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# Deep Gaussian Processes for Angle and Position Discrimination in Active Touch Sensing

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**Abstract.** Active touch sensing can benefit from the representation of uncertainty in order to guide sensing movements and to drive sensing strategies that operate to reduce uncertainty with respect to the task at hand. Here we explore learning approaches that can acquire task knowledge quickly and with relatively small datasets and with the potential to be exploited for active sensing in robots and as models of biological sensory systems. Specifically, we explore the utility of deep (hierarchical) Gaussian Process models (Deep GPs) that have shown promise as models of episodic memory processes due to their low-dimensionality (compactness), generative capability, and ability to explicitly represent uncertainty. Using data obtained in a robotic active touch task (contour following), we show that both single-layer and Deep GP models are capable of providing robust function approximations from tactile data to angle and sensor position, with Deep GPs showing some advantages in terms of accuracy and uncertainty quantification in angle discrimination.

**Keywords:** Active Touch · Deep Gaussian Process · Contour Following · Tactile Sensing

## 1 Introduction

Active and exploratory capabilities of tactile perception can overcome the limitations of acquiring tactile information in a spatially constrained sensory apparatus. The active component of touch involves a modulation of attentional systems, requires decision making and performs purposeful movements to optimally obtain relevant tactile information [8, 11, 20]. In addition, psychophysical studies have characterised the execution of exploratory movement patterns of the sensory apparatus in the extraction of material and geometric properties of objects [9]. According to these studies, the perception of exact shape of an object through tactile sensing relies on a dynamic edge following exploratory procedure. Following the contour of an object depends on correctly perceiving the angle between the edge and the sensory apparatus. An accurate angle perception permits the detection of changes in curvature to maintain contact with the edge of the object whose contour is being explored [10]. Studies on angle perception with the index finger on human subjects have observed the tendency of the execution of movements of the sensory apparatus for the improvement of

angular perception [24, 25]. Thus, the active component of contour following may be contingent on localising the sensory apparatus with respect to the edge of an object and modifying the sensor position to achieve better estimations of the assessed angle. An accurate angle perception would lead to the execution of exploratory movements taking into consideration the relative orientation between the perceiving organ and the edge of the test object.

The utility of exploratory procedures is not limited to specific end-effectors or contact types, highlighting that active touch strategies, as identified in biological systems, could be usefully implemented in robotic systems [21]. The need to deploy robots in unstructured environments requires the integration of multiple modalities to provide the embodied agent with a good understanding of the outside world [5]. Tactile sensing contributes to the direct detection of physical information, for instance surface shape and texture, that can otherwise only be inferred indirectly. A variety of reliable, and small, biomimetic tactile sensors have been developed leading to an upsurge in research on touch in robots [3]. These sensors deliver measurements in the presence of noise which translates into uncertainty that learning models must deal with to make decisions about future actions.

Inspired from the biology of active touch, the execution of the contour following exploratory procedure has relied on methods for localisation of the sensor relative to the object and identification of the edge orientation [12, 16, 21]. Perception of these magnitudes has been subject on applying Bayesian models along with sequential analysis to make decisions under uncertainty to complete the task [14, 15, 17]. Similarly, in [1] it was proved that the implementation of Gaussian process models can result in obtaining more accurate predictions of angle and position magnitudes compared to the use of Bayesian models under the same circumstances. However, the use of Deep Gaussian Process models [6] in the representation of angle and position information from tactile data remains to be assessed. GPs operate to infer the correlation of the training data, which, compared, for instance, to deep learning approaches, can present an advantage in building reliable models with relatively small training data. This can also be useful in understanding biological organisms in which learning can take place rapidly based on limited experience. GPs also grow in complexity to suit the data being therefore robust to overfitting [22]. By explicitly representing uncertainty in the data they can guide exploratory procedures aimed at reducing uncertainty [19], and, as a means of representing the data, they can be more transparent in terms of inspecting the low-dimensional manifold that is acquired by the trained system. In the current paper, we compare GP model with a newer variant, Deep GPs, that exploits hierarchical composition to create a deep belief network based on Gaussian process mappings [6]. Deep GPs can overcome some of the disadvantages of standard GPs, being more robust, analogous to the relationship between deep neural networks and generalized linear models [23].

In the current study, the feasibility of identifying the position and angle of the sensor with respect to the edge of a test object using GP models is evaluated using a tactile dataset consisting of evenly distributed palpations of a

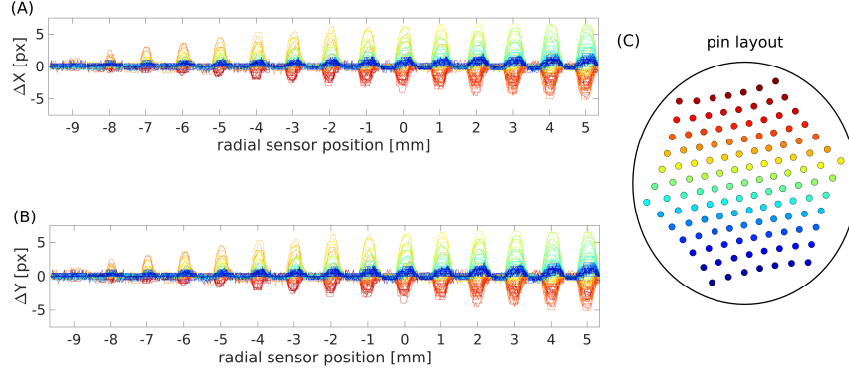
biomimetic fingertip against the surface proximal to the edge of a circular object. The dataset, developed in [2], and using the TacTip sensor [4], is intended for applications that require the discrimination between angle and perceptual classes such as tracing the contour of the object. In the current work, we seek to demonstrate that discrimination of angle and position perceptual classes can be assessed by implementing non parametric models that provide explicit quantification of uncertainty suitable for use in active sensing strategies such as exploratory procedures. Specifically, we train Gaussian Processes and Deep Gaussian Processes models to learn angle and position percepts from tactile information and to assess the accuracy of those predictions. We demonstrate that both methods work effectively with this dataset with the Deep GPs showing some advantages in terms of accuracy and uncertainty quantification.

## 2 Methods

### 2.1 Dataset

Lepora and colleagues have systematically collected sets of tactile data to enable the study of active touch strategies both for biomimetic artificial whiskers and fingertips [13]. These datasets also enable the analysis of regularities in the data when transformed into lower dimensional latent spaces [2]. In the current work, we used a subset of the data obtained with the TacTip biomimetic fingertip mounted on a UR5 robot arm. The sensor is composed of a compliant dome with 127 internal markers whose behavior is captured by a camera located inside the sensor case. The shear displacement of the markers corresponds to the deformation of the compliant component contingent on contacting a surface. The collection procedure followed a series of discrete taps on evenly distributed locations close to the edges of a circular object. The taps were executed along radial frame of reference with respect to the perpendicular angle of the edge. In that sense, position classes consisted of palpations on the surface from -12 mm to 5 mm in increments of 0.5 mm, where the 0 mm position corresponds to a palpation on the edge of the object. Nevertheless, in the present work, we used the data from -9 mm to 5 mm in steps of 1 mm. The use of a batch of the data allowed the assessment the capabilities of GP models to learn from a reduced amount of data, and also for discarding position classes in which the sensor does not provide relevant data due to lack of contact with the object. The angle classes in the original dataset were collected in a range of 0 to 360 degrees with increments of 12 degrees. However, in this work, we used the data from the perceptual classes that can be related to the perception of edges of objects consisting of right angles, i.e: 0, 180, 86, and 264 degrees, being the last two classes utilised as a proxy for classes of 90 and 270 degrees. The tactile data for the 0 degrees perceptual class is depicted in Fig. 1. The data represents the horizontal (Fig. 1a) and vertical (Fig. 1b) displacement of the internal markers, where each tap corresponds to a single radial position class, along with the marker distribution of the tip of the sensory apparatus (Fig. 1c). The data used in this work consists of four angular classes consisting of fifteen taps, where

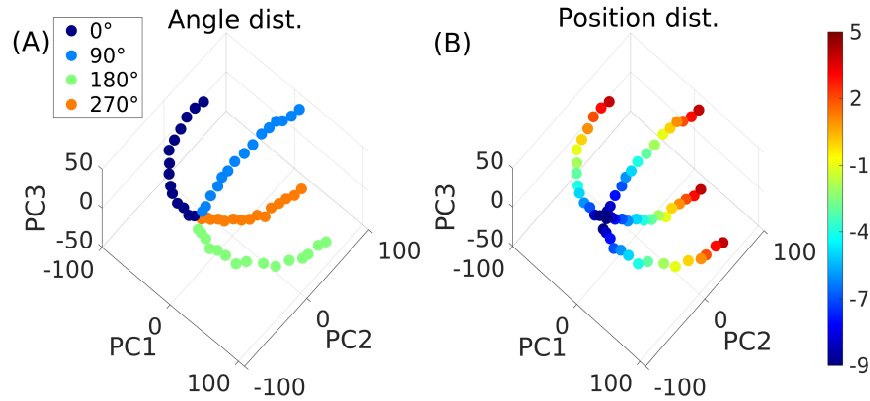
each tap is consistent with a position class for the localisation of the sensor and identification of the angle between the sensing device and the edge of the object.



**Fig. 1.** Tactile data for 15 position classes corresponding to angle perceptual class: 0 degrees. A) Tracking of horizontal marker displacement ( $\Delta X$ ). B) Tracking of vertical marker displacement ( $\Delta Y$ ). C) Layout of 127 internal markers of the TacTip sensor, colours on each plot correspond to the shown marker position

## 2.2 Dimensionality Reduction

Following the work from [2], the transformation of the data from a discrete tap consisting of a stream of the tracking of each of the 127 markers for the  $x$  and  $y$  axes along the duration of the tap can be an effective method to observe the intrinsic invariances and regularities in systematically collected data. As Fig. 2 displays, spatial commonalities in the real world are transferred to the latent space without requiring a supervised learning method. Angular classes occupy specific spaces and follow a clockwise distribution on the manifold (Fig. 2a). Similarly, position classes are evenly distributed in the latent space maintaining the spatial neighbouring from the space of observations. This dimensionality reduction represents a reduction in computational load for the Gaussian Process based models that require the inversion of the covariance matrix of training points to obtain the conditional distribution of test datapoints given the training set. The selection of data reduction to three dimensions was determined due to the 78% of the explained variance contained in the transformed data. The variance explained for each principal component was of a 41.49%, 27.21%, and 10.14%, providing a reasonable representation for the discrimination of angular and position perceptual classes.



**Fig. 2.** Dimensionality reduction of tactile data. (A) Representation of angular classes. (B) Representation of position classes

### 2.3 Gaussian Process Based Models

Gaussian process [22] and Deep Gaussian process [6] models were implemented with the dimensionality-reduced tactile data. Training data consisted of each tap represented in three dimensions. 60 datapoints corresponding to 15 evenly distributed positions for each of the 4 angular percepts were used as an input to the models. Specifically, the non-parametric models were used for regression of angle and position of the sensor with respect to the edges of objects composed of right angles. The Gaussian Process model implementation and optimisation were carried out using the methods provided by the GpFlow library [18]. The model consists of a Matern 5/2 covariance function, replicating the studies performed in [1] with the difference that we used less training samples to train the GP and Deep GP models. The covariance function was used as a kernel for its combination with the data to form the regression model, with  $X^*$  describing previously unseen data, being  $f^*$  the approximation of  $f(X^*)$ :

$$f^*|X^*, X, y \sim \mathcal{N}(m, C), \quad (1)$$

where  $m$  and  $C$ , the mean and covariance of the function approximation for the test data are obtained as follows:

$$m = K_{*x}(K_{xx} + \sigma^2 I)^{-1}y, \quad (2)$$

$$C = K_{**} - K_{*x}(K_{xx} + \sigma^2 I)^{-1}K_{*x}^T, \quad (3)$$

The covariance matrices used to perform the calculation of equations 2 and 3 consist of the covariance between training points:  $K_{xx} = k(XX)$ ; the covariance between training and test points:  $K_{*x} = k(X^*, X)$ , and the covariance between test points  $K_{**} = k(X^*, X^*)$ . In addition, a Deep Gaussian Process regression model with a variational stochastic inference method [23] was implemented with

the offered tools from the GPflux library [7]. The model consisted of two G.P layers, and a Gaussian likelihood layer. Each of the G.P layers were composed of Squared Exponential covariance functions. Deep GP regression represents a hierarchical model, in this case with two layers:

$$y = f_1(f_2(x)), \text{ where } f_1 \sim GP \text{ and } f_2 \sim GP, \quad (4)$$

constructing the function,

$$f : x, \xrightarrow{f_2} z \xrightarrow{f_1} y, \quad (5)$$

Where  $z$  is a latent variable in the hierarchical model, which can be referred as a ‘layer’ in a deep model.

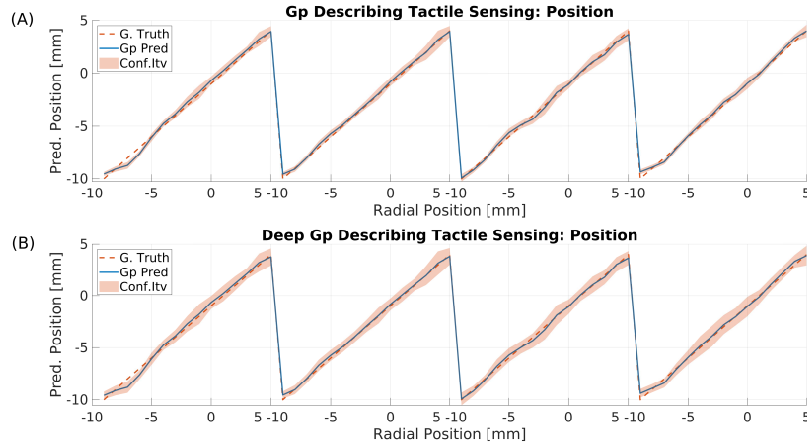
### 3 Results

In this section, a comparison between the implemented models is carried out. We present the learned mean function values along with the 95% confidence interval for the predictions of each model for a given input  $X^*$  representing previously unseen tactile data. Additionally, the performance of the regression using each method is evaluated through the obtention of the mean absolute error, and the coefficient of determination. The MAE provides a quantification of the assessed magnitude corresponding to each regression model, i.e: angle and position. The coefficient of determination, known as  $R^2$  score provides a representation of the proportion of variance of the evaluated variables explained by the independent variables in the model. This coefficient measures goodness of fit and therefore provides an indication of the capacity to predict unseen samples (through the amount of explained variance).

#### 3.1 Position Discrimination

The implementation of Gaussian Process and Deep Gaussian Process regression models for the localisation of the sensor in the radial axis with respect to the perpendicular angle of a right-angled object can serve as a method for implementing movement policies to actively follow the contour of the object. In that sense, as can be seen in Fig. 3, both models provide a similar outcome in predicting the position of the sensor given the tactile data acquired at the specific position relative to the edge. However, it can be observed that the confidence interval, translated into the uncertainty of the prediction tends to be slightly higher with the Deep GP model (Fig. 3b) with respect to the GP model (Fig. 3a). A higher variance in the prediction may result in inaccuracies on the perception of sensor position; nevertheless, the provided variance can be used to determine a fixation point or set of points in which the perception of the respective angle could be predicted with higher accuracy.

The performance of the predictions on test data for each model is presented in Table 1. The coefficient of determination metric indicates that both models



**Fig. 3.** Prediction of radial position of the sensor relative to the edge of a right-angled object ( $y$  axis) using tactile data collected at the corresponding position relative to the edge ( $x$  axis). (A) Predictions from Gaussian Process model. (B) Predictions from Deep Gaussian Process model

will provide accurate predictions in further unseen samples. Even though the mean absolute error of the Deep GP model is 0.013 mm greater than the error from the predictions with the GP model, both models display an error of less than 0.2 mm. This relatively low mean absolute prediction error for the studied models would imply a robust localisation of the sensor with respect to the edge of the object. Consequently displaying a potential to effectively modify the position state parameter for the correct perception of the angle relative to the edge of objects comprised of right angles.

**Table 1.** Performance metrics for Gaussian Process and Deep Gaussian Process for position prediction

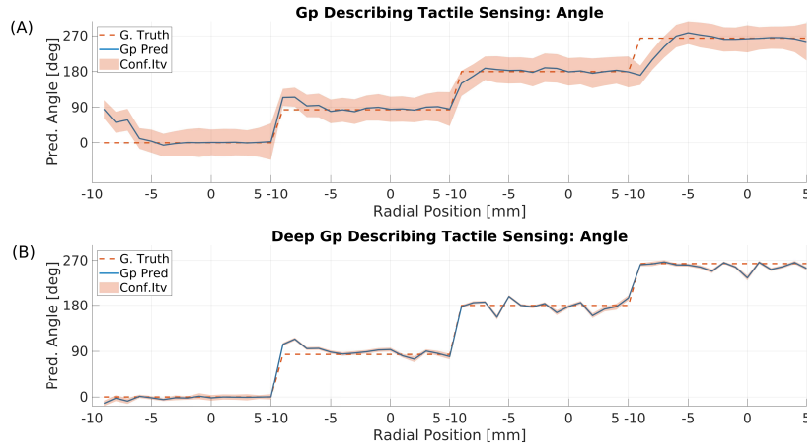
Metric	GP	Deep GP
$R^2$	0.9971	0.9967
MAE	0.1772[mm]	0.1910[mm]

### 3.2 Angle Discrimination

The discrimination of the angle in which the sensor is located with respect to the edge of an object has been proved to be relevant in the execution of exploratory movements in tasks such as following the contour of an object [21]. The predictions provided by both evaluated models are presented in Fig. 4. It



can be observed that the predictive expected value for angle perception tends to be relatively accurate for both models. However, the Gaussian process model presents a higher variance in the prediction (Fig. 4a) with respect to the variance obtained from the Deep model (Fig. 4b). This higher variance shows that a diminished consistency in accurate predictions would be present, inducing a significant impact in the execution of the contour following task. An inaccurate prediction of the angle could lead to the execution of incorrect exploratory movements when the motion policy is set to moving the sensor perpendicular to the perceived angle. In that sense, the Deep GP model presents a better performance and potential to better achieve the task considering the relevance of angle perception in the execution of exploratory movements.



**Fig. 4.** Prediction of perpendicular angle of the sensor relative to the edge of a right-angled object ( $y$  axis) from tactile data obtained at the corresponding radial position relative to the edge ( $x$  axis). (A) Predictions from Gaussian Process model. (B) Predictions from Deep Gaussian Process model

Higher variance in the prediction of test datapoints from the GP model is reflected in the  $R^2$  performance metric as detailed in Table 2, which is lower than the coefficient of determination obtained from the Deep model prediction. The best possible  $R^2$  score is 1, thus the Deep GP model tends to provide more certain predictions than when using the Gaussian Process model. Additionally, through the quantification of the mean absolute error we can demonstrate that the Deep Gaussian process model outperforms its shallow counterpart with an error reduction of approximately five degrees.

**Table 2.** Performance metrics for Gaussian Process and Deep Gaussian Process for angle prediction

Metric	GP	Deep GP
$R^2$	0.9495	0.9912
MAE	11.03°	6.11°

## 4 Discussion and future work

The execution of exploratory movements to maintain the contact with an object in a contour following setting relies on the accuracy of the perception of the angle between the sensing device and the edge of the object. This perception can be enhanced by positioning the sensor in locations where the angle can be perceived with more accuracy (fixation point). In this work, Gaussian Process and Deep Gaussian Process models were implemented for the discrimination of angle and position of the sensor with respect to the edge of right-angled objects with tactile data. For the position discrimination, both models provide a mean absolute prediction error of less than 0,2 mm, which represents an advantage in localising and positioning the sensor in places where the angle can be perceived with more accuracy. With regards to angle discrimination, it was shown that the Deep GP model provided a better performance with respect to the GP model. This outperforming was reflected in the capability to provide less variability in the predictions of previously unseen data. In addition, the deep model presents the potential to produce more accurate predictions as demonstrated in a mean absolute predictive error reduction of five degrees compared to the shallow model. The results from the angle discrimination provide directions about the policy that ought to be followed to perform active touch, i.e. to locate the sensor in a fixation point. It would be straightforward to determine a fixation point in which the sensor needs to be placed by only taking into account the values of the mean function provided by both models. However, the higher variance of the predictions obtained from the GP model suggests that the angle perception is prone to present a higher perception error as opposed to its deep counterpart which indicates that the predictions will be closer to the predicted mean function values. Therefore, the obtained accuracy and reduction in the uncertainty of the predictions of the Deep GP model can directly influence the performance of exploratory movements to successfully follow the contour of objects comprised of right angles.

The application of this type of models have demonstrated the requirement of a relatively small dataset comprised of 60 datapoints to correctly characterise the evaluated magnitudes. Additionally, the quantification of uncertainty becomes a beneficial metric for decision making to perform active touch. Future work will be directed to integrate the studied models in action-perception loops to allow robotic systems actively perform dynamical edge following of objects with different curvatures under practical scenarios where the quantification of uncertainty and accuracy of the predictions could eventually be essential for the completion of the task.

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