Article:

Dye, T.S., Buck, C. orcid.org/0000-0002-0872-9504, DiNapoli, R.J. et al. (1 more author) (2023) Bayesian chronology construction and substance time. Journal of Archaeological Science, 153. 105765. ISSN 0305-4403

https://doi.org/10.1016/j.jas.2023.105765

Title Page

Title Bayesian Chronology Construction and Substance Time
Word count 5,681
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Abstract

Two views of archaeological time are distinguished; an event view that models stratigraphic relations, and a substance view that models genealogical relations among artifacts, including the three modes of change represented by branching, transformation, and reticulation. Chronology construction is more complex in substance time than it is in event time, which only concerns transformation. Allen's interval algebra can be used to specify the chronological relations associated with the modes of change, and these relations can be identified by post-processing the output from Bayesian chronological models. A worked example illustrates how identifying the chronological relations can aid construction of a phyletic seriation of beads recovered from Anglo-Saxon female graves. These results might encourage archaeologists to carry out chronology construction in substance time as an aid to historical inference.

Keywords

- Bayesian chronological modeling
- Phyletic seriation
- Allen's interval algebra
- Anglo-Saxon burials
- Modes of change

1 Introduction

The concept of time central to archaeological inquiry is famously difficult to comprehend; when considered carefully, time “seems to mirror our investigative method back to us, with only the method more deeply clarified” (Helm, 1985, p. 20). Indeed, the characteristics of time—its unity, direction, presence, and independence—depend on the context of inquiry (Rovelli, 2018). “What kinds of time do archaeological materialities produce” (Lucas, 2021, p. 25)?

Archaeologists have long reckoned time in two ways; stratigraphically through observations of superposition (Harris, 1989) and genealogically through artifact correlations based on estimates of similarity (Lyman et al., 1997; Lyman & O’Brien, 2006a). Stratification yields an event view of time as an inseparable part of the space-time nature of the archaeological record. In event time an *event* is a region or volume of space-time recognized by archaeologists as a “single action” interfacial or depositional context (*Archaeological Site Manual*, 1994, p. 5), and these events, which bound different regions of space-time, are the ultimate constituents of the universe (Ramsey, 1991, pp. 68–69). Within archaeology, this view of time is associated with a space-like view of reality (Dunnell, 1982). In contrast, artifact correlation yields a *substance view* of time, where time is absolute to avoid circular reasoning, an *occurrence* (so-called to distinguish it from an event) is for archaeologists a change in the qualities and relations of artifacts at a time, and the universe consists of artifacts with changing qualities and relations (Ramsey, 1991, pp. 68–69). Within archaeology, this view of time is associated with a time-like view of reality (Dunnell, 1982). Ramsey illustrated occurrence with the example of an eclipse, which occurs when the orbital events of sun, moon, and earth coincide in a particular way that eclipses the view of either the sun or moon from earth.

Event time is simpler than substance time. Change in event time is always transformative; one event ends at the stratigraphic boundary where the adjacent event begins. In contrast, change in substance time is genealogical and includes branching and reticulation modes in addition to transformation (Lyman & O'Brien, 2006b). The three modes provide an exhaustive catalog of changes sufficient to construct a phyletic seriation of arbitrary complexity. Although the substance time modes of change are analogous to the phylogenetic modes of cladogenesis, anagenesis, and reticulation (or blending), in what follows the substance time modes of change will be referred to as *branching*, *transformation*, and *reticulation*.

Bayesian chronology construction, a set of practices and software applications actively developed since the late 1980's (e.g., Naylor & Smith, 1988; Buck et al., 1992; Buck et al., 1994; Christen, 1994; Buck et al., 1996; Buck & Millard, 2004; Bronk Ramsey, 2009; Lanos & Philippe, 2018) provides archaeologists with a sophisticated set of statistical modeling and analysis tools capable of carrying out inductive tests in event time, substance time, or a combination of the two to estimate the probability of sequences of events using prior beliefs and chronological information. Each of the Bayesian chronological modeling software applications typically used by archaeologists (e.g. `OxCal`, `BCal`, and `ChronoModel`) is capable of modeling transformation in event time or substance time. They all also allow the export of the Markov Chain Monte Carlo (MCMC) samples used to approximate the posterior distributions of the models, thus facilitating post-processing and close scrutiny of the results produced (Buck et al., 1999; Bronk Ramsey, 2001; Lanos et al., 2015). This makes them more or less complete tools for estimating site chronologies in event time, which is how archaeologists typically apply them. Chronology construction in event time can be considered a solved problem; a Harris matrix record of a systematically excavated and carefully recorded archaeological site can be transformed into a format suitable for use in constructing a Bayesian chronological model either by hand or using a graph theoretic algorithm (Barker, 1986; Harris, 1989; *Archaeological Site Manual*, 1994; Dye & Buck, 2015; Moody et al., 2021). Although the invention of Bayesian calibration was announced with a worked substance time analysis of pottery from the Danebury iron-age hillfort (Naylor & Smith, 1988), archaeologists today rarely carry out chronology construction in substance time. This circumstance is changing somewhat with development of the `ChronoModel` application, which has been used to carry out substance time analyses of transformations among artifact assemblages (e.g., Banks et al., 2019). Nevertheless, best implementation practices for a substance time analysis that includes all three modes of change are not well documented or widely discussed. This paper intends to start the discussion of best practices for Bayesian chronology construction in substance time.

Bayesian chronology construction in substance time is described as follows. Section 2 introduces Allen's interval algebra, which includes a vocabulary for describing the relations of time intervals and a composition function that deduces the relation of two intervals each related to a third interval. Allen's interval algebra is used in Section 3 to explore and illustrate the chronological relations implied by the three modes of change. Section 4 develops an extended example of a substance time analysis that uses Bayesian chronological modelling and Allen's interval algebra to identify instances of each mode of change in a phyletic seriation of beads deposited in the graves of Anglo-Saxon females (Hines & Bayliss, 2013). Section 5 contrasts the substance time analysis with the event time analysis carried out by Bayliss et al. (2013). Section 6 concludes that a properly formulated substance time analysis in combination with an event time analysis of site chronology has potential to contribute to historical inference.

2 Allen's Interval Algebra

The chronological relations of two artifact classes related genealogically can be described with Allen's interval algebra, which identifies 13 basic relations that are distinct, exhaustive, and qualitative (Allen, 1983; Alspaugh, 2019). The interval algebra is defined for the relations between definite time intervals whose endpoints are single values. The thirteen basic relations include every combination of endpoint relations for two such intervals (fig. 1).

It is conventional to express an Allen relation as an *Allen set*, with a notation that indicates two intervals and a set of relations. For example, given definite intervals A and B, where A precedes B, their relation can be represented as the Allen set $A(\mathrm{p})B$ (see fig. 1, *top left*). Due to

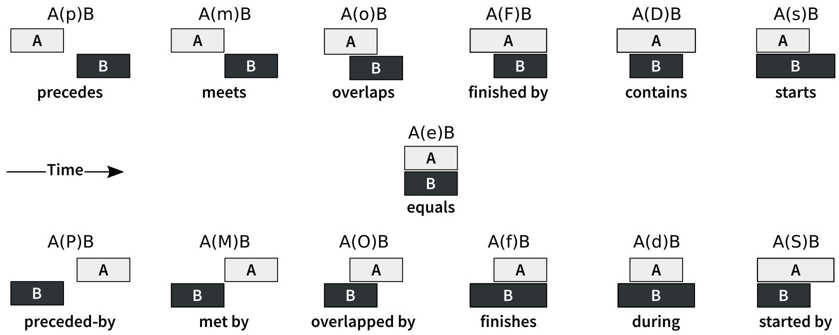


Figure 1: The 13 basic chronological relations of Allen's interval algebra for two time intervals, A and B. Each relation is indicated three ways, from top to bottom: its Allen set; an illustrative graphic, where time runs from left to right; and an English word or phrase that denotes the relation.

the probabilistic nature of the approach, Bayesian chronological modeling rarely yields definite (i.e., precisely known) time intervals. Instead, it typically yields indefinite intervals, whose endpoints are expressed as a range of plausible values, often with a multimodal probability distribution in between. Given indefinite intervals C and D, where C starts before D starts and ends before D ends, an Allen set that expresses this incomplete information is $C(\text{pmo})D$. Note that Allen set $A(\text{p})B$ is a subset of Allen set $C(\text{pmo})B$. In this circumstance, when two Allen sets are related to one another as subset/superset, the relation denoted by the subset is *stronger* and the relation denoted by the superset is *weaker*. Allen sets thus provide a precise vocabulary for characterizing the state of knowledge about the relation between two intervals, regardless of whether the intervals are definite or indefinite.

It is often useful to visualize an Allen set as a Nökel lattice (Nökel, 1991), a graphical representation in which the Allen relations are displayed on a lattice with the equals relation at the center (see fig. 2 below). The vertical dimension of the lattice represents relative age: the interval represented by the first term in the relation is increasingly older than the interval represented by the second term in the relation with distance above the center; below the center the interval represented by the first term in the relation is increasingly younger than the interval represented by the second term in the relation with distance. The horizontal dimension of the lattice represents interval duration: moving left from center increases the duration of the first term of the relation relative to the second term; moving right from center decreases the duration of the first term in the relation relative to the second term. Relations adjacent to one another on the lattice are more similar to one another than to relations farther away.

3 Chronological Relations of Modes of Change

Chronological relations of the modes of change can be established analytically by Allen's interval algebra. The Allen set that describes direct relations between two intervals can be constructed with the aid of the chart of basic relations (see fig. 1) and indirect relations between two intervals, each of which is directly related to a common third interval, can be deduced with a composition operation.

3.1 Branching

In branching mode, ancestor A is directly related to descendant B as $A(\mathrm{oFD})B$; the ancestor starts earlier than the descendant and it persists after the branching event (fig. 2, top row). The composition operation yields a full relation for two descendants of the same ancestor, which indicates that nothing is known about their chronological relation. In the figure, branching is viewed from ancestor A to the future for both descendants B and C. The branching process entails no chronological constraints that might limit the potential of the future; two descendants of a common ancestor might be related to one another according to any one of the thirteen basic Allen relations.

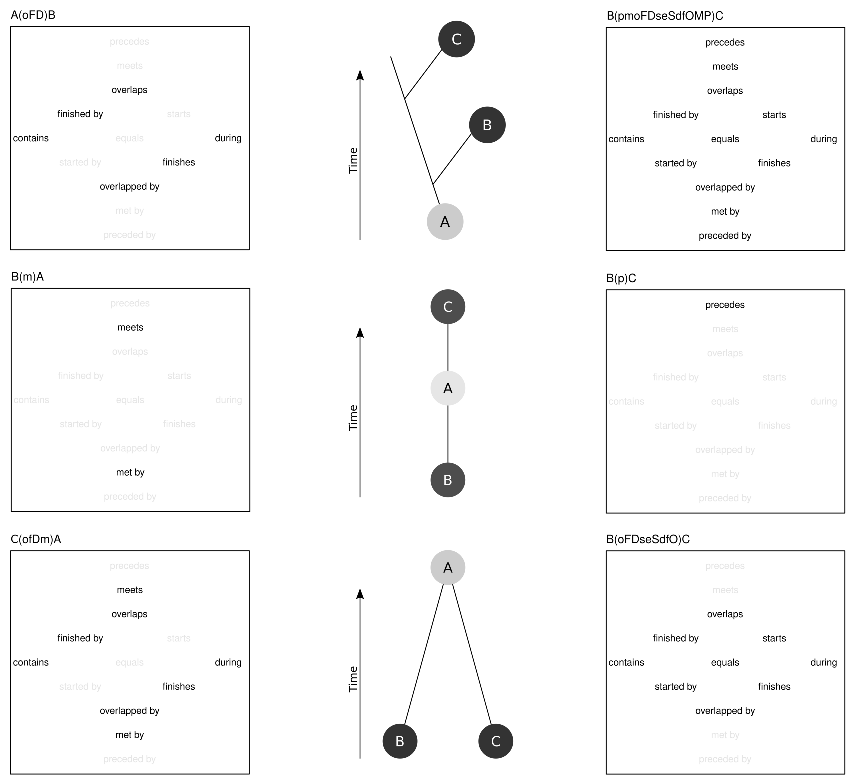


Figure 2: Modes of change: *top row,* branching; *center row,* transformation; *bottom row,* reticulation; *left column,* analytic Nökel lattices of the ancestor-descendant relation and its converse; *center column,* mode illustrations; *right column,* analytic Nökel lattices of the relation of *top,* two descendants, *middle,* an ancestor and a descendant, and *bottom,* two ancestors. Note that the dark nodes, B and C, in the mode illustrations are related to one another through their common relation with the light node, A, and that the temporal position of A relative to B and C is early in branching, intermediate in transformation, and late in reticulation.

3.2 Transformation

The direct relation of a transformative change from ancestor B to descendant A is $B(\mathrm{m})A$; the ancestor meets the descendant at the point of transformation (see fig. 2, middle row). The composition operation yields the *precedes* relation for two intervals, B and C, related by transformation through a third interval, A. In the figure, transformation is viewed from A, and correctly distinguishes the actual past of ancestor B from the potential future of descendant C.

3.3 Reticulation

The direct relation between ancestor B and descendant A in an instance of reticulation is $B(\mathrm{moFD})A$; the ancestor starts before the descendant does and it either ends when the descendant starts or persists (see fig. 2, bottom row). The composition operation indicates the relation of two ancestors in a reticulation is the Allen set $B(\mathrm{oFDseSdfO})C$, which comprises the nine *concurrent* relations that make up the central diamond of the Nökel lattice. This Allen set is stronger than the Allen set for two descendants of the same ancestor in branching mode and incomparable to the Allen set for the ancestor and descendant in transformation mode. In the figure, reticulation is viewed from descendant A to the past for both ancestors B and C, and this focus on an actual past, rather than a potential future, accounts for the relative strength of the Allen set.

4 Phyletic Seriation of Anglo-Saxon Beads

Burials provide much information on the early Anglo-Saxon period in England. Often covered by barrows, they are relatively easy to find and they typically include a wide range of burial goods. This section develops a phyletic seriation of beads recovered from 72 early Anglo-Saxon female graves in cemeteries located in southern and eastern England. Fifty-two of the female graves were investigated as part of a path-breaking project to establish a chronological framework for early Anglo-Saxon graves with high-precision radiocarbon dating of human bone (Hines & Bayliss, 2013). Subsequently, beads from another 20 female graves were added to the corpus after an early Anglo-Saxon cemetery was investigated at Royal Air Force Lakenheath (Hines, 2021).

The bead data were selected for phyletic seriation because they are well documented, with a detailed description of the chronological analysis (see Baxter, 2014), a relational database deposited with the Archaeological Data Service that catalogs the graves and the finds (Hines, 2013), a hierarchical Bayesian chronological model constructed by an experienced chronologist (Bayliss et al., 2013), and bead identifications by an expert (Nielsen, 2013) according to an established classification (Brugmann, 2004).

The phyletic seriation reported here constructs a genealogy of artifact types based on estimates of their similarity and their chronological relations. In this illustrative example, similarity estimates are derived from type descriptions, rather than developed by an expert. Chronological relations among the female graves are based on a chronological prior from the `OxCal` model deposited in the project archive (Hines, 2013). This chronological prior assumes that age determinations on human bone date the interment of the deceased and associated burial goods, including the beads. The chronological prior indicates that most of the graves are not stratigraphically associated with other graves. Nevertheless, pairs of graves at the Castledyke, Dover Buckland, and Edix Hill cemeteries are stratigraphically related, and four graves at the Melbourn cemetery are related stratigraphically to one another (Bayliss et al., 2013, pp. 339–345). In addition to this small amount of a priori relative chronological information, six of the graves yielded a distinctive shield-on-tongue buckle with shoe-shaped rivets, BU2-d/h, with a known origin date that constrains the estimate for these graves to after AD 510 (Bayliss et al., 2013, p. 345). A single grave yielded a coin of the Merovingian king Dagobert I, and was constrained to date after AD 629. These stratigraphic and known age artifact constraints were all retained in our own modeling, but other constraints in the Bayliss et al. (2013) chronological prior were removed including: (i) a sequence that models "all the radiocarbon dates for the female graves as a continuous, uniformly distributed period of burial" (Bayliss et al., 2013, p. 449) because we were uncertain it was useful and preferred to explore evidence for burial tempo a posteriori, rather than impose a uniform distribution a priori; (ii) a sequence based on non-overlapping occurrences of bead-types derived from the occurrence seriation (Bayliss et al., 2013, Table 7.18), which omits several beads believed to be "anomalously old when buried" (Bayliss et al., 2013, p. 450) and introduces model constraints stronger than the underlying analysis; and (iii) a combined age determination for two burials, MaDE1 and MaDE2, which fails a X^2 test for combining age determinations (Ward & Wilson, 1978) at the 5 percent level, apparently due to an outlier age determination for burial MaDE1.

A chronological model shorn of these constraints makes it possible to establish the temporal relations of bead types *a posteriori* by scrutinizing the MCMC output of the Bayesian analysis. Our goal is a phyletic seriation using the posterior information derived from a Bayesian chronological analysis based only on robust and uncontroversial chronological *a priori* assumptions, none of which derives from the beads themselves. The revised chronological model was implemented in `OxCal` and is included in the Supplement. It comprises two sections; the first defines the likelihoods and their constraints, and the second defines occurrences for each of the bead types. The revised model calibrates with `OxCal` version 4.4.2 and the IntCal20 calibration curve (Reimer et al., 2020). As described in the Supplement, five independent runs of the model indicate negligible between-run variability. Results of the calibration are replicable and stable; they provide a suitable basis for exploring MCMC output and, in particular, constructing a phyletic seriation of beads.

Phyletic seriation assumes that artifacts derive from a single tradition (Dunnell, 1970). This assumption appears to be met with the Anglo-Saxon female graves; analysts agree that Anglo-Saxon female burial was a continuous process (Hines & Bayliss, 2013, p. 454; Baxter, 2014, pp. 10, 15). We can check for this in our own analysis by ensuring a smooth and monotone distribution in an occurrence plot. Such a plot displays a substance time estimate of *when* the 1st, 2nd, ..., 72nd interment occurred, without positing an order for the individual interments. It is produced by ordering the date estimates for each grave at each iteration of the MCMC sample and associating the oldest date with interment 1, the second oldest with interment 2, and so on. By collecting the date estimates together in this way, we are not estimating the date of any single grave, but we can make statements about the current state of knowledge of the date of the first, second, third and seventy-second oldest grave in the dataset e.g. by summarizing them using 95% highest posterior density (or credibility) intervals. When we plot such intervals on a graph of the sort shown in fig. 3, we call this an occurrence plot. Such a plot is a straightforward depiction of substance time in that it provides estimates of when relations among interments changed without dating any specific event in our model. That said, in the limiting case, when the order of events is fully specified by the chronological prior of the Bayesian model, an occurrence plot does correspond to the credibility regions on the events, as expected.

Having established via the dating evidence that the graves, and hence the beads, plausibly derive from a single archaeological tradition, we can now move our focus to the deposition history of the beads themselves. Bead type deposition histories are estimated from their occurrences in dated graves and are illustrated with tempo plots. A tempo plot is constructed in the same way as an occurrence plot, but with the addition of a line that follows the mean value of the posterior distribution (fig. 4). The tempo plot yields a graphic where the slope of the mean line directly reflects the pace of change: a period of rapid change yields a steep slope and a period of slow change yields a gentle slope. When there is no change, the line is horizontal. When change is instantaneous, the line is vertical.

The bead deposition histories reveal to the eye two patterns of change in the social practices (Schyfter, 2009) associated with bead acquisition, use, and burial. The polychrome glass bead types—BE1-Dot34, BE1-DotReg, BE1-Koch20 in all colors, BE1-Koch34 in all colors, BE1-Koch49/50, BE1-Melon (Hines, 2021, p. 8), and BE1-Reticella—, amber beads, the monochrome glass bead types BE1-CylPen and BE1-CylRound, and metal bead type BE2-c, were frequently deposited in the fifth and sixth centuries and less frequently in the seventh and eighth centuries, yielding tempo plots that rise early and then bend to the right. In contrast, shell bead types

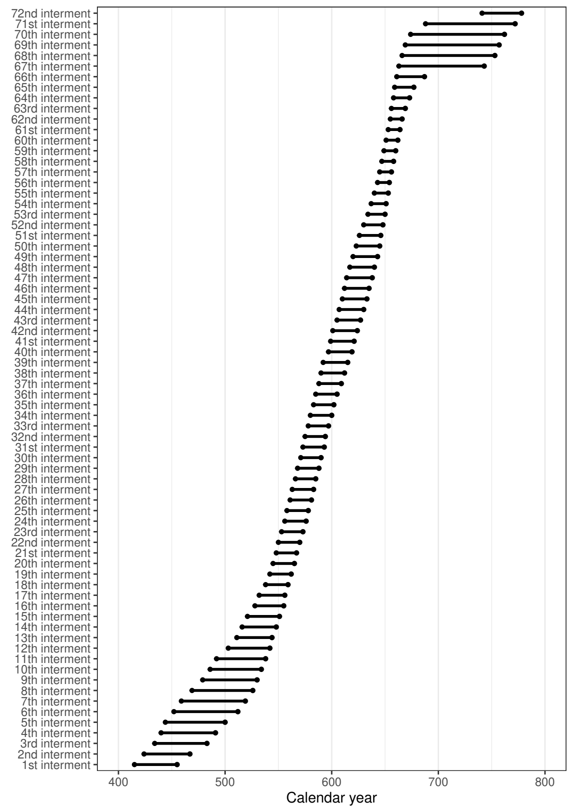


Figure 3: Occurrence plot showing the chronology of interments in 72 Anglo-Saxon female graves from the fifth to eighth centuries. Each symbol represents the 95% credible interval for an interment date estimate. Note that the interment estimates form a continuous sequence starting in the fifth century and extending without evidence of gaps into the eighth century. `ArchaeoPhases` code to reproduce the figure is included in the Supplement.

BE1-Cowrie and BE1-Disc, mineral bead type BE1-Amethyst, and monochrome glass bead types BE1-Dghnt, BE1-Orange, and BE1-WoundSp were mostly deposited in the seventh and eighth centuries, yielding tempo plots that start late or stretch to the right before rising quickly. With the possible exception of the late shell bead type BE1-Disc, which appears to be restricted to the second half of the seventh century and the early eighth century, the bead deposition histories overlap one another. This is another indication of burial continuity, albeit one characterized by change in the types of beads buried with Anglo-Saxon females.

The bead occurrences also provide the raw material for making Bayesian a posteriori statements about modes of change. The probability that two bead types' depositional histories are directly related and consistent with a particular mode can be estimated from the MCMC samples from a Bayesian chronological model. The depositional histories of each pair of beads is observed at each iteration of the MCMC output and a tally kept of all relationships that are consistent with each mode. The tallies are then divided by the total number of MCMC samples to provide the proportion (and hence probability) of all samples that are consistent with the modes of interest. We now consider the evidence for each of transformation, branching and reticulation, in turn.

Given the bead classification, evidence of transformation from one bead type to another appears to be absent in the bead assemblage as it is currently classified; instead transformation is restricted to change over time within individual bead types. In practice, and regardless of how the beads are classified, evidence of transformation from one bead type to another should be difficult to recover from the MCMC output of Bayesian calibration. The relation $\mathit{ancestor}(\mathrm{m})\mathit{descendant}$, characteristic of transformation, requires that the end date of one bead type be equal to the start date of another. The equality relation will occur extremely infrequently in our MCMC output simply because we are modeling with continuous probability distributions that include non-trivial amounts of uncertainty. Instead, one might expect a real-world example to yield a relation such as $\mathit{ancestor}(\mathrm{pmo})\mathit{descendant}$, followed by an argument, perhaps based on the similarity estimate, as to why this weaker relation ought to be interpreted

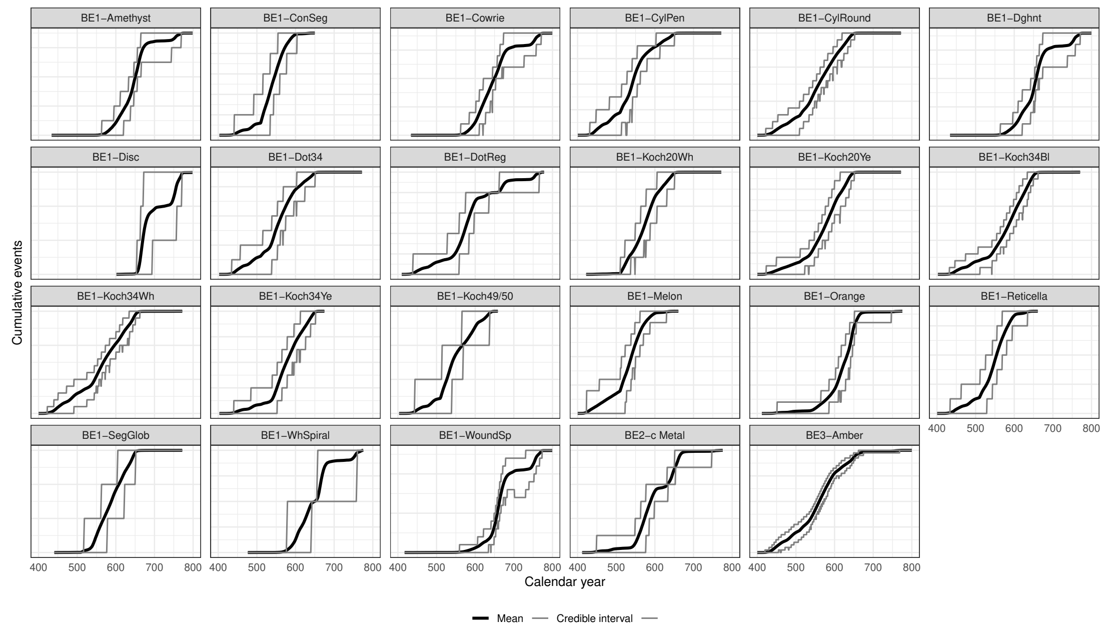


Figure 4: Tempo plots illustrating the mean deposition histories (black lines) of 23 bead types with 95% credible intervals (grey lines). Note that each plot takes one of two shapes: some rise quickly early and then level off while the remainder start slowly and rise quickly at the end. The two tempo plot shapes are typically concurrent. `ArchaeoPhases` code to reproduce the figure is included in the Supplement.

as the stronger one required by the transformation mode.

Inference of branching change is based on the similarity of two bead types, augmented by chronological information that the period of deposition of the ancestor overlaps, is finished by, or contains that of the descendant, *ancestor*(oFD)*descendant*. In the case of the Anglo-Saxon beads, an inference of branching change will imply a relation between an early bead type and one of the later monochrome glass, shell, or mineral bead types. For the purpose of illustration only, we restrict focus to the acquisition stage of the *chaîne opératoire* and change is assumed due to production; in practice change might be introduced at any stage of the *chaîne*. In the case of changes in production, branching inference might be expected to honor the distinction between beads fashioned from stable solids—amber, shell, and mineral—and semi-plastic solid, glass beads, based on the different techniques used in their manufacture (Leroi-Gourhan, 1943).

In the context of this distinction, amber beads—the sole stable solid early bead type—represent the ancestral stable solid bead type from which BE1-Cowrie, BE1-Amethyst, and BE1-Disc might have branched. We can explore these relations in our MCMC output using the tallying approach outlined above to calculate the probability that an observed relation satisfies the expected relation (see Supplement). The results indicate that bead types BE1-Amethyst and BE1-Cowrie are likely descendants of bead type BE3-Amber; their relation always satisfies the expected branching relation. Similarly, bead type BE1-Disc most likely descended from bead type BE1-Cowrie; their relation satisfies the expected branching relation with a probability of 0.87.

The branching inference is more complex for the monochrome glass bead types because there are several early bead types—BE1-CylRound, BE1-CylPen, BE1-Melon, and BE1-SegGlob—that might be ancestral to the late bead types BE1-Orange, BE1-WoundSp, and BE1-Dghnt. In each case, the probability calculations point most strongly to BE1-CylRound as ancestral to the late monochrome bead types (see Supplement).

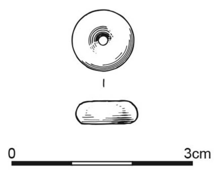
An inference of reticulation is made complex by the fact that it involves three bead types—two ancestors and a descendant—instead of two as in branching and transformation. In this case, two similarity estimates—

between the descendant and each of the ancestors—must be made. One possible instance of reticulation among the Anglo-Saxon beads involves the late monochrome glass bead, BE1-Dghnt. This is an unusual bead, found only in England (Brugmann, 2004, pp. 75–76). The method of creating the hole in BE1-Dghnt differs from other glass beads, where the hole is typically formed by winding the molten glass around a wire, which is then removed, leaving a hole in the bead. Instead, the piercing method used to fashion a BE1-Dghnt bead is similar to how holes are created in the stable solid beads. Bead BE1-Dghnt appears to have evolved by acquiring characteristics from two ancestors, one from among the monochrome glass beads and another from among the stable solid beads.

The procedure to identify potential ancestors in a reticulation can be formalized and illustrated with a refinement of a graph theoretic approach, which is used to order stylistic descriptions of artifacts with a principle of parsimony (Lipo, 2006). The refinement uses a paradigmatic classification based on presence/absence criteria, which can yield an occurrence seriation in certain cases (Lipo, 2006, p. 96). More generally, a paradigmatic classification based on presence/absence criteria has the property that, when represented as a graph, similarity according to the classification is preserved as graph distance (Hage & Harary, 1983, pp. 157–162). If presence and absence are identified as 1 and 0, respectively, then a paradigmatic classification that distinguishes bead type BE1-Dghnt can be constructed and represented as a graph (fig. 5). Potential ancestors in a reticulation that produced bead type BE1-Dghnt, node 101, are its neighbors on the graph of the paradigmatic classification, the monochrome glass beads, node 100, and the stable solid beads, node 001 (fig. 5, *right*).

The likely ancestor to bead type BE1-Dghnt from among the monochrome glass bead types, BE1-CylRound, was established earlier. Unsurprisingly, the most likely ancestor among the stable solid bead types is BE3-Amber (see Supplement).

The phyletic seriation produced by this analysis can be visualized with an illustrative diagram (fig. 6) designed along the lines set out in the early twentieth century by Bashford Dean, an honorary curator of arms and armor at the Metropolitan Museum of Art in New York (Dean, 1915; see



Bead	Glass	Added Color	Pierced	Node
Metal	0	0	0	000
Stable solid	0	0	1	001
Monochrome	1	0	0	100
BE1-Dghnt	1	0	1	101
Polychrome	1	1	0	110

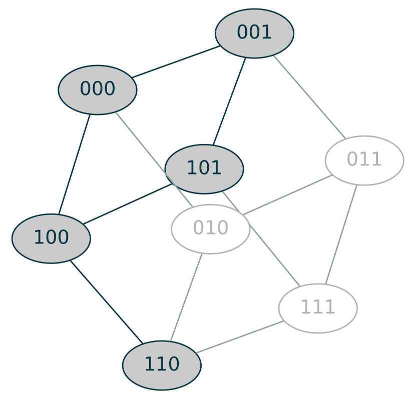


Figure 5: Paradigmatic classification of Anglo-Saxon beads and its graph representation: *top left*, illustration of bead BE1-Dghnt (Hines, 2013); *bottom left*, paradigmatic classification based on binary characters where the rows are the classes and the columns are the attributes; and *right*, graph representation of the paradigmatic classification, where dark nodes represent realized classes and light nodes represent unrealized classes. Note that node 101, which represents BE1-Dghnt, is adjacent to node 100, which represents the monochrome glass beads, and node 001, which represents the stable solid beads.

Lyman & O'Brien, 2006b). The history it illustrates includes all three modes of change, a result that accords with empirical studies of change in musical instrument design (Tëmkin & Eldredge, 2007) as well as general pragmatic considerations (Peirce, 1992; Viola, 2020, pp. 83–88) that indicate every mode is capable of capturing some of the variability exhibited by actual sequences of historical change.

5 Discussion

It is informative to compare the substance time analysis developed here with the event time analysis carried out by Hines and Bayliss (2013). The event time analysis was designed to establish a sequence of phases to which archaeologists might "assign grave-assemblages and a wide range of artifact-types" (Hines & Bayliss, 2013, p. xvii). It is based on occurrence seriations of large numbers of artifact types that were used to produce temporally ordered sequences of graves. The ordered graves were then divided into phases, based on subjective expert opinion, and modeled as abutting one another in the chronological priors for a formal Bayesian analysis. By fitting the model using high-precision radiocarbon data, the authors then explored how well the phases represent "an underlying chronological reality" (Hines & Bayliss, 2013, p. 62).

These explorations yielded mixed results because the occurrence seriation is imprecise. Estimates of its success at temporal ordination range from 62–72% (Baxter, 2014, pp. 12–14), which raises the question whether or not the subjective phase constraints derived from the occurrence seriation are reflected in the radiocarbon data. The non-overlapping occurrences of bead-types based on the occurrence seriation (Bayliss et al., 2013, Table 7.18) are not evident in the deposition histories we produced, which indicate that most bead-types were deposited concurrently with one another. As described in the Supplement to our paper, the probabilities that the 54 non-overlapping relations set out in Bayliss et al. (2013, Table 7.18) occur in the output from our modeling are variable, but typically quite low. Most of the constraints with probabilities greater than 0.9 involve

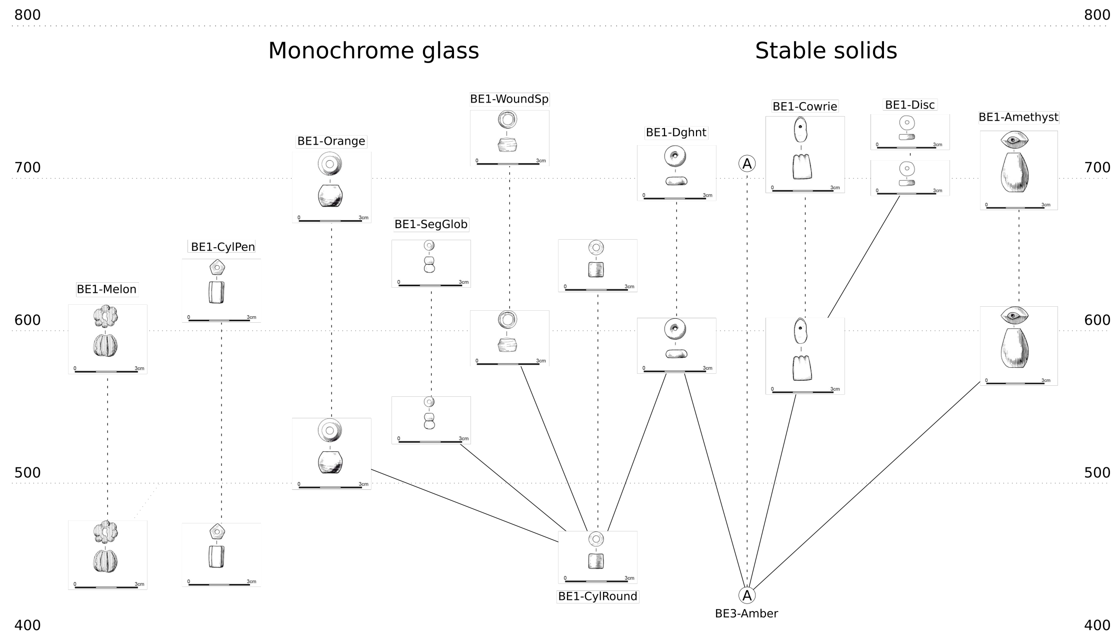


Figure 6: Phyletic seriation of some Anglo-Saxon bead types on a diagram similar to those designed by Dean (1915). The diagram tracks time from the fifth to the eighth centuries on the vertical axis and indicates the range of bead type deposition dates with illustrations (or symbols in the case of BE3-amber beads) centered at the approximate median date with an interval indicating the date of the first and last occurrence. Transformation relationships between bead types are shown with a dashed vertical line, and branching and reticulation with an oblique solid line. Bead illustrations are from Hines (2013).

bead type BE1-Disc, which stands out as one of the later bead types (see fig. 4). These results bolster the conclusion that a "national framework" based on the occurrence seriation will be difficult to apply in practice (Hines, 2021, p. 34).

At a technical level, introduction of phases puts the chronological modeling on ground made familiar by chronology construction in event time. The full range of facilities developed for Bayesian chronology construction in event time is available to the analyst. These facilities include a uniformity assumption on the deposition rate of objects within phases and other modeling assumptions used to counteract the effect of statistical scatter on stratigraphic phase length estimates (Nicholls & Jones, 2001). Such assumptions are a particular concern if we seek to use modeling output to identify modes of change in substance time. First, when the extra modeling assumptions are imposed, as they were by Bayliss et al. (2013), a posteriori interment date estimates are constrained to the sixth and seventh centuries, avoiding calibration curve irregularities in the fifth and eighth centuries that wreak havoc on the precision of age estimates (see fig. 3). The modeling assumptions used by Bayliss et al. (2013) are also responsible for the apparent variation in female interments during the sixth and seventh centuries, which a summed probability distribution shows is bimodal (Bayliss et al., 2013, Fig. 7.3). The peaks near the start of the sixth century and the end of the seventh century, at either end of the interment interval, show the effects of a priori modeling assumptions, in particular the phase deposition model (Nicholls & Jones, 2001). These two characteristics of the phase deposition model, which counteract the statistical scatter on stratigraphic phase length estimates and are integral components of an event time analysis, are not helpful in a substance time analysis. The phase deposition model is based on a physical deposition process for each context followed by thinning due to decay of the archaeological record over time and the archaeologist's selection of samples for dating. It does not extend to the multiphase models (Buck et al., 1992) typical of genealogical change in substance time (Nicholls & Jones, 2001, pp. 514–515), which in archaeology represent a tradition determined by social practice, rather than a physical process. In a substance time analysis, the effects of statistical scatter on depositional histories is not an issue. Rather, the concern is that the temporal range of an observed depositional history *underestimates* the true temporal range due to preservation bias and sampling error (Perreault, 2019, pp. 101–104). Also, in a substance time analysis the absolute age of a change in the qualities and relations of artifacts is best estimated by the age determination(s) associated with it, absent the temporal adjustments of individual age determinations required to estimate stratigraphic phase boundaries.

An effective substance time analysis will ensure that the phase deposition model applies to the event time portion of the chronological prior, but does not operate directly on age estimates for changes in the nature and relations of artifacts. In `OxCal` this means the modeler must make judicious use of the `Boundary` command that introduces the phase deposition model. In `BCal` this means that phases should be restricted to site chronologies and that floating parameters should be used in substance time. The `ChronoModel` application does not implement a phase deposition model and in this way is ideal for substance time analyses.

This is important because substance time plays an indispensable role as one of two factors required for historical inference. Stratigraphy and event time correspond to the repetitive *universal* factor operative at all times and places. Artifact genealogies and substance time correspond to the cumulative and progressive *conditional* factor significant at particular stages (Toulmin & Goodfield, 1965, p. 266). Geologists were the first to use reasoning to make historical inference when, in the early nineteenth century, William "Strata" Smith combined the event time of stratigraphy with substance time change in fossil forms to produce his famous map, *A Delineation of the Strata of England and Wales* (Toulmin & Goodfield, 1965, pp. 162–163). Bayesian chronology building provides archaeologists with the tools needed to carry out historical inference in much the same way, but with the added benefit that the conditional factors are expressed in absolute time, rather than simply ordered chronologically.

6 Conclusion

The two views of time used routinely by archaeologists—event time and substance time—can usefully be distinguished in a Bayesian analysis. Most Bayesian analyses in archaeology are carried out in event time to produce site chronologies that are necessary for historical inference but insufficient by themselves. When they are combined with a Bayesian analysis undertaken in substance time, then the necessary and sufficient conditions for historical inference are met. A substance view that assumes absolute time and is concerned with changing qualities and relations of artifacts can augment expert opinion to establish artifact genealogies that comprise the modes of change represented by branching, transformation, and reticulation. The modes of change can be investigated with the joint posteriors recorded in the MCMC output of a Bayesian model using functions provided by `ArchaeoPhases` software (Philippe & Vibet, 2020).

Substance time analysis potentially adds a temporal strand to the cable of evidential reasoning about artifacts in archaeology (Chapman & Wylie, 2016). Nevertheless, the artifact genealogies yielded by a substance time analysis must be free to take whatever form past social practices might have given them. In a Bayesian analysis, event and substance views of time should be articulated in such a way that methods proper to one do not unduly influence the other. When this is achieved, the resulting chronological model is set to accommodate new observations and update posterior probabilities in a way that potentially benefits historical inference.

Acknowledgments

The authors thank: Keith May, James Taylor, and Steve Roskams for their introduction to Allen's interval algebra; Andrew Millard, Christopher Bronk Ramsey, Ray Kidd, Erik Marsh, and Richard Staff for advice on `OxCal` code; Bo Meson for instructions on using `BCal` floating parameters to model substance time; John Hines for informative correspondence and supplying a copy of the Lakenheath report; Keith May for kindly reading and commenting on an early version of the paper; and Tim Rieth for stimulating discussions of the ideas developed here. Two anonymous reviewers provided extremely helpful comments and suggestions that substantially improved our argument. Errors of fact or interpretation are the authors'.

Author's Contributions

TSD conceived the idea of the paper and then discussed the concepts with the other authors, focusing on their individual areas of expertise. CEB contributed insights on the Bayesian modeling of event and substance time and the handling of substance and event time in the BCal software; she also helped to hone arguments for the more philosophical and model-related parts of the paper. RJD contributed key bibliographic references to the evolutionary archaeology literature and augmented the argument that distinguishes the effects of the phase deposition model. AP maintains the `ArchaeoPhases` software, improved the `R` code for tempo and occurrence plots, and augmented their description in the text. TSD drafted the main manuscript text, the supplementary material and prepared the figures. All authors offered comments on multiple iterations of the draft text and figures and together produced the final manuscript.

Availability of Data and Materials

Data and materials are available in the Supplement.

References

Allen, J. F. (1983). Maintaining knowledge about temporal intervals. *Communications of the ACM*, *26*(11), 832–843.

Alspaugh, T. A. (2019). *Allen's interval algebra*. https://www.thomasalspaugh.org/pub/fnd/allen.html

Archaeological site manual (Third edition). (1994). Museum of London Archaeology Service. London, UK.

Banks, W. E., Bertran, P., Ducasse, S., Klaric, L., Lanos, P., Renard, C., & Mesa, M. (2019). An application of hierarchical Bayesian modeling to better constrain the chronologies of Upper Paleolithic archaeological cultures in France between ca. 32,000–21,000 calibrated years before present. *Quaternary Science Reviews*, *220*, 188–214.

Barker, P. (1986). *Understanding archaeological excavation*. Batsford.

Baxter, M. (2014). *Anglo-Saxon chronology II—the female graves: A commentary on chapter 7 of 'Anglo-Saxon graves and grave goods of the 6th and 7th centuries* AD*: A chronological framework'*. https://www.academia.edu/5990358/Anglo_Saxon_Chronology_II_the_female_graves_A_commentary_on_Chapter_7_of_Anglo_Saxon_Graves_and_Grave_Goods_of_the_6th_and_7th_centuries_AD_A_Chronological_Framework

Bayliss, A., Hines, J., & Nielsen, K. H. (2013). Interpretative chronologies for the female graves. In J. Hines & A. Bayliss (Eds.), *Anglo-Saxon graves and grave goods of the 6th and 7th centuries* AD*: A chronological framework* (pp. 339–458). Society for Medieval Archaeology.

Bronk Ramsey, C. (2001). Development of the radiocarbon calibration program OxCal. *Radiocarbon*, *43*(2A), 355–363.

Bronk Ramsey, C. (2009). Bayesian analysis of radiocarbon dates. *Radiocarbon*, *51*(1), 337–360.

Brugmann, B. (2004). *Glass beads from early Anglo-Saxon graves: A study of the provenances and chronology of glass beads from early Anglo-Saxon graves based on visual examination*. Oxbow Books.

Buck, C. E., Cavanagh, W. G., & Litton, C. D. (1996). *Bayesian approach to interpreting archaeological data*. John Wiley & Sons.

Buck, C. E., Christen, J. A., Kenworthy, J. B., & Litton, C. D. (1994). Estimating the duration of archaeological activity using ^{14}C determinations. *Oxford Journal of Archaeology*, *13*, 229–240.

Buck, C. E., Christen, J. A., & James, G. N. (1999). BCal: An on-line Bayesian radiocarbon calibration tool. *Internet Archaeology*, *7*. http://intarch.ac.uk/journal/issue7/buck/

Buck, C. E., Litton, C. D., & Smith, A. F. M. (1992). Calibration of radiocarbon results pertaining to related archaeological events. *Journal of Archaeological Science*, *19*, 497–512.

Buck, C. E., & Millard, A. R. (Eds.). (2004). *Tools for constructing chronologies: Crossing disciplinary boundaries*. Springer.

Chapman, R., & Wylie, A. (2016). *Evidential reasoning in archaeology*. Bloomsbury.

Christen, J. A. (1994). Summarizing a set of radiocarbon determinations: A robust approach. *Applied Statistics*, *43*(3), 489–503.

Dean, B. (1915). An explanatory label for helmets. *The Metropolitan Museum of Art Bulletin*, *10*(8), 173–177.

Dunnell, R. C. (1970). Seriation method and its evaluation. *American Antiquity*, *35*, 305–319.

Dunnell, R. C. (1982). Science, social science, and common sense: The agonizing dilemma of modern archaeology. *Journal of Anthropological Research*, *38*(1), 1–25.

Dye, T. S., & Buck, C. E. (2015). Archaeological sequence diagrams and Bayesian chronological models. *Journal of Archaeological Science*, *63*, 84–93.

Hage, P., & Harary, F. (1983). *Structural models in anthropology*. Cambridge University Press.

Harris, E. C. (1989). *Principles of archaeological stratigraphy* (Second). Academic Press.

Helm, B. P. (1985). *Time and reality in American philosophy*. University of Massachusetts Press.

Hines, J. (2013). *Anglo-Saxon graves and grave goods of the 6th and 7th centuries* AD*: A chronological framework [data-set]*. Archaeological Data Service. https://doi.org/10.5284/1018290

Hines, J. (2021). The chronological framework of early Anglo-Saxon graves and grave goods: New radiocarbon data from RAF Lakenheath, Eriswell, Suffolk, and a new calibration curve (IntCal20). *The Antiquaries Journal*, 1–37.
Hines, J., & Bayliss, A. (Eds.). (2013). *Anglo-Saxon graves and grave goods of the 6th and 7th centuries* AD*: A chronological framework*. Society for Medieval Archaeology.
Lanos, P., Philippe, A., Lanos, H., & Dufresne, P. (2015). *Chronomodel: Chronological modelling of archaeological data using Bayesian statistics*. http://www.chronomodel.fr
Lanos, P., & Philippe, A. (2018). Event date model: A robust Bayesian tool for chronology building. *Communications for Statistical Applications and Methods*, *25*(2), 131–157.
Leroi-Gourhan, A. (1943). *Évolution et techniques: L'homme et la matière*. Albin Michel.
Lipo, C. (2006). The resolution of cultural phylogenies using graphs. In C. P. Lipo, M. J. O'Brien, M. Collard, & S. J. Shennan (Eds.), *Mapping our ancestors: Phylogenetic approaches in anthropology and prehistory* (pp. 89–107). AldineTransaction.
Lucas, G. (2021). *Making time: The archaeology of time revisited*. Routledge.
Lyman, R. L., & O'Brien, M. J. (2006a). *Measuring time with artifacts: A history of methods in American archaeology*. University of Nebraska Press.
Lyman, R. L., & O'Brien, M. J. (2006b). Seriation and cladistics: The difference between anagenetic and cladogenetic evolution. In C. P. Lipo, M. J. O'Brien, M. Collard, & S. J. Shennan (Eds.), *Mapping our ancestors: Phylogenetic approaches in anthropology and prehistory* (pp. 65–88). AldineTransaction.
Lyman, R. L., O'Brien, M. J., & Dunnell, R. C. (1997). *The rise and fall of culture history*. Plenum Press.
Moody, B., Dye, T., May, K., Wright, H., & Buck, C. (2021). Digital chronological data reuse in archaeology: Three case studies with varying purposes and perspectives. *Journal of Archaeological Sci-*

ence: Reports, *40*, 103188. https://doi.org/10.1016/j.jasrep.2021.103188

Naylor, J. C., & Smith, A. F. M. (1988). An archaeological inference problem. *Journal of the American Statistical Association*, *83*(403), 588–595.

Nicholls, G., & Jones, M. (2001). Radiocarbon dating with temporal order constraints. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, *50*(4), 503–521.

Nielsen, K. H. (2013). Typology. In J. Hines & A. Bayliss (Eds.), *Anglo-Saxon graves and grave goods of the 6th and 7th centuries* AD*: A chronological framework* (pp. 133–229). Society for Medieval Archaeology.

Nökel, K. (1991). *Temporally distributed symptoms in technical diagnosis* (Vol. 517). Springer-Verlag.

Peirce, C. S. (1992). Evolutionary love. In N. Houser & C. Kloesel (Eds.), *The essential Peirce: Selected philosophical writings* (pp. 352–371). Indiana University Press. Repr. of Evolutionary love. (1893). *The Monist*, *3*, 176–200.

Perreault, C. (2019). *The quality of the archaeological record*. University of Chicago Press.

Philippe, A., & Vibet, M.-A. (2020). Analysis of archaeological phases using the CRAN package ArchaeoPhases. *Journal of Statistical Software*, *93*. https://doi.org/https://doi.org/10.18637/jss.v093.c01

Ramsey, F. P. (1991). *On truth: Original manuscript materials (1927–1929) from the Ramsey Collection at the University of Pittsburgh* (N. Rescher & U. Majer, Eds.; Vol. 16). Kluwer Academic.

Reimer, P. J., Austin, W. E. N., Bard, E., Bayliss, A., Blackwell, P. G., Ramsey, C. B., Butzin, M., Cheng, H., Edwards, R. L., Friedrich, M., Grootes, P. M., Guilderson, T. P., Hajdas, I., Heaton, T. J., Hogg, A. G., Hughen, K. A., Kromer, B., Manning, S. W., Muscheler, R., . . . Talamo, S. (2020). The IntCal20 Northern Hemisphere radiocarbon age calibration curve (0–55 kBP). *Radiocarbon*, *62*, 725–757.

Rovelli, C. (2018). *The order of time* (S. Carnell & E. Segre, Trans.). Riverhead Books.

Schyfter, P. (2009). The bootstrapped artefact: A collectivist account of technological ontology, functions, and normativity. *Studies in History and Philosophy of Science*, *40*, 102–111.

Tëmkin, I., & Eldredge, N. (2007). Phylogenetics and material culture evolution. *Current Anthropology*, *48*(1), 146–153.

Toulmin, S., & Goodfield, J. (1965). *The discovery of time.* Harper & Row.

Viola, T. (2020). *Peirce on the uses of history.* de Gruyter.

Ward, G. K., & Wilson, S. R. (1978). Procedures for comparing and combining radiocarbon age determinations: A critique. *Archaeometry*, *20*(1), 19–31.