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The Information in Joint Term Structures of Bond Yields*

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Abstract

While the open-economy macroeconomics literature amply demonstrates the importance of allowing for trade and financial linkages between countries, the finance literature on the term structure of interest rates has largely adopted a "closed-economy" setting in which yields in one country depend only on their own lagged values. This paper examines whether it is in fact necessary to jointly model yields in multiple countries, as proposed by a handful of recent studies. We show that there is little convincing evidence that would point to such a need. The reason is that bond yields are forward-looking

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variables: any relevant information about foreign countries is likely to be already reflected in today's domestic yield curve.

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1. Introduction

The importance of allowing for cross-country linkages in empirical macroecomic models is well-established (see, for example, Pesaran et al. (2004)). In particular, trade and financial linkages mean that shocks hitting one country transmit to others. Thus, closed-economy macroeconomic models may omit important variables and transmission channels. However, the large majority of the finance literature on the term structure of interest rates takes a "closed-economy" approach, in that expected future yields in a single-country depend only on their own lagged values. In this paper, we ask whether those separate, closed-economy term structure models are similarly omitting important information about foreign economies.

We focus on Gaussian no-arbitrage affine term structure models (ATSMs), which are perhaps the most prominent tool for modeling the term structure of interest rates in the finance literature. While the vast majority of previous studies using ATSMs look at yields in a single country in isolation, a literature on joint modeling of interest rates in multiple countries has begun to emerge (examples include Anderson et al. (2010), Egorov et al. (2011), Bauer and Diez de los Rios (2012), Kaminska et al. (2013), and Diez de los Rios (2017)). In these joint ATSMs, expected future yields can depend on lagged values of yields in multiple countries. These studies have typically had two motivations: to model the evolution of yields in multiple countries simultaneously; and / or to predict the exchange rate, which depends on the relative discount factors that price bonds in multiple countries. However, we show there is little evidence that nested separate models omit information that is relevant for either of these purposes.

From the perspective of the open-economic macroeconomics literature, it may appear surprising that we can ignore the information in foreign yields in term structure models despite the trade and financial linkages between countries. To understand it, note first that a standard result in the term structure literature is that three linear combinations of yields can fit the current cross section of yields in a single country with an extremely high degree of precision. Moreover, N principal components extracted from the cross-section of domestic yields must explain domestic yields as least well as any other set of N factors. Thus, replacing any domestic principal components with foreign factors cannot improve the cross-sectional fit. The question of whether joint term structure modeling is necessary therefore comes down to whether there is information in foreign yields that is hidden from (or "unspanned" by) domestic yields but which affects conditional expectations of future domestic yields (in the same way that Joslin et al. (2014) argue that domestic economic variables can). In other words, do foreign yields Granger cause domestic yields? Crucially, even the existence of trade and financial linkages between countries does not guarantee this would be the case; because yields are forward-looking variables, information about foreign economies that matters for the evolution of domestic yields may already be reflected in the current domestic yield curve.

This theoretical justification for considering joint term structure models has been left unclear by previous studies. Moreover, the empirical question of whether there is a need for joint modeling in specific applications has previously been ignored. While the answer is inevitably application-specific, we examine a range of the most popular applications. In particular, we start by looking for evidence of hidden factors in joint models of interest rates in six advanced economies over the period 1990 to 2007. We can test for the presence of unspanned factors fairly straightforwardly, by exploiting the fact that two separate models are equivalent to a restricted joint model. In particular, two standard, separate three-factor models of yields in two countries are equivalent to a restricted six-factor joint model of yields in the two countries. We can therefore test whether the restricted model performs differently from a model that allows for interactions between the two sets of factors. We find that the more flexible models provide no robust, statistically and economically significant benefits relative to the more restricted models, both in terms of their ability to predict future yields and exchange rates. Thus, we conclude that there is no obvious rationale for estimating joint models of yields in the considered applications.

Our finding contrasts with the earlier results of Hodrick and Vassalou (2002), who find evidence that multi-country models can better predict interest rates in four developed economics. However, their study considers models in which the short-term interest rate is driven by only a single factor, which means that single-country models omit substantial information contained in domestic yields. It seems important to us when testing whether there is a need to add information from foreign yields to use a setup which takes into account essentially all of the information in domestic yields. Our results are also related to those of Dahlquist and Hasseltoft (2010), who find that a global factor—constructed as a GDP-weighted average of local factors constructed following Cochrane and Piazzesi (2005)—can help predict bond returns in four developed countries, and also estimate single-country ATSMs that include the global factor alongside domestic yield curve variables. However, their focus is primarily on whether there is a common factor that affects risk premiums across countries (which we discuss further below), and they do not consider whether the ATSMs with the global factor achieve better predictions of interest rates or exchange rates, which is the focus of this paper.[1]

We also show that ATSMs with common factors—that is, factors spanned by yields in both countries—also do not offer significant and robust benefits over standard, separate models. This is important to check because it is possible that allowing for common factors would deliver more precise estimates of those factors and the parameters of the model, and therefore potentially better predictions of future yields. While most previous studies of joint models allow for at least some factors to be common to different classes of yields, they do not consider whether the joint models with common factors offer better predictions of future yields.

[1]They also consider only models that have a particular restricted specification for the prices of risk, whereas we allow for the most flexible specification.

The remainder of this paper proceeds as follows. In Section 2, we set out the separate and joint ATSMs and explain why joint models may have different implications for the dynamics of bond yields. In Section 3, we describe our application to bond yields in six advanced economies. In Section 4, we examine the robustness of our results to allowing for common factors. In Section 5 we summarize our conclusions.

2. Separate and Joint Affine Term Structure Models

Suppose we want to model the yields on two classes of default-risk-free bonds that have payments fixed in different currencies. The "standard" approach would be to model each class of yields using two separate Gaussian ATSMs, which we present in Section 2.1. Alternatively, we can model the two classes of yields jointly using a single ATSM; we present the joint model in Section 2.2, where we also show that two separate models can be written as a restricted joint model.

2.1. Separate Models

The Gaussian ATSM of a single class of yields is entirely standard (see, for example, Duffee (2002)). It makes four assumptions. First, the short-term (that is, one-period) risk-free interest rate relevant for pricing bonds in country j ($r_{j,t}$) is an affine function of an $n_j \times 1$ vector of unobserved pricing factors ($\mathbf{x}_{j,t}$):

$$r_{j,t} = \delta_{j,0,\mathcal{S}} + \delta'_{j,1,\mathcal{S}}\mathbf{x}_{j,t}. \tag{1}$$

Second, there are no arbitrage opportunities from investing in different maturity bonds, which implies that there exists a unique risk-neutral probability measure ($\mathbb{Q}_j$) such that the prices of the j^{th} class of bond ($P_{j,n,t}$) satisfy

$$P_{j,n,t} = \mathbb{E}_t^{\mathbb{Q}_j} \left[\exp\left(-r_{j,t}\right) P_{j,n-1,t+1} \right], \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}_j}$ denotes expectations with respect to the $\mathbb{Q}_j$ measure. Third, the pricing factors follow a first-order VAR under $\mathbb{Q}_j$:

$$\mathbf{x}_{j,t+1} = \mu_{j,\mathcal{S}}^{\mathbb{Q}_j} + \mathbf{\Phi}_{j,\mathcal{S}}^{\mathbb{Q}_j} \mathbf{x}_{j,t} + \mathbf{\Sigma}_{j,\mathcal{S}} \varepsilon_{j,t+1}^{\mathbb{Q}_j}, \tag{3}$$

where $\varepsilon_{j,t+1}^{\mathbb{Q}_j} \sim \mathcal{NID}\left(\mathbf{0}, \mathbf{I}\right)$ is an $n_j \times 1$ vector of Normally distributed shocks. Under these assumptions, the yield on an n-period bond ($y_{j,n,t} \equiv -\frac{1}{n} \log P_{j,n,t}$) is an affine function of the pricing factors, that is, $y_{j,n,t} = -\frac{1}{n}\left(a_{j,n,\mathcal{S}} + \mathbf{b}'_{j,n,\mathcal{S}} \mathbf{x}_{j,t}\right)$ where

$$\begin{aligned} a_{j,n,\mathcal{S}} &= a_{j,n-1,\mathcal{S}} + \mathbf{b}'_{j,n-1,\mathcal{S}} \mu_{j,\mathcal{S}}^{\mathbb{Q}_j} + \frac{1}{2} \mathbf{b}'_{j,n-1,\mathcal{S}} \mathbf{\Sigma}_{j,\mathcal{S}} \mathbf{\Sigma}'_{j,\mathcal{S}} \mathbf{b}_{j,n-1,\mathcal{S}} - \delta_{j,0,\mathcal{S}}, && (4) \\ \mathbf{b}'_{j,n,\mathcal{S}} &= \mathbf{b}'_{j,n-1,\mathcal{S}} \mathbf{\Phi}_{j,\mathcal{S}}^{\mathbb{Q}_j} - \delta'_{j,1,\mathcal{S}}, && (5) \end{aligned}$$

and $a_{j,0,\mathcal{S}} = 0$ and $\mathbf{b}_{j,n,\mathcal{S}} = \mathbf{0}$ (see, for example, Joslin et al. (2011) or the online appendix to this paper for further details).

Finally, the factors follow a first-order vector autoregression (VAR) under the physical probability measure ($\mathbb{P}$):

$$\mathbf{x}_{j,t+1} = \mu_{j,\mathcal{S}} + \mathbf{\Phi}_{j,\mathcal{S}}\mathbf{x}_{j,t} + \mathbf{\Sigma}_{j,\mathcal{S}}\varepsilon_{j,t+1}, \tag{6}$$

where $\varepsilon_{j,t+1} \sim \mathcal{NID}\left(\mathbf{0}, \mathbf{I}\right)$ is an $n_j \times 1$ vector of Normally distributed shocks.

2.2. Joint Model

We next turn to a joint model of yields in two countries. In Section 2.2.1, we set out the assumptions of the joint model and derive expressions for long-term bond yields. In Section 2.2.2, we discuss how to restrict the parameters to ensure that some factors are local to one particular class of yields, and we further show that two separate models are equivalent to a joint model that has only local factors and has over-identifying restrictions on the time-series dynamics of yields. In Section 2.2.3, we explain how we identify the models and estimate them by maximum likelihood.

2.2.1. Bond Pricing and $\mathbb{P}$ Dynamics

The starting point for a joint model is the observation that under the assumption of no arbitrage, the prices of bonds in the first and second countries must satisfy

$$P_{1,n,t} = \mathbb{E}_t^{\mathbb{Q}_1}\left[\exp\left(-r_{1,t}\right) P_{1,n-1,t+1}\right] and \tag{7}$$

$$P_{2,n,t}S_t = \mathbb{E}_t^{\mathbb{Q}_1}\left[\exp\left(-r_{1,t}\right) P_{2,n-1,t+1}S_{t+1}\right], \tag{8}$$

respectively, where S_t is the exchange rate: currency-1 price of one unit of currency 2.

In a joint model, we collect all of the factors that affect yields in either country into a single $n_x \times 1$ vector $\mathbf{x}_t$. The short rate relevant for pricing the first asset class is again affine in these pricing factors:

$$r_{1,t} = \delta_{1,0} + \delta_{1,1}' \mathbf{x}_t. \tag{9}$$

Following Diez de los Rios (2008) and Abrahams et al. (2016), the change in the exchange rate ($\Delta s_t = \log S_t - \log S_{t-1}$) is affine in the factors:

$$\Delta s_t = s_0 + \mathbf{s}_1' \mathbf{x}_t. \tag{10}$$

The factors again follow a first-order VAR under $\mathbb{Q}_1$, that is,

$$\mathbf{x}_{t+1} = \mu^{\mathbb{Q}_1} + \mathbf{\Phi}^{\mathbb{Q}_1} \mathbf{x}_t + \mathbf{\Sigma} \varepsilon_{t+1}^{\mathbb{Q}_1}, \tag{11}$$

where $\varepsilon_{t+1}^{\mathbb{Q}_1} \sim \mathcal{NID}(\mathbf{0}, \mathbf{I})$. Under these assumptions, the pricing of country-1 bonds is directly analogous to the separate model of Section 2.1, with yields given by

$$y_{1,n,t} = -\frac{1}{n}\left(a_{1,n} + \mathbf{b}_{1,n}' \mathbf{x}_t\right), \tag{12}$$

where

$$a_{1,n} \;=\; a_{1,n-1} + \mathbf{b}_{1,n-1}' \mu^{\mathbb{Q}_1} + \frac{1}{2} \mathbf{b}_{1,n-1}' \mathbf{\Sigma\Sigma}' \mathbf{b}_{1,n-1} - \delta_0, \tag{13}$$

$$\mathbf{b}'_{1,n} = \mathbf{b}'_{1,n-1}\mathbf{\Phi}^{\mathbb{Q}_1} - \delta'_1, \tag{14}$$

and $a_{2,0} = 0$ and $\mathbf{b}_{2,0} = \mathbf{0}$. Yields on country-2 bonds are given by

$$y_{2,n,t} = -\frac{1}{n}\left(a_{2,n} + \mathbf{b}'_{2,n}\mathbf{x}_t\right), \tag{15}$$

where

$$a_{2,n} = a_{2,n-1} + s_0 + \left(\mathbf{s}_1 + \mathbf{b}_{2,n-1}\right)' \mu^{\mathbb{Q}_1} + \frac{1}{2}\left(\mathbf{s}_1 + \mathbf{b}_{2,n-1}\right)' \mathbf{\Sigma}\mathbf{\Sigma}' \left(\mathbf{s}_1 + \mathbf{b}_{2,n-1}\right) - \delta_0, \tag{16}$$

$$\mathbf{b}'_{2,n} = \left(\mathbf{s}_1 + \mathbf{b}_{2,n-1}\right)' \mathbf{\Phi}^{\mathbb{Q}_1} - \delta'_1, \tag{17}$$

and $a_{2,0} = 0$ and $\mathbf{b}_{2,n} = \mathbf{0}$ (see Abrahams et al. (2016) or the online appendix to this paper for further details).

Equations (15)-(17) imply that the country-2 short rate takes the form

$$r_{2,t} = \delta_{2,0} + \delta'_{2,1}\mathbf{x}_t \tag{18}$$

where

$$\delta_{2,0} = \delta_{1,0} - s_0 - \mathbf{s}'_1\mu^{\mathbb{Q}_1} - \frac{1}{2}\mathbf{s}'_1\mathbf{\Sigma}\mathbf{\Sigma}'\mathbf{s}_1 and \tag{19}$$

$$\delta_{2,1} = \delta_{1,1} - \left(\mathbf{\Phi}^{\mathbb{Q}_1}\right)' \mathbf{s}_1 \tag{20}$$

(see the online appendix for details). Thus, we equivalently parameterize the joint model in terms of $\delta_{2,0}$ and $\delta_{2,1}$, rather than s_0 and $\mathbf{s}_1$.

Finally, the factors again follow a first-order Gaussian VAR under $\mathbb{P}$:

$$\mathbf{x}_{t+1} = \mu + \mathbf{\Phi}\mathbf{x}_t + \mathbf{\Sigma}\varepsilon_{t+1}, \tag{21}$$

where $\varepsilon_{t+1} \sim \mathcal{NID}\left(\mathbf{0}, \mathbf{I}\right)$.

2.2.2. Common and Local Factors

In a maximally-flexible joint model, all of the factors may be common to yields in both countries; that is, yields in both countries may load on all of the pricing factors. However, when testing for the presence of unspanned factors in foreign yields, it is more convenient to work with models that have only local factors, which have zero loadings for yields in all but one country (previous studies that impose that some factors are local include Egorov et al. (2011) and Kaminska et al. (2013)). These local factors are "hidden" from (or "unspanned" by) yields in all but one country.

Suppose that we want to restrict a joint model with n_x factors such that there are n_c common factors, n_{l_1} factors local to yields in country 1, and n_{l_2} factors local to yields in country 2 (with $n_x = n_c + n_{l_1} + n_{l_2}$). In such a model, $n_c + n_{l_1}$ factors are spanned by the first class of yields and $n_c + n_{l_2}$ factors are spanned by the second class of yields. Such a specification requires that the short rate loadings take the forms

$$\begin{aligned} \delta_{1,1} &= \left[\delta'_{1,1,c}, \delta'_{1,1,l_1}, \mathbf{0}'_{n_{l_2}\times 1}\right]' \textit{and} \qquad (22) \\ \delta_{2,1} &= \left[\delta'_{2,1,c}, \mathbf{0}'_{l_1\times 1}, \delta'_{2,1,l_2}\right]', \qquad (23) \end{aligned}$$

where $\delta_{1,1,c}$ and $\delta_{2,1,c}$ are $n_c \times 1$, $\delta_{1,1,l_1}$ is $n_{l_1} \times 1$, and $\delta_{2,1,l_2}$ is $n_{l_2} \times 1$; and that $\mathbf{\Phi}^{\mathbb{Q}_1}$ takes the form

$$\mathbf{\Phi}^{\mathbb{Q}_1} = \begin{bmatrix} \mathbf{\Phi}_{cc}^{\mathbb{Q}_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{\Phi}_{l_1c}^{\mathbb{Q}_1} & \mathbf{\Phi}_{l_1l_1}^{\mathbb{Q}_1} & \mathbf{0} \\ \mathbf{\Phi}_{l_2c}^{\mathbb{Q}_1} & \mathbf{0} & \mathbf{\Phi}_{l_2l_2}^{\mathbb{Q}_1} \end{bmatrix}, \tag{24}$$

where $\mathbf{\Phi}_{cc}^{\mathbb{Q}_1}$ is $n_c \times n_c$, $\mathbf{\Phi}_{l_1c}^{\mathbb{Q}_1}$ is $n_{l_1} \times n_c$, $\mathbf{\Phi}_{l_1l_1}^{\mathbb{Q}_1}$ is $n_{l_1} \times n_{l_1}$, $\mathbf{\Phi}_{l_2c}^{\mathbb{Q}_1}$ is $n_{l_2} \times n_c$, and $\mathbf{\Phi}_{l_2l_2}^{\mathbb{Q}_1}$ is $n_{l_2} \times n_{l_2}$. Under the zero restrictions in equations (22)-(24) (which we refer to as the "$\mathbb{Q}_1$ restrictions"), we can partition the pricing factors conformably as $\mathbf{x}_t = \left[\mathbf{x}_{c,t}', \mathbf{x}_{l_1,t}', \mathbf{x}_{l_2,t}'\right]'$, where $\mathbf{x}_{c,t}$ are common factors, and $\mathbf{x}_{l_1,t}$ and $\mathbf{x}_{l_2,t}$ are factors local to yields in countries 1 and 2, respectively.

We can test for unspanned factors in yields in other countries by comparing two separate models with a joint model that has only local factors but allows for time-series interactions between two sets of local factors. Note first that we can partition $\mathbf{\Phi}$ as

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_{cc} & \mathbf{\Phi}_{cl_1} & \mathbf{\Phi}_{cl_2} \\ \mathbf{\Phi}_{l_1c} & \mathbf{\Phi}_{l_1l_1} & \mathbf{\Phi}_{l_1l_2} \\ \mathbf{\Phi}_{l_2c} & \mathbf{\Phi}_{l_2l_1} & \mathbf{\Phi}_{l_2l_2} \end{bmatrix}. \tag{25}$$

In general, a model with local factors need not impose any restrictions on the $\mathbb{P}$ dynamics of yields. However, two separate models with n_1 and n_2 factors are equivalent to a joint model that imposes the $\mathbb{Q}_1$ restrictions such that there are no common factors ($n_c = 0$), and n_1 and n_2 local factors ($n_{l_1} = n_1$ and $n_{l_2} = n_2$); and that imposes restrictions on the $\mathbb{P}$ dynamics such that the two sets of local factors are independent, that is, with $\mathbf{\Phi}_{l_1 l_2} = \mathbf{0}$ and $\mathbf{\Phi}_{l_2 l_1} = \mathbf{0}$ in Equation (25) (the online appendix provides further details). If we can accept these $\mathbb{P}$ restrictions, then we would have evidence that there is no relevant marginal information in each class of yields for modeling the time-series dynamics of the other class. Alternatively, if we cannot accept these restrictions, then we would have evidence that the time-series dynamics of yields are misspecified in separate models because they do not allow for interactions between the factors spanned by different classes of yields.

2.2.3. Identification and Estimation

As discussed by, for example, Dai and Singleton (2000), Joslin et al. (2011), and Hamilton and Wu (2012), before we take an ATSM to the data we need to impose a minimum set of identifying restrictions. A maximally flexible model would have only common factors (that is, would not impose the $\mathbb{Q}_1$ or $\mathbb{P}$ restrictions mentioned in the previous section). To identify such a model, we could impose that $\mu^{\mathbb{Q}_1} = \mathbf{0}$, $\mathbf{\Sigma} = \mathbf{I}$, and $\mathbf{\Phi}_{cc}^{\mathbb{Q}_1}$ is a lower triangular matrix with ordered diagonal elements ($\phi_{cc,11}^{\mathbb{Q}_1} \geq \phi_{cc,22}^{\mathbb{Q}_1} \geq ... \geq \phi_{cc,n_c n_c}^{\mathbb{Q}_1}$). In models that have local factors we also require the identifying assumptions that $\mathbf{\Phi}_{l_1 l_1}^{\mathbb{Q}_1}$ and $\mathbf{\Phi}_{l_2 l_2}^{\mathbb{Q}_1}$ are lower triangular with ordered diagonal elements.[2]

[2]We impose that $\mathbf{\Phi}_{cc}^{\mathbb{Q}_1}$, $\mathbf{\Phi}_{l_1 l_1}^{\mathbb{Q}_1}$, and $\mathbf{\Phi}_{l_2 l_2}^{\mathbb{Q}_1}$ have only real eigenvalues. Strictly speaking, a maximally flexible model allows for complex eigenvalues (see Joslin et al. (2011)).

When estimating joint models, we assume that all yields are measured with error. Specifically, we allow for measurement error by assuming that observed yields are given by

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_t^* \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^* \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^* \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \mathbf{w}_t \\ \mathbf{w}_t^* \end{bmatrix}. \tag{26}$$

Here, $\mathbf{y}_t$ is an $n_{y_1} \times 1$ vector of observed yields on the first class of bonds; $\mathbf{y}_t^*$ is an $n_{y_2} \times 1$ vector of observed yields on the second class of bonds; $\mathbf{w}_t \sim \mathcal{NID}\left(\mathbf{0}, \sigma_w^2 \times \mathbf{I}\right)$ and $\mathbf{w}_t^* \sim \mathcal{NID}\left(\mathbf{0}, \sigma_{w^*}^2 \times \mathbf{I}\right)$ are $n_{y_1} \times 1$ and $n_{y_2} \times 1$ vectors of measurement errors; and the definitions of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{A}^*$, and $\mathbf{B}^*$ follow from equations (13), (14), (16), and (17). Equations (21) and (26) form a linear-Gaussian state-space system, and we can therefore estimate the free parameters of the model by maximum likelihood, using the Kalman filter to estimate the latent pricing factors for a given set of parameters.[3]

3. Evidence Regarding Unspanned Information

We now turn to our application to yields in six countries. In Section 3.1, we describe our data set and explain the adopted factor structure of the joint models we consider. In Sections 3.2 and 3.3, we evaluate whether joint models can offer improvements over separate models when it comes to

[3] D'Amico et al. (2018) (among others) provide further details on the estimation of joint models by maximum likelihood using the Kalman filter. Other approaches to estimating joint models exist that assume that some linear combinations of yields are measured without error (for example, Abrahams et al. (2016) and Diez de los Rios (2017)).

predicting yields and exchange rates, respectively.

3.1. Data and Factor Structure

Our data set consists of month-end zero-coupon nominal government bond yields for six countries: Australia, Canada, Germany, Switzerland, the United Kingdom, and the United States. The sample period starts in January 1990 and ends in December 2007. Starting the sample in 1990 is broadly consistent with previous studies of ATSMs of U.S. nominal yields, while ending it in December 2007 largely avoids complications caused by the proximity of nominal bond yields in various countries to the ZLB; we consider the robustness of our results for the U.S.-Germany country pair to extending the sample in a model that enforces the ZLB. At each point in time, we consider a cross section of yields with maturities of six months and one, two, three, five, seven, and ten years.[4]

Table 1 reports the results of principal components analyses conducted on yields in each country separately; specifically, the cumulative proportion of the variation in yields for each country explained by adding additional principal components. For all six countries, three principal components explains essentially all (at least 99.98 percent) of the variation in yields. As is standard in the literature on ATSMs, we therefore assume that three pricing

[4]Data for Australia, Canada, and Switzerland are taken from data set of Wright (2011). Data for Germany are published by the Bundesbank, for the United Kingdom by the Bank of England, and for the United States by the Board of Governors of the Federal Reserve System.

factors extracted from yields span all of the relevant information contained in the yield curve of each country.

[Insert Table 1 here]

To evaluate whether there is any unspanned overseas information that is relevant for predicting domestic yields, we compare two joint models for all possible bilateral pairs of the six countries. Both joint models have six factors, with three factors spanned by yields in one country and the other three factors spanned by yields in the other country, consistent with the standard assumption that three factors are spanned in separately-estimated models. That is, we impose the $\mathbb{Q}_1$ restrictions such that there are no common factors ($n_c = 0$), and 3 local factors spanned by yields in each country ($n_{l_1} = n_{l_2} = 3$). The first joint model is restricted such that there are no interactions between the two sets of spanned factors, that is, $\mathbf{\Phi}_{l_1 l_2} = \mathbf{\Phi}_{l_2 l_1} = \mathbf{0}$. This restricted model is exactly equivalent to two standard, separate three-factor models. The second joint model has unrestricted $\mathbb{P}$ dynamics, that is, $\mathbf{\Phi}_{l_1 l_2}$ and $\mathbf{\Phi}_{l_2 l_1}$ are free parameters. Thus, the only difference between the two models is that second allows for interactions between the spanned factors under $\mathbb{P}$. As discussed above, if the unrestricted model performs significantly better than the restricted model in some dimension, then we would have evidence in favor of joint modeling.

3.2. Predicting Bond Yields

3.2.1. Likelihood-Based Model Selection

We start by evaluating how restricted and unrestricted models compare using standard statistical approaches to model selection based on the value of the optimized likelihood function. Panel $\mathcal{A}$ of Table 2 reports the optimized log likelihoods for each of the models. The numbers above and below the diagonal elements of each panel report the results for the restricted and unrestricted models, respectively. Because the restricted models are nested by the corresponding $\mathbb{P}$-unrestricted models, the log likelihoods are necessarily higher for the unrestricted models. To take into account the greater number of parameters in the unrestricted models, Panel $\mathcal{B}$ of Table 2 reports the Schwarz Information Criterion (SIC) for each of the models. The SIC favors the restricted models in almost all cases, the exception being for joint models of Australian and Canadian yields. Thus, we conclude from the SICs that joint modeling is generally undesirable.

[Insert Table 2 here.]

3.2.2. In-Sample Fit

We next turn to a comparison of in-sample fitting errors. Here, we focus on how well the models predict yields at a 1-month horizon. Although it is more common to consider fitting errors to the current cross section of yields, 1-month-ahead prediction errors may be considered a purer measure of in-sample fit because the maximum likelihood estimation is actually minimizing (weighted) 1-month-ahead prediction errors. One-month-ahead errors are also more interesting because they may be more directly affected by whether or not we allow for time-series interactions between the factors spanned by domestic and foreign yields. In contrast, a comparison of fitting errors to the current cross section of yields is uninteresting because both restricted and unrestricted models have the same factor structure in the cross section.[5]

Table 3 reports the root mean squared errors (RMSEs) between yields at month $t+1$ and the time-t conditional expectations. Panels $\mathcal{A}$ and $\mathcal{B}$ report results for the 1- and 10-year yields, respectively. In each panel, the rows report the fit to yields in different "domestic" countries. The first column reports the RMSE in the restricted joint models, while the remaining columns report how the cross-sectional RMSEs for the domestic country's yields change in unrestricted joint models with yields from different "foreign" countries. We highlight the following results. First, average 1-step-ahead errors are, unsurprisingly, larger than cross-sectional errors, with RMSEs ranging from about 20 to 40 basis points for 1- and 10-year yields in the restricted models reported in the first column. Second, in most cases the reduction in the RMSEs from allowing for time-series interactions between the two sets of local factors are generally statistically insigificant: Diebold-

[5]In our setting with unobserved pricing factors the fit to the current cross section is not *exactly* the same because the factors may differ between restricted and unrestricted models. However, unreported results show that the results are trivial. Other unreported results show that the cross-sectional fit of restricted and unrestricted models to yields not included in the estimation (we consider 4- and 8-year yields) is essentially identical.

Mariano tests of the difference in the squared errors would only lead us to reject the null of equal predictive power of restricted and unrestricted models in a small handful of cases. And third, even in the cases where the difference is statistically significant, they are economically trivial: The largest improvement in the RMSE from joint modeling is just 2 basis points. Thus, we conclude that we cannot meaningfully distinguish between joint models and nested separate models in terms of their in-sample fit. In the following sections we therefore turn to features of the data that are not included directly in the estimation.

[Insert Table 3 here.]

3.2.3. Predicting Yields at a Longer-Horizon

An obvious place to start is to consider the ability of the models to predict yields at longer horizons. Specifically, we consider 12-month-ahead forecasts, which is a fairly standard horizon in yield-forecasting regressions. This provides an test of a feature of the data that is not included directly in the estimation. It is not *a priori* obvious whether relaxing the $\mathbb{P}$ restrictions will bring advantages in matching yields at a 12-month horizon. On the one hand, it is possible that the more flexible specification would allow the model to better predict yields. However, on the other hand, it is possible that the greater number of parameters will result in in-sample over-fitting at the 1-month horizon, resulting in weaker performance at longer horizons.

Table 4 reports the RMSEs between yields at time $t+12$ and time-t conditional expectations. We highlight three results. First, as we would expect, 12-step-ahead prediction errors are substantially larger than the 1-step-ahead errors, with RMSEs from the restricted models ranging from about 75 to 160 basis points. Second, in some cases, the unrestricted models do improve on the restricted models and, at first glance, some of the reductions in RMSEs appear fairly sizeable; for example, the RMSE for the Swiss 1-year yield is reduced by about 20 percent in a unrestricted model compared with a restricted model. However, third, none of the reductions is statistically significant: Diebold-Mariano tests cannot reject the null of equal predictive ability at the 5 percent level. Thus, we conclude that there is little compelling evidence to support joint models in terms of their ability to predict bond yields. We also note that there is little consistency across prediction horizon about which is the best model for a given country pair.

[Insert Table 4 here]

3.3. Predicting Exchange Rates

We next turn to the question of whether unrestricted joint models bring any advantages when it comes to predicting exchange rates.[6] For both the restricted and unrestricted models, we can straightforwardly compute the model-implied change in the exchange rate using Equation (10). Table 5 reports the RMSEs for the differences between the model-implied 1-month

[6]In the online appendix, we also consider another out-of-sample comparison: maximal Sharpe ratios implied by the models. We do not find evidence of a notable difference between joint and separate models.

changes in the exchange rate and the actual changes, for different country pairs. The numbers below the diagonal refer to unrestricted models, while the numbers above the diagonal refer to restricted models. We highlight two results. First, the models do not fit exchange rates closely, with RMSEs that are of the order of 15 to 50 percentage points (annualized). Figure 1 illustrates the problem for one country pair, the United States and Germany: the model-implied changes in exchange rates are far too smooth to match the observed changes.

[Insert Table 5 and Figure 1 here]

Second, the fit to changes in the exchange rate is essentially the same in the restricted and unrestricted joint models. As shown by Chernov and Creal (2019), matching features of the exchange rate requires the inclusion of an additional, dedicated exchange rate factor that is unspanned by bond yields. But of course that does not require a joint model: if there is unspanned information in the exchange rate that is relevant for the dynamics of bond yields, a simpler option would be simply to include the exchange rate as an unspanned factor in a separate three-factor model. Thus, we conclude that a desire to match the exchange rate does not, in itself, motivate joint modeling.

4. Common Factors

The previous section exploits the fact that two separate three-factor models are equivalent to a six-factor joint model with restrictions imposed on the $\mathbb{P}$ dynamics, in order to test for the presence of unspanned information in overseas yields by comparing this restricted model with an equivalent joint model with unrestricted $\mathbb{P}$ dynamics. However, it is possible that this approach may be unfair to joint models more generally. All of the joint models discussed above assume a particular factor structure, in which three factors are spanned only by yields in one country and the other three factors are spanned only by yields in the other country. If some of the six factors are actually common to the cross sections of both sets of yields, then we may obtain more precise estimates of the common factors and their time-series dynamics if we impose that *a priori*.

In this section, we therefore consider whether our results are sensitive to allowing one or two of the factors spanned by the yields in each country to be common to the cross sections of yields in each country, while holding the total number of factors spanned by yields in each country constant, at the standard three.[7] Specifically, we estimate two additional models for each country pair. The first additional model has five factors in total; one common factor and two factors local to each of the two countries. The second additional

[7] In the online appendix, we report the results of various other robustness checks. First, we show that applying restrictions to the time-series dynamics considered in the previous literature on joint models does not affect our core conclusions for joint models of U.S. and German yields. Second, in the only application we find that provides any support for joint modeling, we find a statistically significant improvement in predictions of German yields in a joint model of U.S. and German yields that is extended to cover the more recent period at the zero lower bound. However, even in that case, it appears that the reason relevant information is hidden from German yields is specifically linked to the special case of interest rates being at the zero lower bound. Third, we show that including 3-month yields in a joint models of U.S. and U.K. yields does not affect our main conclusion. Fourth, we show a similar conclusion for joint models of U.S. yields with South African and Singaporean yields (from the data set constructed by Lynch (2019)).

model has four factors in total: two common factors and one factor local to each of the two countries. It is worth stressing an important point about the factor structure of these models: We maintain the standard assumption that three factors are spanned by yields in each country, as in the joint models of Egorov et al. (2011) and Kaminska et al. (2013). Some studies of joint models allow for more than three factors to be spanned by yields in each country. For example, Anderson et al. (2010) estimate two-country models with five common factors. However, if three factors are sufficient to span yields in each country, which is the standard assumption in most of the literature on ATSMs, we also know *a priori* that a specification with more than three spanned factors for yields in each country must be over-parameterized, because it must be possible to rotate the joint model such that only three factors have a non-zero loading on each class of yields. In addition, if we allow more than three spanned factors for each yield curve, then we could not be sure whether any improvements offered by joint models are because we are incorporating unspanned information in overseas yields or because we are incorporating more information from domestic yields (such as the fourth or fifth principal component of domestic yields, as proposed by Duffee (2011)).[8]

[8]If there is some common variation in the two classes of yields, then two separate models with n_1 and n_2 factors may not fit the cross sections of current yields quite as well as a joint model with $n_x = n_c = n_1 + n_2$ factors. Indeed, preliminary (unreported) results showed that a maximally flexible joint model with six common factors fits the cross-section of U.S. and German yields marginally better than two separate three-factor models. But of course two separate six-factor models of U.S. and German yields would achieve an even

Table 6 reports the SIC for the four- and five-factor joint models; the numbers above and below the diagonal refer to four- and five-factor models, respectively. All of the SICs for the models with common factors are clearly smaller in magnitude than those for the six-factor joint models reported in Table 2, with the four-factor model performing worse then the five-factor model. These results suggest that we should not prefer joint models with common factors. This finding is consistent with the previous results of Golinski and Spencer (2018), who show that six common factors mimic two sets of three independent country factors in a joint model of German and U.S. bond yields.

[Insert Table 6 here]

Table 7 reports the RMSEs for 12-step-ahead prediction errors of 10-year yields in models with five and four factors; Panels $\mathcal{A}$ and $\mathcal{B}$ report results for five- and four-factor models, respectively. These RMSEs are generally of the same order of magnitude as for the six-factor models. There are a few cases where the four- and / or five-factor models beat the six-factor model but, with one exception, none of the improvements are statistically significant at the 5 percent significance level according to Diebold-Mariano tests. The one exception is that the prediction errors in joint models of the U.K. yield in a

better cross-sectional fit than a maximally flexible six-factor joint model. This finding is essentially the result reported in Table 2 in Egorov et al. (2011). They show that a given number of principal components of (dollar) Libor and Euribor rates explains less of the pooled data sets than the same number of principal components explains in each of the separate data sets.

joint model of U.K. and Swiss yields are significantly smaller in the four- and five-factor models compared with the six-factor model. However, unreported results for a recursive out-of-sample forecasting exercise show that even this improvement is not robust out of sample. Thus, we conclude that allowing for some of the spanned factors to be common to yields in both countries does not affect our core conclusion that there is little to be gained from joint modeling in the considered applications.

[Insert Table 7 here]

5. Conclusion

This paper shows that there is not strong evidence that it is necessary to jointly model yields in multiple countries in an ATSM framework. The case rests on there being information in the yields of one country that is unspanned by yields in a second country, but which is relevant for explaining conditional expectations of yields in the second country. However, across pairs of six large economies, there is little convincing empirical evidence of that being the case. Predictions of yields from three-factor ATSMs estimated on yields of a single country in isolation are not worse than predictions from six-factor joint ATSMs that have the same cross-sectional specification but that allow for time-series interactions between the factors. The one exception to that conclusion is that we find some gains from joint modeling for a joint shadow rate model of U.S. and German yields, although those gains appear to be related to the better identification of pricing factors at the ZLB, rather than the presence of unspanned information in foreign yields. This leaves open the question as to whether or not international macro factors, like domestic macro factors may be able to, can provide relevant information for forecasting future domestic interest rates, but that remains beyond the scope of the present paper.

While the theory about when joint models are needed is general, the empirical answer to whether this is the case in practice is inevitably application-specific. However, our results cover the large majority of the most popular applications considered in previous literature. It is possible that alternative applications, or even alternative sample periods, would produce different results. But the lesson we draw from our results is that for any considered application, it is always worth asking first whether there is anything to be gained from a joint model; based on our results, we would be surprised if many such applications are easy to find.

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Table 1: Principal Components Analysis

This table reports the cumulative percentage of variation in yields explained by additional principal components. For each country, a panel of 6-month and 1-, 2-, 3-, 5-, 7-, and 10-year yields from January 1990 to December 2007 are used for computing the principal components.

Principal Component	Country					
	Australia	Canada	Germany	Switzerland	United Kingdom	United States
1	99.12	99.10	96.16	99.68	99.02	91.91
2	99.97	99.97	99.74	99.97	99.93	99.76
3	100.00	100.00	99.99	100.00	99.98	99.98
4	100.00	100.00	100.00	100.00	100.00	100.00

Table 2: Log Likelihoods for Joint Models

This table reports the log likelihoods and Schwarz Information Criteria (SIC) for various six-factor joint models. Panel $\mathcal{A}$ reports the likelihoods and panel $\mathcal{B}$ reports the SICs. The joint models are for various pairs of countries and are estimated using a sample from January 1990 to December 2007. The numbers below the diagonals refer to models in which the time-series dynamics of the factors are unrestricted, while the numberrs above the diagonal refer to models in which the time-series dynamics are restricted such the joint models are equivalant to two separate three-factor models.

$\mathcal{A}$: Likelihood	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	4307	4502	4941	3772	4398
Canada	4655	–	3974	4413	3244	3869
Germany	4534	4002	–	4608	3438	4063
Switzerland	4979	4444	4662	–	3877	4502
United Kingdom	3826	3265	3450	3912	–	3333
United States	4416	3902	4086	4534	3357	–
$\mathcal{B}$: SIC	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	−8246	−8635	−9513	−7175	−8427
Canada	−8805	–	−7579	−8457	−6119	−7369
Germany	−8556	−7491	–	−8846	−6508	−7758
Switzerland	−9446	−8375	−8811	–	−7386	−8636
United Kingdom	−7139	−6018	−6387	−7311	–	−6298
United States	−8318	−7291	−7658	−8556	−6200	–

Table 3: One-Month-Ahead Prediction Errors for Joint Models

This table reports the in-sample 1-month ahead fit of various models to yields. Panels $\mathcal{A}$ and $\mathcal{B}$ report the fit for the 1- and 10-year yield, respectively. All figures are root mean squared prediction errors in annualized percentage points. The first column ("None") reports the fit of a standard 3-factor model estimated using only the yields for a single "domestic" country. The remaining columns report the fit to the yields in the domestic country in six-factor joint models with different "foreign" countries. A * or ** indicate that a joint model significantly outperforms the single-country model at the 5 and 1 percent significance levels, respectively. All models are estimated using a sample from January 1990 to December 2007.

$\mathcal{A}$: 1-Year Yield	Foreign Country						
Domestic Country	None	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	0.25	–	0.24	0.24	0.24	0.24	0.24
Canada	0.41	0.39	–	0.40	0.40	0.40**	0.40
Germany	0.25	0.25	0.24	–	0.25	0.25	0.24
Switzerland	0.20	0.20	0.19	0.18	–	0.20	0.19*
United Kingdom	0.30	0.29	0.29	0.29	0.29	–	0.29
United States	0.24	0.24	0.24	0.24	0.24	0.24	–
$\mathcal{B}$: 10-Year Yield	Foreign Country						
Domestic Country	None	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	0.35	–	0.34	0.34	0.34	0.34	0.34
Canada	0.39	0.39	–	0.39	0.39	0.39	0.38
Germany	0.22	0.22	0.21	–	0.21*	0.22	0.21
Switzerland	0.20	0.20	0.20	0.19**	–	0.21	0.20
United Kingdom	0.32	0.31	0.32	0.32	0.32	–	0.31
United States	0.26	0.26	0.26	0.25	0.25	0.26	–

Table 4: Twelve-Month-Ahead Prediction Errors for Joint Models

This table reports the 12-month ahead fit of various models to yields. Panels $\mathcal{A}$ and $\mathcal{B}$ report the fit for the 1- and 10-year yield, respectively. All figures are root mean squared prediction errors in annualized percentage points. The first column ("None") reports the fit of a standard 3-factor model estimated using only the yields for a single "domestic" country. The remaining columns report the fit to the yields in the domestic country in six-factor joint models with different "foreign" countries. None of the differences between the restricted and unrestricted models are significant at the 5 percent significance level according to Diebold-Mariano tests. All models are estimated using a sample from January 1990 to December 2007.

$\mathcal{A}$: 1-Year Yield	Foreign Country						
Domestic Country	None	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	1.10	–	1.16	1.21	1.14	1.05	1.09
Canada	1.57	1.29	–	1.45	1.49	1.40	1.32
Germany	1.09	0.96	0.91	–	1.04	0.95	0.99
Switzerland	1.04	1.09	0.92	0.83	–	0.94	0.86
United Kingdom	1.34	1.20	1.18	1.28	1.25	–	1.20
United States	1.30	1.24	1.35	1.30	1.20	1.31	–
$\mathcal{B}$: 10-Year Yield	Foreign Country						
Domestic Country	None	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	1.12	–	1.18	1.18	1.26	1.15	1.14
Canada	1.34	1.21	–	1.25	1.33	1.26	1.20
Germany	0.89	0.84	0.79	–	0.88	0.85	0.86
Switzerland	0.73	0.66	0.62	0.63	–	0.78	0.68
United Kingdom	1.30	1.27	1.14	1.22	1.23	–	1.16
United States	0.78	0.75	0.76	0.75	0.71	0.77	–

Table 5: Fitting the Exchange Rate

This table reports RMSEs of six-factor joint models to exchange rates. The numbers below the diagonal report results for unrestricted joint models and the numbers above the diagonal refer to models restricted to be equivalent to two separate three-factor models. All results are reported as annualized percentage points. All models are estimated using a sample from January 1990 to December 2007.

Domestic Country	Foreign Country					
	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	27.64	43.67	40.18	42.05	32.19
Canada	27.62	–	40.30	36.65	39.60	21.41
Germany	43.67	40.30	–	13.01	33.04	36.27
Switzerland	40.18	36.65	13.01	–	30.06	33.82
United Kingdom	46.41	40.66	34.37	30.29	–	36.11
United States	32.19	21.40	36.27	33.82	39.03	–

Table 6: Model Selection Criteria for Joint Models with Common Factors

This table reports the Schwarz Information Criteria (SIC) for various joint models with common factors. The joint models are for various pairs of countries. The numbers below the diagonals refer to five-factor models, while the numberrs above the diagonal refer to four-factor models. All models are estimated using a sample from January 1990 to December 2007.

	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	−3729	−391	−6084	−4680	−3700
Canada	−6023	–	−4296	−5454	−3992	−4987
Germany	−6552	−5062	–	−3822	−2876	−2793
Switzerland	−8016	−6956	−5319	–	−4904	−4743
United Kingdom	−5738	−5765	−5475	−5619	–	−2915
United States	−5696	−5969	−5448	−6463	−4180	–

Table 7: 12-Month Prediction Errors for Joint Models: Results for Models with Common Factors: 10-Year Yields

This table reports the 12-month ahead fit of various models to 10-year yields. Panels $\mathcal{A}$ and $\mathcal{B}$, report the fit for 5- and 4-factor joint models, respectively. All figures are root mean squared prediction errors in annualized percentage points. The first column ("None") reports the fit of a standard 3-factor model estimated using only the yields for a single "domestic" country. The remaining columns report the fit to the yields in the domestic country in joint models with different "foreign" countries. A * or ** indicate that a joint model significantly outperforms the single-country model at the 5 and 1 percent significance levels, respectively. All models are estimated using a sample from January 1990 to December 2007.

$\mathcal{A}$: 5-Factor Model	Foreign Country					
Domestic Country	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	1.43	1.49	1.40	1.43	1.33
Canada	1.33	–	1.20	1.24	1.22	1.18
Germany	0.85	0.81	–	0.87	0.87	0.85
Switzerland	0.65	0.69	0.70	–	0.69	0.67
United Kingdom	1.27	1.36	1.26	1.11*	–	1.16
United States	0.84	0.79	0.77	0.71	0.79	–

$\mathcal{B}$: 4-Factor Model	Foreign Country					
Domestic Country	Australia	Canada	Germany	Switzerland	United Kingdom	United States
Australia	–	1.23	1.35	1.78	1.13	1.17
Canada	1.08	–	1.45	1.26	1.26	1.23
Germany	0.85	0.89	–	0.92	0.88	0.86
Switzerland	0.73	0.67	0.70	–	0.96	0.78
United Kingdom	1.35	1.35	1.30	1.08*	–	1.08
United States	0.78	1.01	0.77	0.78	0.79	–

Figure 1: Fitting the Exchange Rate: U.S. and Germany

This figures shows the monthly change in the exchange rate between the U.S. dollar and euro (where the U.S. is defined as the domestic country) and the fitted values implied by separately-estimated three-factor models of yields in each country and by a six-factor joint model with unrestricted time-series dynamics. Both models are estimated using a sample from January 1990 to December 2007.