measure constructed from audit fees (DIFFERENCE), which is designed to capture auditor competition in local U.S. MSA-industry markets. Their empirical findings indicate that, to retain clients, smaller audit firms are pressured into lowering their audit fees to prevent clients from switching to a larger competitor within an MSA-industry market.

Our contribution is threefold. First, we highlight the important issue of endogeneity in extant audit pricing and other accounting studies where mechanically correlated variables are prevalent. Second, using mathematical derivations and simulations in a simplified setting, we assess the bias associated with variables that are mechanically correlated with audit fees, together with their decoupled and company size-based counterparts. Finally, we follow extant research by including alternative specifications for DIFFERENCE in our multivariate regression analysis. Specifically, we sever (decouple) the direct link between an observation's value in the explanatory variable and dependent variable and employ alternative client size-based measures which are not mechanically correlated with audit fees. Using decoupled and sized-based specifications, along with simulations analysis, enables an assessment to be made of the bias associated with

[1]Other terms used in the literature to describe the same phenomenon of mechanical correlation include mathematical coupling (in biomedical research) and endogenous by construction.

mechanically correlated variables in audit pricing studies. We provide the full Stata code and instructions that facilitates the replication of our analysis.

The remainder of this paper is organized as follows. Section 2 provides background information relating to our study, with reference to the issue of mechanical correlation in audit pricing research and the wider accounting and finance literature. The mathematical derivations and simulation results are presented in Section 3, followed by our empirical study in Section 4. The paper concludes in Section 5 with a brief discussion of the salient points to emerge from the analyzes.

2. Background and the Issue of Mechanical Correlation

In this section we describe the endogeneity bias caused by mechanical correlation and its implications for audit pricing studies. In addition, we highlight the different forms of mechanical correlation evident in the wider accounting and finance literature, together with the pros and cons of the methods used to address the consequential bias. We then review a specific stream of audit pricing literature where mechanical correlation is a potential issue. We conclude with a summary of the implications of mechanical correlation bias in audit pricing research and the suggested methods which can be used to address this bias.

2.1. The Problem of Mechanical Correlation

Mechanical correlation between a dependent and explanatory variable can be defined as a situation when the relationship between two variables is induced mechanically, such that the variables are related by a deterministic mathematical link; and at least part of their statistical relationship is an artefact of this link. With reference to medical studies, Archie (1981, p. 296) writes that 'mathematical coupling' (mechanical correlation) occurs when 'one variable either directly or indirectly contains the whole or components of the second variable.' Recognizing the existence of this phenomenon is important when making causal inferences. This is because if a dependent variable is mathematically coupled (mechanically correlated) with an explanatory variable and their relationship is modeled using regression or correlation analysis, biased variable parameters result (Archie, 1981).

In the standard regression case, if variation in the dependent variable (Y) leads to variation of an independent variable (X), then X will be endogenous due to 'reverse causation' (simultaneity) producing biased parameters estimated by the least squares (OLS) method (for accounting examples, see Adams & Ferreira, 2008 and Whisenant et al., 2003). Hence in the standard case, it is *likely* that X is endogenous due to reverse causation; whereas in the case where Y is used to create X, we know Y *must* determine X to some degree because of mechanical correlation.

The standard solution to ameliorate the endogeneity bias caused by mechanical correlation is by employing the IV 2SLS estimator. For instance, Biggar et al. (2018, p. 29) use the estimator when investigating the relationship between a mechanical correlated variable in the context of the pricing policies of utilities. However, as discussed below, it is well known in the accounting literature, that it is often problematic (if not impossible) to obtain a credible instrument (Larcker & Rusticus, 2010); and this especially holds in the presence of mechanical correlation.

2.2. Mechanical Correlation in Accounting Studies

The classical case of mechanical correlation is the common denominator specification, where the dependent variable and explanatory variables are expressed as ratios and share a common denominator, such that Y = A/B and X = C/B. It is interesting to note that, as early as the nineteenth century, Pearson (1896) first identified this form of mechanical correlation, which he referred to as 'spurious correlation'. Lev and Sunder (1979) were the first to highlight this issue in accounting research. More recently, Glasscock et al. (2021) stress that the use of variables with common denominators is still prevalent in the extant accounting literature, including in 'leading' accounting journals. Examples include regression estimates for the following dependent variables, where accounting ratio variables are mechanically correlated: corporate earnings (Sougiannis, 1994), stock market valuation (Han & Manry, 2004), future cash flows (Lee, 2011), market to book value ratio (Trueman et al., 2000), valuation models for internet firms (Keating et al., 2003) and corporate leverage (Sogorb-Mira, 2005).

Another type of mechanical correlation that has received a lot of attention in the literature is the relationship between the change in a variable computed as a difference between the follow-up (X_1) and the initial value (X_0), more specifically, the regression of the change ($Y = X_1 - X_0$) on the initial value (X_0). Tu and Gilthorpe (2012) provide an extensive review of the issue along with the solutions. However, we are not aware of this type of analysis being used in the field of accounting and finance.

A further form of mechanical correlation which occurs in the accounting and finance literature is where X is generated from average values of Y within specific groups. Here, each observation is allocated average values of Y for the group to which the observation belongs[2] and has a constant value for each observation within a group. This has been (incorrectly) used in accounting studies to control for fixed effects in regression models (Gormley & Matsa, 2014, p. 618). The average value of Y is often employed as the explanatory variable in accounting studies[3] that analyze the impact of 'peer' (group) effects on the dependent variable. For instance, Malik et al. (2019) examine the influence of peer groups (X) on Corporate Social Responsibility (CSR) expenditure (Y); where each peer group member (including the company in question) is allocated the average value of CSR expenditure (from Y) for that specific group (within X).

Further examples include Machokoto et al. (2022), who investigate whether a company's working capital management is influenced by its peers, Ouimet and Tate (2020), who examine peer effects in relation to individual investments and Matsunaga (1995) who analyze peer effects related to employee stock options. Wang et al. (2021) use the median value of Y with a view to mitigating the mechanical correlation associated with the mean of Y[4], with reference to peer effects and bank loans. Further, when investigating peer effects on corporate disclosure decisions, Seo (2021, p. 4) employs a decoupled version (below) of the average value of Y 'to avoid a mechanical correlation'. A similar approach is used by Machokoto et al. (2022, p. 8).

In the accounting and finance literature, the problem of mechanical correlation is addressed in a number of studies by severing the direct link between an observation's value in the explanatory variable and its value in the dependent variable. This is achieved by excluding from the calculation of the explanatory variable the value of the dependent variable, for each observation in turn. We term the procedure 'decoupling'. In this context, Durnev et al. (2003, p. 80) state that 'both the market return and broad industry return … are value-weighted averages excluding the firm in question. This exclusion prevents any spurious correlations between firm returns and industry returns in industries that contain few firms'. Moreover, Wahal and Yavuz (2013, p. 144) stress that 'we exclude stock i in calculating the return of the style portfolio to avoid any mechanical correlation between stock i and the style portfolio'. Further examples where decoupling is

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[2]Group averages (X) are clearly mechanically correlated, since variation in Y leads directly to variation in X.

[3]For literature reviews of peer effects on various corporate outcomes, see Seo (2021) and Machokoto et al. (2022).

[4]Given variation in Y should lead to some variation in the median of Y, this approach is unlikely to be bias free. Furthermore, the median has a different interpretation to the mean and hence may be unsuitable theoretically.

implemented, include Matsunaga (1995) who adopt this approach when estimating the value of employee stock options and Adrian and Brunnermeier (2016), who develop a new measure of systemic risk.

Even though the decoupling approach is widely employed, it is not a silver bullet for the mechanical correlation problem. Firstly, even if the decoupled variable could be justified by theory and be employed instead of the original mechanically correlated variable, though it is highly likely to reduce bias substantially, it may not eradicate it completely (Ouimet & Tate, 2020). As illustrated in Section 3, this is because variation in Y may still lead to at least some (residual) variation in decoupled X. Secondly, if on the basis of theory the hypothesized relationship for X is inappropriate for decoupled X, the estimated coefficient for the decoupled X will be biased to some degree due to measurement error.

In attempting to circumvent the endogeneity associated with explanatory variables in panel data, many accounting/finance studies (e.g. Wang, 2012) employ the lag of the variable of interest, in place of its contemporaneous value. Unfortunately, as demonstrated in detail by Reed (2015) and Bellemare et al. (2017), this invariably leads to the lagged variable being endogenous due to correlated error terms which will be the case even if the lagged variable is appropriate on the grounds of theory. If, on the other hand, a lagged structure implies a different causal (and theoretical) relationship, the estimate will be biased because of measurement error. In this context, all the audit pricing studies referenced in this paper specify a hypothesized (and related empirical) contemporaneous relationship.

2.3. Mechanical Correlation in Audit Pricing Research

The manifestation of mechanical correlation in extant (and potential future) audit pricing research, where measures of market competition and industry specialization are based on audit fees, is of high import. In this research, the incumbent auditor's market share (or its function) based on audit fees is used to explain audit fees. However, the mechanical correlation and related issues this presents have not been fully addressed. For instance, many studies have examined whether various concentration measures (including the Herfindahl-Hirschman index) which are derived from aggregations of audit fees, are associated with price premiums (for literature reviews see LSE Enterprise, 2008; and Huang et al., 2016). Examples of market concentration studies include Oxera (2006), LSE Enterprise (2008), Kallapur et al. (2010), Huang et al. (2016) and Eshelman and Lawson (2017).

In a similar vein, a growing number of studies have investigated whether auditor industry specialist variables (based on audit fees) are related to audit fees (for literature reviews see Chu et al., 2018 and Bae et al., 2019). Examples include Dutillieux and Willekens (2009), who examine whether auditor industry specialists earn fee premiums; Bills et al. (2015), who investigate whether auditor specialists in local and national markets are rewarded with fee premiums and Chang et al. (2022), who explore how client product similarity influence audit pricing decisions of industry specialists. In all of these studies, mechanical correlation will result in biased estimates for the variable of interest (and potentially other variables as well), since the measures of auditor specialization and market concentration are based on audit fees.

Some audit pricing studies (Carson, 2009; Francis et al., 2005; Gunn et al., 2019; Minutti-Meza, 2013), use (proxy) client size-based (sales or total assets) variables in place of those based on audit fees. This appears to be an attractive approach since such measures are not mechanically correlated with audit fees. However, size proxies are subject to similar limitations as those pertaining to the decoupling method described above. This will be the case if the theoretical expectations (and causal inferences) relating to explanatory variables derived from audit fees differ from (or are inappropriate for) those obtaining to size-based proxy variables. For instance, when computing industry specialization variables, it has been stated that audit fees capture auditor effort better than client size proxies (Audousset-Coulier et al., 2016).[5] Specifically, relative to the 'true' value of (albeit endogenous) market variables derived from audit fees, proxy ones based on client size are subject to measurement error bias.

Importantly, mechanical correlation is likely to have a larger impact in smaller groups, or when an individual observation is influential. This latter point was made by López-Espinosa et al. (2012, p. 3153) who employ the decoupling procedure to 'prevent' mechanical correlation 'not only when the total number of institutions *n* in the sample is not particularly large, but also when a single institution has a significant weight in relation to the whole system even if *n* is fairly large.' A similar point is made by Minutti-Meza (2013, footnote 24), who states that 'using fees to calculate market share may be problematic in cases where the number of companies in a city–industry combination is small'. Ceteris paribus, bias is likely to be more substantial in smaller market groups, because the weight attached to an individual auditor in a variable's formulation diminishes as market size increases.

In this context, Francis et al. (2005) posit that U.S. auditors compete in relatively small audit markets determined by geographical metropolitan statistical areas (MSA) and 2-digit SIC codes. They found that fees are significantly higher if a Big 5 auditor is jointly the city-level industry leader and the national industry leader. Furthermore, in terms of MSA-industry market share based on audit fees, Numan and Willekens (2012) employ a new variable (DISTANCE) and find that Big 4 auditors increase fees as the distance from their closest competitor lengthens. These authors also include a further variable (industry portfolio share), which is derived from audit fees, in their regression model.[6] Finally, Chu et al. (2018) develop a new market measure constructed from audit fees (DIFFERENCE), which is designed to capture auditor competition in local MSA-industry markets. Consistent with the hypothesized relationship, Chu et al.'s (2018) empirical findings indicate that, to retain clients, smaller audit firms are pressured into lowering their audit fees to prevent these clients from switching to a larger competitor within an MSA-industry market.

2.4. Summary and Implications

The mechanical correlation (endogeneity) bias associated with audit market measures employed in audit pricing studies is likely substantial, given all their elements are comprised of audit fees. Put another way, we cannot say (statistically) that their OLS parameter estimates support hypothesized relationships. The standard approach to 'identify' the 'true' (unbiased) parameters for an endogenous variable (X) is via IV 2SLS estimators. However, locating an appropriate IV - which is (not weakly) correlated with X, but which is uncorrelated with Y, other than via its relationship with X - has proved exacting (if not impracticable) in accounting studies (Larcker & Rusticus, 2010). In the absence of a valid instrument, although not being definitive, we advocate using decoupled and size-based variables as sensitivity measures to gauge the potential bias associated with the original variable.

However, on the assumption that these alternative measures are not theoretically appropriate, we stress that they are subject to limitations. Specifically, neither measure identifies the 'true causal effect', as does the IV estimator if implemented correctly. In the case of client size-based

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[5] 'According to previous theoretical arguments, we could expect that the use of audit fees as a calculation variable to compute ISP (industry specialization) measures could capture auditor effort better than client assets or sales, because audit fees are a function of the client size, complexity, and riskiness, while sales and assets are simple client-size measures that are not as closely associated with audit effort.' (Audousset-Coulier et al., 2016, p. 146). We are grateful to the anonymous referee who brought this point to our attention.

[6] The inclusion of more than one mechanically correlated variable may also lead to multicollinearity issues.

proxies, auditee sales/total assets are substituted for their audit fee components. Clearly, proxy size variables do not suffer from mechanical correlation; but they are subject to measurement error bias. The higher the correlation between the original variable and its size-based proxy, the lower will be the bias.

In the case of decoupled variables, the aim is to eradicate mechanical correlation. However, unlike client size proxies, the formulation of decoupled market measures differs from the original construct (variable), in that the audit fee for each auditor is excluded in turn in its computation. Again, this clearly leads to measurement error. As previously discussed, we expect mechanical correlation of the original market measures to be more pronounced in smaller market groups. However, it may be that the measurement error associated with decoupled variables is more prominent in smaller market groups.

Trying to fill the gap in the empirical audit pricing literature, our study explores the bias associated with mechanically correlated variables derived from market shares that have been developed or employed in very small markets where the bias due to mechanical correlation is likely to be substantial. Assuming that the decoupled and size-based measures are not justified by economic theory, we will explore the extent of the bias due to measurement error when the decoupled and size-based measures are employed instead of the original ones.

3. Mathematical and Simulation Analysis

In this section, we derive the bias emanating from a group average variable (see Section 2), along with its decoupled and size-based counterparts. Employing simulations in an audit pricing framework, we then analyze the mechanical correlation bias associated with a range of market measures, relative to the measurement error bias of their decoupled and size-based alternatives. As stressed above, we assume that the latter alternative measures (constructs) may not be justified by economic theory and hence are subject to measurement error.

3.1. Deriving the Bias of a Mechanically Correlated Variable based on Group Sums

As described previously, mechanically correlated measures based on group sums of the dependent variables are employed in auditing and accounting research as independent variables. In this section, we derive the bias emanating from the simplest function of the group sum, the group average. We examine the performance (biases) associated with decoupled and size-based measures as described above. Note that, unlike market share variables (the ratio of two group sums), the analysis of the group average is mathematically tractable.[7] Our analysis is based on Gormley and Matsa (2014) who analyze the bias arising from the use of group averages of the dependent variable when attempting to eradicate time invariant heterogeneity (fixed effects). Employing the mathematical framework of Gormley and Matsa (2014), we specify the following structure:

$$y_{i,j} = \alpha x_{i,j} + \beta GA_{i,j} + u_{i,j} \tag{1}$$

$$\text{var}(u_{i,j}) = \sigma_u^2, \mu_u = 0$$

$$\text{var}(x_{i,j}) = \sigma_x^2, \mu_x = 0$$

$$\text{cov}(x_{i,j}, u_{i,j}) = \text{cov}(x_{i,j}, u_{i,-j}) = 0$$

Following Gormley and Matsa (2014), we assume that there is a random sample of N groups, the group size k is small, and the number of groups N is large. The first index, i indexes groups,

[7] We are grateful to the Editor for raising this issue.

and runs from 1 to N; whereas the second index, j indexes observations within each group, and runs from 1 to k. u is the random error term with zero mean and constant variance, possibly correlated within groups but independent and identically distributed (i.i.d.) across groups, and x is the additional explanatory variable.[8] Similar to the random error, we assume that its variance is constant; and that the values can be correlated within groups but are i.i.d. across groups. The random error u and the explanatory variable x are assumed to be uncorrelated. As per Gormley and Matsa (2014), we assume the explanatory variable x has zero mean and the intercept in equation (1) is zero to simplify the analysis. The formula for group average ($GA_{i,j}$) is given by:

$$GA_{i,j} = \frac{1}{k}\sum_{g=1}^{k} y_{i,g}$$

Hence equation (1) takes the following form:

$$y_{i,j} = \alpha x_{i,j} + \beta \frac{1}{k}\sum_{g=1}^{k} y_{i,g} + u_{i,j} \tag{2}$$

It is easy to see in equation (2) that the group average $GA_{i,j}$ and $y_{i,j}$ are mechanically correlated, since the group average contains $y_{i,j}$. Thus, variation in $y_{i,j}$ clearly leads to variation in the group average and both the group average and the dependent variable $y_{i,j}$ are determined simultaneously. Consequently, the OLS estimator of the coefficient β will be biased. Note that conceptually (behaviorally), we are not suggesting that there is a continuous deterministic looping relationship between Y and GA, which does not occur in practice. Rather, as per the standard omitted variable issue, we derive the bias associated with the (endogenous) coefficient of GA, viewed as arising from mechanical correlation (GA being correlated with the regression error term). Conceptually, this can be viewed as peer group effects (GA) on audit fees (Y), where Y is in levels, rather than logarithms.

It is instructive to express the group average and the dependent variable as a function of the explanatory variable x and the random error u. As illustrated in Appendix 1 in the online supplementary materials, if the absolute value of the coefficient β is less than unity ($|\beta| < 1$) then it can be shown that:

$$y_{i,j} = \alpha x_{i,j} + \frac{\alpha\beta}{(1-\beta)}\frac{1}{k}\sum_{g=1}^{k} x_{i,g} + \frac{\beta}{(1-\beta)}\frac{1}{k}\sum_{g=1}^{k} u_{i,g} + u_{i,j} \tag{3}$$

And the group average is equal to:

$$GA_{i,j} = \frac{1}{k}\sum_{g=1}^{k} y_{i,g} = \frac{\alpha}{(1-\beta)}\frac{1}{k}\sum_{g=1}^{k} x_{i,g} + \frac{1}{(1-\beta)}\frac{1}{k}\sum_{g=1}^{k} u_{i,g} \tag{4}$$

The bias of the OLS estimator of the group average coefficient is caused by its correlation with the contemporaneous error term $u_{i,j}$ and the correlation is apparent from equation (4).

[8]Even though the additional explanatory variable x makes the analysis more complicated, it facilitates the analysis of the OLS estimator for the coefficient β in equations (1) or (2), and the corresponding estimators when the decoupled group average or size-based group average are employed instead of the group average, all within a single framework.

The OLS estimator will yield upwardly biased and inconsistent estimates of the parameter β[9] (see Appendix 2 of the online supplementary materials for the derivation):

$$\hat{\beta}^{GA} = \beta + \frac{[1+(k-1)\rho_{u_j,u_{-j}}](1-\beta)}{\alpha^2 \frac{\sigma_x^2}{\sigma_u^2} \frac{k-1}{k}[1+(k-1)\rho_{x_j,x_{-j}}](1-\rho_{x_j,x_{-j}}) + [1+(k-1)\rho_{u_j,u_{-j}}]} \tag{5}$$

As discussed in Section 2, if the explanatory variable is based on group sums of a dependent variable, one of the most widely used methods to address the mechanical correlation is decoupling. In case of the group average, if we exclude the contemporaneous value of the dependent variable y for each observation in turn in the explanatory variable (the deterministic link between the dependent variable and the group average), we get the decoupled group average (*GADC*):

$$GADC_{i,j} = \frac{1}{k-1} \sum_{g=1, g\neq j}^{k} y_{i,g} \tag{6}$$

Obviously, the decoupled group average measures the original group average with error; and hence induces measurement error bias. Moreover, and consistent with expectations (above), the decoupled group average exhibits some residual mechanical correlation with the contemporaneous error term. To see this, we need to substitute the true expression for $y_{i,j}$ from equation (2) into formula (6):

$$GADC_{i,j} = \alpha \frac{1}{k-1} \sum_{g=1, g\neq j}^{k} x_{i,g} + \frac{\alpha\beta}{(1-\beta)} \frac{1}{k} \sum_{g=1}^{k} x_{i,g} + \frac{\beta}{(1-\beta)} \frac{1}{k} \sum_{g=1}^{k} u_{i,g} + \frac{1}{k-1} \sum_{g=1, g\neq j}^{k} u_{i,g} \tag{7}$$

Hence, like the group average, the decoupled group average also covaries with the error term, although the covariance is smaller.[10] Thus, the OLS estimator will be biased even in the absence of measurement error. Assuming the data-generating process is given by equation (1), equation (8) reveals that the formula for the decoupled estimator is relatively complicated and in general the estimator is biased and inconsistent[11] (see Appendix 3 of the online supplementary materials for derivations):

$$\hat{\beta}^{GADC} = \beta + \frac{\frac{-\alpha^2\beta(k-\beta)}{k(k-1)} \frac{\sigma_x^2}{\sigma_u^2} + \frac{\alpha^2\beta(-k^2+2k+k\beta-2\beta)}{k(k-1)} \frac{\sigma_x^2}{\sigma_u^2} \rho_{x_j,x_{-j}} + \frac{\alpha^2\beta(k-\beta)}{k} \frac{\sigma_x^2}{\sigma_u^2} (\rho_{x_j,x_{-j}})^2 + \frac{\beta(k+\beta-2)}{k-1} + \frac{k^2-k-k\beta-\beta^2+2\beta}{k-1} \rho_{u_j,u_{-j}}}{\frac{1}{1-\beta}\left[\frac{\alpha^2(k-\beta)^2}{k(k-1)} \frac{\sigma_x^2}{\sigma_u^2} + \frac{\alpha^2(k-2)(k-\beta)^2}{k(k-1)} \frac{\sigma_x^2}{\sigma_u^2} \rho_{x_j,x_{-j}} - \frac{\alpha^2(k-\beta)^2}{k} \frac{\sigma_x^2}{\sigma_u^2} (\rho_{x_j,x_{-j}})^2 + \frac{(\beta^2-2\beta+k)}{k-1} + \frac{k^2-2k-\beta^2+2\beta}{k-1} \rho_{u_j,u_{-j}}\right]} \tag{8}$$

As discussed above, to avoid mechanical correlation in the audit pricing studies, client size (total assets or sales) was employed instead of audit fee in formulas for market shares. Consequently, the size-based market share measures were used instead of the original ones to make inference

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[9]Even though it is not the main focus of this study, the OLS estimate of the coefficient α is biased towards zero (see Appendix 2 of the online supplementary materials).

[10]As shown in Appendix 1 of the online supplementary materials, the term standing in front of the contemporaneous error term in equation (7) is $\frac{\beta}{(1-\beta)k}$ and since $|\beta| < 1$, this is smaller than the term $\frac{1}{(1-\beta)k}$ in equation (4).

[11]In a similar vein, the decoupled estimator of the coefficient α is biased if the true value of the coefficient is different from zero (see Appendix 3 of the online supplementary materials for details).

about the original parameters using OLS[12] as substitutes. The size-based group average (GAX) has the following formula:

$$GAX_{i,j} = \frac{1}{k}\sum_{g=1}^{k} x_{i,g} \tag{9}$$

Even though it is clearly free from correlation with the error term, it quantifies the original group average with imprecision; and so its estimated parameter will be biased due to measurement error. It turns out that the size-based estimator for the coefficient β is indeed biased and inconsistent, whenever the term $\frac{\alpha}{1-\beta}$ is different from unity (i.e. $\alpha+\beta=1$)[13], or the true value of the coefficient β is different from zero (see Appendix 4 of the online supplementary materials for derivations), so that:

$$\hat{\beta}^{GAX} = \frac{\alpha}{1-\beta}\beta = \beta + \frac{\beta(\alpha+\beta-1)}{1-\beta} \tag{10}$$

Looking at the above formulas, under general conditions, inferences relating to any of these variables – the original, decoupled or size-based group average – are biased. However, the estimated parameters for the decoupled and size-based constructs are likely to be closer to the true values compared to those obtained using the original mechanically correlated variables. Having derived these formulas, we now explore the comparative bias of the three estimators using simple charts.

As our derivations show, the estimators depend on six parameters: the true value of the coefficient α, the true value of the coefficient β, group size (k), the ratio of variance x to variance u (σ_x^2/σ_u^2), the within-group correlation of variable x ($\rho_{x_j,x_{-j}}$), and the within-group correlation of the random error u ($\rho_{u_j,u_{-j}}$). As displayed graphically in Figure 1, we vary these parameters one at a time to demonstrate how the bias changes for the group average and its decoupled and size-based counterparts. We use the following default values: $\alpha=0.5$, $\beta=0.1$, $k=4$, $\sigma_x^2/\sigma_u^2=25$, $\rho_{x_j,x_{-j}}=0.3$ and $\rho_{u_j,u_{-j}}=0.1$.[14] The charts are presented in Figure 1.

In accord with expectations (see Section 2), Panel A in Figure 1 reveals that the variation in group size (k) has a large impact on the bias associated with the original group average variable. Specifically, if the group size is small, the original group average estimator exhibits substantial positive bias and deviates the most from the true value of the β coefficient. In contrast, the decoupled estimator, though also exhibiting positive bias, is relatively close to the true value of β, though the biases converge as the group size increases.[15] As shown in Panel A, the size-based estimator is unaffected by the group size because it depends solely on the true values of parameters α and β. It exhibits negative bias since the sum of the default true values of the parameters α and β is less than unity (see equation (10)). Importantly, in absolute terms, for given default true values of the coefficients α and β, the bias associated with the original estimator is substantially larger than that obtaining to its size-based counterpart.

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[12] In the following text, we term this situation 'size-based' estimator.

[13] A potential reason why the estimator for the coefficient β is the $\frac{\alpha}{1-\beta}$ multiple of its true value (see equation (10)), could lie in the fact that the group average for the variable x appears in formula (4) for the normal group average multiplied by the term $\frac{\alpha}{1-\beta}$. Secondly, and quite interestingly, the estimator for the coefficient α is unbiased (see Appendix 4 of the online materials for derivations).

[14] Our choice of the default values reflects (to a large extent) the values in the dataset used in our empirical study (see section 4). However, we have prepared an interactive tool where an interested reader can experiment with various values of the six underlying parameters and observe how the relative performance of the estimators change (see Appendix D for details).

[15] In unreported analysis we found that, if the within-group correlation of the random error u is zero, both estimators converge to the true β value, so their biases decrease with growing group size. The interested reader can verify this using the interactive tool that we provide (see Appendix D for details).

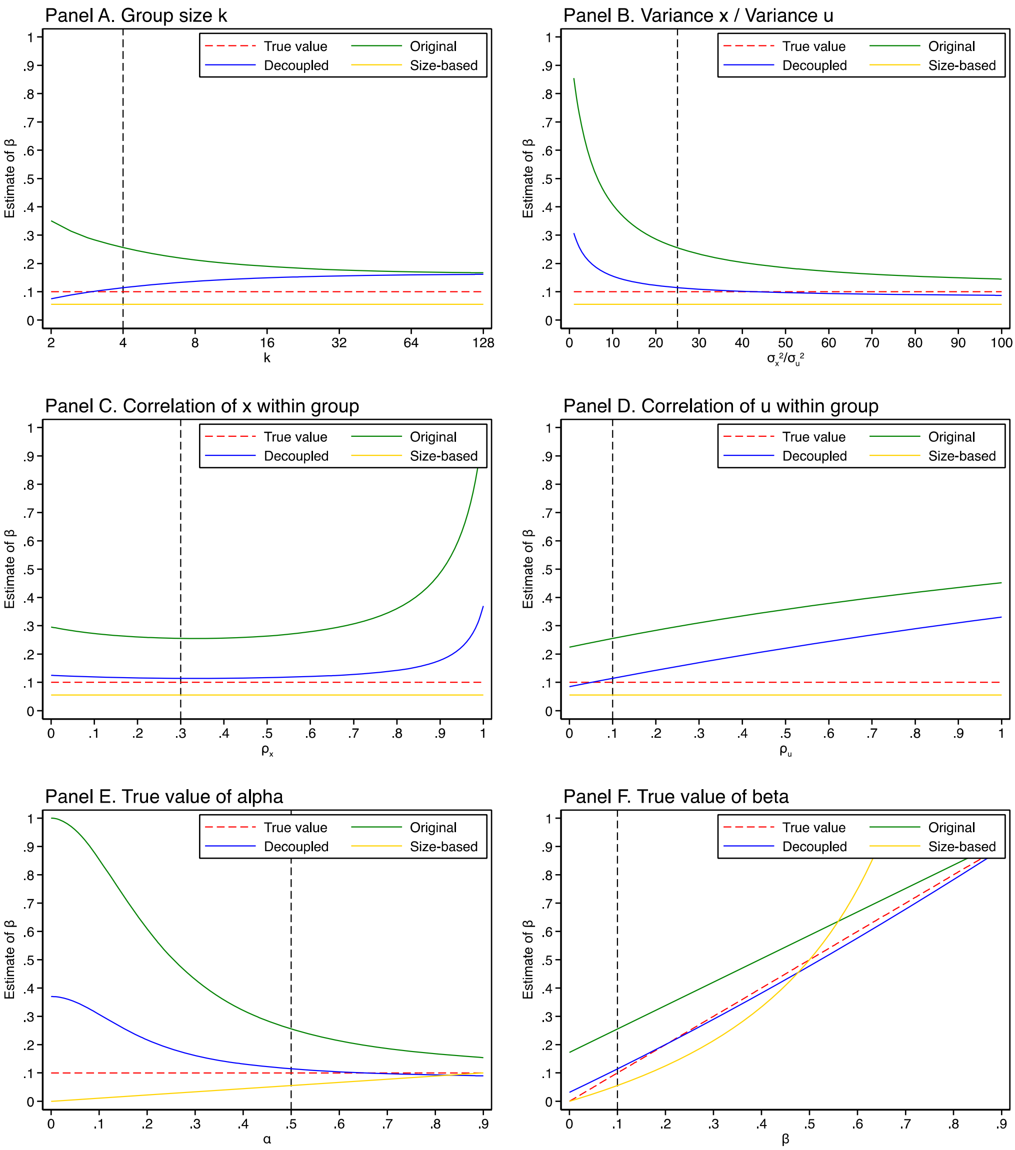

Figure 1. Comparative statics of the bias for original (mechanically correlated), decoupled and size-based group average. This figure shows the analytical solutions for the mean estimated values of the parameter beta from the equation $y_{i,j} = \alpha x_{i,j} + \beta GA_{i,j} + u_{i,j}$ (equation (1)) for the original group average (GA); and where the decoupled group average or the x-based (size-based) group average are employed in place of GA. The estimates are functions of six parameters: group size k (default value 4), the ratio of variances of the variable x and the error term u (default value 25), the within-group correlation of the variable x (default value 0.3), the within-group correlation of the error term u (default value 0.1), the true value of the parameter alpha (default value 0.5) and the true value of the parameter beta (default value 0.1). The individual panels show how the estimated values change by varying a specific parameter value while holding the remaining parameters constant at their default values. In each chart, the vertical dashed line indicates the default parameter value.

As displayed in Panel B, the relative variation of variable *x* and the error term *u* affects the original and decoupled estimators adversely when the proportion of variances is small. Then, both the original and the decoupled estimators are biased but the value of the decoupled estimator is much closer to the true value, especially when the ratio of variances exceeds 20. Again, the size-based estimator is unaffected by changes in the relative variation of x and u.

Panel C reveals that the degree of within-group correlation of x affects the bias of the original and the decoupled estimators adversely when the degree of correlation is relatively high (greater than 0.7). Nevertheless, the bias of the decoupled estimator is much smaller than the original estimator. On the other hand, (as shown in Panel D), the within-group correlation of the random error u affects the bias of both estimators adversely across the whole range of values. The increase in bias is monotone for both estimators and again the decoupled estimator exhibits less bias than the original estimator. The bias of the size-based estimator is unaffected by the within group correlation of either variable x or the random error u, it is constant and remains negative.

The bias of all three estimators is affected by the true values of the parameters α and β as shown in panels E and F respectively. Panel E reveals that the bias associated with the three estimators decreases as the true value of α increases. In addition, all three estimators converge to the true value of β as α increases; but while the original and the decoupled estimators converge to the true value from above (their bias is positive), the size-based estimator converges from below (its bias is negative). Panel E also shows that the absolute value of bias for both the decoupled and the size-based estimators is smaller than the absolute value of bias of the original group average.

As far as the dependence of the estimators on the true value of β is concerned, Panel F reveals that the larger is its true value, the closer the original estimator is to it. The decoupled estimator, though exhibiting slight non-linearity, is relatively close to the true value across all range of values. Again, the decoupled estimator is significantly less biased than the original one. The behavior of the size-based estimator is more complex, in that its bias is highly non-linear (curvy). Specifically, it is close to the true value if β is very small, exhibits the negative bias for values below 0.5, but then the bias flips sign once the true value of β reaches 0.5. As the true value of β increases further, the value of the estimator explodes. As shown in equation (10), this is because the denominator of the bias moves closer to zero.

In summary, for a wide variety of scenarios, the analysis demonstrates that the bias of the decoupled and size-based variables is substantially smaller than that of the original construct. Hence these findings provide some support for the use of size-based and decoupled variables as sensitivity measures in empirical research to deal with the problem of mechanical correlation. Second, the bias of the original estimator is at its largest for the smallest group size. Third, the decoupled estimator is consistently closer to the true value than the original estimator. Finally, the size-based estimator is only affected by the true values of parameters α and β, with the sign of the bias being determined by whether their sum is greater or smaller than unity.

3.2. Simulation Analysis

We now turn our attention to mechanically correlated variables based on market shares that are employed in audit pricing research, which contain smaller market groups, as described previously. These are RATIO, LEADER, DISTANCE and DIFFERENCE.[16] The first variable, RATIO, is a measure of the market share of an incumbent auditor within a given group. It is defined as the total value of *Audit_fee* attributed to *Auditor_id* within a group, divided by the total *Audit_fee* for that group. The second measure, LEADER, follows Francis et al. (2005) by

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[16]We have not included the well-known concentration measure HHI in this analysis because it differs from the analyzed measures in several important respects. Firstly, HHI is the same for all auditors in an audit market, whereas the analyzed measures usually differ for each auditor. Secondly, unlike the analyzed measures, the squared value of market share is employed in its formulation. Thirdly, we are unaware of any study that employs HHI as the main independent variable within the framework of small MSA-industry markets.

generating a binary variable which indicates the *Auditor_id* with the highest market share in each group.[17] The third measure, DISTANCE, follows Numan and Willekens (2012) by quantifying the smallest absolute difference between the *Audit_fee* market share attributed to *Auditor_id* and the *Audit_fee* market share attributed to its closest competitor within each group. Finally, we follow Chu et al. (2018) and compute DIFFERENCE as the difference between the market share of the largest *Auditor_id* based on *Audit_fee* and the market share of *Auditor_id* within each group. The full formulas for all measures along with numeric examples are presented in Appendix A.

As noted above, given they are expressed as a ratio of two group sums, these market measures are more complex than a simple group average. At the same time, in audit pricing models, the dependent variable (along with the auditee size one) is usually in the form of natural logs. Finally, there are maximum, minimum, or absolute value operators in the formulas of some of these variables. These features make mathematical derivations equivalent to those of the group average unrealistic and even intractable.

However, though the mathematical derivations may be unfeasible, random simulations can be employed to explore the bias associated with employing the mechanically correlated measures that are functions of market share, in the context of an audit pricing model. Using randomly generated data, we demonstrate that mean bias of the mechanically correlated market share variables is substantial and persistent, especially in small groups - where it is easy to obtain statistically significant results even if the true effect (coefficient) is zero. We also show that the decoupled and size-based measures do not suffer from the latter. Moreover, we find that these alternative measures are closer to the true value when the true value is different from zero.

We simulate the following data generating process:

$$\ln(y_{i,j}) = \alpha \ln(x_{i,j}) + \beta\, MEASURE_{i,j} + u_{i,j} \tag{11}$$

Where $\ln(y)$ is the natural log of audit fees, $\ln(x)$ is a client size variable[18], *MEASURE* is one of four market variables described above and u is the error term. Firstly, when the true value of the coefficient β is zero, we explore the bias associated with original, decoupled and size-based market variables for increasing group sizes. To this end, in a fixed group structure with group sizes ranging in multiples of 2 from 4 to 128 observations per group[19], we randomly generate a dependent variable $\ln(y_i)$ and a subgroup indicator (*Auditor_id*) to compute the market share measures for each group..[20] In this part, we generate simulated samples where the true effect (coefficient) of the original variable is zero[21], so that values for y would be computed only as a function of the explanatory variable x and the error term u:

$$\ln(y_{i,j}) = \alpha \ln(x_{i,j}) + u_{i,j} \tag{12}$$

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[17]In unreported analyzes, we also used the variable SPECIALIST - indicating an auditor with market share greater than 10%, 20% or 30%, following the definitions used in Craswell et al. (1995) and Numan and Willekens (2012); and the results were consistent with those for LEADER.

[18]We use both variables x and y in the form of natural logs, because (as is standard) this specification was employed in the studies of Francis et al. (2005), Numan and Willekens (2012) and Chu et al. (2018).

[19]We provide simulations across this wide range of group sizes, to examine bias in a number of market scenarios. Inter alia, it enables an assessment to be made of whether (as predicted) bias is lower in larger markets.

[20]This is repeated 1000 times for each group size, assuming there are at most 4 subgroups (auditors) in each group. The setup reflects the situation where Big 4 auditors compete for clients. The detailed description of the simulation framework is presented in Appendix B.

[21]Our procedure is in principle similar to the famous Granger and Newbold (1974) study; in that the data are simulated with no effect and hence the OLS estimate equals the bias.

Hence, to assess the bias associated with the market variables, we estimate the OLS regression parameters for the following model[22]:

$$\ln(y_{i,j}) = \mu + \alpha \ln(x_{i,j}) + \beta\ MEASURE_{i,j} + u_{i,j} \tag{13}$$

and observe the coefficient β and its standard error; noting again, that for an unbiased estimator, the mean estimated coefficient (β) would be zero.

Panels A to D in Table 1 present the simulation results. Each panel presents the statistics for the estimates of the coefficient β by group size for each of the four measures described above. The statistics include the mean bias[23] in the coefficient β and the percentage of simulations where the estimated coefficient β was positive or negative and statistically significant at the 5% significance level. We first examine the ratio of group sums (RATIO). These groups are usually nested whereby the denominator is the sum of all fees for the whole audit market, while the numerator is the sum of audit fees collected by the incumbent auditor in that same audit market. In audit pricing studies, focused on auditor specialization or competition, the variable corresponds to the market share of the incumbent auditor or share of its returns generated in a specific group. Even though the market share itself is rarely used as a standalone explanatory variable, the share of the incumbent auditor's returns generated in a specific group has been employed.[24] Nevertheless, it is instructive to explore its bias, given it is an important component of other relevant market measures.

As shown in Table 1, all four original market measures are subject to substantial bias. Consistent with our derivations for the group average variable (above), the degree of bias is most pronounced in smaller groups; but dissipates as group size increases (as do percentages of samples where the coefficients are statistically significant). In contrast, the bias associated with the decoupled and size-based variables exhibits bias near to zero for all group sizes.

More specifically, Panel A shows that the mean bias of RATIO decreases monotonically with group size, ranging from 0.323 in the smallest group (n = 4) to 0.257 in the largest one (n = 128). Even though the true effect of RATIO is zero, its OLS coefficient is positive and statistically significant in all simulated samples up to and including where the group size is 16. Moreover, for the largest group, it is positive and statistically significant in 64.8% of the random samples. On the other hand, the mean bias associated with the decoupled and size-based measures is close to zero for all group sizes; and the percentage of random samples where the estimated coefficients are statistically significant fluctuates around 5%.

Panel B reveals that the OLS coefficient of LEADER gradually decreases from 0.155 for group size 4 down to 0.028 for group size 128. Reflecting this, its coefficient is statistically significant in 100% of the simulated samples for the smallest group, falling to 35.1% for the largest one. For both the decoupled and size-based versions of LEADER, their bias is again very close to zero and the percentage of random samples where the estimated coefficients are statistically significant ranges from 3.3% to 7.5%.

However, against the trend, Panel C reveals that mean bias for DISTANCE initially increases with the group size, reaches a maximum for group size 16 and then decreases as group size increases. For group sizes from 4 to 16, the estimated coefficient for DISTANCE is positive and statistically significant in more than 92% of the simulated samples. For the largest

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[22] As noted in tables 1 and 2, constant terms are included in the model, but their parameters are unreported for brevity.

[23] The mean bias is computed as a difference between the mean estimated coefficient in 1000 random samples and its true value.

[24] For instance, Numan and Willekens (2012) used industry portfolio share - computed as the ratio of returns of the incumbent auditor generated in an MSA-industry to the returns generated in the MSA as one of their independent variables of interest. Therefore, the analysis of RATIO is informative regarding industry portfolio share variable.

group, it is positive and statistically significant in only 26.9% of them. As with the other variables, the bias associated with decoupled and size-based measures is close to zero for all group sizes. For both these variables, the percentage of random samples when the coefficients are statistically significant increases with group size, ranging from about 6% to 30%; and is

Table 1. Estimation results using simulated data for group sizes.

Panel A: RATIO							
Group size (k)		4	8	16	32	64	128
Original	Mean bias in β	0.323	0.318	0.303	0.283	0.271	0.257
	β is positive & significant*	100.0%	100.0%	100.0%	99.8%	92.1%	64.8%
	β is negative & significant**	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Decoupled	Mean bias in β	– 0.001	– 0.001	– 0.000	– 0.002	– 0.001	– 0.008
	β is positive & significant*	2.3%	3.0%	3.0%	2.4%	2.6%	3.1%
	β is negative & significant**	3.6%	3.3%	3.2%	3.2%	2.6%	3.2%
Size-based	Mean bias in β	– 0.000	– 0.001	– 0.000	– 0.000	– 0.000	– 0.002
	β is positive & significant*	1.8%	2.2%	2.1%	2.1%	2.8%	2.1%
	β is negative & significant**	4.1%	2.6%	2.2%	2.7%	3.0%	3.0%
Panel B: LEADER							
Group size (k)		4	8	16	32	64	128
Original	Mean bias in β	0.155	0.114	0.082	0.057	0.040	0.028
	β is positive & significant*	100.0%	100.0%	99.9%	92.0%	65.9%	35.1%
	β is negative & significant**	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Decoupled	Mean bias in β	– 0.001	– 0.000	– 0.000	0.000	– 0.000	0.000
	β is positive & significant*	2.7%	2.6%	3.0%	2.6%	3.0%	3.7%
	β is negative & significant**	3.4%	2.8%	4.5%	2.4%	3.7%	3.7%
Size-based	Mean bias in β	0.000	– 0.000	– 0.000	– 0.000	– 0.000	– 0.001
	β is positive & significant*	2.0%	2.6%	1.5%	1.6%	1.9%	2.7%
	β is negative & significant**	2.4%	2.7%	3.1%	1.7%	2.0%	2.6%
Panel C: DISTANCE							
Group size (k)		4	8	16	32	64	128
Original	Mean bias in β	0.125	0.208	0.217	0.185	0.159	0.120
	β is positive & significant*	98.0%	99.7%	92.6%	55.1%	34.3%	26.9%
	β is negative & significant**	0.0%	0.0%	0.0%	0.2%	2.0%	9.9%
Decoupled	Mean bias in β	– 0.001	– 0.000	– 0.001	– 0.006	0.006	– 0.007
	β is positive & significant*	3.1%	4.6%	4.9%	5.7%	11.2%	13.8%
	β is negative & significant**	3.7%	4.8%	4.8%	5.7%	9.6%	16.3%
Size-based	Mean bias in β	– 0.001	0.000	0.000	– 0.000	– 0.002	0.002
	β is positive & significant*	3.1%	3.2%	3.7%	4.5%	6.3%	12.9%
	β is negative & significant**	4.1%	3.0%	4.4%	6.2%	7.0%	11.6%
Panel D: DIFFERENCE							
Group size (k)		4	8	16	32	64	128
Original	Mean bias in β	– 0.340	– 0.285	– 0.244	– 0.215	– 0.197	– 0.186
	β is positive & significant*	0.0%	0.0%	0.0%	0.0%	0.3%	3.2%
	β is negative & significant**	100.0%	100.0%	100.0%	95.3%	70.7%	49.0%
Decoupled	Mean bias in β	0.000	0.001	0.001	0.001	0.004	0.008
	β is positive & significant*	2.8%	3.4%	3.1%	5.4%	12.4%	16.9%
	β is negative & significant**	2.0%	3.0%	3.2%	5.9%	11.2%	16.2%
Size-based	Mean bias in β	– 0.000	0.001	0.000	0.000	– 0.000	0.005

(*Continued*)

Table 1. Continued

Panel D: DIFFERENCE						
Group size (k)	4	8	16	32	64	128
β is positive & significant*	2.6%	2.6%	5.0%	7.4%	12.5%	22.0%
β is negative & significant**	2.0%	3.3%	4.6%	6.3%	12.5%	17.9%

The table shows summary of estimation results from models using 1000 simulated samples with 4196 observations in each sample. The details of the simulations' setup are described in Appendix B. The following model specification is being estimated: $\ln Y = \mu + \alpha^* \ln X + \beta^* MEASURE + u$, where lnY is the logarithm of the simulated audit fee, lnX is the logarithm of the randomly generated auditee size and MEASURE is a competition or specialization measure (RATIO, LEADER, DISTANCE and DIFFERENCE); for the original, decoupled and size-based measures. RATIO is the ratio of group sums and corresponds to the market share of the incumbent auditor. LEADER, DISTANCE and DIFFERENCE are measures of auditor specialization or competitive position based on the incumbent auditor's market share and are employed by Francis et al. (2005), Numan and Willekens (2012) and Chu et al. (2018), respectively. All four variables (RATIO, LEADER, DISTANCE and DIFFERENCE) are computed using simulated audit fees as described in Appendix A. The columns show results for simulations with different (increasing) MSA-industry market sizes (number of clients), ranging from 4 to 128, where the true effect of the coefficient β is zero. * Indicates the proportion of times from 1000 simulations that the coefficient is positive with *p*-value of its t–statistic ≤ 0.05. ** Indicates the proportion of times from 1000 simulations that the coefficient is negative with *p*-value of its t–statistic ≤ 0.05.

distributed approximately equally to cases when it is positive and negative and statistically significant.

The statistics for DIFFERENCE in Panel D reveal that the mean bias is most pronounced (-0.340) for the smallest group and then decreases in absolute value as group size increases, reaching -0.186 for the biggest group. While the estimated coefficient is negative and statistically significant in all of the simulations for group sizes up to 16, it then declines and falls to 50% of the simulations for the largest group. The mean bias for the decoupled and size-based versions of DIFFERENCE is close to zero. The percentage of samples when the coefficients are statistically significant increases with the group size, starting from below 5% in the smallest group and increasing to nearly 40% in the largest groups. Again, these total percentages are distributed approximately equally between estimated coefficients which are positive and negative.

In summary, the results presented in the Table 1 convey a relatively clear picture. Firstly, in all the simulations the *true* size of effect was zero. Yet, the mean estimated coefficients (bias) for the three measures LEADER, DISTANCE and DIFFERENCE are generally substantive having the same sign as those reported in prior studies. This suggests that the size and significance of the causal effect in empirical studies may be overstated. Secondly, the pattern of statistical significance is similar for each measure. Specifically, the estimated coefficients are statistically significant in a very high percentage (up to 100%) of the random samples for the smallest group sizes, but decrease as the group size increases. Ergo, we are more likely to observe a spurious statistically significant relationship if the group size is small. This is the case for relatively small groups determined by geographical MSA and 2-digit SIC levels.[25] Finally, the results for the decoupled and size-based measures demonstrate that they are useful for uncovering potentially spurious relationships. The mean bias associated with their coefficients was close to zero and the percentage of samples with statistically significant coefficients is relatively small, especially for smaller groups.

[25] Francis et al. (2005) report an average group size of 6 observations for MSA-industry markets, Numan and Willekens (2012) 7.46 observations and Chu et al. (2018) 4.6 observations.

We next explore the bias of the three estimators if the true value of the coefficient β is different from zero. We do so in the smallest markets (group size 4), because the measures under analysis are frequently employed in samples where the mean audit market size is relatively small. More specifically, for each mechanically correlated measure we generate simulated samples where the dependent variable is computed based on equation (11). We employ coefficient (β) values of 0.1, 0.2, 0.3 and 0.4 for RATIO, LEADER and DISTANCE; and values of -0.1, -0.2, -0.3 and -0.4 for DIFFERENCE.[26]

Table 2 present OLS regression statistics for the estimated values of β as a function of its true value. For perspective, the first of the five columns report findings when the true value of the coefficient β is zero. The estimates show that the mean bias associated with the original variables changes little as the true value of the coefficient β increases. In contrast, for their decoupled and size-based counterparts, the absolute value of the bias tends to increase. More specifically, the mean estimated values are between zero and the true value of the coefficient β for all the measures, the bias ranging between around 19% and 38% of the true effect; with lower bias being associated with the decoupled measures of RATIO and LEADER and higher for the decoupled versions of DISTANCE and DIFFERENCE.

However, even though bias increases, for the highest analyzed true values of the coefficient β it is still much smaller (excepting DISTANCE) than the bias of the original measures. For DISTANCE, the bias is approximately equal for the largest true value of β, though this exceeds the value of the estimated coefficient published in prior literature. Moreover, in terms of the statistical significance of the decoupled and size-based variables, Table 2 shows that, even for the smallest non-zero true effect analyzed (0.1), the estimated coefficients exhibit their expected signs and are statistically significant in a substantial proportion of the random samples. Overall, the simulation results reported in Table 2 give further credence to the employment of decoupled and size-based variables as sensitivity measures in empirical studies; in that even when the true values of β are different from zero, they still provide estimates closer to the true values than do the original variables.

Having been informed by the results of mathematical derivations and simulations, we next present our empirical study, where we examine whether Chu et al.'s (2018) empirical findings for DIFFERENCE are robust to the use of decoupled and size-based variables. Details of how to replicate the analysis in this section are provided in Appendix D.

4. Empirical Study

As described above, in this section we analyze the potential bias associated with a mechanically correlated variable in an audit pricing framework. To do so, we use a recent sample of US companies to analyze the relationship between audit pricing and the competition measure (DIFFERENCE) introduced in Chu et al. (2018). At the outset we should state that we are not criticizing Chu et al.'s (2018) study (nor any other studies referenced in this paper), nor do we seek to (exactly) replicate[27] Chu et al.'s (2018) results. Rather, following Chu et al.'s (2018) sample selection procedure and variables' specification, we examine the issue of mechanical correlation using more recent data.[28]

[26] The range of values is selected so that it covers and exceeds the values reported in the literature.

[27] This is not possible because the dataset is unavailable for re-analysis and the historical location of companies is not available in the Audit Analytics or Compustat databases. In addition, because in any archival database which reports variables for a calendar or financial year, at any point in time some companies will not have filed their accounts for that year but may subsequently file them and then be included later for the given year.

[28] Of course, it is possible that our findings/inferences may not (at least) fully hold in Chu et al.'s (2018) original data.

4.1. Variables, Data and Summary Statistics

Chu et al. (2018) define an audit firm's competitive position (DIFFERENCE) as the difference between the market share of the dominant auditor and an incumbent auditor in an MSA-industry

Table 2. Estimation results using simulated data as the true value of beta changes.

Panel A: RATIO						
True value of β		0	0.1	0.2	0.3	0.4
Original	Mean bias in β	0.323	0.318	0.312	0.306	0.300
	β is positive & significant*	100.0%	100.0%	100.0%	100.0%	100.0%
	β is negative & significant**	0.0%	0.0%	0.0%	0.0%	0.0%
Decoupled	Mean bias in β	– 0.001	– 0.021	– 0.041	– 0.060	– 0.077
	β is positive & significant*	2.3%	86.6%	100.0%	100.0%	100.0%
	β is negative & significant**	3.6%	0.0%	0.0%	0.0%	0.0%
Size-based	Mean bias in β	0.000	– 0.026	– 0.049	– 0.069	– 0.086
	β is positive & significant*	1.8%	88.6%	100.0%	100.0%	100.0%
	β is negative & significant**	4.1%	0.0%	0.0%	0.0%	0.0%
Panel B: LEADER						
True value of β		0	0.1	0.2	0.3	0.4
Original	Mean bias in β	0.155	0.155	0.155	0.155	0.155
	β is positive & significant*	100.0%	100.0%	100.0%	100.0%	100.0%
	β is negative & significant**	0.0%	0.0%	0.0%	0.0%	0.0%
Decoupled	Mean bias in β	– 0.001	– 0.027	– 0.049	– 0.069	– 0.088
	β is positive & significant*	2.7%	98.4%	100.0%	100.0%	100.0%
	β is negative & significant**	3.4%	0.0%	0.0%	0.0%	0.0%
Size-based	Mean bias in β	0.000	– 0.027	– 0.054	– 0.081	– 0.107
	β is positive & significant*	2.0%	99.2%	100.0%	100.0%	100.0%
	β is negative & significant**	2.4%	0.0%	0.0%	0.0%	0.0%
Panel C: DISTANCE						
True value of β		0	0.1	0.2	0.3	0.4
Original	Mean bias in β	0.125	0.128	0.130	0.132	0.134
	β is positive & significant*	98.0%	100.0%	100.0%	100.0%	100.0%
	β is negative & significant**	0.0%	0.0%	0.0%	0.0%	0.0%
Decoupled	Mean bias in β	– 0.001	– 0.036	– 0.071	– 0.104	– 0.137
	β is positive & significant*	3.1%	63.9%	99.2%	100.0%	100.0%
	β is negative & significant**	3.7%	0.0%	0.0%	0.0%	0.0%
Size-based	Mean bias in β	– 0.001	– 0.036	– 0.07	– 0.102	– 0.133
	β is positive & significant*	3.1%	77.6%	99.9%	100.0%	100.0%
	β is negative & significant**	4.1%	0.0%	0.0%	0.0%	0.0%
Panel D: DIFFERENCE						
True value of β		0	– 0.1	– 0.2	– 0.3	– 0.4
Original	Mean bias in β	– 0.340	– 0.332	– 0.324	– 0.316	– 0.308
	β is positive & significant*	0.0%	0.0%	0.0%	0.0%	0.0%
	β is negative & significant**	100.0%	100.0%	100.0%	100.0%	100.0%
Decoupled	Mean bias in β	0.000	0.038	0.074	0.108	0.139
	β is positive & significant*	2.8%	0.0%	0.0%	0.0%	0.0%
	β is negative & significant**	2.0%	80.5%	100.0%	100.0%	100.0%
Size-based	Mean bias in β	0.000	0.034	0.066	0.095	0.121

(*Continued*)

Table 2. Continued

Panel D: DIFFERENCE					
True value of β	0	− 0.1	− 0.2	− 0.3	− 0.4
β is positive & significant*	2.6%	0.0%	0.0%	0.0%	0.0%
β is negative & significant**	2.0%	82.4%	100.0%	100.0%	100.0%

The table shows summary of estimation results from models using 1000 simulated samples with 4196 observations in each sample. The details of the simulations' setup are described in Appendix B. The following model specification is being estimated: $\ln Y = \mu + \alpha^*\ln X + \beta^*MEASURE + u$, where lnY is the logarithm of the simulated audit fee, lnX is the logarithm of the randomly generated auditee size and MEASURE is a competition or specialization measure (RATIO, LEADER, DISTANCE and DIFFERENCE); for the original, decoupled and size-based measures. RATIO is the ratio of group sums and corresponds to the market share of the incumbent auditor. LEADER, DISTANCE and DIFFERENCE are measures of auditor specialization or competitive position based on the incumbent auditor's market share and are employed by Francis et al. (2005), Numan and Willekens (2012) and Chu et al. (2018), respectively. All four variables (RATIO, LEADER, DISTANCE and DIFFERENCE) are computed using simulated audit fees as described in Appendix A. The five columns show results for simulations, where the MSA-industry market size is constant (equal to 4) and the true value of the coefficient β is changing. * Indicates the proportion of times from 1000 simulations that the coefficient is positive with p-value of its t–statistic ≤ 0.05. ** Indicates the proportion of times from 1000 simulations that the coefficient is negative with p-value of its t–statistic ≤ 0.05.

market as follows:

$$DIFFERENCE_{at} = \frac{\sum_{j\in \text{ dominant auditor in MSA}-industry} AF_{jt} - \sum_{j\in \text{ auditor } a \text{ in MSA}-industry} AF_{jt}}{\sum_{j\in MSA-industry} AF_{jt}} \quad (14)$$

The decoupled version of DIFFERENCE (DIFFERENCE_DC), using the analogous definition in equation (14) is specified as:

$$DIFFERENCE\_DC_{it} = \frac{\sum_{j\in \text{ dominant auditor in MSA}-industry} AF_{jt} - \left(\sum_{j\in \text{ incumbent auditor in MSA}-industry} AF_{jt} - AF_{it}\right)}{\sum_{j\in MSA-industry} AF_{jt} - AF_{it}} \quad (15)$$

As shown, DIFFERENCE_DC is calculated in the same way as DIFFERENCE, but with one important distinction: client i's audit fee is *excluded* (decoupled) from all elements of equation (14) at time t. Hence, even though DIFFERENCE_DC is derived from audit fees, the direct empirical link with a company's audit fee is severed. The auditee size-based proxy variables we employ replace all the audit fee terms in DIFFERENCE with client total assets (TA) or client sales (SA); and are labeled DIFFERENCE_TA and DIFFERENCE_SA.

As is standard, the dependent variable is the natural log of audit fees (LAF). Control variable definitions and labels are reported in Appendix C and are the same as those used by Chu et al. (2018). These are: auditee size (LTA); complexity, as proxied by the number of business segments (LBSEG) and the number of geographical segments (LGSEG); an indicator variable for foreign sales (FOREIGN); the ratio of current assets to total assets (CATA); the ratio of quick assets to current liabilities (QUICK); the ratio of long-term debt to total assets (LEV); the ratio of earnings before interest and tax to total assets (ROI); an indicator variable for losses (LOSS); an indicator variable for busy period (YE); an indicator variable for a going concern qualification (OPINION); and an indicator variable for a Big 4 auditor (BIG). As per Chu et al. (2018), the regression specification controls for fixed effects (FE) for 2-digit SIC industry sectors (Industry FE), MSAs (MSA FE) and data year (Year FE).

As shown in Panel A of Table 3, we generate the data using the sample selection criteria specified by Chu et al. (2018). But there was an issue regarding the identification of markets. We

Table 3. Description of Data.

Panel A: Sample selection	
Observations in the U.S. with positive audit fee for 2017–2019 on Audit Analytics	23,148
Less:	
Observations not on Compustat or missing information for MSA assignment	– 11,458
Financial sector (SIC 6000–6999)	– 2,795
MSA-industry markets with only one auditor	– 2,483
Market sample (calculation of competition measures)	**6,412**
Less:	
Missing values for control variables	– 177
Audit engagements in the first or second year	– 951
Estimation sample	**5,284**

Panel B: MSA-industry-year market statistics (n = 6,412)	
Number of markets	1,128
Median number of observations per market	3
Mean number of observations per market	5.68

Panel C: Descriptive statistics (n = 5,284)

	mean	sd	min	p25	p50	p75	max
LAF	14.008	1.361	10.722	13.171	14.154	14.947	17.094
FEE	2,713,091	4,099,123	45,334	524,750	1,402,922	3,100,000	26,540,000
DIFFERENCE	0.255	0.268	0.000	0.000	0.198	0.446	0.938
DIFFERENCE_DC	0.472	0.353	0.000	0.131	0.450	0.774	1.000
DIFFERENCE_TA	0.371	0.343	0.000	0.000	0.383	0.663	0.992
DIFFERENCE_SA	0.365	0.338	0.000	0.000	0.373	0.643	0.990
LTA	6.391	2.521	– 0.573	4.738	6.610	8.196	11.667
LSALES	5.785	2.878	– 6.908	4.031	6.286	7.786	12.540
LBSEG	0.092	0.336	0.000	0.000	0.000	0.000	1.609
LGSEG	0.131	0.413	0.000	0.000	0.000	0.000	1.946
CATA	0.482	0.275	0.033	0.257	0.458	0.710	0.992
QUICK	2.368	2.848	0.052	0.862	1.402	2.635	17.934
LEV	0.249	0.264	0.000	0.019	0.205	0.370	1.481
ROI	– 0.090	0.568	– 3.842	– 0.055	0.082	0.135	0.402
FOREIGN	0.548	0.498	0.000	0.000	1.000	1.000	1.000
OPINION	0.102	0.302	0.000	0.000	0.000	0.000	1.000
YE	0.756	0.429	0.000	1.000	1.000	1.000	1.000
LOSS	0.452	0.498	0.000	0.000	0.000	1.000	1.000
BIG	0.673	0.469	0.000	0.000	1.000	1.000	1.000

Panel D: Correlation matrix of primary variables (n = 5,284)

		1	2	3	4	5
1	LAF	1.00				
2	DIFFERENCE	– 0.58	1.00			
3	DIFFERENCE_DC	– 0.19	0.51	1.00		
4	DIFFERENCE_TA	– 0.52	0.83	0.40	1.00	
5	DIFFERENCE_SA	– 0.52	0.82	0.39	0.94	1.00

Variables are defined in Appendix C. All correlation coefficients are statistically significant at the 5% level of significance or better.

identified the audit markets using client location. However, the historical location of companies is unavailable in the Audit Analytics and Compustat databases. Since Compustat supplies only the current location of companies, we use a relatively short sample period of three years (2017-2019) to limit potential miscoding. As shown in Panel A, the final estimation sample comprises 5,284 observations.[29] As highlighted in Panel A, we use the sample of 6,412 firm-year observations (referred to as the market sample) to compute the competition measures. Full instructions to replicate the data and results of our empirical study are provided in Appendix D.

Panel B presents descriptive statistics for the market sample. On average, there are 5.68 companies in each MSA-industry market, with the median being 3. Panel C shows that the mean of DIFFERENCE (0.255) is a little higher than that (0.233) reported by Chu et al. (2018). As shown, average audit fees are around $2.71 million, substantially more than Chu et al. (2018) reported ($1.47 million). Of course, inflation will largely contribute to this disparity.[30] Panel D reports a correlation matrix for audit fees (LAF) and the primary variables of interest. It shows there are sizeable correlations between DIFFERENCE_TA (DIFFERENCE_SA) and LAF, with correlation coefficients of -0.52 (-0.53) respectively. These compare to correlation coefficients of -0.58 (-0.19) between DIFFERENCE (DIFFERENCE_DC) and LAF. Noteworthy is that DIFFERENCE_TA (DIFFERENCE_SA) are highly correlated with DIFFERENCE, with correlation coefficients of 0.84 (0.83) respectively; and hence appear strong surrogates for DIFFERENCE. However, though sizeable, the correlation (0.51) between DIFFERENCE and DIFFERENCE_DC is substantially lower than for the size-based variables.

4.2. Empirical Results

Using the same control variables as Chu et al. (2018), Table 4 presents regression estimates for DIFFERENCE together with its decoupled and size-based counterparts. Model 1 reports regression parameters for DIFFERENCE. It reveals its coefficient is -0.399 - which is somewhat larger in absolute terms than that (-0.340) reported in Chu et al.'s (2018) study - and is highly significant. Hence, Model 1 provides strong support for the original findings of Chu et al. (2018) and the hypothesized relationship between DIFFERENCE and audit fees. We also note that all control variables exhibit their expected signs and (with exception of LGSEG and YE) are statistically significant at the 1% level. The model is well determined with reference to its adjusted R^2 (0.897); which is a little higher than that (0.866) reported by Chu et al. (2018). Hence the inferences for DIFFERENCE in our new data are highly congruent with those of Chu et al. (2018).

Model 2 includes the decoupled variable DIFFERENCE_DC. It shows that its coefficient is close to zero, statistically insignificant and changes sign. It appears that when the elements of a company's audit fee are excluded from DIFFERENCE, it is no longer associated with audit fees. Other things equal, the estimates for DIFFERENCE_DC show that the bias associated with DIFFERENCE is likely to be substantial.

Employing client size-based versions of market measures in audit pricing studies appears plausible, since they are not mechanically correlated. Of course, auditee size is not a perfect substitute for audit fees in constructing market measures. However, as reported above, DIFFERENCE_TA and DIFFERENCE_SA are both highly correlated with DIFFERENCE, making them viable surrogates for the latter. Models 3 (4) show that, though exhibiting the correct signs,

[29]Note that we delete 11,458 companies which are not on Compustat, or which have missing information on auditors' city location, which is required to assign each firm year to a given MSA. In this context, we used the list of Core Based Statistical Areas (CBSAs) and Combined Statistical Areas (CSAs) from March 2020. If an audit firm's city did not correspond to an MSA, we looked up a county or county equivalent for the city and assigned the MSA accordingly. The list was available at: https://www.census.gov/geographies/reference-files/time-series/demo/metro-micro/delineation-files.html, accessed on 27 July 2021.

[30]This follows, given the more recent data employed in the current study (2017-2019), relative to that (2000-2011), of Chu et al. (2018); whose earliest data year is 2000, some 17 years before that in the current data.

Table 4. Estimation results for models with DIFFERENCE and its alternatives.

	(1) LAF	(2) LAF	(3) LAF	(4) LAF	(5) LAF
DIFFERENCE	−0.399***				
	(−8.44)				
DIFFERENCE_DC		0.024			
		(0.70)			
DIFFERENCE_TA			−0.012		
			(−0.35)		
DIFFERENCE_SA				−0.054	−0.028
				(−1.51)	(−0.79)
LSALES					0.106***
					(9.17)
LTA	0.448***	0.465***	0.464***	0.462***	0.369***
	(56.46)	(59.24)	(57.24)	(56.65)	(27.85)
LBSEG	0.083***	0.089***	0.088***	0.088***	0.084***
	(4.20)	(4.47)	(4.44)	(4.40)	(4.38)
LGSEG	0.0143	0.008	0.008	0.009	0.006
	(0.84)	(0.45)	(0.48)	(0.49)	(0.33)
CATA	0.406***	0.424***	0.423***	0.421***	0.298***
	(7.19)	(7.36)	(7.37)	(7.33)	(5.48)
QUICK	−0.047***	−0.048***	−0.048***	−0.047***	−0.027***
	(−12.87)	(−12.76)	(−12.79)	(−12.72)	(−7.12)
LEV	0.091**	0.087**	0.088**	0.088**	0.070*
	(2.55)	(2.39)	(2.40)	(2.42)	(1.90)
ROI	−0.097***	−0.108***	−0.106***	−0.106***	−0.184***
	(−4.74)	(−5.24)	(−5.14)	(−5.12)	(−8.56)
FOREIGN	0.277***	0.289***	0.289***	0.289***	0.267***
	(11.14)	(11.49)	(11.49)	(11.48)	(11.10)
OPINION	0.156***	0.166***	0.166***	0.166***	0.178***
	(3.86)	(3.99)	(4.00)	(4.00)	(4.46)
YE	0.034	0.042*	0.041*	0.041*	0.047**
	(1.47)	(1.78)	(1.74)	(1.71)	(2.04)
LOSS	0.095***	0.101***	0.102***	0.102***	0.139***
	(4.82)	(5.07)	(5.08)	(5.12)	(7.00)
BIG	0.365***	0.456***	0.449***	0.442***	0.453***
	(11.80)	(15.40)	(15.01)	(14.83)	(15.46)
Constant	10.720***	10.420***	10.440***	10.480***	10.440***
	(140.56)	(146.62)	(136.79)	(137.19)	(139.53)
Industry FE	Yes	Yes	Yes	Yes	Yes
MSA FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
N	5284	5284	5284	5284	5284
Adjusted R^2	0.897	0.894	0.894	0.894	0.898

The variables are described in Appendix C. The standard errors are clustered in companies.
*, **, and *** indicate coefficients are statistically significant at the 10%, 5% and 1% levels, respectively. Corresponding t-statistics are displayed in parentheses.

the coefficients[31] of DIFFERENCE_TA (DIFFERENCE_SA) are close to zero and statistically insignificant.

Typically, audit pricing studies use client total assets as their size variable. However, some studies (e.g. Clatworthy & Peel, 2007; Pong & Whittington, 1994) employ both total assets and

[31] Although not the focus of our study, regarding control variable coefficients, recall from our analysis in Section 3 (footnote 9), that for the group average, the coefficient of the explanatory variable *x* was biased in the opposite direction of its sign. On the other hand, for the size-based group average, its coefficient was unbiased. Even though the setting is much more complicated, when we compare the coefficients of the control variables in models 1 and 3, the coefficients in Model 1 (except for LEV) are indeed smaller in absolute value than the coefficients in Model 3.

sales in their regression specifications. In this regard, Pong and Whittington (1994, p. 1075) stress that there are two dimensions to an audit, 'an audit of transactions and a verification of assets. The former will be related to turnover and the latter to total assets'. This implies there may be an omitted variable problem, if sales (LSALES) is not employed as an additional control variable. This would appear particularly germane when sales is used as a proxy to construct market competition/industry specialization measures.[32] In this context, Model 5, replicates Model 4, but includes both size variables. It shows that the size of the coefficient for DIFFERENCE_SA reduces by around 51%, with its associated t-value declining by a similar amount (about 52%).[33]

In summary, the inferences drawn from the client sized-based variables are identical to the decoupled one, together indicating that (in our data), bias emanating from mechanical correlation is likely to be large in magnitude.[34] Note, however, it may be the case that for other audit pricing studies, mechanically correlated variables exhibit expected (and statistically significant) signs when employing decoupled variables and/or client size-based proxies. This would provide some support for the hypotheses relating to mechanically correlated variables.

In Section 3 we presented simulations which demonstrated that when the true values of the coefficients of market measures is non-zero, decoupled and size-based variables are not bias-free. However, within the feasible range of true coefficient values, we show that (generally) they provide estimates closer to the true coefficient values than do the original variables. Further, when the true value of the coefficient is zero, we report that the estimated coefficients for the decoupled and size-based variables are close to zero and statistically insignificant. Ceteris paribus, this might imply that the true value of the coefficient for DIFFERENCE is either zero or relatively small. So, to a large extent, the findings reported by Chu et al. (2018) might be explained by mechanical correlation.

5. Summary and Conclusion

In this paper we highlight the important issue of mechanical correlation in audit pricing research, when explanatory variables of interest are expressed in audit fee terms. Mechanical correlation produces biased (endogenous) estimates for such variables. The issue is of high import since many audit pricing studies use market share and industry specialist variables which are mechanically correlated.

After employing mathematical derivations to establish the bias associated with mechanically correlated variables, together with their decoupled and size-based counterparts, we use simulation analysis in a simplified audit pricing setting to provide further evidence on this issue. Our mathematical analysis demonstrates that, under a wide range of scenarios, the coefficients of decoupled and size-based measures are closer to the 'true' (unbiased) value of the coefficient for

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[32]For instance, to examine the robustness of their results, Francis et al. (2005) use total assets as their size variable, but report results using LEADER based on client sales. They find that its coefficient remains statistically significant with its expected sign, but the magnitude of its effect decreases by approximately 50%, relative to the original variable derived from audit fees which is consistent with our simulations results. However, Francis et al. (2005) do not control for client sales. That is why we cannot exclude the possibility that the sales-based LEADER variable manifests (at least to some extent) the effect of sales omitted from the model specification.

[33]In contrast, when (in unreported analyzes) the log of sales (LSALES) is added as an additional control variable in models to 1 to 3, the coefficients and t-values for DIFFERENCE, DIFFERENCE_DC and DIFFERENCE_TA change only marginally in all cases.

[34]Given certain assumptions (see Wooldridge, 2013), the measurement error associated with a proxy variable may lead to 'attenuation bias', such that its coefficient is biased towards zero. However, the attenuation bias assumption does not hold if the original variable is endogenous, as is the case here; and in relation to the variables analyzed in Section 3.

the group average than the original group average coefficient; providing support for use of these approaches in the literature.

Moreover, via simulations of commonly used audit market competition and auditor specialization variables, we show that (concordant with our mathematical derivations), due to mechanical correlation, these measures will likely have statistically significant coefficients even if the true effect does not exist (is zero). The issue is exacerbated when these measures are based on a relatively small number of observations in each market group, which is typically the case in the literature. We also show that the decoupled and size-based measures are unbiased if the true effect is zero, and if it is not zero, their coefficients are in general closer to the true value than are the original measures. For the decoupled and size-based measures, we document the bias in the range of $20\% - 40\%$ of the true effect. However, this was obtained using a simplistic setting and future work could explore whether this is the case in more complex multivariate settings.

We further illustrate the nature of the problem by replicating the regression specification of Chu et al. (2018) on a more recent sample and find our parameter estimates for DIFFERENCE are congruent with theirs. Our principal regression findings are as follows: when we decouple the current auditor's audit fee from DIFFERENCE its coefficient is close to zero and statistically insignificant. When we employ client size (sales or total assets) in place of audit fees to compute DIFFERENCE, the coefficients of these proxy variables are also close to zero and statistically insignificant. Although these findings and inferences may not fully hold in Chu et al. (2018) original data, our results suggest that the true effect of DIFFERENCE is likely to be much smaller than that reported in their study.

Of course, the use of decoupled and size-based market variables is not without limitations. In particular, they may alter the original market variable to such an extent, that it does not correspond to the original construct or underlying theory. Hence, if the decoupled and size-based measures are inconsistent with the theoretical underpinnings of the original mechanically correlated measures, their coefficients will be biased because of the measurement error. Moreover, and consistent with our findings in Section 3.1, empirical studies report that decoupling may not remove the whole bias (Gormley & Matsa, 2014), or may induce a negative correlation with the dependent variable when a positive one is expected (Ouimet & Tate, 2020).

Nevertheless, we demonstrate that despite these shortcomings, the inferences based on the decoupled and size-based measures are more reliable across a wide range of scenarios; in that the bias due to mechanical correlation of the original measures is greater than the bias due to measurement error of their decoupled and size-based alternatives. Our findings indicate that this occurs predominantly in smaller markets with fewer auditor observations. Hence in future audit pricing studies - in the absence of a credible instrumental variable - we advocate that, alongside presenting findings for market variables constructed from audit fees, results for their decoupled and size-based counterparts should also be reported as sensitivity measures to gauge potential bias. If these measures exhibit their expected signs and are statistically significant in regression models, though not being conclusive, they may be viewed as providing some comfort to the researcher regarding the validity of the hypothesized relationship for the original variable. Our analyzes suggest this is especially important where smaller markets are analyzed, such as at the city level or within MSA-industry sectors. Given our findings, we recommend that the results of extant audit pricing studies which employ mechanically correlated variables (particularly those based on smaller markets) should be re-evaluated using decoupled and size-based measures.

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Supplemental data

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Appendix A. Formulas and Examples for the Simulated Measures

Similar to Gormley and Matsa (2014), in simulations we use two indices, i and j. The first indexes groups (audit markets), and runs from 1 to N. The second indexes observations within each group, and runs from 1 to k, where k is the fixed group size. The total sample size is equal to $N \times k$. The variable $y_{i,j}$ is the dependent variable. The variable $a_{i,j}$ (AUDITOR) takes on values from 1 to 4 and identifies a subgroup.

Table A. Example of the computation for one market

i	*j*	*AUDITOR*	*FEE*	*AUD_SUM*	*RATIO*	*LEADER*	*DISTANCE*	*DIFFERENCE*
1	1	1	130	750	0.250	0.00	0.087	0.310
1	2	1	620	750	0.250	0.00	0.087	0.310
1	3	2	140	1,680	0.560	1.00	0.310	0.000
1	4	2	650	1,680	0.560	1.00	0.310	0.000
1	5	2	890	1,680	0.560	1.00	0.310	0.000
1	6	3	290	490	0.163	0.00	0.087	0.397
1	7	3	200	490	0.163	0.00	0.087	0.397
1	8	4	80	80	0.027	0.00	0.137	0.533
		Total	3,000					

Table A provides an example for one group (audit market) with 8 clients. AUD_SUM is the sum of fees collected by a specific auditor in the audit market. RATIO is a ratio of two group sums and corresponds to the market share of the incumbent auditor. It has the following formula:

$$RATIO_{i,j} = \frac{\sum_{g:a_{i,g}=a_{i,j}} y_{i,g}}{\sum_{g=1}^{k} y_{i,g}}$$

As shown in Table A, the group sum (denominator) comprises the total fees (3000) of the whole audit market, so in the example given it is 3000. The group sum in the numerator is the sum of audit fees collected by the incumbent auditor in the audit market. For auditor 1 in Table–A, it is 750; so $RATIO_{1,1} = RATIO_{1,2} = \frac{750}{3000} = 0.25$.

LEADER is the indicator of market leader, so it is equal to one if the auditor has the highest market share, zero otherwise. It has the following formula,

$$LEADER_{i,j} = I\left[\max_{a\in\{1..4\}}\left(\sum\nolimits_{g:a_{i,g}=a} y_{i,g}\right) = \sum\nolimits_{g:a_{i,g}=a_{i,j}} y_{i,g}\right]$$

Where $I[.]$ is the indicator function, equal to unity if the condition in brackets is fulfilled and zero otherwise. In Table A, auditor 2 has the highest market share, and hence it is the market leader. Auditor 1 is not the market leader and therefore $LEADER_{1,1} = LEADER_{1,2} = 0$.

The variable DISTANCE denotes the absolute distance in terms of market share from the closest competitor. It has the following formula:

$$DISTANCE_{i,j} = \frac{\min_{a\in\{1..4\}\wedge a\neq a_{i,j}} \left|\sum\nolimits_{g:a_{i,g}=a} y_{i,g} - \sum\nolimits_{g:a_{i,g}=a_{i,j}} y_{i,g}\right|}{\sum\nolimits_{g=1}^{k} y_{i,g}}$$

In the example given in Table A, the closest competitor for auditor 1 in terms of market share is auditor 1. The absolute difference in terms of market share between the auditor 1 and auditor 3 is 0.087, hence $DISTANCE_{1,1} = DISTANCE_{1,2} = |0.250 - 0.163| = 0.087$.

Finally, DIFFERENCE denotes the difference in terms of market share between the market leader and the incumbent auditor. It has the following formula:

$$DIFFERENCE_{i,j} = \frac{\max_{a\in\{1..4\}}\left(\sum\nolimits_{g:a_{i,g}=a} y_{i,g}\right) - \sum\nolimits_{g:a_{i,g}=a_{i,j}} y_{i,g}}{\sum\nolimits_{g=1}^{k} y_{i,g}}$$

As shown in Table A, auditor 1 has a market share 0.250 whereas the market leader has a market share of 0.560. Hence $DIFFERENCE_{1,1} = DIFFERENCE_{1,2} = 0.560 - 0.250 = 0.310$.

Appendix B. Description of Simulation Setups

Setting:

We simulate the following data generating process:

$$\ln(y_{i,j}) = \alpha \ln(x_{i,j}) + \beta MEASURE_{i,j} + u_{i,j}$$

Where $\ln(y)$ is the dependent variable, $\ln(x)$ is the natural log of size, MEASURE is one of the variables RATIO, LEADER, DISTANCE or DIFFERENCE, u is the random error, i is the group number running from 1 to number of groups N and i is the observation number within groups running from 1 to group size k.

Establishing the parameters of simulations

The choice of $N \times k = 4096$ observations for each simulation was chosen to facilitate a systematic examination of the effects of mechanical correlation across group sizes which are multiples of 2. Also, this number is fairly close to the number of observations in the final estimation sample (5284) in our empirical study.

The choice of the values of parameters needed for the generation of the explanatory variable x are: the standard deviation $\sigma_{\ln(x)} = 2.5$ and the within-group correlation $\rho_{\ln(x)_j,\ln(x)_{-j}} = 0.3$.

For the random error u the standard deviation $\sigma_u = 0.5$ and the within-group correlation $\rho_{u_j,u_{-j}} = 0.1$. These parameters were based approximately on the final estimation sample in our empirical study.[35] These parameters, along with the value of the coefficient $\alpha = 0.5$, remain constant in each simulated sample.

Since the first objective of the simulations is to explore the behaviour of the estimators as a function of group size k if the true value of the coefficient β is zero, for each group size we run six sets of simulations with the true value of the coefficient $\beta = 0$ and the fixed group sizes $k = 4, 8, 16, 32, 64, 128$. Thus, for each simulation and each group size, we run the following steps:

Step 1: Allocation of group identifiers

In each random sample, the structure of groups was fixed. For example, if there were groups with group size 4, the first four observations (i.e., observation number 1 to observation number 4) belonged to group 1, the following four observations (i.e., observation number 5 to observation number 8) belonged to group 2, and so on; such that that the last four observations (i.e., observation number 4093 to observation number 4096) belonged to group 1024 (4096/4). The structure remained the same for each simulated dataset.

Step 2: Generating of the random error and the explanatory variable x

Next, we randomly generated the random error. We used normal distribution with the mean $\mu_u = 0$, standard deviation $\sigma_u = 0.5$, and the within group correlation $\rho_{u_j,u_{-j}}$ is constant and equal to 0.1.

Then we generated the explanatory variable $\ln(x)$. We used normal distribution with mean $\mu_{\ln(x)} = 0$ and standard deviation $\sigma_{\ln(x)} = 2.5$, and the within group correlation of the values for the explanatory variable $\rho_{\ln(x)_j,\ln(x)_{-j}}$ is constant and equal to 0.3.

Step 3: Assigning of subgroup identifiers

We demonstrate the existence of the bias using measures that are based on ratio of two group totals where the group in the numerator is smaller than the group in the denominator. We report simulation results where, in each group, there were at most 4 subgroups[36]. The subgroup identifier was generated randomly using a uniform discrete distribution with values 1 to 4, each with probability of 1/4.

Step 4: Generating of the dependent variable

The dependent variable was generated under the assumption that the true value of the parameter β is zero. Hence the following equation was used:

$$\ln(y0_{i,j}) = \alpha \ln(x_{i,j}) + u_{i,j}$$

[35] With regard to our empirical study, we have employed the parameters derived from the final estimation sample to keep important distributional properties intact, but except for scale associated with the ratio of variances of x and u, the results were relatively insensitive to the choice of the parameters.

[36] This setup reflects scenario where Big 4 auditors compete for clients.

Step 5: Calculation of measures of industry specialization, competition and market concentration

Finally, we computed measures RATIO, LEADER, DISTANCE and DIFFERENCE using the baseline value of the dependent variable $y0_{i,j}$.

Step 6: Calculation of size-based measures of industry specialization, competition and market concentration

We computed measures RATIO_X, LEADER_X, DISTANCE_X and DIFFERENCE_X using the values of the independent variable x.

Step 7: Calculation of the decoupled measures of industry specialization, competition and market concentration

We computed measures RATIO_DC, LEADER_DC, DISTANCE_DC and DIFFERENCE_DC using the values of the dependent variable y0.

In the simulations, we repeated the whole process for each group size 1000 times. Therefore, in total, we generated 6000 random samples with 4096 observations in each sample.

Then, since the second objective of the simulations was to explore the performance of the estimators as a function of the true value of the coefficient β in small groups (markets), for samples randomly generated with a group size with 4 observations, for each measure and value of the coefficient β (we used values 0.1, 0.2, 0.3 and 0.4 for measures RATIO, LEADER and DISTANCE, and values -0.1, -0.2, -0.3 and -0.4 for DIFFERENCE) we did the following:

Step 8: Generating of the dependent variable

The dependent variable was generated using formula:

$$\ln{(y_{i,j})}^{(k)} = \alpha \ \ln{(x_{i,j})} + \beta MEASURE_{i,j}{}^{(k-1)} + u_{i,j}$$

In the first iteration, the measure assuming the zero true value of β is employed.

Step 9: Calculation of the new value of the measure

In this step, the new value for the measure is computed using the newly computed value of the dependent variable:

$$MEASURE_{i,j}{}^{(k)} = f[\mathrm{y}_{\mathrm{i,j}}^{(k)}]$$

Step 10: Calculation of the error:

The error is computed as a sum of absolute differences between the iterations across all observations:

$$error^{(k)} = \sum_{i=1}^{N}\sum_{j=1}^{k} |\ln(y_{i,j})^{\ (k)} - \ln(y_{i,j})^{\ (k-1)}|$$

If the error is greater than or equal to 10^{-12} then return to step 8, otherwise continue to step 11.

Step 11: Calculation of the decoupled measures of industry specialization, competition and market concentration

We computed measures RATIO_DC, LEADER_DC, DISTANCE_DC and DIFFERENCE_DC using the values of the last iteration of variable y.

We repeated the whole process 1000 times. Therefore, in total, we generated for each measure 4000 random samples with 4096 observations in each sample.

Finally, for each sample we estimated regression models for each market measure.

Appendix C. Variable Definitions

Dependent variable		
LAF	=	Natural logarithm of audit fee of client i in year t
Test variables		
DIFFERENCE	=	The difference of the total audit fees in an MSA-industry market between the largest audit firm in the market and the incumbent auditor of client i in year t ÷ total audit fees in the MSA-industry market. An MSA-industry market is defined as a two-digit SIC industry in a U.S. Metropolitan Statistical Area (MSA, U.S. Census Bureau definition) in year t.
DIFFERENCE_DC	=	Is calculated using the same formula as for the DIFFERENCE but the audit fee of client i in year t is not included in the formulation.
DIFFERENCE_TA	=	DIFFERENCE computed using total assets instead of audit fees
DIFFERENCE_SA	=	DIFFERENCE computed using sales instead of audit fees
Control variables		
LTA	=	Natural logarithm of total assets in million $
LSALES	=	Natural logarithm of sales in million $
LBSEG	=	Natural logarithm of the number of unique business segments
LGSEG	=	Natural logarithm of the number of unique geographic segments
CATA	=	Ratio of current assets to total assets
QUICK	=	Quick assets, i.e., ratio of current assets excluding inventory to current liabilities
LEV	=	Ratio of long-term debt to total assets
ROI	=	Ratio of earnings before interest and tax to total assets
FOREIGN	=	Indicator of foreign operations, equals one if revenue from foreign operations is reported, and zero otherwise
OPINION	=	Indicator of going concern audit report, equals one for a going-concern audit report, and zero otherwise
YE	=	Indicator of busy season, equals one for December 31 year-end, and zero otherwise
LOSS	=	Indicator of accounting loss, equals one if there is a loss in the current year, and zero otherwise
BIG	=	Big auditor indicator variable that equals one for Big 4 auditors, and zero otherwise
Industry FE		2-digit SIC industry sectors' indicators (fixed effects)
MSA FE		MSA indicators (fixed effects)
Year FE		Indicators (fixed effects) for years 2018 and 2019

Appendix D. Replication Files and Interactive Tool

The purpose is to facilitate the replication of the steps taken to download the sample data and estimate all the results which are contained in the paper. It also provides details of the interactive tool referred to in Section 3.1. All files described below are available online (https://doi.org/10.5281/zenodo.7922326).

Step 1: Download the necessary data from WRDS for the empirical study

a. The first step downloads the data from WRDS, execute the Stata do-file "**do1_wrds_download.do**" This do-file will download the full datasets directly from WRDS platform.
b. For this do file to correctly execute, an account on WRDS is necessary and the ODBC driver needs to be installed, see: https://wrds-www.wharton.upenn.edu/pages/support/programming-wrds/programming-stata/stata-from-your-computer/
c. This do file was executed on 27th July 2021 and the associated database saved as a static file as at that date.
d. Since the data is regularly updated by the data providers, it is practical to download all the information and make use of the static download file. As a result, running this script later may result in minor variations between the downloaded data and the data which was used in the manuscript.

Step 2: Save a reduced form of the dataset with the necessary variables

a. The second step saves a reduced form of the dataset downloaded in Step 1 above, which only contains the necessary variables for the subsequent analysis by executing the Stata do-file **"do2_reduce_datasets.do"**

Step 3: Dataset for city-MSA cross-mapping

a. We have provided the Stata dataset **"msa_lookup.dta"** containing the CBSA (core based statistical areas) code for a city if the city lies in a CBSA and the CBSA is a metropolitan statistical area. There are two types of CBSAs – micropolitan statistical areas and metropolitan statistical areas. The term MSA used in the manuscript stands for a core based statistical area which is a *metropolitan* statistical area.
b. To assign the CBSA code, the most recent census delineation file was used from March 2020. Core based statistical areas (CBSAs), metropolitan divisions, and combined statistical areas (CSAs), are available at: https://www.census.gov/geographies/reference-files/time-series/demo/metro-micro/delineation-files.html, (accessed on 27 July 2021). This dataset links counties to CBSA codes.
c. Some cities can be matched directly to CBSA codes using the CBSA title. However, if it was not possible to match a city with a given MSA (CBSA code) based on the census file, we used further information downloaded from:
   - https://simplemaps.com/data/us-cities and,
   - https://github.com/grammakov/USA-cities-and-states/blob/master/us_cities_states_counties.csv
d. If it was still not possible to find the MSA (CBSA code), we found the county manually (using mostly Wikipedia, but also webpages of the U.S. cities and states) and assigned the CBSA code based on the census file. If the city spans over several counties, we used the county where the biggest part of the city lies.

Step 4: Generating the market sample, and final estimation samples

a. The fourth step requires the execution of the Stata do-file **"do3_sample_and_tables.do"** to create the market sample and final estimation samples based upon the reduced dataset created in Step 2. Executing this do-file will result in the automated generation of Table 3 and Table 4.

b. Since the datasets obtained from WRDS change over time, datasets downloaded at a later date may differ from those obtained earlier. Therefore, for transparency, we include the identifiers of observations (company identifier and financial year) for both the market sample (sample_market_identifiers.dta) and the final estimation sample (sample_final_identifiers.dta).

Step 5: Performing the simulations

a. The fifth step requires the execution of the Stata do-file **"do4_simulations.do"** to estimate the results of the simulations reported in Table 1 and Table 2.
b. Executing this do-file may require several hours of computer run time depending on the specifications of the machine being used and flavour of Stata. We ran this file using an iMac 2020 with 3.6GHz i9 processor and Stata MP4, and this took about 8 hours to complete.
c. This do-file creates separate datasets for each measure and market size.

Step 6: Generating Figure 1

The last step requires script **"do5_figure1.do"** to be run to generate Figure 1.

Interactive tool

As referred to in Section 3.1, we have prepared an interactive tool in R-studio that can be used to explore the comparative statics of the three estimators for the coefficients α and β in equations (1) and (2), by changing the values of the underlying parameters. The script r6_shiny.R needs to be run in R-studio. After running the script, the output window needs to be maximized and the app works well if the screen resolution is at least 1920 $\times$ 1080 pixels (Full HD). Alternatively, the tool can be run online (https://marekkacer.shinyapps.io/group_average/).